STATIC, CYCLIC AND POST LIQUEFACTION

UNDRAINED BEHAVIOUR OF FRASER RIVER SAND

by

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ABSTRACT

An experimental study of static, cyclic and post cyclic undrained behaviour of Fraser river sand in the triaxial test is presented. Static undrained behaviour over a range of deposition densities, from loosest to dense, and a range of confining stress is studied. Compression as well as extension loading paths are included. It is shown that for identical density and stress state the sand is dilative in triaxial compression but contractive in triaxial extension loading. It is shown that at a given confining stress static shear stress can change a dilative sand into a contractive one.

Cyclic loading leading to liquefaction is studied at specific targeted densities from loose to dense at a range of confining stress and the liquefaction resistance determined. It is shown that the sand can undergo large deformation even before a state of 100% pore pressure ratio occurs. There is no effect of the level of confining stress on liquefaction resistance of loose sand. The resistance of denser sand, however, decreases with increase in confining stress.

During post liquefaction undrained monotonic loading, the sand initially deforms with an essentially zero stiffness which then increases with the level of strain. The effect of maximum shear strain due to cyclic loading, relative density, mode of loading and the level of confining stress prior to cyclic loading on the post liquefaction monotonic undrained response of the sand is investigated. Volumetric strain due to dissipation of excess pore pressure until liquefaction is assessed as a function of maximum shear strain, relative density and confining stress prior to inducing liquefaction. The similarity of post liquefaction behaviour (both stress-strain response and volumetric strain) between sand brought to the liquefied state by cyclic loading and by static load/unload cycle is also investigated.
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List of Symbols

CSR -critical effective stress ratio

D_{rc} -relative density after consolidation

D_{ri} -relative density after set-up and consolidation to 30 kPa

D_{50} -effective grain size of soil sample; 50% by dry weight of sample is smaller than this grain size

e_c -void ratio after consolidation

e_i -void ratio after set-up and consolidation to 30 kPa

K_c -consolidation stress ratio = \frac{\sigma'_{VC}}{\sigma'_{HC}}

N -number of loading cycles

N_L -number of loading cycles to initial liquefaction (2.5% axial strain)

\Delta U -excess pore water pressure

\varepsilon_a -axial strain

\varepsilon_{\text{max}} -maximum axial strain

\varepsilon_V -volumetric strain

\varepsilon_{V\text{max}} -maximum volumetric strain

\gamma_{\text{max}} -maximum shear strain

I_B -brittleness index

K_{\sigma} -correction factor due to effect of confining stress

\phi_{\text{CSR}} -effective friction angle at critical stress ratio

\phi_{PT} -effective friction angle at phase transformation
PT - phase transformation

$\sigma_d$ - deviator stress

$\sigma'_{H}, \sigma'_{V}$ - horizontal and vertical effective stress

$\sigma'_{1}, \sigma'_{3}$ - maximum and minimum effective stress

$\sigma'_{3c}$ - effective confining stress

$\sigma'_{HC}, \sigma'_{VC}$ - horizontal and vertical effective consolidation stress

$S_{uss}$ - undrained steady state strength

$S_{uP}$ - peak shear strength

$S_{uPT}$ - undrained PT state strength

$\tau_{cy}$ - cyclic shear stress

$\tau_s$ - static shear stress
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To those children who are the light of the world
Chapter 1

INTRODUCTION

Liquefaction of saturated sands has been the topic of extensive laboratory research over the past 25 years. Under static loading the term liquefaction is associated with a strain softening type of undrained response. Under cyclic undrained loading, however, liquefaction is usually defined in terms of strain criterion. The sand is deemed to have liquefied if it develops a specified level of strain, regardless of the manner in which the strain develops (NSERC 1985). The strain development under cyclic loading can either be on account of strain softening in the manner similar to that under static loading, or on account of cyclic mobility. The latter is associated with excursions of the stress state of the sand through transient states of zero effective stress ($\sigma^'_{3}=0$). The first time occurrence of this $\sigma^'_{3}=0$ has been termed initial liquefaction (Seed 1979). At the conclusion of cyclic loading following liquefaction the residual conditions in the sand normally correspond to $\sigma^'_{3}=0$.

Until recent past the major concern during earthquake loading of saturated sands has been to safeguard against the occurrence of liquefaction. If liquefaction was a possibility depending on the initial stress and density state of the sand together with the characteristics of cyclic strains imposed by the earthquake, remedial measures in the form
of densification was specified. No attempt was generally made to estimate earthquake induced displacement. During the last few years, however, several researchers have emphasized a great need for estimation of such displacements. As a consequence some empirical and analytical techniques towards this goal have been proposed (Finn et. al., 1986; Hamada et. al., 1987; Byrne, 1990).

One of the key information required in estimating earthquake induced displacement is the post liquefaction stress-strain response of sand. Specifically, the response of sand when it undergoes excursions through states of $\sigma'_{3}=0$ is needed when modelling spacial progress of liquefaction in a given earth structure. Little research has been carried out on this aspect of sand behaviour. If the sand is contractive under static loading, it has been assumed that its steady state (or residual) strength remains unaltered on monotonic loading following liquefaction induced by cyclic loading (Byrne et. al., 1992). No experimental evidence exists in support of such a contention.

Dissipation of excess pore pressures induced by earthquake loading is the cause of post earthquake settlements. Post cyclic loading compressibility of sand controls the magnitude of these settlements. The conclusions from the few experimental studies that have carried out on this topic (Lee and Albaissa, 1974; Tatsuoka et. al., 1984; Tokimatsu and Seed, 1987; Nagasse and Ishihara, 1988). are often contradictory. Whereas the magnitude of shear strain and relative density have been identified as the major factors that control volumetric deformations on reconsolidation, there are conflicting view points on the effects of the level of confining pressure and the amplitude of cyclic stress.

This thesis presents an experimental study of static, cyclic (leading to liquefaction)
Chapter 1  Introduction

and post cyclic behaviour of a sand in the triaxial test. The primary focus is intended on
the post cyclic behaviour, both in relation to stress-strain response as well as volumetric
deformation on dissipation of excess pore pressures following liquefaction. This
necessitated a comprehensive investigation of the static and cyclic behaviour which takes
the sand to the liquefied state prior to assessment of its post cyclic behaviour.

The study encompasses static undrained behaviour over a range of deposition
densities, from loosest to dense, and a range of confining stress. Both isotropic and
anisotropic initial consolidation states are considered and compression as well as
extension loading paths are included, in order to assess path dependence of behaviour.
Cyclic loading leading to liquefaction is studied at specific targeted densities from loose
to dense, and again a range of confining stress level is used. Finally post liquefaction
monotonic undrained response is studied as it is influenced by factors such as the
maximum shear strain due to cyclic loading, relative density, mode of loading and the
level of confining stress prior to cyclic loading. Similarly, post liquefaction volumetric
behaviour is assessed as a function of maximum shear strain, relative density and
confining stress prior to inducing liquefaction.

Possible similarity of post cyclic behaviour between sand brought to the liquefied
state by cyclic loading and by static load/unload cycles is also investigated.
For several decades, research interest has focused on the undrained behaviour of sand. This was mainly because of the concern that arose from flow slides and damage that occurred during the 1964 Alaska and Niigata earthquakes. Undrained response of saturated sand is traditionally considered separately under monotonic and cyclic loading conditions. Interest in monotonic loading has generally been related to undrained failure associated with flow slides. The characteristic feature of such behaviour is extremely large deformations under very small shear resistance. Conditions which could bring about such a response could be rapid increase in stresses due to earthquake loading, shock loading or even static loading. Interest in cyclic undrained loading behaviour has been related to the susceptibility of sand to accumulate undesirable deformation during earthquake shaking.

2.1 Monotonic loading behaviour

The range of typical undrained triaxial compression behaviour of isotropically consolidated saturated sand is shown in Figure 2.1 (Castro, 1969; Castro et al., 1982; Chern, 1985). The variations in stress-strain curves from type 1 to type 3 is associated
Figure 2.1 Characteristic monotonic undrained loading stress–strain response of sandy soils (Kuerbis, 1989)
Chapter 2  Literature Review

with increasing relative density. Type 1 and 2 are characteristic response exhibited by loose sand and sand at higher effective confining stress. Type 1 response is generally associated with flow failure. Type 3 is a characteristic dilative response exhibited by dense sand and sand at lower effective confining stresses.

Types 1 and 2 are strain softening or contractive response- a behaviour associated with loss of shear resistance after the occurrence of a peak. Type 1 response has been called liquefaction by Castro (1969) and Seed (1979) and true liquefaction by Chern (1985). It is a strain softening response with unlimited unidirectional strain. The characteristic feature of this type of response is continued deformation at constant void ratio, confining stress and shear resistance, (called steady state or residual strength) which has been called steady-state deformation or flow deformation, since it resembles the flow of a fluid (Castro, 1975; Vaid and Chern, 1983). However, the shear resistance during such deformation is of a frictional nature. Type 2 response represents strain softening with limited unidirectional strain and has been called limited liquefaction (Castro, 1969). In limited liquefaction, strain hardening follows strain softening after a minimum in undrained strength. This strain hardening that is accompanied by increasing effective confining stress and decreasing pore pressure limits the amount of shear strain possible under constant shear stress. Type 3 response represents the strain hardening behaviour reflecting no loss of shear resistance. Sand showing such a behaviour is called dilative.

2.1.1 Critical stress ratio

The peak deviator stress in Figure 2.1 indicates the initiation of strain softening behaviour. On the effective stress path (ESP), such as shown in Figure 2.2, the effective
Figure 2.2 Characteristic monotonic undrained loading stress-strain response of sandy soils (Kuerbis, 1989)
stress ratio corresponding to the peak deviator stress has been called critical stress ratio (CSR) by Vaid and Chern (1983) and Chern (1985). CSR has been shown to be unique for a given strain softening sand in triaxial compression loading (Chern, 1985; Chung, 1985; Kuerbis, 1989) but dependent upon deposition void ratio (Chung, 1985), soil gradation and sample preparation technique (Kuerbis, 1989) in extension loading. Other researchers such as Castro (1982) and Sladen et al. (1985), show that CSR varies with void ratio and confining stress level in triaxial compression loading of moist tamped specimens. This difference can be attributed to the difference in specimen preparation techniques. Vaid and Chern and Chung adopted water pluviation technique which guarantees a uniform specimen for a poorly graded sand while specimen uniformity and fabric are difficult to control by the moist tamping method. Chung (1985) and Kuerbis (1989) show that triaxial extension CSR values are considerably lower than compression CSR values, implying stress path dependent undrained behaviour.

2.1.2 Phase transformation state

The arrow in Figure 2.1 indicates the termination of strain softening response, i.e., the start of increase in shear resistance and decrease in pore pressure with further straining. The point at which the induced pore pressure stops increasing and starts decreasing in response of limited liquefaction or dilative type has been termed as phase transformation state by Ishihara et al. (1975). The termination of strain softening is characterized by a sharp turnaround in effective stress path diagram (Figure 2.2). The friction angle at phase transformation state has been shown to be unique for a given sand (Chern, 1985), but among different sands it is dependent upon soil mineralogy (Negussey
et. al., 1986). Under undrained loading, the angle at phase transformation state equals the friction angle mobilized at steady state and under drained loading it equals constant volume friction angle $\phi_{CV}$ (Chern, 1985; Negussey et. al., 1986). For type 2 and 3 response, after passing the phase transformation state, the effective stress path moves up and follows the undrained failure envelope that represents the line of maximum obliquity. For type 1 response, the terminal effective stress state stays on the PT or SS line while steady state deformation continues indefinitely.

2.1.3 Ultimate failure envelope

On further straining beyond the PT state, the effective stress path follows the undrained failure envelope or the line of maximum obliquity (Ishihara et. al., 1975). The angle of maximum obliquity has a general dependence upon soil mineralogy and the intrinsic angle of friction between mineral grains. Castro et. al. (1982) reported that the angle at maximum obliquity varies with stress and strain level, first increasing with increasing strain and then decreasing to phase transformation angle at very large strain. Miura and Toki (1982) showed that this angle increases with increase in relative density and varies with loading mode, triaxial extension or compression and variation in sample preparation technique. However, Vaid et. al. (1990) and Kuerbis (1989) showed that loading mode, sample preparation technique and gradation do not affect the angle at maximum obliquity, and thus it is a unique property of a given sand.

2.1.4 Steady state concepts

Predominant interest has generally focused on the study of true liquefaction behaviour. Most of the understanding of this phenomena has come from monotonic
loading of conventional undrained triaxial compression tests (Castro, 1969; Castro et. al.,1982 and Casagrande, 1975). Such behaviour has been studied in relationship to the problem of flow slides. It has been shown by Castro (1969) that if sand undergoes true liquefaction, the effective confining stress and shear resistance during steady state deformation are uniquely related to void ratio, regardless of the initial consolidation stress conditions. This unique line within the three dimensional space of void ratio, shear stress and effective confining stress has been called the steady state line (Castro et. al., 1982; Chern, 1985). Data from rounded to angular sands under a wide range of confining pressures and consolidation stress ratios have been presented in support of the steady state concept (Castro, 1969; Castro et. al.,1982; Chern, 1985).

Steady state concepts imply that a soil is either dilative or contractive to failure, depending on the state of the soil relative to the steady state line. The type 2 response shows that a soil may be both contractive and dilative, depending upon strain level. Castro (1982) suggests that steady state is only achieved after all soil dilation is complete, which may occur at very large undrained strength if a soil is substantially dilative past phase transformation state. The degree of dilation of different soils which have been strained past phase transformation state dependent upon soil type as well as soil density.

Many researchers who have used steady state concepts have treated phase transformation state as steady state. This results in conservative design as the strength gain due to dilation after phase transformation state is ignored. Chern (1985) considered stress condition at phase transformation state for limited liquefaction and found a unique relationship between void ratio and effective confining stress in triaxial compression
loading tests. This line also encompassed the steady state data if the true liquefaction type of response was observed in undrained loading.

At a given initial void ratio and stress state, undrained response of sand may be a function of sample fabric. Sand fabric is governed by the technique of sample formation and strain history. Miura and Toki (1982) show that there is a large difference in stress strain response between samples prepared by pluviation through air, moist tamping and moist rodding. Undrained response is softened if the sand is pre-strained in a direction opposite to subsequent undrained loading (Ishihara and Okada, 1982; Chung, 1985). Chung (1985) reported that a large pre-strain in a direction opposite to subsequent shearing may transform an initially dilative sand into a contractive sand.

The undrained response of soils is particularly sensitive to the direction of loading. Chung (1985) shows that undrained water pluviated sand is considerably weaker in triaxial extension than in compression loading. At a given initial void ratio and stress state, compression behaviour could be dilative and extension contractive. Miura and Toki (1982) showed that the degree of dilatancy in extension mode is very different from that under compression. For identical initial consolidation stress and void ratio states, the volumetric strain ($e_v$) under drained loading in the extension mode was found to be much higher than that under the compression mode. Very different effective stress paths were observed in undrained extension and compression tests. These differences were more profound for sands prepared by pluviation methods, and increased with decreasing density states. Fabric anisotropy inherent in loose pluviated specimens has been identified as the primary cause for the observed direction dependent behaviour.
2.2 Cyclic loading behaviour

Initial interest in cyclic loading behaviour of sand was triggered by the extensive failures associated with saturated sands during the Alaska and Niigata earthquakes of 1964. Consideration centred predominantly on the response of saturated sand under level ground, which are subjected to reversing shear stresses on horizontal planes (Seed and Lee, 1966). The stress conditions on such soil elements are simulated in the laboratory by undrained cyclic simple shear or cyclic triaxial test on isotropically consolidated samples. The sand is subjected to constant amplitude cyclic shear stresses on horizontal planes in the simple shear test or constant pulsating deviator loads in the triaxial test. Continued cyclic loading results in the development of large strains and the soil is said to have liquefied. This strain development during cyclic loading may be due to true liquefaction, limited liquefaction or cyclic mobility (Castro, 1969; Seed, 1979). Due to the different mechanisms that are responsible for strain development during cyclic loading (i.e., true liquefaction, limited liquefaction or cyclic mobility), the results of cyclic loading are generally assessed in terms of the strain criterion. The cyclic strength or resistance to liquefaction of a sample is thus defined as the cyclic stress amplitude required to cause a specified level of strain (2.5, 5, 10%) in a fixed number of cycles.

Cyclic loading leads to a gradual softening of response as pore pressure and shear strain develop. In general, very low shear strain occurs until a pore pressure in excess of about 60% of the initial effective confining stress has developed (Seed, 1979). During some stage of the cyclic loading with shear stress reversal, a transient state of zero effective stress is reached when the applied shear stress in sand is zero. Zero effective
stress in sand would imply zero shear resistance for a frictional material and hence its equivalence with a liquid and the corresponding phenomenon, liquefaction. Castro (1969) has shown cases in which true liquefaction developed during cyclic loading much in the same manner as that observed under monotonic loading (Figure 2.3a). Vaid and Chern (1983) and Chern (1985) however, have shown cases of cyclic loading of sand wherein limited liquefaction developed in the same way as that observed in type 2 response under monotonic loading (Figure 2.3b).

Vaid and Chern (1983) and Chern (1985) showed that strain softening associated with limited liquefaction in triaxial compression is also initiated at the critical value of effective stress ratio (CSR) at which strain softening associated with true liquefaction initiates in static tests, regardless of the void ratio or consolidation stress conditions of sand. Following the arrest of strain softening in cyclic tests, the subsequent unloading from the peak amplitude of cyclic deviator stress causes large increase in pore pressure bringing the sand close to a state of zero effective stress, but with very little change in deformation (Figure 2.3b,c). Reloading in the extension region of the stress cycle causes the sand to undergo large deformation with its stress state moving along the undrained failure envelope. Subsequent unloading from the peak amplitude of deviator stress on extension side, once again brings the sand to a state of zero effective stress, and further reloading into the compression region again causes the stress state to move along the failure envelope with large deformation developed. Repetition of this loading and unloading process causes a progressive increase in cyclic deformation following limited liquefaction. The relationships between strain amplitude and number of loading cycles
Figure 2.3 Undrained cyclic loading behaviour of contractive sand—true liquefaction and limited liquefaction (Chern, 1985)
when liquefaction or limited liquefaction develops are shown in Figure 2.3d.

In the cyclic mobility type of response, the pore pressure and cyclic deformations increases progressively without strain softening. Vaid and Chern (1983) and Chern (1985) have noted that the sand develops very small deformation as long as its effective stress state stays below the stress ratio corresponding to the phase transformation line (Figure 2.4a,b). Significant amount of deformation is accumulated only when the stress state crosses the PT line during the loading phase. Unloading causes large increase in pore pressure bringing the sand close to the state of zero effective stress, but with very little change in deformation. Repetition of this phenomenon of stress state moving alternatively into the region beyond the PT lines with cycles of loading ultimately results in a transient state of zero effective stress, which is responsible for further accumulation of deformation at a much faster rate. Strain accumulation with cycles of loading in this type of response is shown in Figure 2.4c.

There are many similarities between cyclic and monotonic loading behaviour (Castro, 1982; Chern, 1985; Chung, 1985). Castro (1969) and Castro et.al. (1982) showed that the steady state line for true liquefaction is unique under monotonic and cyclic loading conditions. This implies that the undrained loading path (monotonic or cyclic) has no effect on the steady state line. Chern (1985) showed that the initiation of strain softening under monotonic and cyclic loading occurs at a unique value of effective stress ratio (CSR). Also the arrest of strain softening during limited liquefaction occurs at the unique effective stress ratio corresponding to the phase transformation line. Chern (1985) presented a unified approach for determining the type of undrained response for both
Figure 2.4 Undrained cyclic loading behaviour of dilative sand—cyclic mobility (Chern, 1985)
monotonic and cyclic loading conditions from a knowledge of the initial state of the sample ($e_c, \sigma'_{3c}, k_c$) and the superimposed shear loading.

2.3 Post liquefaction behaviour of sand

Displacements induced by liquefaction of soil can be very large and result in severe damage to earth and earth supported structures including embankment dams and general life line facilities. Following cessation of the earthquake the sand undergoes settlements due to the dissipation of the excess pore pressure developed during cyclic loading. Little systematic experimental research has been carried out on the post liquefaction undrained stress response and post cyclic volumetric deformations of sand. Such information is needed in analysis procedures for estimating earthquake induced displacements and settlements after the earthquake.

2.3.1 Post liquefaction stress-strain response

A state of residual $\sigma'_3=0$ is commonly realized once the sand liquefies. However this may not always be true. The level of strain defined as the occurrence of liquefaction may be realized without the isotropically consolidated sand having experienced a state of $\sigma'_3=0$ or 100% pore pressure ratio (Finn and Vaid, 1978). Post liquefaction undrained response will therefore depend on the magnitude of residual $\sigma'_3$ remaining after cyclic loading. Byrne (1990) proposed a simple model based on a single degree of freedom system that incorporates the post liquefaction stress-strain response of sand.

Kuerbis (1989) presented typical cyclic loading stress-strain response of a tailings sand that developed strain as a consequence of limited liquefaction (Figure 2.5). Prior to liquefaction the strains are small, but upon liquefaction (defined as $e_a \geq 2.5\%$) large strains
Figure 2.5 Stress–strain response of sand, pre and post liquefaction (Kuerbis, 1989)
occur. Unloading of shear stress following limited liquefaction brought about a transient state of $\sigma'_3=0$. On further loading after liquefaction the sand initially deforms at an essentially zero stiffness which then increases progressively with level of strain. This unusual stress-strain response for the liquefied soil results from the fact that, upon shearing from the transient $\sigma'_3=0$ state the soil dilates all the way causing the effective stress to increase and hence modulus to stiffen. Each excursion through transient $\sigma'_3=0$ state makes the sand to behave as a material that stiffens with strain regardless of the mode of loading.

It has been suggested that the steady state (or residual) strength of a contractive sand is not altered during monotonic loading following liquefaction induced by cyclic loading (Byrne et. al., 1992). This assumption has been used in estimating earthquake induced displacements in sand. No experimental evidence as to its validity has been demonstrated by any researcher.

2.3.2 Post liquefaction volumetric deformations

Excess pore pressures are generated in saturated sands under the undrained conditions by cyclic stresses caused by earthquake motions. When these pore pressures dissipate after the earthquake motions cease that the volumetric strains occur, cause settlements at the ground surface.

Lee and Albaissa (1974) using the cyclic triaxial apparatus showed that the amount of reconsolidation volumetric strain for non liquefaction conditions (pore pressure ratio $<100\%$) increases with increasing grain size of the soil, decreasing relative density, and increasing excess pore pressure generated during undrained cyclic loading, but is
independent of the amplitude of cyclic loading. They also pointed out that the effect of confining stress on volumetric strain is significant only when the developed pore pressure ratios are greater than about 0.6. If liquefaction does not develop, the resulting volumetric strains are always likely to be less than 1%. Tatsuoka et. al. (1984) and Nagasse and Ishihara (1988) studied reconsolidation volumetric strains after initial liquefaction (pore pressure ratio=100%) under cyclic undrained simple shear loading and found that their amount though settlement can be significantly influenced by the maximum shear strain developed in the soil and the soil density is relatively insensitive to level of effective pressure prior to cyclic loading. Tokimatsu and Seed (1987) based on available test data concluded that the primary factors controlling earthquake induced settlement are the cyclic shear strain, relative density of the sand and the magnitude of the earthquake that controls the amplitude of cyclic stress. Ishihara and Yoshimine (1992) proposed a series of curve that relate volumetric strain to the density of sand and shear strain amplitude, but independent of cyclic stress level and confining pressure. These relationships are based on data published by Nagasse and Ishihara (1988) and some limited triaxial test data by other researchers.

2.4 Research needs

The foregoing review suggests that little systematic research has been carried out on the post liquefaction undrained monotonic response of sands. Post liquefaction stress-strain response is used in situations where earthquake induced displacements need to be estimated. The dependence of this strain on factors such as relative density, effective confining pressure, amplitude of cyclic stress and strain and mode of loading,
compression or extension is not known. Similarly, the limited data that exists on factors controlling post liquefaction volumetric deformations is, in general, conflicting and does not address influence of some other key factors that have bearing on these deformation.

This thesis is an attempt to address some of the above gaps in our knowledge of the post liquefaction behaviour of sand. Clearly, a necessary complement to such a study is the systematic study of static and cyclic behaviour of the sand. In view of the potential earthquake risk in the Fraser delta, Fraser river sand was selected for a comprehensive assessment of its static, cyclic and post cyclic response.
CHAPTER 3

EXPERIMENTAL WORK

3.1 Material tested

Tests were performed on Fraser river sand dredged from the Fraser river in British Columbia. This sand was selected since it underlies the populated Fraser delta which is located in the region of high seismicity. The sand is collected from several places along the Fraser river. The original sand was found to have 1% clay fraction. For testing purposes it was decided to clean the sand of this clay fraction and remove particles above 1 mm size.

Fraser river sand is a uniform grey coloured medium grained sand with subangular to subrounded particles. The average mineral composition of the sand is 40% quartz, quartzite and chert, 11% feldspar, 45% unstable rock fragments (mainly volcanics) and 4% miscellaneous detritus (Garrison et. al., 1969). The grain size distribution of the test sand is shown in Figure 3.1. The average particle size ($D_{50}$) is 0.3 mm. The maximum and minimum void ratio in accordance with ASTM D2049, are 1.0 and 0.68 respectively. The specific gravity is determined as 2.72.

3.2 Testing apparatus

All tests were conducted using the triaxial apparatus. A schematic diagram of the
Figure 3.1  Grain size distribution curve of Fraser river sand
test equipment is shown in Figure 3.2. Triaxial specimens were approximately 12.8 cm high and 6.4 cm in diameter. Axial load, cell pressure, pore pressure and axial displacement were measured using electronic transducers coupled to a data acquisition system interfaced with a microcomputer. Volume changes were measured with a pipette.

The testing system consists of a triaxial cell and a loading system. The loading system is capable of strain controlled monotonic loading and load controlled cyclic loading. A double acting piston was used for a smooth transfer from load controlled cyclic loading to subsequent strain controlled monotonic loading. Strain controlled loading has been adopted to investigate the post cyclic undrained monotonic behaviour of sand.

3.2.1 Consolidation

Samples were isotropically consolidated by step wise increase of cell pressure in reservoir A and using a back pressure of 100 kPa by opening valves 1,3 and 6. Prior to initiating isotropic consolidation, both chambers of the piston were pressurized to an approximately equal base pressure. To compensate the uplift on the loading ram during setting up of back pressure, the pressure in the lower piston was reduced appropriately. Anisotropic consolidation was done by step wise increase of cell pressure as well as the appropriate axial stress to follow a constant $K_c$ stress path.

3.2.2 Cyclic loading

Cyclic loading under load controlled mode was applied by means of an electropneumatic transducer. The low pressure output from the electropneumatic transducer is amplified by a ratio relay before admission to one chamber of the double acting air piston. A sinusoidal cyclic load pulse was applied at 0.1 Hz and calibrated prior
Figure 3.2 Schematic layout of load controlled cyclic and strain controlled monotonic loading system
to setting up of the test specimen by use of a dummy rod in place of the soil sample. Generally the 0.1 Hz cyclic loading frequency is slower than anticipated in most earthquake loading conditions. The slower rate is chosen for a better resolution of measurements with the data acquisition system.

During cyclic loading there can be fluctuations in cell pressure due to the displacement of water by the loading ram moving in and out of the cell when large deformations develop. To avoid this, the cell pressure was maintained through reservoir B rather than reservoir A.

3.2.3 Monotonic loading

After the cyclic loading, the strain drive was lowered and connected to the upper portion of the double acting piston through a load connecting ring. This ensures no disturbance to the existing stress condition of the sample after liquefaction. The double acting piston acts as an extension of the strain controlled system. Axial extension as well as compression loading can be performed by this system. Pre cyclic monotonic loading was carried out in a similar manner.

Tests were conducted at an axial strain rate of 0.5 percent per minute. Strain controlled tests provide a better record of stress-strain and pore pressure response than load controlled triaxial tests, especially if the soil undergo limited liquefaction or liquefaction. Several workers (Chang et. al.,1982; Castro et. al.,1982;Chern et. al.,1985) report that undrained shear properties of liquefaction or limited liquefaction type are unaffected if pneumatic loading system is used.
3.3 Resolution of measurements

Axial stress was corrected for: 1) membrane loads, as recommended by Kuerbis and Vaid (1990), 2) uplift force due to cell pressure which does not act on sample cap rod area, 3) friction on the rod, depending on strain direction, 4) buoyant weight of loading rod and top cap, 5) LVDT spring force on sample, and 6) half total weight of the sample. The sample mid height was the reference point of cell pressure, pore pressure and axial stress measurement. The measured stresses were accurate to ±0.5 kPa for low pressures. For the high pressure testing above 800 kPa, the measured stresses were accurate to ±0.75 kPa. The resolution of axial and volumetric strains was 0.01%.

3.4 Sample preparation and set up

Several researchers (Lee and Seed, 1967; Finn et. al., 1971; Vaid and Negussey, 1984) have described the water pluviation technique for sample preparation. This technique simulates the deposition of sand through water found in alluvial soils and mechanically placed hydraulic fills. This technique ensures sample saturation. The pluviated sands have lower energy of deposition because of the lower terminal velocity of sand falling through water. This results in a looser deposit compared to air pluviated sands. Vaid and Negussey (1984) reported that the water pluviation technique produces uniform samples of poorly graded sand. Water pluviation of well graded and silty sands results in particle segregation and thus non uniform samples.

Saturated test samples were prepared by pluviating boiled sand in de-aired water which filled the sample cavity formed by a membrane lined split mould. The tip of the pouring nozzle was always kept submerged in water during deposition. In order to keep
the sedimented sand approximately level, the pouring tip was traversed laterally during deposition. To achieve minimum possible disturbance while levelling, the excess sand was siphoned off by using a small rubber tube. All samples were prepared loose in this manner and the depositional relative density was determined after the loading cap was in place. Higher initial densities, if required, were obtained by densification. Densification was achieved by tapping on the base of the triaxial cell with a soft hammer while maintaining a gentle pressure on the loading cap. Following sealing of the membrane with the top cap, a vacuum of about 30 kPa was applied to the drainage line in order to provide a small confinement to the sample prior to dismantling the split mould. The initial relative density \( D_{ri} \) of the sample refers to this effective confining stress \( \sigma'_{3c} \). The lowest \( D_{ri} \) prepared by this technique was approximately 5%. The detailed sample preparation technique has been described by Negussey (1984) and Chern (1985). The careful sample preparation technique, which involved sedimentation by mutual transfer of sand with water without contacting air, resulted in virtually saturated samples with B value greater than 0.98.

The triaxial cell was assembled, and the drainage line was connected to the volume change and pore water pressure measuring devices after centring on the loading platform and connecting the cell pressure line. After the sample loading ram was connected to the loading piston rod, consolidation of sample was carried out by step wise increase of cell pressure. At each increments of consolidation pressure, the drainage line was kept open for a short period of time (10 to 15 minutes) until secondary consolidation,
if any, was completed. After the last increment of consolidation pressure, the drainage line was closed and the sample was ready for undrained monotonic or cyclic loading.

3.5 Test program

3.5.1 Static tests

Compression and extension tests were carried out on

a) Loosest deposited sand

b) Sand prepared at specific targeted densities

Both isotropically and anisotropically consolidated states and a range of confining pressure was used. This was intended to delineate the domains of contractive and dilative behaviour.

3.5.2 Cyclic tests

These were performed on isotropically consolidated specimens at targeted densities identical to those for the static tests. Again a range of confining pressures was used at each selected density. This was intended to assess cyclic loading resistance of Fraser river sand and its dependence on the level of confining stress.

3.5.3 Post liquefaction tests

Following cyclic loading, the sand was either loaded statically in an undrained mode or the excess pore pressure due to cyclic loading was allowed to dissipate until effective stress prior to cyclic loading were restored. Static loading in compression or extension yielded the post cyclic undrained stress-strain response of the sand. Pore pressure dissipation tests enabled assessment of volumetric deformations which control
post cyclic (or post liquefaction) settlements.

3.5.4 Other tests

In addition, static load/unload tests with one or more load/unload cycles were carried out in order to induce liquefaction ($\sigma'_{3}=0$) statically. Post liquefaction stress-strain response as well as volumetric deformation due to excess pore pressure dissipation were then determined in a manner similar to that for cyclically liquefied sand. This was intended to examine if the behaviour was similar regardless of the manner of inducing liquefaction.
CHAPTER 4

TEST RESULTS

In this chapter, the undrained behaviour of Fraser river sand during pre-cyclic static loading, both in triaxial compression and extension modes, is discussed. This is followed by the results of cyclic loading liquefaction tests. Post liquefaction undrained monotonic response following cyclic loading is presented for both compression and extension loading modes. Finally volumetric deformations ensuing on reconsolidation following liquefaction to stresses prior to cyclic loading are presented and discussed.

Under conventional triaxial compression the major principal stress $\sigma'_1$ is the vertical stress ($\sigma'_V$) and minor principle stress $\sigma'_3$ is the horizontal stress ($\sigma'_H$). However, under triaxial extension loading, the vertical stress ($\sigma'_V$) is the minor principal stress while the horizontal stress ($\sigma'_H$) is the major principal stress. In this thesis, test data is presented in terms of $\sigma'_V$ and $\sigma'_H$ instead of $\sigma'_1$ and $\sigma'_3$. The shear stress is defined as $(\sigma'_V-\sigma'_H)/2$. Thus, positive $(\sigma'_V-\sigma'_H)/2$ represents compression loading whereas negative $(\sigma'_V-\sigma'_H)/2$ implies extension loading.

The sand was always deposited at the loosest possible state resulting the void ratio approximately equal to the maximum void ratio obtained by the ASTM method. The void
ratio after densification, if needed, and after the application of a confining stress of about 30 kPa is referred to as $e_i$, the placement void ratio and the void ratio following consolidation to the desired confining stresses is referred to as $e_c$, the consolidated void ratio.

### 4.1 Pre cyclic monotonic behaviour

The behaviour of loosest deposited sand was investigated in a comprehensive manner. Confining stresses ranging from 40 kPa to 1200 kPa were used and both isotropic and anisotropic consolidation states were considered. The relative density after consolidation, $D_{rc}$, ranged from 8% to 27%. The sand behaviour was then investigated at specific targeted relative densities of 19%, 40% and 59%. Again a range of confining stress was used at each targeted density to investigate the domains of contractive and dilative behaviour as influenced by the initial state characterized by $D_{rc}$, $\sigma'_3$, and $K_c$ ($=\sigma'_{VC}/\sigma'_{HC}$).

#### 4.1.1 Behaviour of loosest deposited sand

Figure 4.1 shows compression and extension response of isotropically consolidated sand at several levels of confining pressure. Compression behaviour may be seen to be dilative except under the highest $\sigma'_3=1200$ kPa when it manifests a slightly contractive response. In contrast, the behaviour in extension at each confining stress is contractive.
Figure 4.1 Undrained static behaviour—loosest deposited
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At lower levels of $\sigma'_3$ including 200 kPa, the response is of the true liquefaction type with steady state conditions realized at axial strain of 2 to 3%. At higher $\sigma'_3$, the behaviour changes to the limited liquefaction type. The material did not show any tendency towards necking even after loaded to a strain level of 9% in extension.

The test results shown in Figure 4.1 and 4.2 indicate the effect of confining stress on response in compression is opposite to that in extension. In compression increasing $\sigma'_3$ results in a less dilative behaviour, eventually turning into contractive response at the highest $\sigma'_3$ used, whereas the effect of increasing $\sigma'_3$ in extension is to cause a less contractive response. The effect of increasing $\sigma'_3$ at a given placement density $D$ is to densify the sand. Increased density promotes less and increased $\sigma'_3$ more contractive response. The results in Figure 4.1 and 4.2 thus suggest that whereas the effect of increasing $\sigma'_3$ offsets the effect of densification in compression, the opposite is true during extension loading.

That pluviated sand is contractive in extension over a much larger range of $D$ than in compression has been demonstrated by several investigators (Bishop et. al., 1965; Miura and Toki, 1982; Chern, 1985; Chung, 1985; Kuerbis, 1989). Loosest Fraser river sand is dilative in compression except under very high confining stress. Inherent anisotropy in pluviated sand has been considered responsible for the differences in compression and extension behaviour for a given initial state.

4.1.1.1 Effect of static shear

Compression and extension behaviour at $\sigma'_3=400$ kPa for $K_c=1$ and $K_c=2$ is
Figure 4.2 Undrained static behaviour—loosest deposited
Chapter 4  Test Results

illustrated in Figure 4.3. The presence of static shear does not seem to influence extension response. In compression loading, however, static shear turns an otherwise dilative sand into a contractive one. This emphasizes the dominant role of static shear in promoting contractive behaviour at constant confining stress, even though the \( K_c = 2 \) state is somewhat denser than the \( K_c = 1 \) state. Similar behaviour has been reported by Chern (1985) for a tailings sand, and by Chung (1985) for Ottawa sand.

4.1.2 Behaviour at fixed density states

A given void ratio \( e_c \) prior to shear can be achieved either by consolidating sand from a loose \( e_i \) to high confining stresses, or from a dense \( e_i \) to low confining stresses. Considerable densification occurred in Fraser river sand under the application of large confining pressure. This implies that certain relative densities may not be accessible to a certain level of confining stress. Compressibility of the sand decreases with decrease in \( e_i \) as shown in Figure 4.4 for different \( e_i \) values. Using this information as a guideline, the consolidated relative density could be targeted to any desired level.

The undrained static response of the sand at three fixed consolidated relative densities of 19%, 40% and 59% is shown in Figure 4.5, 4.6 and 4.7. This enables isolation of the effect of confining stress alone on stress-strain response. At low \( D_{rc} = 19\% \) (Figure 4.5) the dilative response in compression does not appear to be significantly altered with increase in confining stress. In extension however, the contractive response of the true liquefaction type at lower \( \sigma'_{3c} \) changes to limited liquefaction type with increase in confining stress. This amounts to a decreased contractive tendency not
Figure 4.3 Effect of static shear on the behaviour of sands at the loosest state
Figure 4.4 Compressibility at various initial void ratios
Figure 4.5 Undrained static behaviour—$D_{re}=19\%$
Figure 4.6 Undrained static behaviour - $D_{re}=40\%$
Figure 4.7 Undrained static behaviour - $D_r=59\%$
commonly associated with increasing confining stresses. At \( D_{rc} = 40\% \) (Figure 4.6) there is a clear evidence of lesser dilative behaviour in compression with increasing confining stress. In extension, however, increase in confining stress does not significantly much affect the degree of contractive response as evidenced by an approximately constant value of brittleness index, \( I_B = \frac{(\sigma_d)_{peak} - (\sigma_d)_{min}}{(\sigma_d)_{peak}} \). At dense \( D_{rc} = 59\% \) (Figure 4.7) the effect of increase in confining pressure is to cause an increase in contractive tendency. Clearly the effect of a given increase in confining pressure on the behaviour of Fraser river sand depends upon the level of \( D_{rc} \), and for a given \( D_{rc} \) it depends in addition on the mode of loading, compression or extension.

4.1.3 Behaviour at fixed \( \sigma'_3c \)

Behaviour at a fixed confining stress at several density states enables the effect of relative density alone to be isolated. Figure 4.8a shows undrained behaviour at \( \sigma'_3c = 100 \) kPa as it is affected by the density state. Similar behaviour at higher \( \sigma'_3c = 400 \) and 1200 kPa is illustrated in Figure 4.8b and 4.8c. As expected, increasing relative density makes an already dilative behaviour more dilative and an already contractive behaviour less contractive or even transforms it into dilative regardless of the mode of loading, compression or extension.

4.1.4 Triggering of contractive deformation

The friction angle \( \phi_{CSR} \) mobilized at the peaks of deviator stress for the states that showed contractive response is shown in Figure 4.9 as a function of placement void ratio \( e_i \). Contractive response in compression was associated with only the loosest \( e_i \) states.
Figure 4.8a Effect of relative density on the undrained static behaviour

$D_{rc}$

- 19%
- 40%
- 59%

$\sigma'_{3e}=100$ kPa

Axial Strain, $\varepsilon_a$ (%)

$\left(\sigma'_V-\sigma'_H\right)/2$ (kPa)

$\left(\sigma'_V+\sigma'_H\right)/2$ (kPa)
Figure 4.8b Effect of relative density on the undrained static behaviour
Figure 4.8c Effect of relative density on the undrained static behaviour
Figure 4.9 Relationship between friction angle at CSR and placement relative density
\( \phi_{\text{CSR}} \) in compression is essentially constant at about 26 degrees, not dependent on initial stress conditions. Extension loading CSR-friction angles however are much smaller and tend to increase with increase in placement density. Like compression, static shear stress prior to straining does not influence \( \phi_{\text{CSR}} \). Lower \( \phi_{\text{CSR}} \) in extension loading implies that triggering of liquefaction will occur in the extension phase during cyclic loading. The contractive behaviour is exhibited by Fraser river sand over a large range—loosest to more than 50% placement density in extension loading. But in compression loading, the contractive response was associated with only the loosest depositional density.

Chern (1985) also showed that the CSR-friction angle in compression is independent of \( \sigma'_3 \), \( K_C \) and \( D_{\text{rc}} \) for contractive Ottawa and a tailings sand. Chung (1985) and Kuerbis (1989) showed that extension \( \phi_{\text{CSR}} \) values are much lower than compression values for Ottawa and tailings sands. These values increase with increase in initial sample preparation density (Chung, 1985) much in the same manner as observed for Fraser river sand.

Figure 4.10 shows variation of peak shear strength \( S_{\text{up}} \) in extension with confining stress \( \sigma'_3 \) for the loosest placement density. Just as in normally consolidated clays, \( S_{\text{up}}/\sigma'_3 \) may be noted to be a constant up to \( \sigma'_3 \) of about 800 kPa. This ratio does not seem to be affected by the anisotropic consolidation ratio \( K_C \) prior to straining. This is in contrast to contractive behaviour in compression where the ratio \( S_{\text{up}}/\sigma'_1 \) and not \( S_{\text{up}}/\sigma'_3 \) is found constant (Chern, 1985). The peak strength ratio is plotted against consolidated relative density in Figure 4.11. A unique linearly increasing relationship may
Initial effective confining stress, $\sigma'_{3c}$ (kPa)

$e_i = 0.96$

Figure 4.10 Effect of confining stress on peak shear strength
Figure 4.11 Peak shear strength ratio of the sand
be noted, that is independent of the initial placement density and initial stress state $\sigma'_{3c}$ and $K_c$.

The degree of strain softening or contractive response can be conveniently characterized in terms of the brittleness index, $I_B$. Britteness index is considered indicative of the flow potential of a contractive sand. The variation of brittleness index in extension with effective confining stress is shown in Figure 4.12. Two relationships are shown- for the loosest placement density $D_{ri}$ and a targeted $D_{rc}$ implying increasing $D_{ri}$ with $\sigma'_{3c}$. The largest brittleness index may be seen to be associated with the loosest deposited sand under low levels of confining stresses. Increasing $\sigma'_{3c}$ results in lower $I_B$. Although increase in $\sigma'_{3c}$ should cause increased contractive behaviour, the opposite effect of the associated increase in relative density apparently causes a net decrease in contractive behaviour and hence a reduced $I_B$. It is interesting to note that at a constant $D_{rc}$ of 40%, $I_B$ does not appear to depend on the level of $\sigma'_{3c}$.

4.1.5 Steady state and phase transformation state

Figure 4.13 shows the effective stress conditions at PT or steady state in extension and compression. The effective stress conditions at PT or steady state in extension and compression may be seen to lie on unique straight lines passing through the origin regardless of relative density, initial Stress state ($\sigma'_{3c}$ and $K_c$), type of response (contractive or dilative) and mode of loading. These lines have equal slope implying that the friction angle at steady state and PT state for both contractive and dilative response are equal and independent of the mode of loading. The friction angle at PT or steady state
Figure 4.12 Brittleness Index—Fraser river sand

Brittleness Index, $I_B$

Effective confining stress, $\sigma''_{3e}$ (kPa)

$e_l=0.96$
$e_c=0.87$

Extension
Figure 4.13 Angle of friction at phase transformation state
is found to be a constant at 32 degrees. Chern (1985) and Chung (1985) also reported that
the friction angle at PT state is unique for a given sand regardless of confining stress,
relative density and mode of loading.

Figure 4.14 shows the relationship between shear strength at PT or steady state
and consolidated void ratio for a range of initial confining stresses for contractive
response in extension loading. At a given $e_c$, the shear strength at PT or steady state
increases with initial confining stress. This is contrary to the common belief based on
compression loading that the shear strength at PT or steady state is a function only of the
consolidated void ratio. For Fraser river sand no such unique relationship between $S_{uPT}$
or $S_{uss}$ and $e_c$ exists but different relationships exist each characteristic to a given $\sigma'_3c$.
For a given $\sigma'_3c$ the initial static shear stress however does not influence $S_{uPT}$ or $S_{uss}
versus e_c$ relationship. To the writer’s knowledge, only Kuerbis (1989) notes that $S_{uPT}$
is not only a function of $e_c$ but also of $\sigma'_3c$ in the extension mode of loading for Brenda
mine tailings sand.

As previously pointed out, limited liquefaction occurred in compression only for
samples consolidated from the loosest depositional state to very high confining stress.
Hence relationships similar to those in Figure 4.14 do not exist for compression loading.

4.1.6 Ultimate failure state

Effective stress conditions at maximum obliquity are shown in Figure 4.15. These
are defined by two straight lines of equal slope passing through the origin implying an
averaged mobilized friction angle of about 36 degrees regardless of relative density,
Figure 4.14 Effect of confining stress on shear strength at PT state.
Figure 4.15 Angle of friction at maximum obliquity of Fraser river sand
confining stress, $K_c$ level and mode of loading—compression and extension. Chern (1985) and Chung (1985) also reported that the angle of friction at maximum obliquity is unique for a sand. But Miura and Toki (1982) noted that this angle increases with increase in relative density and varies somewhat between triaxial extension and compression loading.

### 4.1.7 Unloading behaviour

Figure 4.16 shows undrained loading-unloading behaviour of sand at $D_{rc}=19\%$ and $\sigma'_{3c}=100$ kPa. The response is dilative in compression and contractive of the true liquefaction type in extension. Compression loading has been carried to well past the PT state and the extension loading represents contractive steady state deformation over strain range of 2 to 3\%. It may be noted that on unloading of the shear stress both specimens liquefy, i.e end up in a state of $\sigma'_3=0$. This type of behaviour will later be referred to a liquefaction induced by a static load/unload cycle as opposed to liquefaction induced by cyclic loading.

A static loading/unloading cycle did not always result in a state of $\sigma'_3=0$, upon unloading. This situation occurred in dense sands ($D_{rc}=59\%$) consolidated to $\sigma'_{3c}=400$ kPa and higher in the compression mode of loading and was apparently due to an insufficient level of strain prior to unloading. However upon a second cycle of load/unload the sand ended with a state $\sigma'_3=0$. A virgin specimen when strained sufficiently prior to unloading did liquefy on unloading. Hence it can be concluded that not only the material should be strained past phase transformation state but also the straining should surpass a minimum level to realize a state of zero effective stress, on
Figure 4.16 Undrained static loading and unloading results
4.2 Cyclic Loading behaviour

Cyclic loading tests were carried out on isotropically consolidated specimens over a range of effective confining stresses. For the purpose of this investigation, liquefaction in undrained triaxial loading is defined as the development of axial strain in excess of 2.5% single amplitude (5% peak to peak axial strain between extension and compression loading phases). This is the usual definition of liquefaction adopted in literature (NRC 1985). The strain could develop as a result of contractive deformation during a particular cycle or cyclic mobility depending on the initial state of sand and the amplitude of cyclic loading. The behaviour was investigated at three targeted relative densities, $D_{rc}$ of 19%, 40% and 59%.

4.2.1 Criterion for contractive deformation

From the investigations of Castro et. al. (1982) and Chern (1985), the criteria for contractive deformation to occur during cyclic loading can be summarised as follows:

1) Sand must be contractive under monotonic loading

2) The maximum shear stress (static or cyclic) must be greater than the undrained PT or SS shear strength, i.e. $\tau_{cy} + \tau_s > S_\text{uPT}$ or $S_\text{uss}$. Since all tests on Fraser river sand were carried out on isotropically consolidated state ($\tau_s = 0$), this reduces to $\tau_{cy} > S_\text{uPT}$ or $S_\text{uss}$. 

unloading.
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3) There must be sufficient loading cycles to lead the effective stress path to the CSR lines.

All three criteria must be satisfied simultaneously for the occurrence of liquefaction.

The implicit assumption in the above criteria is that the steady state and CSR line are not affected by the stress path (compression or extension) or the manner of loading—static or cyclic. The results obtained from the undrained static loading reveal that the sand is contractive over a wide range of relative densities in the extension mode of loading but is mildly contractive in the compression mode of loading only at high confining stress consolidated from the loosest depositional state. In addition CSR in extension is much smaller than in compression for a given initial state. Consequently, contractive deformation would always occur in the extension phase of the cyclic loading, regardless of the initial density or $\sigma'_{3c}$ state.

4.2.2 Strain development due to cyclic loading

Typical cyclic loading response of a loose specimen is shown in Figure 4.17. With the continued cycling, the state of effective stress was drawn closer and closer to the CSR line (defined from static tests). During this period the cumulative axial strain developed was between 0.3 to 0.6% depending on the relative density and cyclic stress ratio to which the samples were subjected. As soon as the stress path hit the CSR line, very large strains, about 11% developed as a result of contractive deformation. Since the amplitude of $\tau_{cy} = 24$ kPa was greater than the undrained phase transformation strength, $S_uPT = 14$
Figure 4.17 Undrained cyclic loading response—liquefaction
kPa (see Figure 4.14) contractive deformation was expected. As the Fraser river sand shows strain softening response only in the undrained extension mode, this deformation developed during the extension phase of the last cycle. The residual strain following the termination of the cyclic loading was always on the extension side. This was because the cyclic loading was initiated with the compression pulse and concluded with an extension pulse.

Figure 4.18 shows the stress path and axial strain development in a dense sample prepared at 59% relative density. In this case the cyclic shear stress $\tau_{cy}=133.6$ kPa is less than the phase transformation strength, $S_{up}=150$ kPa (see Figure 4.14). Thus, contractive deformation can not develop. Under this condition only cyclic mobility can be induced. The stress path hit the PT line (defined by static tests) in extension first, because of its characteristic inclination to the right. This event caused an axial strain of about -2%. The strain prior to this was very small, confirmed Chern's (1985) findings that significant strains develop only after PT line is crossed during cyclic loading. During the compression phase of the next cycle, the later part of the stress path traverses along the line of maximum obliquity (defined from static tests) and during the extension phase of this cycle, the maximum axial strain, $\varepsilon_{max}$ developed was -4%, more than the specified 2.5% axial strain for the development of liquefaction. It may be noted that this strain developed without any excursion through a state of transient $\sigma'_3=0$ (100% pore pressure ratio).

The occurrence of contractive deformation or cyclic mobility is a function of $D_{rc}$.
Figure 4.18  Undrained cyclic loading response – cyclic mobility
\( \sigma'_3c \) and \( \sigma_d/2*\sigma'_3c \) level. At loose relative density of 19\%, the sand always developed large contractive deformation anywhere between 10\% to 16\%. The axial strain before the stress state hit the CSR line was less than 0.4\%. Very large axial strain, more than 10\% occurred during the last half cycle. For the medium dense and dense sand, the maximum axial strain developed either due to contractive deformation or cyclic mobility, was between 2.5\% to 5\%, dense sand experiencing somewhat lesser axial strain, in general. Even though these samples underwent axial strain in excess of 2.5\% during the cyclic loading, some did not reach a state of zero effective stress at the conclusion of cyclic loading.

### 4.2.3 Cyclic resistance data

Cyclic tests were performed at relative densities of 19\%, 40\% and 59\% and at each relative density several \( \sigma'_3c \) were used. Figures 4.19 through 4.21 show cyclic stress ratio versus number of cycles to liquefaction for Fraser river sand at various relative densities. The test data for the sand at 19\% relative density and for a range of confining stresses varying from 100 kPa to 400 kPa may be seen to lie on a single resistance curve. Apparently, there is no effect of confining stress level on liquefaction resistance. The cyclic shear stress amplitude was always greater than the phase transformation strength for each specimen and hence liquefaction was associated with the development of contractive deformation. The \( D_{rc}=19\% \) state is not accessible to the sand under a confining stress of 800 kPa and above and hence there is no resistance data.

At higher relative densities of 40 and 59\%, the resistance to liquefaction decreases
Figure 4.19 Liquefaction resistance curve — $D_{rc}=19\%$
Figure 4.20  Liquefaction resistance curves - $D_{rc}=40\%$
Figure 4.21  Liquefaction resistance curves – $D_{rc}=59\%$
with increase in confining stress. The decrease of resistance to liquefaction with increase in confining stress increases with increase in relative density. At both 40 and 59% relative densities, strain until liquefaction developed due either to contractive deformation or cyclic mobility (solid data points) depends upon the relative values of cyclic stress ratio and phase transformation strength. The resistance curves at each $\sigma'_{3c}$ are however, smooth and no indication of the mechanism of strain development during cyclic loading is apparent.

4.2.4 Effect of confining stress on cyclic resistance

Fig.4.22 shows cyclic stress ratio versus effective confining stress at a fixed number of cycles to liquefaction ($N_L=10$) at relative densities of 19%, 40% and 59%. The curves were interpolated from the raw test data presented in Figure 4.19 through 4.21. As pointed out earlier the confining stress does not have any influence on the resistance to liquefaction at the loose density state. But at higher relative densities, the cyclic stress ratio needed for liquefaction at a fixed number of cycles may be seen to decrease with increase in initial effective confining stress, the highest decrease being associated with the densest state. The rate of decrease in resistance to liquefaction with increase in confining stress increases with increase in relative density, and the largest decrease is associated with the densest state in the range of low confining stresses. This is consistent with the known dilatancy characteristic of sands where increasing confining stress makes only marginal change in dilatancy on loose sand. In dense sand however, similar changes in confining stress cause progressive suppression of dilatancy or increased contractancy.
Figure 4.22 Liquefaction resistance curves of Fraser river sand at $N_L=10$ cycles
If the effective confining stress is greater than one atmospheric pressure (100 kPa), the cyclic stress ratio causing liquefaction is related to the cyclic stress ratio causing liquefaction under a confinement of 100 kPa through a factor $K_\sigma$ (Seed and Harder, 1990).

$$\frac{\tau_{cy}}{\sigma'_{3c}}\left(\sigma'_{3c}=100 \text{kPa}\right) = \frac{\tau_{cy}}{\sigma'_{3c}}\left(\sigma'_{3c}=100 \text{kPa}\right) \times K_\sigma$$

The $K_\sigma$ values deduced from the cyclic resistance data presented in Figure 4.19 to 4.21 is shown in Figure 4.23. The correction factor $K_\sigma$ corresponding to 10 stress cycles may be seen to be a function of both confining stress and relative density. The correction increases with increase in confining stress and relative density. The resistance to liquefaction reduces by a maximum of about 20% at the highest confining stress 1200 kPa for the dense 59% relative density. Most of the reduction occurs between $\sigma'_{3c}=100$ to about 600 kPa, and it seems that not much further drop in $K_\sigma$ is likely for confining stresses in excess of 600 kPa. Although no tests were carried out on very dense sand, it is not expected that the form of relationship between $K_\sigma$ and $\sigma'_{3c}$ would differ from that at lower density states. In conformity with increase in contractiveness with a given increase in $\sigma'_{3c}$ at higher relative density levels, $K_\sigma$ would be expected to be somewhat larger than the values observed at the maximum test relative density of 59%.

The $K_\sigma$ data in the literature is compared with the laboratory data obtained on Fraser river sand in Figure 4.23 (Seed and Harder, 1990). There is a wide range of $K_\sigma$ values at a given confining stress. Lumping data without regard to relative density state may contribute to this large $K_\sigma$ range. For granular materials for which relative density
Figure 4.23 Relationship between effective confining stress and $K_\sigma$. 

![Graph showing the relationship between effective confining stress and $K_\sigma$.]
has been specified, such as Sacramento river sand, a clear decrease in $K_{\sigma}$ with relative density may be seen, although $K_{\sigma}$ is presented at a single value of the confining stress. $K_{\sigma}$ values for this sand at similar relative density are very comparable to those for the sand tested.

It may be pointed out that for loose sands which have the largest susceptibility to liquefaction, $K_{\sigma}$ is approximately unity regardless of the confining stress level. Adoption of lower values in design based on same average value using the body of data in Figure 4.23 would lead to a conservative design.

The data from the literature in Figure 4.23 is restricted to confining stress levels not exceeding 600 kPa. The results from tests on Fraser river sand indicate that $K_{\sigma}$ at higher stresses may not suffer much further degradation than it has already experienced until about 600 kPa.

4.2.5 Effect of relative density

The relationship between cyclic stress ratio to cause liquefaction in ten stress cycles versus relative density is shown in Figure 4.24. As is common to other sand, the resistance to liquefaction increases with relative density at all levels of confining stresses. The rate of increase in resistance with relative density however is more pronounced at low than at high confining stresses. The converging of the resistance curves towards the loose relative density is an indication of the independence of cyclic stress ratio with confining stress as evidenced by the test data.
Figure 4.24  Cyclic loading liquefaction resistance curves
4.2.6 Residual condition at the conclusion of cyclic loading

When cyclic loading was terminated, a state of zero effective stress ($\sigma'_3=0$ or 100% pore pressure ratio) was realized in most of the cases except for specimens at medium and dense relative densities (40% and 59%) under confining stresses of 400 kPa and larger. This state of $\sigma'_3=0$ occurred for the first time following conclusion of the last loading cycle in which the specified $\varepsilon_a \geq 2.5\%$ developed. A typical example is shown in Figure 4.25. The last half cycle of extension loading may be seen to cause liquefaction and the unloading phase brings the specimen to the $\sigma'_3=0$ state. Thus excursion through a transient state of $\sigma'_3=0$ did not occur for the specified level of strain development defined as liquefaction. Even for cyclic tests (Figure 4.26) in which a residual finite effective stress remained at the conclusion of cyclic loading, the strain development as a result of cyclic loading was not due to any excursion through a transient $\sigma'_3=0$ state. Since cyclic mobility is commonly associated with strain accumulation on account of excursions through transient $\sigma'_3=0$ states during loading cycles, the strain development of the type illustrated in Figure 4.26 can not strictly be called cyclic mobility. Cyclic mobility would have been achieved only if further loading cycles were applied. In that case $\varepsilon_a$ much larger than the specified 2.5% would have developed.

It may be pointed out that a state of zero effective stress was also not realized by a static load/unload cycle in medium and dense sand unless strain during loading exceeded a certain minimum value. At the conclusion of cyclic loading, dense and medium dense specimens at $\sigma'_3 \geq 400$ kPa developed a strain level of only 2.5% to 5%.
Figure 4.25 Cyclic loading behaviour - $D_{rc}=19\%$
Figure 4.26 Cyclic loading behaviour - $D_{rc}=40\%$
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Apparently an axial strain development of much larger than this value would be needed to achieve a state of zero effective stress at the conclusion of cyclic loading. Static load/unload tests indicated that a minimum strain level of about 7% was needed during loading in order to cause a $\sigma'_{3}=0$ state on unloading.

4.3 Post cyclic undrained monotonic behaviour

The residual strain at the end of cyclic loading was always extensional. This was because the specified strain $\varepsilon_a \geq 2.5\%$ during the last cycle developed on the extension side at the instant of peak stress amplitude in extension, and little strain recovery occurred when the shear stress was unloaded to zero. Most of the post cyclic monotonic loading tests were carried out in the compression mode. This involved strain reversal with respect to strain direction due to cyclic loading. A few tests were however performed in the extension mode which did not involve strain reversal. This was done to investigate possible differences in response between compression and extension modes. As stated in an earlier section, most of the test samples ended with a state $\sigma'_{3}=0$ at the conclusion of cyclic loading. Their post cyclic monotonic loading response will be discussed first followed by the response of samples which ended with states $\sigma'_{3}\neq0$, even though they were deemed to have liquefied based on the criterion of a specified level of strain development.
4.3.1 Stress strain response

Figure 4.27 shows the response during the last loading cycle and post cyclic monotonic stress-strain response of loose sand under $\sigma'_3=100$ kPa prior to cyclic loading. Axial strains shown are based on the sample configuration prior to cyclic loading. Cyclic loading clearly shows the development of contractive deformation causing an axial strain of about -14%. Unloading of shear stress to zero brings the sand to the $\sigma'_3=0$ state, but results in little strain recovery. During post liquefaction undrained loading, the sand deformed at virtually zero stiffness over a large range of axial strain (almost 20%). With further straining, the stiffness increases with increase in strain. This stress-strain response in which the modulus increases with increase in axial strain is opposite to the usual response of soil. The unusual stress-strain response of the liquefied soil results from the fact that, upon shearing the soil dilates all the way causing the effective stress to increase. The deformation progresses at a mobilized friction angle that equals the angle of maximum obliquity. The stress-strain curve after some axial strain becomes essentially linear and there is no tendency towards approaching a residual strength even after a post liquefaction strain of about 32%. Under static loading, this sand would behave contractive in extension with a residual (steady state) strength of only 5 kPa. In compression, however it would be dilative.

4.3.1.1 Dependence on relative density

Figure 4.28 shows post liquefaction monotonic response at three relative densities for a fixed $\sigma'_3=100$ kPa prior to cyclic loading. To facilitate comparison the response
Figure 4.27  Post cyclic monotonic response
Figure 4.28 Effect of relative density on post cyclic undrained monotonic response
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of dense sand is taken as the reference curve and stress-strain curves at other relative densities are horizontally shifted so as to match \( \sigma_d = 5 \) kPa point on each curve. It may be noted that the rate of build up of \( \sigma_d \) beyond this 5 kPa point increases as the relative density increases and the axial strain at which the curves become linear decreases with an increase in relative density. The post liquefaction deformation proceeds along the average line of maximum obliquity observed under static loading regardless of the relative density state. The magnitude of strain, \( \Delta e \), required to mobilize \( \sigma_d = 5 \) kPa commencing from the instant of maximum strain during cyclic loading, however, decreases with increase in density. In other words, the region of strain over which the stiffness is close to zero is less for dense than for loose sand.

4.3.2 Comparison with behaviour following \( \sigma'_3 = 0 \) induced by static loading and unloading

In Figure 4.29, the response of the sand following the state of \( \sigma'_3 = 0 \) induced by static loading and unloading is compared with that of sand which was brought to a state of \( \sigma'_3 = 0 \) by cyclic loading. Comparisons are shown for relative densities of 19%, 40% and 59% under a confining stress of 100 kPa prior to loading. As in the previous section, the response of cyclically loaded sample is taken as the reference curve and the response curve of statically loaded sample was horizontally shifted so as to match \( \sigma_d = 5 \) kPa point on each curve. This was necessary because of different amplitudes of maximum strain developed in extension prior to post liquefaction loading. The post liquefaction responses may be noted to be essentially similar at each relative density regardless of the manner
Figure 4.29  Comparison of post liquefaction response of statically and cyclically liquefied sands
by which the state $\sigma'_3=0$ was brought about. Thus a convenient way of obtaining post liquefaction response would be to use static loading and unloading instead of cyclic loading to induce liquefaction ($\sigma'_3=0$).

### 4.3.3 Comparison of pre cyclic and post cyclic behaviour

Pre liquefaction and post liquefaction stress-strain response of the sand at relative densities of 19%, 40% and 59% consolidated to an effective stress of 100 kPa are compared in Figure 4.30 through 4.32. The stiffness of the sand decreases with increase in strain until the phase transformation state for the sand subjected to pre liquefaction static loading, but it continuously increases with strain during post liquefaction monotonic loading. The contractive behaviour during pre cyclic loading has been eliminated as a consequence of cyclic loading (Figure 4.31 and 4.32). The post cyclic stress-strain response is always dilative with deformation occurring all the time at a mobilized friction angle equal to the friction angle at maximum obliquity obtained for the pre liquefaction static loading. The post cyclic stiffness is very small during the initial phase of loading, but with increase in strain level, the stiffness essentially becomes equal to the pre cyclic value in the post phase transformation region. No indication of any residual strength condition on post cyclic loading is apparent regardless of density state or the mode of loading.

### 4.3.4 Range of post liquefaction behaviour

The post liquefaction undrained stress-strain curve can be characterized into three distinct regions as shown in Figure 4.33. Region 1 spans from the state at which $\sigma'_3=0$
Figure 4.30 Undrained static response – $D_{re}=19\%$

$\sigma_{3c}^i=100$ kPa  \quad D_{re}=19\%

---

Pre liquefaction

Post liquefaction

---

Axial Strain, $\varepsilon_a$ (%)
Figure 4.31 Undrained static response – $D_{re}=40\%$
Figure 4.32 Undrained static response – $D_{rc} = 59\%$
Figure 4.33 Characterization of post cyclic curve

$D_{rc} = 59\%$

$\sigma'_{3c} = 100 \text{ kPa}$

Axial Strain, $\varepsilon_a \text{ (\%)}$

Cyclic loading

Post cyclic mono. loading
at the end of cyclic loading until a measurable $\sigma_d = 5$ kPa develops on some shearing. This is the region with very small stiffness. The size of this region decreases with increase in relative density. Region 2 commences from $\sigma_d = 5$ kPa and can be approximated as a parabolic curve representing continuously increasing stiffness with axial strain. Region three corresponds to the linear segment of the stress-strain curve, and thus a constant modulus.

An approximate length of region 1, may be taken as the axial strain increment $\Delta \varepsilon$ from the point at which $\varepsilon_{\text{max}}$ developed in bringing about liquefaction until the point at which a measurable $\sigma_d = 5$ kPa is recorded. As shown in Figure 4.34, $\Delta \varepsilon$ is a function of the maximum strain developed during cyclic loading. As pointed out earlier, the maximum strain development on liquefaction is a function of relative density and decreases with increase in relative density. Thus, the change in strain, $\Delta \varepsilon$ is highest for the loose density. Average values of $\Delta \varepsilon$ for the sand tested are respectively 20%, 3.5% and 2% for 19%, 40% and 59% relative density states.

Figure 4.35 shows the range of post liquefaction stress-strain response at relative densities of 19%, 40% and 59%. Only the curved portions of the response that start at $\sigma_d = 5$ kPa and extend to the beginning of the linear segments are shown. As before, all curves are translated horizontally so as to merge at $\sigma_d = 5$ kPa. It may be noted that there seems a definite trend for the response to become stiffer with increase in confining stress for the sand at loose relative density. But as the relative density increases, the effect of confining stress is not so apparent. The observed range of post liquefaction behaviour at
Figure 4.34 Maximum strain due to cyclic or static load/unload cycle
Figure 4.35 Range of post liquefaction behaviour
each relative density can be approximated by the average expressions given in Table 4.1.

In this table regions 1 and 3 are treated as linear. In the curved region 2, $\epsilon_a$ refers to axial strain in percent measured from the state $\sigma_d = 5$ kPa. The expressions given disregard any dependence of response on confining stress prior to liquefaction. The average strain levels over which the curved portions manifest are 5.8%, 3.8% and 2.5% for 19%, 40% and 59% relative density states.

<table>
<thead>
<tr>
<th>Relative density $D_{rc}$ (%)</th>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Young's modulus (kPa)</td>
<td>Parabolic</td>
<td>Young's modulus (MPa)</td>
</tr>
<tr>
<td>19</td>
<td>25</td>
<td>$\sigma_d = 2.5 \times \epsilon_a^2 + 2 \times \epsilon_a$</td>
<td>3.70</td>
</tr>
<tr>
<td>40</td>
<td>125</td>
<td>$\sigma_d = 10 \times \epsilon_a^2 + 10 \times \epsilon_a$</td>
<td>8.50</td>
</tr>
<tr>
<td>59</td>
<td>250</td>
<td>$\sigma_d = 20 \times \epsilon_a^2 + 6 \times \epsilon_a$</td>
<td>14.50</td>
</tr>
</tbody>
</table>

The stress-strain response in region 3 is a straight line in all cases. Its slope increases with increase in relative density. For the loose sand the slope of the line is somewhat larger for higher confining stress, but no such dependence seems to exist at higher relative densities.
4.3.5 Effect of multiple loading cycles

The dotted curves in Figure 4.36 show the range of post cyclic stress-strain responses at all confining stresses at two density states. Also plotted in the figure are the stress-strain response of the sand during second cycles of the static loading and unloading. Theses responses fit very well with the responses obtained from cyclic and one cycle load/unload static tests. With each additional cycle following liquefaction the sample accumulates larger maximum and residual strain. But the $\Delta e$ versus $e_{\max}$ relationship for the samples subjected to multiple post liquefaction cyclic loading is essentially identical to that for the sand that has undergone single post liquefaction loading cycle (Figure 4.34). This suggests that, during each additional cycle of loading, and the stress state passes through $\sigma'_{3}=0$, it will increase $e_{\max}$ and $\Delta e$, but the post liquefaction stress-strain response will not get significantly altered.

4.3.6 Effect of loading mode

Post liquefaction monotonic tests were also carried out in the extension mode. These included loading following liquefaction by either cyclic loading or static load/unload cycles. Moreover both single and multiple loading cycles were used. Post liquefaction monotonic loading of loose sand immediately following cyclic loading could not be carried out. This was because cyclic loading induced extensional strain of the order of 15% and continuing extensional strain beyond this level caused the specimen to neck leading to nonuniform specimen deformation. Post liquefaction behaviour of loose sand therefore refers to only that determined following liquefaction induced by static
Figure 4.36 Effect of multiple loading cycles
load/unload cycle, single or multiple.

Figure 4.37 shows the range of post liquefaction monotonic behaviour in extension at relative densities of 19, 40 and 59%. Unlike the compression behaviour, loose sand does not appear to show specific dependence of behaviour as the level of confining stress prior to liquefaction. The spread of stress strain curves at each relative density is of the same magnitude as observed under compression loading (Figure 4.35). The compression behaviour has also been superimposed in Figure 4.37 that illustrates the extensional behaviour, and it may be noted that at each relative density, extensional response is substantially softer than compressional response.

The average extensional response at these relative densities can be approximated, as in the case of compression response, by the three characteristic regions. The parameters describing each region are given in Table 4.2.

Table 4.2 Extension

<table>
<thead>
<tr>
<th>Relative density</th>
<th>Region 1</th>
<th>Region 2</th>
<th>Region 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{rc}$ (%)</td>
<td>Young’s modulus (kPa)</td>
<td>Parabolic</td>
<td>Young’s modulus (MPa)</td>
</tr>
<tr>
<td>40</td>
<td>75</td>
<td>$\sigma_d+5=-2*\varepsilon_a^2+2*\varepsilon_a$</td>
<td>4</td>
</tr>
<tr>
<td>59</td>
<td>400</td>
<td>$\sigma_d+5=4.5*\varepsilon_a^2+7*\varepsilon_a$</td>
<td>6.3</td>
</tr>
</tbody>
</table>
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Figure 4.37  Range of post liquefaction response in extension mode
4.3.7 Behaviour if residual state $\sigma'_3 \neq 0$

Figure 4.38 shows typical post cyclic stress-strain response of the sand at $D_{rc} = 40\%$ consolidated to a confining stress of 1200 kPa prior to cyclic loading. During the last cycle, the sample developed $e_{\text{max}} = -3.85\%$, and hence was deemed to have liquefied but without an excursion through a transient $\sigma'_3 = 0$ state. At the conclusion of cyclic loading, a residual confining stress $\sigma'_3 \approx 100$ kPa remained. On subsequent post cyclic monotonic compression loading, the pore pressure first increases without attaining a state of pore pressure ratio equal to unity before dilation starts. The behaviour is similar to precyclic response, wherein the modulus first decreases before it starts increasing with strain as dilation starts. As evident from the stress path, the dilation commences when the mobilized friction angle equals the angle of phase transformation and the dilation occurs along the line of maximum obliquity. Even after the sample was unloaded to $\sigma_d = 0$ following first cycle of compression loading, it did not reach a state of $\sigma'_3 = 0$. The sample was now loaded on the extension side, and when unloaded back to $\sigma_d = 0$ following an extensional strain of about 7%, it attained $\sigma'_3 = 0$.

Clearly the post cyclic behaviour for a given $\sigma'_{3c}$ prior to cyclic loading will depend on the magnitude of residual $\sigma'_3$ remaining after cyclic loading. This is illustrated in Figure 4.39 for medium dense sand, $D_{rc} = 40\%$, at $\sigma'_{3c} = 400$ kPa. For post cyclic residual states of $\sigma'_3 = 8, 25$ and 45 kPa, the sand had liquefied (developed axial strain between 3.5 to 3.7%). However, the maximum strain developed for the residual...
Figure 4.38 Undrained behaviour at high confining stress
Figure 4.39 Effect of residual $\sigma'_3$ on stress–strain response
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$\sigma'_3$ states of 105 and 175 kPa was less than 0.4\% and hence, according to the definition the sand did not liquefy. An exploded view of the stress strain response in the earlier stage is shown in the inset.

It may be noted in Figure 4.39 regardless of the level of $\sigma'_3$ the post cyclic stress response is similar to the pre cyclic, in that the modulus first decreases with strain before it starts to increase following the initiation of dilation corresponding to the phase transformation state. The stress strain curves move progressively higher and axial strain until phase transformation state decreases as the level of residual $\sigma'_3$ increases. Thus the undrained stress strain behaviour of sand at deformation levels typical of concern during earthquakes does not correspond to moduli that continually degrade with strain as assumed by some researchers while carrying out effective stress analysis of earthquake problems (Finn et. al.). At larger strain amplitudes even loose sand would correspond to a cyclic behaviour in which the modulus does not always decrease during the increasing phase of the shear stress. Depending on the residual effective stress state, the modulus on loading does decrease initially, but it starts increasing once the dilation commences.

4.4 Post cyclic volumetric deformations

As pointed out earlier, cyclic loading did not always conclude with a residual $\sigma'_3=0$ state. In general, dense sand and sand under higher $\sigma'_3c$ regardless of relative
density gave rise to $\sigma'_3 \neq 0$ at the end of cyclic loading. In the following section, volumetric deformation on reconsolidation to pre cyclic $\sigma'_3$ are discussed separately for the two residual $\sigma'_3$ conditions.

4.4.1 Residual $\sigma'_3 \neq 0$ condition

Figure 4.40 shows volumetric strain due to reconsolidation of a specimen at $D_{rc}=19\%$ with an initial $\sigma'_3 = 100$ kPa. It developed an $e_{\text{max}} = -11\%$ during cyclic loading. The volumetric strain that occurred during virgin consolidation is also shown in the figure. The volumetric strain during reconsolidation may be noted to be much larger than that during virgin consolidation. The increase in compressibility during reconsolidation is clearly due to the undrained shear strain the sand has undergone. The volumetric strain in an identical sample in which liquefaction was brought about by a static load/unload cycle is also shown in the figure. The results from both tests are essentially the same. Hence in the subsequent discussions, no distinction is made between liquefaction induced due to cyclic or static load/unloading.

The dramatic increase in compressibility of sand following undrained shear strain is in sharp contrast to similar behaviour of clays. In clays, undrained shearing strains give rise to some increase in compressibility on recompression. However the increased compressibility is at the most 50% larger than the recompression and not virgin compressibility of sand that has not been cyclically loaded (Yasuhara and Anderson, 1992).
Figure 4.40 Volume change behaviour of sand liquefied by cyclic and load/unload tests
4.4.1.1 Dependence on $\varepsilon_{\text{max}}$

Figure 4.41 shows volumetric strains of two identical specimens at an initial $\sigma'_{3c}=100$ kPa and $D_{rc}=19\%$, which underwent maximum shear strains of 6% and 16.5% ($\gamma_{\max}=1.5*\varepsilon_{\max}$). The volumetric strain increases with increase in maximum strain development. The post liquefaction volume change is larger than the pre liquefaction volume change eventhough the relative density is higher in the former case. Tatsuoka et. al. (1984), Tokimatsu and Seed (1987) and Ishihara (1992) also observed that the compressibility of sand increases with increase in maximum shear strain which the sand has undergone during cyclic loading.

4.4.1.2 Dependence on $\sigma'_{3c}$

Reconsolidation volumetric strains of two identical specimens that have undergone identical $\gamma_{\max}=16.5\%$ and at $D_{rc}=19\%$, but consolidated to pre cyclic $\sigma'_{3c}$ equal to 100 kPa and 400 kPa are shown in Figure 4.42. The volumetric strain increases with increase in pre cyclic consolidation stress. This is contrary to the findings by Tatsuoka (1984) and Tokimatsu and Seed (1987) who report that volumetric strain donot depend on the level of $\sigma'_{3c}$. Volumetric strain for consolidation over a large range of confining stress would be expected to be larger as observed in Figure 4.42 since the recompression curve should be similar, being controlled by the maximum shear strain induced.

4.4.1.3 Dependence on relative density

Figure 4.43 shows reconsolidation volumetric strains that occurred in sand at relative densities of 19%, 40% and 59% consolidated to a pre cyclic $\sigma'_{3c}=100$ kPa. The
Figure 4.41  Effect of maximum shear strain on volume change behaviour
Figure 4.42 Effect of confining stress on volume change behaviour

Effective confining stress, $\sigma'_3$ (kPa)

Volumetric strain ($\%$)

$\gamma_{\text{max}} = 16.5\%$

$D_{rc} = 19\%$

- 400 kPa
- 100 kPa
Figure 4.43 Effect of relative density on volume change behaviour

Effective confining stress, $\sigma'_3 \text{ (kPa)}$

Virgin compression

$D_{rc}=19\% \quad \sigma'_{3e}=100 \text{ kPa}$

Virgin compression

$D_{rc}=40\% \quad \sigma'_{3e}=100 \text{ kPa}$

Virgin compression

$D_{rc}=59\% \quad \sigma'_{3e}=100 \text{ kPa}$

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volumetric strain increases as the relative density decreases, although some difference can be attributed to the different level of maximum shear strain. These results are in accordance with the findings of Tatsuoka (1984), Tokimatsu and Seed (1987) and Ishihara (1992).

4.4.1.4 Volume change behaviour

Figure 4.44 shows reconsolidation volume change data from several tests. The data includes several levels of $\sigma'_{3c}$ and $\varepsilon_{\text{max}}$ developed at several relative densities. Approximate curves have been drawn through data points to show relationship of maximum volumetric strain, $\varepsilon_{V\text{max}}$ with $\varepsilon_{\text{max}}$ at fixed levels of $D_{\text{rc}}$ and $\sigma'_{3c}$. Although the data is not extensive, a clear trend as to the dependence of $\varepsilon_{V\text{max}}$ versus $\varepsilon_{\text{max}}$ relationship on both $D_{\text{rc}}$ and $\sigma'_{3c}$ may be noted.

It may be pointed out that the data in Figure 4.43 for loose sand in the domain of small $\varepsilon_{\text{max}}$ cannot be developed from specimens in which liquefaction was induced by cyclic loading. Loose initial states undergo contractive deformation during cyclic loading and thus develop very large $\varepsilon_{\text{max}}$ in the cycle in which such deformation occur. Hence statically induced liquefaction was used to relate $\varepsilon_{V\text{max}}$ to $\varepsilon_{\text{max}}$. In such tests, $\varepsilon_{\text{max}}$ can be controlled using strain controlled loading. In the field however, if contractive deformation does develop, large $\varepsilon_{\text{max}}$ will ensue and hence $\varepsilon_{V\text{max}}$ associated with small $\varepsilon_{\text{max}}$ may not be of any relevance. In their presentation of results, Tokimatsu and Seed (1987) and Ishihara (1990) do show domain of small $\varepsilon_{\text{max}}$ for loose sand. It is not clear how such a state could be realized under stress controlled loading.
Figure 4.44 Relationship between maximum volumetric strain and maximum axial strain

\[ * \sigma'_3 \neq 0 \]
\( (\sigma'_3, D_{rc}) \)

![Graph showing the relationship between maximum volumetric strain and maximum axial strain.](image-url)
4.4.2 Residual $\sigma'_3 \neq 0$ condition

Figure 4.45 shows typical $\varepsilon_V$ on recompression for several initial $\sigma'_3$ and $D_{rc}$ states. The compressibility over a given range of $\sigma'_3$ during recompression is somewhat less than that during virgin compression at higher levels of confining stress but is of the same order of magnitude at the lower, 400 kPa confining stress. This is in contrast to the $\sigma'_3=0$ residual conditions which lead to a drastic increase in compressibility, noted earlier. It appears that liquefaction together with the occurrence of $\sigma'_3=0$ residual condition is responsible for making the sand very compressible on reconsolidation. Liquefaction with residual conditions $\sigma'_3 \neq 0$ will clearly result in lesser volumetric strains than those occur with residual $\sigma'_3=0$ condition. The magnitude of the differences between the two cases will clearly depend on the level of residual $\sigma'_3$, the larger magnitude leading to smaller volume changes. A typical comparison is shown in Figure 4.44 for the sand at $D_{rc}=40\%$ and $\sigma'_3=400$ kPa. For a given $\varepsilon_{\text{max}}$, the $\sigma'_3 \neq 0$ residual condition may be noted to result in smaller volumetric strain than that occurs for the $\sigma'_3=0$ case.
Figure 4.45 Volume change behaviour when residual $\sigma'_3 \neq 0$
CHAPTER 5

CONCLUSIONS

Undrained static, cyclic and post liquefaction loading response of saturated Fraser river sand has been studied under triaxial conditions over a widely ranging confining pressures and relative densities under isotropic as well as anisotropic consolidated conditions. Cyclic loading behaviour leading to liquefaction was assessed at targeted relative densities of 19%, 40% and 59% at several levels of isotropic confining stresses. Post liquefaction undrained monotonic response studied out to delineate the effects of relative density, confining stress, maximum shear strain, manner of shear strain development and the mode of loading (compression and extension). In addition volumetric deformation ensuing on dissipation of excess pore pressure due to cyclic loading was studied as influenced by the level of confining stress, relative density and maximum shear strain due to cyclic loading. Based on the test results, the following conclusions can be drawn.

1. a. The static compression response of the sand even at the loosest density state is dilative except under the highest $\sigma'_3c=1200$ kPa when it manifests a slightly contractive response. In extension however, the sand is contractive over a range of densities, loose to a density just under $D_{rc}=59\%$. The contractive response in extension loading decreases
with increase in relative density at a given confining pressure and a given relative density it decreases with increase in confining stress.

b. In compression loading, static shear stress may turn a dilative sand into a contractive one, at a given confining stress despite additional densification due to shear stress. The presence of static shear stress does not appear to influence extension response significantly.

c. $\phi_{CSR}$ in compression is essentially constant at about 26 degrees, not dependent on initial stress conditions. Extension loading CSR- friction angles are much smaller and tend to increase with increase in placement density. Static shear stress prior to straining does not influence $\phi_{CSR}$.

d. The peak shear strength ratio $S_{up}/\sigma'_{3c}$ versus relative density in extension is a unique linearly increasing relationship that is independent of the initial placement density and initial stress state $\sigma'_{3c}$ and $K_c$.

e. The effective stress conditions at PT state in extension and compression lie on unique straight lines ($\phi_{PT}$ or SS =32°) regardless of relative density, initial stress state ($\sigma'_{3c}$ and $K_c$), type of response (contractive or dilative) and mode of loading. At a given consolidated void ratio, the shear strength at PT state or steady state increases with initial confining stress in extension loading. For a given $\sigma'_{3c}$, the initial static shear stress does not influence $S_{uPT}$ or $S_{uss}$.

f. The average mobilized friction angle at maximum obliquity is about 36 degrees regardless of relative density, confining stress, $K_c$ level and mode of loading-
Chapter 5 Conclusions

compression and extension.

g. The rate of decrease in resistance to liquefaction with increase in confining stress increases with increase in relative density and the largest decrease is associated with the densest state. The confining stress does not have any influence on the resistance to liquefaction at the loose density state, i.e. $K_\sigma = 1$ at loose states of density.

2. h. During cyclic loading, excursion through transient states of $\sigma'_3 = 0$ did not occur for the specified level of strain ($e_a \geq 2.5\%$) deemed to have caused liquefaction. When cyclic loading was terminated after the sand developed the specified strain level ($e_a \geq 2.5\%$), a state of zero effective stress ($\sigma'_3 = 0$ or 100% pore pressure ratio) was realized in most of the cases except for specimens at medium and dense relative densities (40% and 59%) under confining stresses 400 kPa and larger. A state of zero effective stress was also not realized by a static load/unload cycle in medium and dense sand unless strain during loading exceeded a minimum level of about 7%.

3. i. During the post liquefaction monotonic loading of sand that developed 100% pore pressure ratio, the sand initially deformed with essentially zero stiffness which then increased with the level of strain. The unusual behaviour results from the fact that, upon shearing the soil dilates causing the effective stress to increase. The deformation progresses at a mobilized friction angle that equals the angle of maximum obliquity observed under static loading. The stress-strain curve after some axial strain becomes essentially linear. The rate of build up of deviator stress during increases as the relative density increases and the axial strain at which the curves become linear decreases with
increase in relative density.

j. The volumetric strain during reconsolidation following liquefaction is even larger than that during virgin consolidation for the sand that realized a state of $\sigma'_3=0$ at the end of cyclic loading or static load/unload cycle. The volumetric strain of identical samples in which the liquefaction was brought by cyclic loading and a load/unload cycle was identical. The maximum volumetric strain increases with increase in undrained shear strain, and $\sigma'_3$ level but decreases with increase in relative density.
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