RELIABILITY-BASED DESIGN FOR HIGHWAY HORIZONTAL CURVES

by

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Abstract

For more than fifty years the notion of comfortable lateral acceleration has governed the horizontal curve design procedure in North America. With new road and vehicle technology, new methods of design have to replace the old procedures to provide consistent and safer roads to the users. The “limit state design” concept, taken from structural engineering, has already shown to provide a meaningful value of safety to highway design, see Navin (1990-1992).

With the cooperation of the British Columbia Ministry of Transportation and Highways, controlled experiment and field observation were performed to develop the “limit state design” concept for highway horizontal curves. These measurements allowed to accumulate actual statistical information of the basic variables involved in the driving process of horizontal curves. During the experiment, designed with a Latin Square, eight regular drivers and two expert drivers drove four different curves at two speed levels and two pavement conditions. The response variables from this experiment were the lateral acceleration, the speed and the level of comfort. During the observation, in addition to the geometric characteristics of the four horizontal curves selected on the “Sea to Sky” Highway, the speed, lateral acceleration and lateral placement of the free moving passenger cars traveling through these curves were gathered.

The computer program RELAN was used to perform First Order Reliability Method (FORM) analysis for passenger cars subjected to skidding by comparing the expected lateral acceleration
Abstract

supplied by the road to the expected lateral acceleration demanded by the vehicle-driver. RELAN was also used to compute the reliability index $\beta$ and to provide the probability of non-compliance for existing highway horizontal curves, by comparing the expected radius supplied by the highway to the expected radius demanded by the car/driver system. From data collected during the empirical studies, results show an increased probability of non-compliance with a decrease of radius.

Using reliability-based design method, transportation engineers can adjust the design of horizontal curves to fulfill a desirable probability of non-compliance or a desirable reliability index $\beta$. Designers have also a representation of the main variables involved in the process of driving in a horizontal curve and, therefore, have a better control of their designs. With reliability analysis, transportation engineering is provided with a highway design method which better responds to the actual driving demand on the road, and is supported with a measure of ‘safety’ or non-compliance.
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Introduction

How safe is a particular design method? is a question that concerns highway engineers. For example, are the limit state methods of structural engineering more reliable than handbook methods of highway geometric design? The answer is not easy since the design variables and the measures of success for the two disciplines are different.

Several authors (Lamm: 1995, Badeau: 1995, Harwood: 1994) involved in transportation have criticized the geometric road design standards for not providing safety for the user under certain conditions. Curiously, given the century and a half of transportation engineering, you would expect the experience and knowledge of design and operation to give transportation, risks similar to risks from structural failure. Blockley (1980) compared different engineering systems and estimates of the probabilities of death. For the United Kingdom in 1980 he computed an approximate annual risk of death per person of $2 \times 10^{-4}$ due to car travel and of $10^{-7}$ due to structural failure (buildings, bridges).

The changes of transportation through the ages is by itself an indication of the importance for the society to be mobile. Figure 1 illustrates the transportation developments over time. The invention of the automobile in 1886 revolutionized the concept of personal transportation. The highway network has evolved to satisfy the growth of private car with efficient and safe routes.
Figure 1. Development of transportation.

[Adapted from: Navin: 1993]
Introduction

If one can safely cross Canada by car within six days, it is because highway design allows one to adopt safe speeds, which usually correspond to an accepted level of comfort of the driver.

The North American standard for horizontal curves set limiting value of lateral acceleration based on the level of comfort experienced by drivers during experiments completed prior to 1940 (Moyer: 1934, Stonex et al.: 1940, Moyer et al.: 1940). In North America, the present advisory speeds for horizontal curves is based on this early notion of comfort (Merrit: 1988). Ministry of Transportation and Highway’s current observations found an 85th percentile speed of 80 km/h for a section posted at a speed limit of 60 km/h along the “Sea to Sky” highway in British Columbia. Today’s drivers may take more risks, or the vehicles they drive are more efficient. The latter is the most probable alternative. Today, most drivers drive on highways in a relatively comfortable way. Therefore, the standard values for the lateral acceleration obtained by Moyer (1934) are no longer representative of what drivers feel. Consequently, the current design procedures may not properly combine the geometry of the highway elements with modern vehicles and drivers’ performances, and therefore may not provide the optimum safety for the users.

In 1993, in British Columbia, 46,952 persons were injured and 512 were killed in 93,819 reported vehicle accidents (Motor Vehicle Branch: 1993). Ferrandez (1993) had the following comments on road accidents:

“The general population still believe that accidents are random and unavoidable events, where all attempts to stop them is illusory, … , in most cases it would have been possible to foresee a few seconds earlier the events if the trajectory or the speed
had not been modified. It is a given that accidents have multiple causes. An accident is the result of several factors related to the personality and behavior of the driver, to the design and construction of the road, and its immediate environment and to the design and maintenance of the vehicle. [The accident] is the symptom of highway dysfunction in which the components are in constant interaction.” (translated from Ferrandez: 1993).

Research summarized in TBR Special Report 214 (1987) shows the correlation between accident rates and the different geometric elements that composed the road. For a two lane rural highway, the degree of curvature has been found to be the strongest geometric variable to be related to accident rates. The accident rate in curves is roughly six times greater than on a corresponding straight highway.

The question for the design engineers is; how do elements of the roads geometry influence the safety of the overall design, and what is the cost effectiveness of possible improvements? The current highway design procedures themselves are not able to provide a meaningful measure of the reliability of the road. If a particular design of a road follows the requirements of the geometric design standards, then, there is an implicit assumption that the design provides a good level of safety to the driver-vehicle system (Navin: 1992).

Transportation engineers, when designing highway horizontal curves balance variables that represent the driver, the car, and the environment. The variables obey the laws of physics that surround the tire-car-driver system in horizontal curves. A significant difficulty for the transportation engineer is to model the behavior of the tire-car-driver system by joining the psychological aspect of the driver with the mechanical and physical factors of the vehicle.
traveling along horizontal curves. Many research programs (Moyer et al.: 1940, Taragin: 1954, Emmerson: 1969, McLean: 1981, Wong et al.: 1992, and others), have focused on finding the best correlation between the horizontal curves and the behavior (speed and path) of the tire-car-driver system. These research programs are essential for setting design policies.

Highway design engineers use the American Association of State Highway Transportation Officials (AASHTO) A Policy... (1990) and, on the Road and Transportation Association of Canada (RTAC) (1986) as geometric design standards. RTAC (1986) presents the minimum radius for horizontal curve as:

\[ R_{\text{min}} = \frac{V^2}{127(e_{\text{max}} + f_{\text{max}})} \]  

The design is controlled by the maximum lateral acceleration \( f_{\text{max}} \) allowed at a particular design speed \( V \) and a maximum superelevation \( e_{\text{max}} \) (side slope of the road). This official procedure, simply represent the basic dynamic and geometric variables: design speed, radius, coefficient of friction, and superelevation. The design standards derived from these variables do not explicitly consider the influence of the driver and vehicle. Also, especially in unfavorable economic situations, horizontal curve designs tend to adhere to the minimum radius allowed by the standards (R. J. Voyer, personal communication, June 1995) and this lead to minimum design of highways.
In structural design, the probabilistic approach is used to ensure that a reasonable margin of safety is achieved. A properly designed structure has a very low probability of a partial or total collapse due to an extreme load. This probability corresponds to the probability of having the applied loads greater than or equal (limit state) to the resistance of the structure. An absolutely safe structure cannot be designed since there are always some uncertainties about the nominal constraints during the lifetime of the structure. For example, a local earthquake may create higher forces on the structure that may damage it.

For a structural system, the probability of failure is computed by comparing the two main, random variables: the resistance $S$ (strength of a structural element), and the demand $D$ (loads applied on the element) in the performance function $G$. The function $G$ represent the difference of the supply and demand:

$$G = S - D$$

(2)

Figure 2 is a graphical representation of the system of Equation 2 with the corresponding probability of failure or non-compliance and the safety margin. The margin of safety is defined as:

$$M = E(S) - E(D)$$

(3)

where: $M$ = Margin of safety.

$E(S)$ = Expected value of Supply.

$E(D)$ = Expected value of Demand.
The two random variables, $S$ and $D$, are a set of many other random variables. The multi-variables analysis suggest the use of approximate methods such as: Monte Carlo or the Iterative Fast Monte Carlo simulations to compute the probability distributions. Monte Carlo simulation involves the generation of random numbers. An application of this method is presented by Navin (1992), when he calculated the safety margin distribution for stopping sight distance of cars and trucks. The First and Second Order Methods (FORM and SORM) have been developed in order to make the probability calculation accessible when simulation is not possible. If $S$ and $D$ are normal, independent random variables, then, $G$ is a normal variable, and the FORM and SORM method can compute the reliability index $\beta$. 

Figure 2. Generalized margin of safety definition.
The reliability index $\beta$ allows engineers to evaluate the probability of non-compliance ($P_{nc}$) by using the standard normal probability distribution function (see figure 1.3). The reliability index $\beta$ is a relative measure of safety that is chosen to give the desired degree of reliability to the engineering element being designed. In structural engineering design the value of $\beta$ usually range from 2 to 3.5. In the limit state design equation for a structural element: $\phi S \geq \sum \alpha_i Q_i$, the loads factors $\alpha_i$ and the resistance factors $\phi$ are safety parameters which assure the desired degree of reliability. $\sum \alpha_i Q_i$ represents the Demand and $\phi S$ represents the Supply of this particular engineering system. Consequently, a portion of the margin of safety is in the load factors and the other portion is in the resistance factor.

![Figure 3. Definition of the safety index $\beta$.](image-url)
The idea of using the limit state concept for a more “consistent” road design was introduced and presented by Navin (1990, 1992). It is known that a “consistent” design of a highway, as when designing a structural system, is done by considering the whole of the highway as a unit. The reliability of a whole structure is a function of the reliability of each element that composes the structure. Therefore, a good knowledge of the reliability of each geometric element is essential to design a consistent highway. For the reliability analysis of highway horizontal curve, Navin (1992) proposed to use the radius as being the basic element involved in the performance function $G$. The performance function $G$ would be a representation of the radius supplied by the design $R_s$, and the radius demanded by the users $R_D$. $R_s$ refers to the geometric characteristics of the curve. If this element is considered to have negligible variability, then it can be treated as constant. However, this radius may not represent a single value since the trajectory of all the given cars in a curve, often, do not follow the designed centre lane. $R_D$ is defined in terms of the driver speed $V_D$ and the lateral acceleration capacity of the road-tire-vehicle as:

$$R_D = \frac{V_D^2}{a_y}$$  \hspace{1cm} (4)

The basic concept of horizontal curve design, then, becomes $\phi R_s \geq \alpha_R R_D$, where $R_s$ and $R_D$ are the two main random variables, and $\phi$ and $\alpha_R$ are the safety factors. This concept takes into account the variability and the uncertainty of the variables involved in the system.

Navin (1992), adapted the structural terminology for highway by designating probability of failure as probability of non-compliance ($P_{nc}$). Probability of non-compliance is an intuitive
measure of the non-performance of the studied element and may not necessarily correspond to an event that could be associated to the risk of an accident.

This thesis investigates the use of reliability analysis to measure the level of safety of a horizontal curve. The main goal of this research is to obtain new basic data from which the margin of safety and the probability of non-compliance of a horizontal curve may be estimated.

Because the most complex variable to consider in highway geometric design is the driver, many transportation researchers have simplified their field studies by combining the driver with the vehicle. The car-driver system (C/D) is the studied variable during the thesis. A driving experiment was made with eight regular and two expert drivers at the Boundary-Bay Airport. Also, driving habits along four curves were observed on Highway 99 between Horseshoe Bay and Squamish in British Columbia.

A Modified Latin Square was used to design the driving experiment. Four radii (16, 25, 60, and 100 m), two level of speed (comfortable, fast), two pavement conditions (wet, dry) and four group combinations (driver, passenger) with female and male subjects, were the controllable variables distributed among the 16 treatment combinations of the Latin Square. The same car was used for all tests. Because it is a good tool for understanding and providing insight into the behavior of the C/D system, the notion of comfort was investigated during the analysis. The lateral acceleration and the speed are the two elements that drivers adjust in a horizontal curve to
achieve their comfort. In addition, for all tests, a passenger accompanied the drivers and recorded on a subjective scale his or her own level of comfort.

Four curves with different radii were selected on Highway 99. The speed, the lateral acceleration and the path of the free moving passenger cars going through the curves were measured in both directions. From the individual speed and lateral acceleration, an effective path radius was also computed.

From these empirical studies, statistical information was collected from the basic variables: velocity, lateral acceleration, lateral placement of the car, the level of comfort and the geometric characteristics of the curves such as the radius and the superelevation.

A program entitled RELAN or RELiability ANalysis, which uses FORM/SORM methods, was developed at the Civil Engineering Department of the University of British Columbia by Foschi et al. (1993). The first exercises using this software was to compute the probability of non-compliance based on the skidding criterion. The two variables used for this calculation are the lateral acceleration developed by the C/D subjects (the demand), and the maximum lateral acceleration, provided by expert drivers (the supply). The analysis is made for the horizontal curves of the experiment and for the Highway 99. A second exercise was to develop the performance function $G$ based on $R_s$ and $R_d$, and to use the data collected on Highway 99 to compute the reliability index $\beta$ and the corresponding probability of non-compliance.
This report is structured in the following way: Chapter 1 discusses different road aspects such as design, economy and safety. Throughout this chapter, accident studies are used to support the safety problems encountered on roads and horizontal curves. Different approaches to design horizontal curves are also introduced. Chapter 2 is a literature review of the driver behavior at horizontal curves. Variables which mostly influence the speed in the curves are then presented. Chapter 3 and 4 are the procedures and the analyses of the experiment and observation. Chapter 5 discusses the reliability concept in more detail, including the basic principles of First and Second Order Reliability Methods. Two types of reliability calculations follow: One is for measuring the risk of skidding when people drive at comfortable and fast speed, and the other is for measuring the non-compliance for simple horizontal curves. Finally the last section includes conclusion and recommendation for further studies.
CHAPTER 1
Road, Design and Safety

1.1 Road

1.1.1 Classification

People drive on different types of highways with different expectation of comfort and safety. The design characteristics of a specific road is set to correspond to the importance of the route. The level of service is defined by the flow on the road and for two lane rural highways is the percentage of traffic following. The speed of travel does not vary greatly.

The North American standards, as defined by the American Association of State Highway and Transportation Officials’ (AASHTO’s) *A Policy on Geometric Design of Highways and Streets* (1990) and Road and Transport Association of Canada (RTAC) (1986), classify the road system into two major groups: rural and urban roads. For each of these groups, the roads are designated by their function: local, collector, arterial or freeway. Each of these sub-classifications satisfies a “service function” and a “traffic characteristic” (RTAC: 1986).

1.1.2 Design Speed

The design speed is the primary determinant when designing the vertical and horizontal alignment of any class of road. The North American standards define design speed as “the
maximum safe speed that can be maintained over a specified section of highway when conditions are so favorable that the design features of the highway govern" (AASHTO: 1990).

For rural roads, the RTAC (1986) manual proposes a design speed for each specific classification as presented in Table 1.1.

**Table 1.1. Rural Road classification and corresponding design speed, RTAC (1986).**

<table>
<thead>
<tr>
<th>Design speed</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>km/h</td>
<td>Local</td>
</tr>
<tr>
<td>50</td>
<td>RLU50</td>
</tr>
<tr>
<td>60</td>
<td>RLU60</td>
</tr>
<tr>
<td>70</td>
<td>RLU70</td>
</tr>
<tr>
<td>80</td>
<td>RLU80</td>
</tr>
<tr>
<td>90</td>
<td>RLU90</td>
</tr>
<tr>
<td>100</td>
<td>RLU100</td>
</tr>
<tr>
<td>110</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td></td>
</tr>
<tr>
<td>130</td>
<td></td>
</tr>
</tbody>
</table>

R = Rural; U = Undivided; D = Divided.

Horizontal curve is one geometric element of the horizontal alignment which refers to this design speed.

### 1.1.3 Law of Mechanics for Horizontal Curves

The law of mechanics of a point mass travelling at a constant speed in a horizontal curve with a given radius is defined as:
\[
\frac{mgV^2}{gR} = mge + mgf_y
\]  

(1.1)

The left part of equation 1.1 represents the centrifugal force created by the speed \( V \) of the vehicle of mass \( m \) in a curve with radius \( R \). On the right side, the superelevation \( e \) (side slope of the lanes) and the coefficient of friction of the pavement \( f_y \) are centripetal forces which counteract the centrifugal force at the tires' point of contact. Figure 1.1 illustrates the components acting on a vehicle in a horizontal curve.

\[ m = \text{vehicle mass (kg)} \]
\[ V = \text{vehicle speed (m/s)} \]
\[ R = \text{curve radius (m)} \]
\[ g = \text{gravitational acceleration (m/s}^2\text{)} \]
\[ \alpha = \text{atan e (superelevation) (degree)} \]
\[ ay = \text{lateral acceleration (measured by accelerometer) (g's)} \]
\[ fy = \text{friction force (N)} \]
\[ N = \text{normal force (N)} \]

Figure 1.1. Vehicle in superelevated horizontal curve.

Hereafter, it is assumed that the lateral acceleration as measured by the accelerometer corresponds to the side friction coefficient between the tires and the pavement.
1.1.4 Vehicle

In a car, the probability of skidding is higher than the probability of rolling over when driving in normal conditions in a horizontal curve (rollover threshold of 1.2g). The opposite is valid for trucks, which are more likely to rollover than to skid (rollover threshold of 0.4g and skidding threshold of 0.8g). The experiment, which measured the performance of the vehicle/driver system when driving in horizontal curves, only involved cars. The work thesis is based on the performance of such vehicles.

1.1.4.1 Car maximum lateral acceleration

A road test digest was published in Car and Driver (1995) which presents the performance of 126 cars and car-vans. The road-holding data provides the maximum lateral acceleration that the tested vehicles could generate when cornering on a 300 ft (91.4 m) diameter skid pad. The distribution of the maximum lateral acceleration of the car population is presented in Figure 1.2. Hereafter, this population's data is assumed to follow a normal distribution.

The conditions for the cornering friction test were good. The asphalt pavement was in dry condition and cars with good tires were driven by experts. However, these ideal conditions are sometimes not encountered on the roads. The side friction coefficient is a function of the condition of the pavement and of the tires. On the road, the ice supplies a coefficient of friction from 0.1 to 0.2, and the packed snow supplies one from 0.3 to 0.5 (Hutchinson et al. (1989) in Stang (1993)). In the case of wet pavement, the lateral friction coefficient for a given speed is a
Figure 1.2. Distribution of the maximum lateral acceleration that a car experienced on a 300 ft diameter skid pad.

function of the water depth and the quality of the tires’ tread. Figure 1.3 shows a distribution of the coefficient of friction for a new tire for five different depths of water. Figure 1.4 presents, for three types of pavement textures with five millimeters of water depth, the ranges of coefficient of friction for tires with tread and for tires without tread.
Figure 1.3. Typical set of Lateral Force Coefficients for a standard new tire for different water depths (indoor tire testing facility). [Source: Williams et al. (1983)]

The quality of the pavement as well as the quality of the tires contribute to the performance of the road-tire-car-driver system when cornering in horizontal curves and, therefore, to the driver's safety.
1.2 Road and Safety

The geometric design for highways has always been studied in order to improve their economic efficiency and safety aspects. Due to the increase in vehicles on the road and the inevitable resulting increase in accidents and fatalities, people’s safety in the immediate environment of the road has become the main concern of governments.

Research on safety is generally developed by doing accident studies, based on statistical analysis or based on the analysis of the accident typology at a particular location. Two-lane and four-lane
undivided rural highways have a higher proportion of accidents than all other road classes (Choueiri et al.: 1994)

The accident event involves, to a certain degree, several factors. Choueiri et al. (1994) presented them as:

- Human factors: misjudgment of the road alignment and traffic, speeding, alcohol or drugs, experience, handicaps, and gender.
- Physical features of the site: horizontal and vertical alignments.
- Presence and action of traffic: traffic volume, traffic mix, and seasonal and daily variations.
- Legal issues: federal and provincial laws, traffic control devices at the sites and degree of enforcement.
- Environmental factors: weather and pavement conditions.
- Vehicle deficiencies: tires, brakes, and age of the vehicle

The multi-factor nature of accident events is now understood by transportation engineers. Because of the difficulties in collecting data at an accident site, it is difficult to assess the degree to which each factor played a part in an accident. Similarly, the combination of two geometric variables (e.g. low radius with low sight distance) makes the differentiation even more difficult (Wong et al.: 1992:425).

Treat (1980) and Sabey et al. (1980) in Wong et al. (1992) presented human factors, which are involved in 95% of accident cases, as being the most influential. This is supported by Figure 1.5,
presented by Sayed (1994), which reveals that human factors are involved in 93% of the cases in British Columbia.

![Venn diagram showing accident responsibility percentages](source: Sayed (1994))

The same figure also shows that the road environment is involved in 30% of the cases. The overall safety of a road section is the result of the safety of the road section’s elements, as shown in Figure 1.6:

![Flowchart showing success and non-compliance modes](source:Sayed (1994))

**Figure 1.6.** Success and non-compliance modes for road section
On a two-lane rural highway, a driver is at higher risk of having an accident in a horizontal curve than on a straight portion of the same highway.

1.2.1 Safety in Horizontal Curves

From accident studies, the degree of curvature (degree of angle variation per 100 ft (33 m)) has been found to be the strongest geometric variable to be related to accident rate. (TRB report 214: 1987). The smaller the radius, the higher the accident rate (Choueiri et al.: 1994; Zegeer et al.: 1992). There is a strong negative relationship between accident rates and radii up to 500 m. For radii over 500m, the accident rate is constant or decreases slightly with increasing radius.

The radius is not the only design element that affects the safety of a curve: lane width, roadsides and side-slopes, spiral transitions, superelevation, gradient, pavement surface, delineation and other traffic control devices, shoulder width and slope, sight distance, have also been studied jointly or alone. From their study on the effect of road improvements, Zegeer et al. (1992) found that curve flattening reduces accident frequency by as much as 80%, depending on the length of the curve and the degree of flattening. Widening lanes, widening paved shoulders, adding unpaved shoulders, adding spiral and improving superelevation were also improvements which would could reduce accidents from 5% for the spiral transition up to 21% for lane widening.

In the thesis, the radius of simple curve, the superelevation, and the pavement surface are the sole physical parameters used in the analysis of the probability of non-compliance of a horizontal curve.
There are contradictory opinions on the efficiency of spiral transition curves. From 138 spiral
transition curves and 152 standard curves, Tom (1995) proceeded to an accident study which
revealed that the fatal and injury accident rate for spiral curves was significantly higher than for
standard curves. In spiral transition curves, Tom (1995) found that the accident rate was
significantly higher for the right-hand turn than for the left hand-turn in short to moderate radii.
His recommendation, addressed to the California Department of Transportation, was to avoid
adopting spiral curves as standard practice. Zegeer et al. (1992) found that adding “spiral to a
new or existing curve will reduce total curve accident by approximately 5 percent.” Brenac
(1996) explained that the French standards do not use the compound curves because “[they] are
considered likely to impair the perception of the curvature and generate accidents.”

A limitation of those studies is that the curves are considered isolated from the road section.
“The safety effect of an individual curve is influenced not only by the curve’s geometric
characteristics, but also by the geometry of the adjacent highway segments” (TRB report 214:
1987).

The inconsistency of the alignment is also a cause of accidents. The inconsistency of a road
section is a “Violation of driver expectancy.” For example, a short curve after a long tangent will
result in a high reduction of speed of the car/driver system before entering the curve. In his
research review, Johnston (1982) summarized the outcome of the Peter Casey and Associates
(1979) research, where the speed reduction required before driving into the curve was shown to
be one factor responsible for fatal accidents.
Two early studies quoted in Brenac (1996) showed the correlation between alignment and levels of safety of horizontal curves. One study revealed that there is a high risk of accident in curves with a high degree of curvature, when the average curvature on the road section is low. This risk decreases when the average curvature increases (Research on Road Traffic: 1965). The second study presented the same idea by saying that the accident rate does not differ between high and low curvature, if there is an increase of bend per kilometer (Baldwin: 1946).

The consistency of the road alignment can be measured, as presented by Lamm et al. (1995), with safety criterion based on the speed distribution along the road section as presented in Table 1.2.

<table>
<thead>
<tr>
<th>SAFETY CRITERION</th>
<th>GOOD</th>
<th>FAIR</th>
<th>POOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>[V85_i - V85_{i+1}] ≤ 10 km/h</td>
<td>&lt; [V85_i - V85_{i+1}] ≤ 20 km/h</td>
<td>20 km/h &lt; [V85_i - V85_{i+1}]</td>
</tr>
<tr>
<td>II</td>
<td>[V85-V_D] ≤ 10 km/h</td>
<td>&lt; [V85-V_D] ≤ 20 km/h</td>
<td>20 km/h &lt; [V85-V_D]</td>
</tr>
</tbody>
</table>

V85 = 85th Percentile Speed; V_D = Design Speed; i = Geometric elements.

The physical parameters of a curve are often used as criteria to provide a meaningful measure of the safety of a road section or of its geometric elements. Lin (1990) used the degree of curvature as a safety indicator of a road section, and presented a linear relationship between accident rates and curvatures:
where $A$ is accident rate in Acct./MVM (1MVM = 1.6 Million Vehicle Kilometers). Craus et al. (1979) used a superelevation coefficient $\beta^2 = \frac{a_s}{a_c}$, where $a_s$ is the 'superelevation acceleration' and $a_c$ is the 'centrifugal acceleration', to express the degree of safety for a vehicle travelling along a curve. Navin (1990) proposed to use the radius as the physical parameter to provide safety information on horizontal curves, by using reliability analysis. This method is investigated in the thesis.

1.3 Horizontal Curve Design

Literature that relates to engineering design usually defines the role of the engineer and his design as a balance between performance and safety:

"One of the principle aims of engineering analysis and design is the assurance of the system performance within the constraint of economy" [Jonsson; 1992].

"Structural engineers serve basic needs of the community in that the product of their efforts help society to run more efficiently and create greater wealth, prosperity and stability." [Blockey D.; 1986].

"Highway design involves reaching an optimal compromise between safety, efficiency and economy." [Kanellaidis; 1996]

The authors agree that, in a design process, there is a trade-off between safety and economy. This trade-off corresponds to the failure rate accepted by the designers.
The economical aspects in transportation engineering are major factors influencing the design of a highway. Because of the "complex engineering-political-financial enterprise" (Navin: 1992) that is transportation, costs attributed to highway construction and improvement have to be justified by some return value measured in reduced travel time, reduction in pollutants, and increase in safety. Seneviratne (1994 and 1995) proposed to model highway design based on the different cost functions involved in the road environment. With regard to a horizontal curve, an optimum radius or degree of curvature can be derived from two basic cost functions: accident cost and construction cost as presented in Figure 1.7.

![Figure 1.7. Hypothesized cost functions [for an optimum degree of curvature].
[Source: Seneviratne (1994)]](image)

The advantage of the cost-effectiveness model, if the input data are valid, is that it provides a direct cost information of the designed horizontal curve. The problem lies in the fact that, after
reaching the optimum curvature, there is a risk that safety will be viewed as a secondary variable. The predominant consideration of the engineer must always be the safety of the driver/car system and the people in the immediate environment of the road. The reliability-based design method could also involve a cost parameter to account for the economic limitations, in which case it would intervene as a secondary variable. The economic aspect of highway design, construction and performance is not considered in this thesis.

1.3.1 North American Design Standards

Using Equation 1.1, North American standards propose a simplified curve equation to design a minimum radius:

\[ R_{\text{min}} = \frac{V^2}{g(e_{\text{max}} + f_y)} \]  \hspace{1cm} (1.3)

where

- \( R \) = radius of the curve (m),
- \( V \) = design speed (Table 1.1) (m/s),
- \( e \) = superelevation (m/m),
- \( f_y \) = road coefficient of friction,
- \( g \) = gravitational acceleration (9.81 m/s\(^2\))

The maximum superelevation allowed by Canadian standards ranges from 0.02 m/m for urban local roads to 0.08 m/m for freeways. A higher superelevation, combined with a low road friction factor, would result in a skid toward the inner shoulder for slow vehicles.

The North American design standards assign to each design speed, a corresponding level of lateral acceleration that is tolerable (assumed comfortable) to a reasonable driver. Table 1.3
presents the levels of lateral acceleration recommended by the standards of AASHTO (1990) and RTAC (1986) as the maximum value to be used in Equation 1.2.

Table 1.3. Coefficient of friction for different design speeds.  
[Source: AASHTO (1990)]

<table>
<thead>
<tr>
<th>Design Speed mph (km/h)</th>
<th>f value recommended</th>
</tr>
</thead>
<tbody>
<tr>
<td>80 (129)</td>
<td>0.08</td>
</tr>
<tr>
<td>70 (113)</td>
<td>0.1</td>
</tr>
<tr>
<td>60 (97)</td>
<td>0.12</td>
</tr>
<tr>
<td>50 (81)</td>
<td>0.14</td>
</tr>
<tr>
<td>40 (64)</td>
<td>0.15</td>
</tr>
<tr>
<td>30 (48)</td>
<td>0.16</td>
</tr>
</tbody>
</table>

The design speed concept and the maximum side friction adopted by these standards have been criticized by several authors.

Badeau et al. (1995), Choueiri et al. (1994), Lamm et al. (1991), and Navin (1991) have underlined the inconsistencies between the manuals’ reference values, based on the 1930’s studies, and the new vehicle technologies which are found on today’s roads.

Lamm et al. (1991) found that side friction demand exceeded the side friction assumed when the degree of curvature is greater than 6.5 degrees, or when the operating speed in the curve is less than 80.4 km/h.

The posted advisory speed is commonly selected in North America by referring to the lateral acceleration listed in Table 1.3. From data gathered in the province of Quebec, Badeau et al.
(1995) found that drivers travel existing curves above the posted speed limit. Assuming that today's drivers still select comfortable and safe speeds in horizontal curves, Badeau et al.'s observation shows a certain inconsistency between the actual performance of the car/driver system and the methods used in designing horizontal curves and in selecting appropriate advisory speeds.

In evaluating the adequacy of the geometric design policy AASHTO (1990) for horizontal curve, Harwood et al. (1995) mentioned that the notion of a comfortable level of lateral acceleration provides a great margin of safety that minimizes the risk of accident. This would be true if people were driving at the maximum safe speed that corresponds to "the minimum speed at which the driver or passenger will feel a side pitch towards the exterior of the curve." (Barnett: 1936). As presented by Badeau et al. (1995), running speed in curves is not based on the advisory speed but on a driver's judgment, which may increase the rate of accidents.

The 85th percentile speed (maximum speed that 85 per cent of drivers would adopt when traveling in good weather and low traffic (Lay: 1986)) is useful for setting speed limits because it exceeds the speed of most approaching vehicles. This technique, rather than the aforementioned procedure, has traditionally been used by traffic engineers in some American states (TRB Report 214: 1987).

Within the scope of their analysis, Harwood et al. (1994) concluded that "the existing [High-Speed Horizontal curves Design Criteria] provide adequate margin of safety against skidding and
rollover by both passenger cars and trucks...” However, for the Low Speed Horizontal Curves Design Criteria, “the minimum radius, ..., may not provide adequate margin of safety for trucks [in poor environment] or with low rollover thresholds.” For their analysis, they assumed that all vehicles were driven at the design speed. Based on observation, McLean (1988) showed that, in a curve, the car free speed is higher than the design speed when the later is below 100 km/h, and that the car free speed is lower than the design speed when the later is over 100 km/h.

Navin (1992) used a probabilistic approach to compute the expected “critical cornering speed” for trucks by assuming a driver’s risk acceptance in curves was 3 in 1,000. For a properly loaded trucks, the expected speed was 65.9 km/h “or roughly 9 km/h less than the speed at which rollover actually occurred.” With a poorly loaded truck, the rollover threshold speed was 81.7 km/h, “or 7 km/h greater than the rollover speed.” The advantage of this method over Harwood’s traditional one is that the expected speed defines, or is close to, the real limit of the vehicle/driver performance, which is better defined by the impending skid or the impending rollover of the vehicle.

1.3.2 Other methods

The Australian design practice uses the 85th percentile of the desired speed (the speed adopted by the driver on the least constrained element of the road) as the design speed to be used in Equation 1.3 for a minimum radius. The maximum side friction in Table 1.4 is a function of this design speed.
Table 1.4. Maximum design values of side Friction Demand For Sealed Pavements. [Source: ARRB]

<table>
<thead>
<tr>
<th>Design Speed km/h</th>
<th>Coefficient of Side Friction $f_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.35</td>
</tr>
<tr>
<td>60</td>
<td>0.33</td>
</tr>
<tr>
<td>70</td>
<td>0.31</td>
</tr>
<tr>
<td>80</td>
<td>0.26</td>
</tr>
<tr>
<td>90</td>
<td>0.18</td>
</tr>
<tr>
<td>100</td>
<td>0.12</td>
</tr>
<tr>
<td>110</td>
<td>0.12</td>
</tr>
<tr>
<td>120</td>
<td>0.11</td>
</tr>
<tr>
<td>130</td>
<td>0.11</td>
</tr>
</tbody>
</table>

For the same design speed, the maximum side frictions in Tables 1.3 and 1.4 are different. For a design speed of 80 km/h, AASHTO’s side friction is 0.14, whereas ARRB’s side friction is 0.26. This means that, by using Equation 1.3, for a non-superelevated curve, AASHTO’s minimum radius is two times longer than what ARRB design procedures would provide for the same design speed. Using Lin’s (1992) linear relationship between accident rate and curvature (see Equation 1.2), the risk of having an accident on the Australian curve would be two times greater than on the curve designed in North America. This simple example shows that for the same design speed there is some discrepancy between the radii adopted by these countries.

Brenac (1996) presented a review of the British, German and French design procedures. The 85th percentile speed is also used by the three countries for designing roads. In the UK and in Germany, this speed refers to a journey speed for the road section studied (2 km minimum in UK), whereas in France the 85th percentile speed is defined for each particular point studied on the road. "The local characteristics of the point (radius of the horizontal curvature, cross-section,
gradient) explain more than 70% of the between site [speed] variation.” (Brenac: 1996). The three countries’ standards have others policies which cover the consistency of the road alignment.

It is a given that a sharp curve after a long tangent is an accident-prone location. Lamm et al. (1994), after highlighting the importance of the human aspect on road design, criticized the Green Book (1990) manual for not providing any guidelines on highway consistency: “no mention is made of, nor is guidance given on, any design procedure comparable to curvilinear alignment design for achieving better consistency and safety on two-lane rural roads.” (Lamm et al.: 1994).

Brenac (1990 and 1996) criticized the design speed concept for not representing the actual speed of the drivers in the curves. In 1990, Brenac added to his concluding remarks that even the 85th percentile speed should not be considered for the design, but that the “speed which is a function of the perception drivers have of the bend during their approach, ...” should rather be used. This speed would be the almost perfect variable to be used in a “limit state design” equation for horizontal curves.

Limit state design for transportation engineering, of the same form as the limit state design model used to design structural elements, was presented by Navin (1990). The expected value and the variance for stopping sight distance, horizontal curve, decision sight distance, passing sight distance, and vertical curve were presented. The values for the intervening random variables were selected from literature or estimated. For each design parameter, the safety margin, safety
index, and chance of failure (chance of non-compliance for horizontal curve) were computed. Navin (1992) showed the adequacy of using the probabilistic approach when he computed the expected critical cornering speed for trucks. In addition to providing the expected value of a multiple, uncertain random variable system, reliability analysis provides a value of safety.

The existing design procedures and the variants target a unique goal: Render the roads reliable and safe. The methods are different among the countries, but all deal with the same and most uncertain variable, the driver and his vehicle.
CHAPTER 2
Horizontal Curve and Drivers.

2.1 Car/Driver Behavior

The art of highway design has evolved toward safer road because of a better understanding of the car/driver systems behavior. From prior observations and experiments, transportation engineers can almost tell the average speed that car/driver systems would adopt on a new road. A car/driver system has a human component which learns, memorizes and modifies his behavior according to his temperament and the changing environment, hence, the engineers cannot exactly predict the speed of this variable.

According to Wong et al. (1992), there is a considerable variation among drivers’ speed, path radius and accelerations. A person might drive at a comfortable speed of 60 km/h whereas another might adopt 80 km/h as a comfortable speed. The speed drivers undergo is also a function of the trip’s purpose and circumstances. It is likely that somebody late for an important interview will select a higher speed than somebody in a leisurely Sunday drive.

2.2 Car/Driver Behavior in Horizontal Curve

The variables influencing the speed and the car/driver system in a horizontal curve have been studied since the early 1930’s. In 1954, Taragin presented these variables, for the first time, after an extensive observation of the behaviors of the vehicle-driver systems in curves. Studies about
CHAPTER 2: Horizontal curve and drivers

driver behaviors in horizontal curve have mostly focused on the speed and the path that
car/drivers systems select around curves. Then, regression equations to explain the influence of
the dependent variables have been developed.

2.2.1 Drivers’ speed

2.2.1.1 Variables

Taragin (1954) identified the degree of curvature as the most important factor in determining
speeds on curves. All following research have agreed that the radius of a curve is the important
geometric features which influence the driver’s speed. For compound curves, Badeau et al.
(1995) found a high correlation between the driving speed in the curve and the “curves’ first
encountered radius”. Lin (1990) explain that curvatures account for 87% of the variation in the
following 85th percentile speed equation:

\[ V_{85} = 61.9 - 1.906D + 0.032D^2 \]  \hspace{1cm} (2.1)

where D is the degree of angle variation for 100 ft.

Taragin (1954) found that pavement width also produced significant effect. Lin (1990) found
that lane width can explain 4% of the variation of the 85th percentile speed (equation 2.1). The
same study revealed little influence due to the superelevation rate and shoulder width. Badeau et
al. (1995) mentioned that the superelevation does not seem to be perceived by the drivers. The
same observation was made by McLean (1978) and Kanellaidis et al. (1990), who explained that
superelevation had no effect on observed speed. Another factor which could affect the driving
speed in the curve is the grade. However, very little information that examines the influence of the combination of horizontal curves and grades is available (Badeau et al.: 1995).

Wong et al. (1992) explained that side frictions provided by roads vary considerably with time because of different factors: weather, season variations and the wearing caused by the tires’ friction. Therefore, drivers cannot estimate the real side friction when negotiating a curve. McLean’s (1983) study revealed that “there was no empirical evidence that drivers respond to actual or subjectively predicted side friction in selecting their speed around the curve.” The same conclusion was adopted by Taragin (1954). When the road surface conditions are ‘readable’ as snow or ice, heavy rain or dry, unpaved or paved roads, drivers roughly adapt their speed in a curve according to a perceived adherence. However, the provided visual information is generally insufficient for drivers to select a speed according to the available side friction factor.

The expected value of the speed in a curve is not only dependent on geometry variables, but also on the driver’s desired speed (McLean: 1981). Desired speed is defined as the 85th percentile speed that drivers would maintain in the least constraining portion of the road (tangents). After an extensive research at 120 curve sites, McLean (1981) proposed the following regression for the 85th percentile car speed in curves:

\[ V_{c85} = 53.8 + 0.464V_F - \frac{3260}{R} + \frac{85000}{R^2} \]  

(2.2)

where \( R \) is the curve radius in m and \( V_F \) is the desired speed 85th percentile car. 92% of the variance of the dependent variable is explained by the regression. To better represent the 85th
car speed in the curve, McLean partitioned the data into groups of about same desired speed and
developed a “family of curve speed prediction relations” as shown in Figure 2.1.

![Figure 2.1. Curve speed prediction relationships [source: McLean (1981)]](image)

### 2.2.1.2 Speed variation

The common conclusion, since the first study on variation of speed around curves, is that speed
adjustment is made prior to the curve, and speed in the curve is constant thereafter. Other studies
have concluded that speed adjustment also occurs in the curve.

One of Taragin’s (1954) suggestions was that drivers do their speed adjustments before entering
a road curve. He concluded that the speed in the curve is mostly constant. Mintsis (1988) and
Badeau (1995) found that minimum speeds were at the middle of the curve. Mintsis (1988) explained that speed variations are highly dependent on the degree of curvature, and that it is for high curvature that the speeds were changing considerably. Mintsis’s (1988) study could only concur Taragin’s suggestion for the good vehicles. In general, truck drivers know that their vehicles are more likely to rollover in a curve, and that they do not drive a vehicle which has the acceleration performance of a car.

2.2.1.3 Speed distribution

Mintsis (1988) investigated the probability distribution which best represents the speed population in horizontal curves. He concluded: “the normal distribution was found to provide a statistically significant description of the speed, ..., on the approach to curves, ..., as well as the entry, middle and exit points and for cars and goods vehicles.” McLean (1970) also suggested that passenger car and heavy goods vehicle speed distributions at the curve center showed no significant departures from the normal distribution, with a coefficient of variation of about 0.14.

2.2.2 Lateral acceleration/speed relationship

Derived from the study of Ritchie et al. (1968), Leeming et al. (1944), Moyer et al. (1940) and Kneebone (1964), Craus et al. (1979) presented in their Figure 2 a linear decreasing lateral acceleration with increasing travel speed in the curve. They explained that this decrease is the result of the “driver being overcautious when evaluating driving risk, which increases with increasing travel speed.” (Craus et al.: 1979). In the relationship between the lateral acceleration and the curve speed, the low speed driver type of the entire population is closer to the mean than
the fast driver type (Craus et al.: 1979: 8). In their study, they adopted the following relationship between the lateral acceleration and the speed design:

\[ \frac{a_r}{g} = 0.262 - 0.00182V_D \]  \hfill (2.3)

where the first expression is the ratio of the lateral acceleration to the gravity acceleration (9.81 m/s\(^2\)), and \( V_D \) is the design speed in km/h. This correlation is valid up to a speed of 100 km/h.

The linear regression is also used by McLean (1974) to represent a decrease of lateral acceleration with an increase of speed for the data of DMR and Taragin. Even if the regressions are statistically significant, only small proportions of the variances could be explained by the regressions.

Herrin et al. (1974) proposed to model the trade-off between lateral acceleration \( a_y \) and speed \( V \) by using an ‘aspiration velocity’ \( V_A \), a ‘lateral acceleration tolerance’ \( A_T \), and an expedience parameter \( \beta \) in the following form:

\[ \frac{a_y}{A_T} = 1 - \exp[\beta(V_A - V)] \]  \hfill (2.4)

Large negative values of \( \beta \) indicate an expedience strategy (Unwillingness to trade more speed for more lateral acceleration). The model is used in chapter 3 and 4 to represent the side friction-
speed relationship for the data collected during the experiment at the Pacific Traffic Education Center (PTEC) and the observations on Highway 99. Good (1978) criticized the use of the parameter $\beta$ in a literature review, and gave the concluding remark:

Although speculation about individual driver strategies [expedience parameter], based on averaged side friction/speed data, is likely to be fruitless, such data are nevertheless useful from the road designer’s point of view. They allow comparison of friction demands at various speeds with the friction supply capabilities of the tire-road system.

2.2.3 Drivers’ Path

Wong et al.’s (1992) study revealed “no statistically significant correlation between path radius and speed [of the car/driver system].” In their study, Wong et al. (1992) showed that path radius is different than the design center line radius. Therefore, the demanded side friction varies from lanes and drivers’ path.

Herrin et al. (1971) found that the curvature, the lane width and the length of the curve were significantly influential to the effective curvature.

2.2.4 Difference Between Drivers

Vaniotou’s study (1991) on drivers behaviors revealed that on average, men give a higher value than women when asked what would be their speed to negotiate bends that were showed on video film. Male drivers gave a speed between 8 to 12 km/h higher than female drivers.

An earlier study by Bezkorovainy (1966) concluded that there was a significant speed difference between male and female drivers at the center of the observed curves. Male drivers drove 3 to 4
mph (4.8 to 6.4 km/h) faster than female drivers. This comparison shows that women, with respect to speed, takes less risk than men when driving horizontal curves.

Fildes et al.'s (1982) laboratory studies showed that drivers are more accurate in estimating the exit direction of the bends when drivers are on the outside lane of the curve. Also, they revealed that drivers tend to overestimate the curvature when approaching a curve. It was found that there is a correlation among the sight distances or ‘preview’, the curve angle, the driving direction of the curve, and the accuracy of the driver in estimating the angle of the curve.

Milosevic et al.'s (1990) study revealed that drivers underestimate their speed in the middle of the curve. The same study concluded that experienced people drive on a difficult curve at a speed greater than the speed they would estimate as safe. They added that “drivers with greater experience are able to drive faster with less risk. An experienced driver believes that his experience can compensate for faster speed.”

2.3 Selection of The Variables

Prior studies have shown that the selection of a speed when driving a curve is influenced, to a certain degree, by a great number of factors: the driver, the curvature, the approach speed or the desired speed, the lane width, and the traffic volume. Even if the skid resistance factor and the superelevation play an important role in the safety of a car going around a curve, some studies have stipulated that they are not really influential in the driver’s selection of the speed when
driving a horizontal curve. For the experiment and the observations, the following variables are covered: the radius, the friction factor, the superelevation of the curves as well as the path radius, the speed and the lateral acceleration of the drivers.
CHAPTER 3

PTEC Experiment

3.1 Purpose of The Experiment

The probabilistic approach in reliability engineering requires knowing the values and the distributions of the variables. The purpose of the field experiment was to collect statistical information on the variables to describe the behavior of the Car-Driver (C/D) system when negotiating curves of different radii. Driving speed and the corresponding lateral acceleration that people experience in a specific curve on wet and dry pavement were, of particular interest. Glennon et al. (1985) explained that the best descriptor of C/D behavior is the lateral acceleration, because it is felt by the driver and affects the tire-pavement friction forces.

The AASHTO (1990) design standard has, for each design speed, a corresponding level of lateral acceleration that is tolerable and assumed comfortable to a reasonable driver. In the author experiments, for each speed and lateral acceleration, the driver and the passenger, provided a subjective value of their level of comfort. This information is needed to estimate the drivers perception of the dynamic parameters and performance of the car when going through a horizontal curve.
Expert drivers, who drove the curves at their maximum speed, provided the threshold of the dynamic parameters for all conditions used in the experiment.

3.2 Design of The Experiment

3.2.1 Variables

Two types of variables were involved in the experiment: the controllable and the uncontrollable. The five controllable variables were the curves’ radius (R), the skid resistance factor (wet, dry), the gender (M, F) of the subjects drivers and passengers, and the speed level (comfortable, fast) required of the drivers. Two categories of uncontrollable factors were those associated with the drivers, such as their mood during the day of the experiment, and those associated with the environment, such as the scenery or the weather conditions.

The response variables to the test were the speed of the C/D-passenger system, the level of comfort indicated by the passengers, and the lateral acceleration, which is the result of the combination of the velocity of the vehicle and the radius of the curves.

3.2.2 Latin Square design

A modified Latin square design was used to conduct the experiment. This model is presented by Myers (1972: p272), and a more detailed explanation of Latin Square design is provided in Appendix 1. The advantage of this model over a complete factorial design, is that it allows us to realize an experiment which involves few subjects and which investigates several variables with an acceptable level of power. Therefore, it was possible to do the experiment in a short time.
period and on a restricted budget, without biasing the analysis of the variances. Figure 3.1 is a random selection of the 576 possible 4 x 4 Latin Squares for the five variables.

<table>
<thead>
<tr>
<th>Level Bi</th>
<th>Level Ri</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
</tr>
</thead>
<tbody>
<tr>
<td>B₁</td>
<td>V₂D</td>
<td>V₁W</td>
<td>V₁D</td>
<td>V₂W</td>
<td></td>
</tr>
<tr>
<td>B₂</td>
<td>V₁D</td>
<td>V₂W</td>
<td>V₂D</td>
<td>V₁W</td>
<td></td>
</tr>
<tr>
<td>B₃</td>
<td>V₂W</td>
<td>V₂D</td>
<td>V₁W</td>
<td>V₁D</td>
<td></td>
</tr>
<tr>
<td>B₄</td>
<td>V₁W</td>
<td>V₁D</td>
<td>V₂W</td>
<td>V₂D</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.1. Latin Square used for the experiment.

Figure 3.1 is a Latin Square because each of the four $V_iD/W$ treatments appears exactly once in each column and once in each row. This square represents 16 different $B_i-R_i-V_iD/W$ treatment combinations. The controllable variables $B$, $R$ and $V_iD/W$ with their respective level are defined in Table 3.1.

Table 3.1. Definition of the four levels of the four variables.

<table>
<thead>
<tr>
<th>Level Bi</th>
<th>Driver</th>
<th>Passenger</th>
<th>Level Ri</th>
<th>Radius</th>
<th>Level V_iD/W</th>
<th>Speed/pavnt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B₁</td>
<td>Male</td>
<td>Female</td>
<td>R₁</td>
<td>16 m</td>
<td>V₁D</td>
<td>Comfort/Dry</td>
</tr>
<tr>
<td>B₂</td>
<td>M</td>
<td>M</td>
<td>R₂</td>
<td>26 m</td>
<td>V₁W</td>
<td>Comfort/Wet</td>
</tr>
<tr>
<td>B₃</td>
<td>F</td>
<td>F</td>
<td>R₃</td>
<td>60 m</td>
<td>V₂D</td>
<td>Fast/Dry</td>
</tr>
<tr>
<td>B₄</td>
<td>F</td>
<td>M</td>
<td>R₄</td>
<td>100 m</td>
<td>V₂W</td>
<td>Fast/Wet</td>
</tr>
</tbody>
</table>
For organizational purposes, each treatment had a number. Table 3.2 presents the treatment numbers. These numbers correspond also to the order of achievement of the test. For example, treatment T5 is the fifth treatment which corresponds to the radius R3 driven at scenario V1 (comfortable speed) on dry D pavement by male M driver with Female F as passenger.

Table 3.2. Treatment number.

<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>T1</td>
<td>T11</td>
<td>T5</td>
<td>T15</td>
</tr>
<tr>
<td>B2</td>
<td>T2</td>
<td>T12</td>
<td>T6</td>
<td>T16</td>
</tr>
<tr>
<td>B3</td>
<td>T9</td>
<td>T3</td>
<td>T13</td>
<td>T7</td>
</tr>
<tr>
<td>B4</td>
<td>T10</td>
<td>T4</td>
<td>T14</td>
<td>T8</td>
</tr>
</tbody>
</table>

3.2.3 Number of subject

The power (probability of rejecting a false null hypothesis) of the $F$ test was used to evaluate the number of subjects to be included in the experiment. Myers (1972: p89) presents the method.

The power of the $F$ test is plotted against an alternative $\phi$, where $\phi^2 = \frac{n \sum_j (\mu_j - \mu)^2}{\sigma_e^2/a}$ is the ratio of $n$ times the variance among the treatment population means where:

- $n$ = number of subject by group $a$,
- $a$ = number of groups,
- $\mu_j$ = mean of the treatment,
- $\mu$ = mean of the treatment population,
- $\sigma_e^2$ = to the population error variance.

In our case, Ritchie's data (1968), was adopted. In this data a scenario A (late for an important interview) and C (leisurely Sunday drive) was taken from his experiment, and used to simulate the 4 x 4 Latin Square for the PTEC experiment. When
these values are plotted in the Latin Square, the population error variance is $\sigma_e^2 = 3.69$ and the variance among the treatment population means is $\frac{\sum (\mu_j - \mu)^2}{a} = 2.97$ where $a = 16$. Using $\phi' = \sqrt{\phi^2}$ as the critical value, we can write $\phi' = 0.90\sqrt{n}$. Turning to the chart presented in Appendix 2, for $df_1 = 8$, and with $df_2 = a(n - 1)$ then we can derive the level of the power of the $F$ test which is the probability of rejecting a false null hypothesis. If one chooses $n = 4$, then $\phi' = 1.8$ and $df_2 = 48$, and with $\alpha = 0.01$ the power is 0.86. Based on the assumptions that the pavement condition influences the driver's speed in the curve and that Ritchie's data (1968) is a good representation of the population, then an $n$ of 4 does quite nicely. Four subjects are required in each cell of the Latin Square of Figure 3.1.

### 3.3 Procedure for The Experiment

The experiment was carried out at the Pacific Traffic Education Center (PTEC) site at Boundary Bay Airport. The area is an abandoned runway used by the Royal Canadian Mounted Police and PTEC for driving practice. The runway is composed of two sections approximately 600 meters long and 60 meters wide. The tests were carried out during three hot, sunny days.

#### 3.3.1. Controllable Variables

3.3.1.1. Scenarios, (level $V_1$)

The designed lateral acceleration proposed by the North American design standards, refers to comfortable speed for a reasonable driver. Hence, for the experiment, the subject drivers were
asked to be sometimes reasonable and sometimes not. Following the treatment, the drivers had to choose their speed based on the guideline for each speed scenario presented in Table 3.3.

Table 3.3. Definition of the two scenarios presented to the drivers.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Uniform</th>
<th>Guideline</th>
</tr>
</thead>
</table>
| V1       | Comfortable speed| • You are on a highway and you drive the curve at such a speed that you feel comfortable.  
           |                   | • Could be a leisurely Sunday drive.                                       |
| V2       | Very difficult speed | • You are on a highway and you drive the path at the maximum speed that consider to be safe.  
           |                   | • Could be a higher speed than the speed adopted when you are late for an important interview. |

Throughout the experiment, the drivers was accompanied by a passenger. The idea is that driver and passenger may not have the same perception of the lateral acceleration, because one is active and the other is passive in the driving process. The driver and the passenger were recording information on their comfort, based on their feeling of the side pitch toward the outside of the curve: the driver by selecting his speed, and the passenger by scoring his comfort on the subjective scale. The scale presented four sets in the following way:

| very difficult | uncomfortable | comfortable | easy |

The speed selected for scenario V1 is assumed to correspond to a comfortable ride of the driver, whereas the speed under scenario V2 is assumed to be at least an uncomfortable ride from the driver’s point of view.
3.3.1.2. Gender, (level $B_1$)

Two groups of four Subjects, participated to the experiments. With two women and two men the following combinations were possible:

Male No = 1 and 2; female No = 3 and 4.

<table>
<thead>
<tr>
<th>Level $B_i$</th>
<th>Driver-passenger</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$</td>
<td>1-3, 1-4, 2-3 and 2-4</td>
</tr>
<tr>
<td>$B_2$</td>
<td>1-2 and 2-1</td>
</tr>
<tr>
<td>$B_3$</td>
<td>3-4 and 4-3</td>
</tr>
<tr>
<td>$B_4$</td>
<td>3-1, 3-2, 4-1 and 4-2</td>
</tr>
</tbody>
</table>

With four people, the above combinations allow two couples per treatment. To satisfy $n = 4$, two days of test with four subjects per day were needed. The subjects mostly came from the Civil Engineering Department at the University of British Columbia and from the Ministry of Transportation and Highways. A summary of driving history is provided in the Table 3.4.

**Table 3.4. Subjects information.**

<table>
<thead>
<tr>
<th>subject</th>
<th>Age</th>
<th>Driving experience</th>
<th>Frequency (km/week)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20-30</td>
<td>31-50</td>
<td>&lt; 5</td>
</tr>
<tr>
<td>day 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>2</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>3</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>4</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>day 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>2</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>3</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>4</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

The urban environment is the road network frequently used by the subjects.
3.3.1.3 Curves, (level R_i)
Two short curves with $R_{1\text{car center lane}} = 16$ m and $R_{2\text{car center lane}} = 26$ m, and two longer curves with $R_{3\text{car center lane}} = 60$ m and $R_{4\text{car center lane}} = 100$ m were the four bends. All radii were treated independently (i.e. in a different path) to avoid interactions that would have happened between two radii, if they had been designed in the same path. The two short curves were laid out in a figure-eight and the two longer curves were laid out in a S-shaped figure as presented in Figure 3.2. The lane, 4 m wide, was delineated by small 30 cm high cones. Even if the lane width was higher than 3.7 m (standard highway width), the presence of the cones, and the absence of the shoulder, gave the sensation that the lane was narrower than 4 m.

3.3.1.4 Skid resistance factor, (level D/W)
All curves were tested for Dry and Wet surface pavement. This provides the safety margin under lower skid resistance of the road. The ‘wet’ pavement condition was involved in the experiment to see how people evaluate the risk of driving curves on wet pavement. The treatments made on wet pavement were grouped in the afternoon. A water truck was used to wet the pavement between the tests. The water depth was visually evaluated as being between 0.5 and 1 mm.

3.3.2 Material
A camera video 8 mm was mounted on the right rear side window of the car. The video was pointed down at the pavement, on which marks were equally spaced in the curves. A representation of the position and denomination of the marks is presented in Figure 3.2. The laps of time over the distance between two marks, provided the speed of the vehicle. The figure
eight loop was a closed loop. Therefore, it was possible to record the mean speed between all the intervals, and between the two curves C1 and C2 as well. For the figure “S” loop, which was not a closed loop, it was not possible to collect the speed between drives.

An accelerometer type “g-analyst” was calibrated on the floor between the driver and the passenger and between the front and rear sites. The “g-analyst” is a commercial package which includes a display and control device and an acceleration transducer unit. The transducer
contains a three-axis accelerometer which measures instantaneous acceleration in g. The information was downloaded simultaneously into a lap-top computer. Because the acceleration is recorded versus the time history, the time information collected from the video allows us to match the acceleration data with the position of the vehicle on the path.

The vehicle was a 1994 Mazda 323 Protégé SE 4 doors Sedan model. The car was equipped with standard tires of good condition. The vehicle was loaded approximately the same for each test. In the vehicle during the tests were the subject driver, the subject passenger, and one person on a rear seat who gave the scenario instruction and controlled the different devices.

3.3.3 Expert Drivers

Two expert drivers drove the four curves at the maximum safe speed before skidding. Their maximum speeds are assumed to be the maximum safe speed for all radii and all types of pavement treatment. The expert drivers were R.C.M.P. officers from Delta in British Columbia and a driving instructor from the Accident Research Team at the University of British Columbia.

3.3.4 Procedures

- The subjects were assigned with a number from 1 to 4 drawn at the beginning of the day.
- All drivers, each accompanied by one passenger, first did five runs in one of the two figure-eights and in one of the two ‘S’ figures, in order to familiarize themselves with the car and curve environment.
• The drivers and the passengers were called, according to their number, to perform a particular treatment from T1 to T16. Table 3.5 presents the treatment assignment for each subject number.

Table 3.5. Treatment assignment.

<table>
<thead>
<tr>
<th>TREAT.</th>
<th>DRIV.</th>
<th>PAS.</th>
<th>TREAT.</th>
<th>DRIV.</th>
<th>PAS.</th>
<th>TREAT.</th>
<th>DRIV.</th>
<th>PAS.</th>
<th>TREAT.</th>
<th>DRIV.</th>
<th>PAS.</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1</td>
<td>3</td>
<td>T5</td>
<td>1</td>
<td>4</td>
<td>T9</td>
<td>4</td>
<td>3</td>
<td>T13</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td></td>
<td>2</td>
<td>3</td>
<td></td>
<td>3</td>
<td>4</td>
<td></td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>T2</td>
<td>1</td>
<td>2</td>
<td>T6</td>
<td>1</td>
<td>2</td>
<td>T10</td>
<td>4</td>
<td>1</td>
<td>T14</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1</td>
<td></td>
<td>2</td>
<td>1</td>
<td></td>
<td>3</td>
<td>2</td>
<td></td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>T3</td>
<td>3</td>
<td>4</td>
<td>T7</td>
<td>4</td>
<td>3</td>
<td>T11</td>
<td>2</td>
<td>4</td>
<td>T15</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3</td>
<td></td>
<td>3</td>
<td>4</td>
<td></td>
<td>1</td>
<td>3</td>
<td></td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>T4</td>
<td>3</td>
<td>2</td>
<td>T8</td>
<td>4</td>
<td>2</td>
<td>T12</td>
<td>2</td>
<td>1</td>
<td>T16</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1</td>
<td></td>
<td>3</td>
<td>1</td>
<td></td>
<td>1</td>
<td>2</td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

• Drivers did four loops for each treatment - two in one direction and two in the opposite direction - at a speed that they selected according to scenario V1 or V2.

• During the tests, the passengers indicated their comfort level for each bend driven. It is assumed that their scores, reported on a subjective scale, are mainly based on their own feeling of the lateral acceleration and the speed.

3.4 Result of The Experiment

3.4.1 Errors

Three sources of errors were recorded. The first two errors are related to the speed. Errors could have been made when collecting speed data. The VCR reading provided a precision of 30 frames
per second. An error of plus or minus 2 frames was evaluated during the counting. This error corresponds to 3 to 7 km/h depending on speed and radius.

The departure regarding the lateral position of the car from the center lane was different among drivers, and radii, and between speeds and the two directions of driving. Therefore, the distance between two marks was not a constant. However, based on the tire prints left on the pavement, an average lateral position of the car was measured and, consequently, the lateral position of the camera was corrected.

The third source of error concerns the acceleration. It was assumed that there was no superelevation. A crown was present at the center lane of the runway, which created light slopes on each side of the axle of the runway lane. Therefore, an error of plus or minus 0.02g was recorded by the g-analyst. When the car was at the center of the small curves, the vehicle was above the crown and the superelevation was zero. When the vehicle traveled on the bigger radii, a negative superelevation was experienced at the middle of the curves.

### 3.4.2 Latin Square analysis

It is assumed that the critical values of speed/lateral acceleration is at the center of the curves. The collected variable used to run the algorithm of the Latin Square was the average speed of the car for each treatment. Each value within each treatment, represents the driver’s speed average from the speeds taken at the center of the curve, between the marks B and D, of the two curves.
C1 and C2 (see Figure 3.2). Table 3.6 is a representation of the 4 X 4 square with the data and the subtotals.

Table 3.6. Observed data* and subtotal.

<table>
<thead>
<tr>
<th>Level ( \mathbf{B} )</th>
<th>Level ( \mathbf{R} )</th>
<th>( \text{R1} )</th>
<th>( \text{R2} )</th>
<th>( \text{R3} )</th>
<th>( \text{R4} )</th>
<th>Sum of rows</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{B Sum} )</td>
<td>127.1</td>
<td>122.3</td>
<td>160.09</td>
<td>235.23</td>
<td>644.72</td>
<td></td>
</tr>
<tr>
<td>( \text{B1} )</td>
<td>30.78</td>
<td>25.84</td>
<td>41.39</td>
<td>54.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>29.78</td>
<td>28.91</td>
<td>36.06</td>
<td>64.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>33.71</td>
<td>35.47</td>
<td>42.14</td>
<td>61.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>32.83</td>
<td>32.08</td>
<td>40.5</td>
<td>55.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{B1 Stdev} )</td>
<td>1.81</td>
<td>4.14</td>
<td>2.73</td>
<td>4.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{B1 Mean} )</td>
<td>31.78</td>
<td>30.58</td>
<td>40.02</td>
<td>58.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{B2 Sum} )</td>
<td>96.21</td>
<td>151.06</td>
<td>212.34</td>
<td>195.02</td>
<td>654.63</td>
<td></td>
</tr>
<tr>
<td>( \text{B2} )</td>
<td>24.4</td>
<td>34.73</td>
<td>54.66</td>
<td>42.77</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>20.87</td>
<td>37.52</td>
<td>53.82</td>
<td>46.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>24.92</td>
<td>38.76</td>
<td>51.83</td>
<td>53.29</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>26.02</td>
<td>40.05</td>
<td>52.03</td>
<td>52.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{B2 Stdev} )</td>
<td>2.23</td>
<td>2.27</td>
<td>1.38</td>
<td>5.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{B2 Mean} )</td>
<td>24.05</td>
<td>37.77</td>
<td>53.09</td>
<td>48.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{B3 Sum} )</td>
<td>122.79</td>
<td>155.98</td>
<td>159.17</td>
<td>177.56</td>
<td>615.5</td>
<td></td>
</tr>
<tr>
<td>( \text{B3} )</td>
<td>30.31</td>
<td>39.77</td>
<td>32.8</td>
<td>42</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>30.7</td>
<td>39.4</td>
<td>34.52</td>
<td>46</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>31.08</td>
<td>36.16</td>
<td>46.42</td>
<td>45.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>30.7</td>
<td>40.65</td>
<td>45.43</td>
<td>44.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{B3 Stdev} )</td>
<td>0.31</td>
<td>1.96</td>
<td>7.13</td>
<td>1.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{B3 Mean} )</td>
<td>30.70</td>
<td>39.00</td>
<td>39.79</td>
<td>44.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{B4 Sum} )</td>
<td>103.46</td>
<td>120.05</td>
<td>210.48</td>
<td>239.72</td>
<td>673.71</td>
<td></td>
</tr>
<tr>
<td>( \text{B4} )</td>
<td>23.67</td>
<td>26.63</td>
<td>48.91</td>
<td>55</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>23.28</td>
<td>26.86</td>
<td>51.88</td>
<td>65</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>29.34</td>
<td>33.81</td>
<td>58.56</td>
<td>66.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>27.17</td>
<td>32.75</td>
<td>51.13</td>
<td>53.12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{B4 Stdev} )</td>
<td>2.90</td>
<td>3.80</td>
<td>4.16</td>
<td>6.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{B4 Mean} )</td>
<td>25.87</td>
<td>30.01</td>
<td>52.62</td>
<td>59.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Sum of columns</strong></td>
<td>449.56</td>
<td>549.39</td>
<td>742.08</td>
<td>847.53</td>
<td>2588.56</td>
<td></td>
</tr>
<tr>
<td><strong>Level VD/W</strong></td>
<td>579.95</td>
<td>719.56</td>
<td>553.91</td>
<td>735.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Subtotal</strong></td>
<td>V1W</td>
<td>V2W</td>
<td>V1D</td>
<td>V2D</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* mean speed at the middle of the curve between marks B and D
The analysis of the variance is resumed in Table 3.7.

### Table 3.7. Latin squaring treatment combinations.

<table>
<thead>
<tr>
<th>SV</th>
<th>df</th>
<th>SS</th>
<th>EMS</th>
<th>F</th>
<th>F.9</th>
<th>F.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>a×2d×2n-1</td>
<td>63</td>
<td>8613.97</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>ad-1</td>
<td>3</td>
<td>110.56</td>
<td>36.85</td>
<td>2.51</td>
<td>2.80</td>
</tr>
<tr>
<td>C</td>
<td>ad-1</td>
<td>3</td>
<td>6110.17</td>
<td>2036.72</td>
<td>138.54</td>
<td>2.80</td>
</tr>
<tr>
<td>Treat. Comb</td>
<td>ad-1</td>
<td>3</td>
<td>1637.19</td>
<td>545.73</td>
<td>37.12</td>
<td>2.80</td>
</tr>
<tr>
<td>A</td>
<td>a - 1</td>
<td>1</td>
<td>1608.41</td>
<td>1608.41</td>
<td>109.41</td>
<td>4.04</td>
</tr>
<tr>
<td>D</td>
<td>d - 1</td>
<td>1</td>
<td>1.71</td>
<td>1.71</td>
<td>0.12</td>
<td>4.04</td>
</tr>
<tr>
<td>AD</td>
<td>(a - 1)(d - 1)</td>
<td>1</td>
<td>27.07</td>
<td>27.07</td>
<td>1.84</td>
<td>4.04</td>
</tr>
<tr>
<td>B.cells res.</td>
<td>(ad - 1)(ad - 2)</td>
<td>6</td>
<td>50.41</td>
<td>8.40</td>
<td>0.57</td>
<td>2.30</td>
</tr>
<tr>
<td>S/cells</td>
<td>a×2d×2(n - 1)</td>
<td>48</td>
<td>705.64</td>
<td>14.70</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The first important result is that the interactions among the variables are negligible, which confirms the validity of the model presented in Appendix 1. The partial test of the hypothesis that variables do not interact is given by

\[
F_{(6,48)} = \frac{EMS_{\text{res.}}}{EMS_{\text{S/cells}}} = \frac{8.40}{14.70} = 0.57
\]

For a .05-level test, the critical value is \( F_{.05} (6,48) = 2.30 \). Hence, the experimental data tends to confirm the hypothesis that the interactions are negligible.

Under these conditions, it is obvious that the speed variation is affected by the change of request between the two scenarios (variable V), and the change of radius (variable R). However, the experimental data indicates that the interaction due to the presence of water on the pavement, is not statistically significant. Drivers did not significantly reduce their speed on wet pavement.
Also, the variation between the groups is not statistically significant. This last information proved that there is a certain consistency between drivers.

3.4.2.1 Difference between gender.

From the analysis we cannot say that there is a significant difference between female and male drivers. Probably due to the reduced group size of male and female drivers, we cannot reach the conclusion of Vaniotou (1991) and Bezkorovainy (1966) who observed significant differences of speed between genders.

Table 3.8. Mean speed * for men and women for the eight radii and speed scenario combinations.

<table>
<thead>
<tr>
<th>n = 4</th>
<th>MALE</th>
<th>FEMALE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>stdev</td>
</tr>
<tr>
<td>R1</td>
<td>V2</td>
<td>31.78</td>
</tr>
<tr>
<td>R1</td>
<td>V1</td>
<td>24.05</td>
</tr>
<tr>
<td>R2</td>
<td>V2</td>
<td>37.77</td>
</tr>
<tr>
<td>R2</td>
<td>V1</td>
<td>30.58</td>
</tr>
<tr>
<td>R3</td>
<td>V1</td>
<td>40.02</td>
</tr>
<tr>
<td>R3</td>
<td>V2</td>
<td>53.09</td>
</tr>
<tr>
<td>R4</td>
<td>V1</td>
<td>48.73</td>
</tr>
<tr>
<td>R4</td>
<td>V2</td>
<td>58.81</td>
</tr>
</tbody>
</table>

* mean speed at the middle of the curve between marks B and D

3.4.3 Speed

3.4.3.1 Speed distribution.

At PTEC, the speed adjustment in horizontal curves is proportional with the curvature. This result corresponds to Mintis (1988) observations. Figures 3.3 to 3.6 show for all radii, the mean
speed distributions of the subjects when driving for both scenarios, and the mean speed distribution of the expert drivers when driving at the maximum speed before sliding. In the small radii R1 and R2, the subjects reduced their speed to a minimum at about the center of the curves, then, accelerated towards the exit. This phenomenon was representative for both scenarios V1 and V2. However, this behavioral pattern was not observed for the larger radii R3 and R4. For those radii, once drivers had selected their speed, they tended to maintain it. As this study is based on experiments, comparisons with prior studies on real road environments and conclusions can be made only with reservation.

Figure 3.3. Mean speed distribution for radius R1 = 16m.
Figure 3.4. Mean speed distribution for radius $R_2 = 25m$.

Figure 3.5. Mean speed distribution for radius $R_3 = 60m$. 
The expert drivers were always accelerating in the first bend encountered in the figure ‘S’. The performance of the car limited the performance of the expert drivers and, therefore, they could only approach the capacity of the tire-pavement in the second bend.

In Chapter 1, it was explained that the speed in a horizontal curve follows a normal distribution. In addition, when viewing the dispersion of the individual speeds around the mean, one can see an apparent symmetry in each cell of Table 3.6 (average coefficient of skewness = 0.4). Therefore, for further computation, it is assumed that the speeds selected in the curves at PTEC follow a normal distribution.
3.4.3.2 Maximum speed

3.4.3.2.1 Coefficient of friction available

A skid test was performed at PTEC by the Royal Canadian Mounted Police in 1992 (King and McIsaac 1993). The runs were carried out on the asphalt portion of the runway. The skid test consisted of accelerating the vehicles within a predetermined speed range, and then making a hard brake application to skid the vehicle. Four different type of equipment were used on three different makes of car, to analyze the acceleration. The weather was approximately the same as the conditions for the horizontal curve experiment. The results for the mean deceleration were 0.92 g for a 1992 Chevrolet Caprice, 0.84 g for a 1992 Dodge Caravan and 0.75 g for a 1989 Ford Mustang. The mean value of the cars is 0.84 g, which is assumed to be the quality of the PTEC pavement in braking.

The curves were layed out on the concrete pavement, which has a higher roughness than asphalt. If these skid tests had been made on the concrete surface, the mean value would have approached 0.90 g of deceleration. The distribution of the load on the four car's tires, is different when braking than when cornering. Therefore, the skid pattern when cornering, happens usually with a lower lateral acceleration $a_y$ than the longitudinal acceleration $a_x$ needed to skid when braking hard. The value of $a_y$ can be approximated as: $a_y = 0.90 \, a_x$. Then, referring to the value of the longitudinal deceleration, the corresponding maximum capacity of the side friction factor on the concrete pavement would be 0.81. The Mazda Protégé, developed 0.76 g of lateral acceleration for the road-holding test published by Car and Driver (1995) (see Figure 1.2).
3.4.3.2.2 *Theoretical maximum speed*

From the basic equilibrium equation for horizontal curves, the theoretical maximum speed in the curve before the sliding event is defined as \( V_{\text{max}} = \sqrt{g(f_{\text{max}} + e)}R \). If there is no superelevation, then \( V_{\text{max}} = \sqrt{gf_{\text{max}}} R \). Assuming a lateral skid resistance factor for the vehicle used of 0.80 for dry conditions at PTEC, and assuming this value constant for the four curves, then the theoretical maximum speeds should be 40, 50, 78 and 100 km/h for R1, R2, R3 and R4 respectively.

These theoretical speeds have been approached by the expert drivers. Table 3.9 shows that both drivers were at the theoretical speed when driving R1 and R2. They could not get to the maximum speed on R3 and R4 on dry pavement because they were limited by the car’s performance. However, when they were driving on wet pavement they did reduce their speed, which shows that they were at the maximum speed before sliding.

**Table 3.9. Expert driver average speeds * in km/h for the middle of the curves compared to the theoretical speeds.**

<table>
<thead>
<tr>
<th>Radius</th>
<th>( V_{\text{max}} = \sqrt{gf_{\text{max}}} R )</th>
<th>dry pavement</th>
<th>wet pavement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Expert 1</td>
<td>Expert 2</td>
</tr>
<tr>
<td>R1</td>
<td>40</td>
<td>39.60 99%</td>
<td>39.63 99%</td>
</tr>
<tr>
<td>R2</td>
<td>50</td>
<td>49.56 99%</td>
<td>48.04 96%</td>
</tr>
<tr>
<td>R3</td>
<td>78</td>
<td>69.20 89%</td>
<td>68.64 88%</td>
</tr>
<tr>
<td>R4</td>
<td>100</td>
<td>82.74 83%</td>
<td>84.24 84%</td>
</tr>
</tbody>
</table>

* mean speed at the middle of the curves when taking the curves counter-clockwise.
3.4.3.3 Curve to the right and curve to the left

It was found that the speeds selected by the drivers were higher when the small curves were driven counter-clockwise -i.e. left curve- than clockwise -i.e. right curve-. 

Table 3.10 Mean speed km/h of subject drivers when driving clockwise and counter-clockwise.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Left curve</th>
<th>Right curve</th>
<th>T</th>
<th>t.99</th>
<th>t.95</th>
<th>Sta.</th>
<th>Sign.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>stddev</td>
<td>mean</td>
<td>stddev</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T1: R1-V2</td>
<td>34.39</td>
<td>1.49</td>
<td>29.25</td>
<td>2.09</td>
<td>5.31</td>
<td>2.62</td>
<td>1.76</td>
</tr>
<tr>
<td>T2: R1-V1</td>
<td>25.72</td>
<td>2.43</td>
<td>22.32</td>
<td>1.89</td>
<td>2.92</td>
<td>2.62</td>
<td>1.76</td>
</tr>
<tr>
<td>T3: R2-V2</td>
<td>41.01</td>
<td>2.36</td>
<td>37.07</td>
<td>1.72</td>
<td>3.58</td>
<td>2.62</td>
<td>1.76</td>
</tr>
<tr>
<td>T4: R2-V1</td>
<td>31.46</td>
<td>3.97</td>
<td>28.60</td>
<td>3.33</td>
<td>1.46</td>
<td>2.62</td>
<td>1.76</td>
</tr>
<tr>
<td>T5: R3-V1</td>
<td>41.39</td>
<td>3.11</td>
<td>38.68</td>
<td>3.20</td>
<td>1.61</td>
<td>2.62</td>
<td>1.76</td>
</tr>
<tr>
<td>T6: R3-V2</td>
<td>54.11</td>
<td>2.97</td>
<td>52.06</td>
<td>3.23</td>
<td>1.24</td>
<td>2.62</td>
<td>1.76</td>
</tr>
<tr>
<td>T7: R4-V1</td>
<td>46.08</td>
<td>1.94</td>
<td>43.50</td>
<td>2.41</td>
<td>1.02</td>
<td>3.14</td>
<td>1.94</td>
</tr>
<tr>
<td>T8: R4-V2</td>
<td>63.08</td>
<td>9.65</td>
<td>56.69</td>
<td>6.77</td>
<td>0.66</td>
<td>3.14</td>
<td>1.94</td>
</tr>
<tr>
<td>T9: R1-V2</td>
<td>32.88</td>
<td>0.73</td>
<td>28.57</td>
<td>1.10</td>
<td>8.63</td>
<td>2.62</td>
<td>1.76</td>
</tr>
<tr>
<td>T10: R1-V1</td>
<td>27.53</td>
<td>3.04</td>
<td>24.22</td>
<td>2.58</td>
<td>2.20</td>
<td>2.62</td>
<td>1.76</td>
</tr>
<tr>
<td>T11: R2-V1</td>
<td>31.66</td>
<td>3.62</td>
<td>29.49</td>
<td>4.25</td>
<td>1.03</td>
<td>2.62</td>
<td>1.76</td>
</tr>
<tr>
<td>T12: R2-V2</td>
<td>39.44</td>
<td>1.74</td>
<td>36.11</td>
<td>2.78</td>
<td>2.69</td>
<td>2.62</td>
<td>1.76</td>
</tr>
<tr>
<td>T13: R3-V1</td>
<td>40.39</td>
<td>6.87</td>
<td>39.19</td>
<td>6.46</td>
<td>0.34</td>
<td>2.62</td>
<td>1.76</td>
</tr>
<tr>
<td>T14: R3-V2</td>
<td>53.84</td>
<td>4.94</td>
<td>51.42</td>
<td>3.06</td>
<td>1.10</td>
<td>2.62</td>
<td>1.76</td>
</tr>
<tr>
<td>T15: R4-V2</td>
<td>59.52</td>
<td>5.69</td>
<td>58.09</td>
<td>4.14</td>
<td>0.54</td>
<td>2.62</td>
<td>1.76</td>
</tr>
<tr>
<td>T16: R4-V1</td>
<td>48.94</td>
<td>4.59</td>
<td>48.57</td>
<td>5.20</td>
<td>0.14</td>
<td>2.62</td>
<td>1.76</td>
</tr>
</tbody>
</table>

This is statistically significant for the small radius R1 at both scenario V1 and V2, and significant for R2 only when the curve was driven for scenario V2. This behavior may be explained by a better evaluation of the curvatures when the driver is on the inside of the bend. Inside the bend of high curvature, drivers can more easily rely on the inside delineation. As soon as the radius is over 25 meters, then the difference of speed between a left and right curve is not statistically significant.
No difference of speed was noticed for all radii and directions between the two bends C1 and C2.

3.4.3.4 Speed radius relationship

Figure 3.7 shows that the logarithmic model is a good descriptor of the correlation between mean speed selected in the curves and the radius of the PTEC curves. The difference of speed between regular driver and expert is increasing with increase of radius. Even if this difference is very small for the sharpest curve, none of the subjects drove at the experts’ speed.

![Graph showing speed vs radius for expert and regular drivers, with logarithmic models for each scenario.](image)

Figure 3.7. Mean speed and radius for scenario V1 and V2 on dry and wet pavement, and for expert drivers on dry pavement only.
3.4.4 Lateral Acceleration

The result, presented in Figure 3.8, indicates that 0.76 g, being the maximum lateral acceleration published by Car and Driver (1995) for the Mazda Protégé, is a reasonable value for small curves only. For the PTEC path environment, the lateral friction factor threshold provided by the experts is decreasing with the increase of radius.

One can see that the mean lateral friction of 0.2, for R4 at scenario V1, corresponds to a mean speed of 46 km/h. If 46 km/h is considered as a design speed, the recommended coefficient of friction by AASHTO is 0.16. There is consistency between what is observed on this particular radius and what the AASHTO design procedure would propose for a design speed of 46 km/h. However, the AASHTO recommended coefficient of friction is not consistent with the way the subjects drove the path for scenario V2.

In general, with the increase of radius, the speed selected by drivers increases and the corresponding coefficient of friction decreases.

3.4.5 Level of Comfort

The eight figures presented in Appendix 3 show, for all the passengers, that a comfortable or an easy ride is under the region of 0.40g of lateral acceleration. Figures 3.10 and 3.11 present the felt lateral acceleration and the range of comfort, scored by the passengers, for the PTEC radii.
When drivers performed the curves under scenario V1, the passengers agreed with the drivers' rides by scoring them at least, as comfortable rides (Figure 3.9). The mean lateral accelerations of the vehicle corresponding to this scenario is 0.32g for R1, 0.29g for R2, 0.21g for R3 and 0.20g for R4. When drivers performed the curves under scenario V2, which is assumed to be an uncomfortable or very difficult ride, then passengers consider the rides as uncomfortable for the small radii, but tend to consider the rides as comfortable for the bigger radii (Figure 3.10). Mostly based on the lateral acceleration, the passengers did not feel any discomfort in R4 whereas the drivers were assumed at their maximum speed.
After a certain degree of curvature, drivers seemed to adjust their speed not on the felt lateral acceleration but on other factors which are probably related to the alignment of the road, the vehicle, the drivers, and the immediate environment of the road. The drivers interpret these factors in a global way and estimate the risk of the road for a particular speed. The speeds adopted by drivers are a function of the drivers' risk perception which was not measured.

Figure 3.9. Radius and passenger level of comfort (left) and radius and lateral acceleration (right) for scenario V1.
CHAPTER 3: PTEC Experiment

Figure 3.10. Radius and passenger level of comfort (left) and radius and lateral acceleration (right) for scenario V2.

This information reflects the trade-off concept between lateral acceleration and speed. For a small radius, drivers select their speed based on the felt lateral acceleration and for a large radius, drivers select their speed based on the speed itself.

3.4.6 Speed and lateral acceleration

In Figure 3.11. and 3.12. the plain curves represent the lateral acceleration versus the speed $V$ of the subjects when plotted in equation $a_y = V^2/gR$. These curves are a perfect representation of
the individual values of speed versus acceleration, and they illustrate the degree of accuracy obtain from the g-analyst and the video camera. Furthermore, it confirms the assumption that the superelevation was negligible.

For both scenarios, the drivers seemed to trade lateral acceleration for speed. Because the subjects never reach the limit of the car-tire capacity, the linear regression presented in Figures 3.11 and 3.12 can be interpreted as being the trade-off between speed and lateral acceleration as the radius increases.

Figure 3.11. Lateral acceleration and Speed for scenario V1.
With the assumption that drivers selected their speed based on the level of comfort for scenario V1, and on the level of safety for scenario V2, then it seems that the decision factors used by drivers to select their speed in the curves, are the felt lateral acceleration for the small radii and the speed itself for the bigger radii.

The linear relationship between lateral acceleration and speed, does not seem to be the best representation. It seems more appealing to hypothesize an exponential relation between speed and lateral acceleration. Based on the trade-off idea, Herrin et al. (1971) proposed to model car-driver behavior using an 'Aspiration Velocity' $V_A$ and a lateral 'Acceleration Tolerance' $A_T$ as variables in the following exponential expression:
CHAPTER 3: PTEC Experiment

\[ \frac{a_y}{A_T} = 1 - e^{[\beta(v_A - V)]} \]  

(4.2)

where:

- \( a_y \) = lateral acceleration
- \( V \) = speed
- \( A_T \) = lateral Acceleration Tolerance
- \( V_A \) = Aspiration Velocity
- \( \beta \) = ‘expedience’ parameter

\( \beta = 0 \rightarrow \) Linear speed acceleration trade-off.
\( \beta << 0 \rightarrow \) Do not want to trade speed for more lateral acceleration.

In their study, they refer the ‘Aspiration velocity’ as the speed that a driver would like to maintain on a particular road section. The lateral acceleration tolerance represents the maximum value that a driver would not exceed to satisfy either his comfort or his safety. The trade-off relation between lateral acceleration and speed for a negative \( \beta \) is illustrated graphically in Figure 3.13.

![Figure 3.13. General Expedience.](source: Herrin et al, 1971)
This model suggests that on a highway with no curvature, drivers control their ride based on their “Aspiration Velocity” $V_A$. The drivers reduce their speed as the radius of the curve decreases. In curves with low radius, the speed adopted by the drivers, satisfies the “acceleration tolerance” $A_T$ of the drivers.

In Figure 3.14, Herrin et al. (1971) model is used to represent the trade-off between speed and lateral acceleration for the PTEC data. The value for the “Aspiration Velocity” $V_A$ and the lateral “Acceleration Tolerance” $A_T$ are assumed, and the value for the expedience parameter $\beta$ is chosen in order to have the best fit for the acceleration data collected at PTEC.

For scenario V1 (comfortable) the maximum $A_T$ is assumed as 0.40g. $V_A$ is taken as the maximum speed developed in R4: 55 km/h. This maximum speed is assumed to represent the desired speed for the PTEC environment. For scenario V2 (maximum safe speed), the value $A_T$ is chosen as 0.76g, corresponding to the maximum lateral acceleration that a Mazda Protégé driven by an expert can handle in sharp curves (Car and Driver: 1995). $V_A$ corresponds to the maximum speed of the expert driver in R4 and is taken as 85 km/h. For a good fit with the data presented in Figure 10b, the value for $\beta$ is assumed and presented in Table 3.11.

The two average curves, representing the Herrin et al. model, seem to be a good representation of the matching pairs of acceleration versus speed, for a given level of comfort. $\beta$ for the comfortable scenario is smaller than $\beta$ for high speed scenario. As describe by Herrin, decreases
Figure 3.14. Lateral acceleration and speed a) V1 b) V2, with the use of Herrin et al. Model.
in β indicate a change in the lateral acceleration-velocity trade-off toward more of an expedience
criterion. On curves with small radii drivers adjust their comfortable speed according to their
lateral acceleration threshold tolerance $A_T$ (between 0.35 and 0.4g), and on curves with longer
radii drivers are approaching their ‘Aspiration Velocity’; the speed they would travel at on a
straight section of same environment. This optimum speed or aspiration velocity could be
compared to the desire or speed environment adopted by the Australian design methods (see

3.5 Conclusion

During the experiment, regular drivers drove two scenarios (comfortable speed V1 and fast speed
V2) into four non-superelevated horizontal curves on dry and wet pavement. For each rides, a
passengers provided information on his level of comfort. Expert drivers drove the four curves at
their maximum speed for dry and wet conditions. From these tests, the following information
was collected:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>limits</th>
<th>$A_T$</th>
<th>$V_A$</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>Upper</td>
<td>0.40</td>
<td>55</td>
<td>-0.12</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>0.40</td>
<td>55</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td>Lower</td>
<td>0.40</td>
<td>55</td>
<td>-0.02</td>
</tr>
<tr>
<td>V2</td>
<td>Upper</td>
<td>0.76</td>
<td>85</td>
<td>-0.032</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>0.76</td>
<td>85</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>Lower</td>
<td>0.76</td>
<td>85</td>
<td>-0.01</td>
</tr>
</tbody>
</table>
CHAPTER 3: PTEC Experiment

- Average and standard deviation of speeds and lateral accelerations at the center of the curves.

- Trade-off between lateral accelerations and speeds:
  
  - For scenario V1, people tend to drive at a speed which corresponds to a lateral acceleration of 0.35 - 0.40g in sharp curves. However, on flatter curves, the lateral acceleration seems to not influence the speed selection. The drivers selected their comfortable speed mostly based on the speed itself.
  
  - For scenario V2, the regular drivers' speeds in sharp curves are almost the experts' speeds. The gap between the two set of speeds increased thereafter with the increase of radius. Regular drivers, adjusted their maximum 'safe' speed in flatter curves mostly based on the speed itself.

- Difference between female and male drivers.

- Difference of behavior between dry and wet pavement. The experts' speeds and lateral accelerations on wet pavement are used in Chapter 5 as the supply of the system.

- Average and standard deviation of the maximum tire-pavement performance with the car Mazda Protégé driven by experts. Hereafter, it is assumed that the average performance of the cars traveling on the today's highways is represented by the average performance of the Mazda Protégé.

At PTEC, the traffic flow was not represented for the experiment, nor was the scenery that may be viewed from the highway lanes. The main influential factors for the selection of the speed, were the horizontal curvatures for the geometric design side and the drivers for the human side. The volume of the traffic stream is known as being an influential factor for speed on highway 75.
horizontal curves, but this factor would had been difficult to develop for the experiment. In order to compare some of the data gathered at PTEC with what really happens on existing horizontal curves therefore four curves on Highway 99 were surveyed between Horseshoe-Bay and Squamish.
CHAPTER 4

Curve Observations: “Sea to Sky” Highway

To complete the Car/Driver behavior study, four curves on Highway 99 between Horseshoe-Bay and Squamish, were observed. This highway section corresponds to the Landmark Kilometer Inventory (LKI) of the segment 2920 (library reference for the photologue of the Ministry of Transportation and Highways), and corresponds to a mountainous alignment type. Going toward Squamish, the road generally follows Howe Sound inlet on the left and is cut on the hillside on the right.

The speed, lateral placement and lateral acceleration were the three variables collected during the observation. Speed and lateral placement were collected with the use of a 8 mm video camera. The lateral acceleration was measured with the g-analyst mounted in a Mazda Protégé model SE.

Because of the mountainous profile of the highway, the speeds adopted in the curves were probably dependent on the overall alignment of the road. Nevertheless, three of the curves had high degrees of curvature (small radius) so that a speed reduction was noticeable.

4.1 Curves

The highway has two lanes each with a width of 3.7 m. The curves were selected based on several criteria. The first criterion was to find curves beside an accessible elevated location at a suitable distance, to set up the video camera. The second criterion was to select curves which
were isolated from prior curves, that would have influenced the speed of the C/D system before entering the surveyed curves. The third criterion was to select curves for which horizontal curvature was the most determinant factor in the speed of the C/D system. For example, the curves with vertical grade were not considered. Table 5.1 presents the geometric characteristics of the four curves.

They are referred to the LKI of the section 2920, north bound direction, going from Horseshoe-Bay to Squamish. The radius of the curves were measured from blue-prints, whereas the other information was collected from the MoTH photolog data.

<table>
<thead>
<tr>
<th>Table 4.1. Curves characteristics.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Section 2920</strong></td>
</tr>
<tr>
<td><strong>Radius (m)</strong></td>
</tr>
<tr>
<td>measured on blue-print</td>
</tr>
<tr>
<td><strong>Length (m)</strong></td>
</tr>
<tr>
<td><strong>Superelevation</strong></td>
</tr>
<tr>
<td><strong>Grade (%) +/-1%</strong></td>
</tr>
<tr>
<td><strong>North B</strong></td>
</tr>
<tr>
<td><strong>South B</strong></td>
</tr>
<tr>
<td><strong>location (LKI in km)</strong></td>
</tr>
</tbody>
</table>

4.2 Procedures

A video camera was located either on the hillside of the mountain or on a natural rock outcrop. The camera was orientated so that it covered the maximum length before and after the center of the curves. The camera was hidden either by trees or branches, in order to be out of the view of the drivers and passengers. Marks used for measurement were prominently displayed on each
side of the road. The marks were spaced every 10 or 20 meters. Those marks were not visible to the driver and passengers.

While the video recorded the vehicles' path and speed going through the curves, a subject driver drove the Mazda Protégé equipped with the g-analyst in both directions through the curves. Sometimes the subject drove at his comfortable speed, sometimes at his "uncomfortable" speed, and sometimes the driver tracked a free-moving passenger car (no obstruction vehicle ahead). When following a car, the Mazda Protégé was close enough to the tracked vehicle so that both speeds were assumed equal.

4.3 Observation

4.3.1 Lateral placement

Figure 4.1 shows the lateral positions in the curves of the free moving vehicles. The shade areas represent the proportion in percent of the cars' position in the curves. It is in curve #2, with the biggest radius, that car-driver systems mostly follow the center of the lane for each direction. For the other radii, the drivers in both directions tended to cut the curves. The search in minimizing the change of speed when driving in the curve, might explain this behavior. In order to minimize this speed change, drivers flattened the bends either by driving on the shoulder or, in some cases, by driving partly in the opposite lane. Possibly the drivers search to optimized their level of comfort by being closer to their 'Aspiration Velocity' \( V_A \) and their lateral 'Acceleration Tolerance' \( A_T \).
CHAPTER 4: Curves Observation: "Sea to Sky" Highway

Figure 4.1. Lateral placements in the four Highway 99 curves.
As one can see, the path for a given C/D system does not follow the center of the lane nor does it follow the center line of the curve. Each system selects its own path which corresponds to a unique radius. Using the matching pairs of speed-lateral acceleration and the superelevation, an effective path radius of the C/D system is computed for each horizontal curve and lanes. The mean and the standard deviation of the effective radius is compared to the designed center line radius in Figure 4.2.

![Graph showing mean and standard deviation of effective radius and the radius center line.](image)

**Figure 4.2. Mean and standard deviation of effective radius and the radius center line.**

The variation of paths selected within a particular radius is a function of several factors related to:

- The geometric characteristics: The designed center line radius, lane width and number of lanes, length of the curve, horizontal and vertical sight distance, and shoulder. (TRB Report 214: 1887)
• The traffic: Opposing traffic on two lane highway and traffic volume (taragin: 1944 in Stang: 1993).

• The drivers: An experience driver might take more risk, by flattening the curve, than unexperienced one. Someone familiar with the road might also choose a different path than somebody unfamiliar.

4.3.2 Speed

Table 4.2 presents the mean speed of the free-moving cars and trucks for the four radii. In general, trucks drive slower than cars. This is statistically significant for all radii expect in southbound direction for curve #3.

<table>
<thead>
<tr>
<th>curves</th>
<th>North bound</th>
<th>South bound</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean speed</td>
<td>mean speed</td>
</tr>
<tr>
<td></td>
<td>car</td>
<td>stddev</td>
</tr>
<tr>
<td>#1</td>
<td>69.98</td>
<td>6.39</td>
</tr>
<tr>
<td>#2</td>
<td>83.75</td>
<td>5.19</td>
</tr>
<tr>
<td>#3</td>
<td>77.53</td>
<td>8.15</td>
</tr>
<tr>
<td>#4</td>
<td>69.95</td>
<td>6.19</td>
</tr>
</tbody>
</table>

Figure 4.3 is the distribution and the 85th percentile speed for the four curves and for the two directions. As expected, there is a direct relationship between the speed selected by drivers and the radius of the bends. Smaller is the radius and lower is the speed of the C/D system. The 85th
percentile speed in curve #4 differs from north-bound and south-bound. In this particular small curvature, the C/D systems going north-bound could easily flatten the bends, whereas the systems going toward Horseshoe-Bay were limited by a side barrier, and thus, limited in their cutting. Therefore, drivers had to reduce their speed significantly, in order to maintain their level of comfort.

Figure 4.3. Speed distribution in the four H99 curves and for the inner and outer lane distinctively.
A long range spot speed study from the MoTH is ongoing at two locations on the “Sea to Sky” Highway. The data is collected with the use of detection loops. The first loop is located at 0.8 km after Horseshoe-Bay on a two lane section, and the second is located 10 km after Squamish toward Whistler on a three lane section. The posted speed is 60 km/h and 80 km/h respectively. The 85th percentile speed for October 1995, is 84 km/h for the first check point and 97 km/h (slow lane) to 108 km/h (fast lane) for the second location. The 85th percentile speed in the four curves presented in Figure 4.3, varies from 70 to 92 km/h, depending on the degree of curvature.

Figure 4.4 indicates that speeds on Highway 99 curves follow McLean (1974) models, which use DMR (1969), Taragin (1957), and Emmerson (1970) data. Even if modern cars are more efficient than the 1957 cars, it seems that the speed selected by the C/D system is constant over time. The overall alignment of the mountainous Highway 99, is probably an influential factor in the selection of speed. It is possible that if the observations had been made on an isolated curve of a straight and level rural road, the speed would have been slightly different.

It seems that people drive faster in curves of same radii, on Highway 99 than on the PTEC paths. The difference of ‘Aspiration Velocity’ between drivers on Highway 99 and drivers on PTEC paths, is the factor which can explain this offset.
4.3.3 Lateral acceleration.

Figures 4.5 to 4.6 present the lateral accelerations experienced along the four curves. The gray plain lines correspond to comfortable speeds selected by the subject. The thin lines represent the lateral acceleration experienced by the Mazda Protégé when tracking a free-moving passenger car. The lateral acceleration experienced by the driver of the preceding vehicle, is assumed to be the same as the lateral acceleration experienced by the subject. The dotted lines correspond to uncomfortable speeds selected by the subject. The uncomfortable rides resulted in curves #1 and #3 and in one direction only.
CHAPTER 4: Curves Observation: "Sea to Sky" Highway

Figure 4.5. Lateral acceleration collected in curves #1. Negative acceleration values for a left curve and positive values for a right curve.

Figure 4.6. Lateral accelerations collected in curves #2. Negative acceleration values for a left curve and positive values for a right curve.
CHAPTER 4: Curves Observation: "Sea to Sky" Highway

Figure 4.7. Lateral accelerations collected in curves #3. Negative acceleration values for a left curve and positive values for a right curve.

Figure 4.8. Lateral accelerations collected in curves #4. Negative acceleration values for a left curve and positive values for a right curve.
The comfort level is a very subjective. Based on the dotted lines and gray lines the accelerations collected from the tracked C/D systems seem to fall in a comfortable speed range.

Figure 4.9 displays the lateral acceleration developed on the Highway and at PTEC. For the small radii, the lateral accelerations collected, fall in the range represented by the two PTEC scenarios V1 and V2. Therefore, drivers tend to tolerate higher lateral acceleration at a given comfortable speed on the highway curves, than in the ones designed for the experiment. The difference between the geometric characteristics of the road and the difference of the immediate environment of the road, are elements which could explain this offset.

Figure 4.9. Lateral acceleration and radius for Highway 99 data and PTEC data.
4.3.4 Lateral Acceleration and Speed

The friction factor versus the speed is presented in Figure 4.10. The linear correlation is compared with the data of Taragin and DMR used by McLean (1974), and with the data collected at PTEC. Scenario V1 regression is close to the regression using Taragine and DMR data, however, drivers on Highway 99 seem to tolerate higher lateral accelerations.

![Figure 4.10. Speed and friction factor for McLean model, PTEC data, and Highway 99 data.](image)

The lower and upper limits of the lateral acceleration versus speed data, are defined with the use of the Herring et al. model. The values for $A_T$, correspond to the maximum comfortable lateral
acceleration defined during the PTEC experiment. The value for $V_A$ approximately corresponds to the 85th speed in the curves. The value of $\beta$ is arbitrary chosen for the best representation of the lower and upper bounds. The values are summarized in Table 4.2 and the limits are presented in Figure 4.11.

Table 4.2. Empirical data for the Herrin et al. model.

<table>
<thead>
<tr>
<th>$A_T$ (g)</th>
<th>$V_A$ (km/h)</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper bound</td>
<td>0.40</td>
<td>95</td>
</tr>
<tr>
<td>Lower bound</td>
<td>0.35</td>
<td>80</td>
</tr>
</tbody>
</table>

Figure 4.11. Lateral acceleration and speed with the use of Herrin et al. Model.
Herrin et al. model is a good representation of the scatter of speed/lateral acceleration points. In the last Figure, the lower and upper bounds do not share the same ‘aspiration velocity’ $V_A$. In Figures 3.15 a) and b) the two bounds have the same ‘aspiration velocity’ $V_A$. Despite this difference, it seems that there is similitude between the values of the expedience parameters for scenario VI at PTEC, and the values of the expedience parameters for Highway 99. The expedience parameters range from -0.045 to -0.100 for Highway 99’s environment, and from -0.020 to -0.120, for the ‘comfortable’ scenario, for PTEC’s environment. This similitude is used to say that drivers on Highway 99 drove at comfortable speeds.

The relationship between lateral acceleration and speed in Figure 4.11 is a characteristic of the alignment of Highway 99. Each road alignment should have a typical lateral acceleration-speed relationship, since the car/driver systems have different behaviors for different highway alignments. Consequently, each types of road alignment, i.e. flat, undulating, hilly and mountainous (McLean: 1995), should be related to a typical $V_A$ and $A_T$ values.

4.4 Conclusion

From the data collected on Highway 99 were computed:

• The mean and standard deviation of the effective path radius selected by the driver population.

• The mean and standard deviation of the car/driver system speed and lateral acceleration at the center of the curves.
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The statistical information (mean, standard deviation and distribution) of the participating variables are needed to run the algorithm which computes the probability of non-compliance.
5.1 Fundamentals of Reliability Theory

The aim of highway design is to provide adequate performance and safety for the highway elements. According to the North American standards, if the procedures are followed properly, the design element of the road will provide a good level of safety. Measuring the level of safety is a complex task due to the nature of the variables involved in the road design. One popular solution to estimate safety is to compare accident rates with the road's geometric variables, and to develop correlation. The reliability approach using safety margin has been used to characterize the risk in highway design. The safety margin for a given situation, compares the demand for some engineering requirement to the supply. In fact, the problem for any engineering system design is the determination of the supply ($S$), also referred as the capacity of some engineering characteristic, to satisfy the demand ($D$) for that characteristic.

5.1.1 Limit State Design

Limit state design is now used by structural engineers to design buildings. The structural engineering design standards are built around the randomness of the different variables involved in the supply and the demand of any structural system. For example, the basic structural limit state design equation, based on the resistance of the element under design, is given as:
where $R$ and $\phi$ are respectively, the nominal resistance and the performance factor of the designed element. $\phi$ represents the uncertainty around the nominal resistance of the material. $D$, $L$, $Q$ and $T$ are the different types of loads applied to the element. These loads are increased by respective load factors $\alpha$, which refer to the uncertainty on the nominal or average load during the lifetime of the system. $\gamma$ and $\psi$ are respectively the importance and load combination factors, and they refer to the randomness of getting different load combinations. In Equation 5.1, both the supply and the demand are random variables involved in the system. The factors on both sides of Equation 5.1, have a direct influence on the performance and safety of the system. In the design process, the engineer has, through the different factors present in the design equation, a visualization of the level of safety.

### 5.1.2 Definitions of Safety Measures

The basic measure of safety is the ratio of the nominal values of the supply $\tilde{S}$ and the demand $\tilde{D}$ called the factor of safety $SF$:

$$SF = \frac{\tilde{S}}{\tilde{D}}$$  \hspace{1cm} (5.2)

where: $\tilde{S}$ = nominal value of the supply; $\tilde{D}$ = nominal value of the demand.
Both supply and demand are functions of uncertain variables: loads, earthquakes, and construction, or, related to highways: construction, vehicle, driver, and environment. Therefore, \( \tilde{S} \) and \( \tilde{D} \) cannot be estimated with certainty, and the factor of safety, which is a function of random variables becomes itself, a random variable (Harr, 1987). The central factor of safety \( CFS \) is then defined as:

\[
CFS = \frac{E[S]}{E[D]} \tag{5.3}
\]

where:
- \( E[S] \) = expected value of supply
- \( E[D] \) = expected value of demand

Usually, to take into account the uncertainties of the system, the designers shrink the safety margin of their design in order to provide higher design values. They decrease the expected supply value and increase the expected demand value by a number of their respective standard deviations. The safety factor now becomes:

\[
FS = \frac{E[S] - k_s \sigma_s}{E[D] + k_d \sigma_d} \tag{5.4}
\]

where:
- \( E[S] \) = expected value of supply;
- \( E[D] \) = expected value of demand;
- \( k_s, k_d \) = multiples of sigma unit of their respective function (Usually 1);
- \( \sigma_s, \sigma_d \) = standard deviation of supply and demand.

Therefore, one can observe that the central factor of safety exceeds the factor of safety. Another practical measure to assess the safety of a system, is to define the margin of safety, as the difference between the expected value of the supply and the expect value of the demand:
CHAPTER 5: Reliability analysis

\[ M = E[S] - E[D] \]  (5.5)

where
\[ E[S] = \text{expected value supply} \]
\[ E[D] = \text{expected value demand} \]

The safety margin, presented in Figure 5.1 has started to be used by some researchers (Navin: 1990, ESRA: 1987) in transportation to assess the safety level of some designed elements.

Figure 5.1. Definition of demand, the supply and the margin of safety

5.1.3 Reliability and Probability of Failure \( P_f \)

The most appropriate design of a particular system should involve all the variables that play a role in the life of the system. Some variables are still unknown and some are not measurable, so that the measure of the reliability usually refers to the basic and most influential variables. The
chance of non-performance or failure for a system is again a function of the variables supply and demand and is defined by the engineer. The safety margin takes the form of a performance function, $G$, which can be written in terms of the difference of the supply, $S$, and the demand, $D$. The equation is:

$$G = S - D$$  \hspace{1cm} (5.6)

The probability of failure $P_f$ of the system corresponds to the probability of the event $(G = S - D) < 0$. The limit state situation is when $G = 0$. The probability of failure provides information on the level of performance of the system and may not necessarily correspond to the collapse of a structural element. If the probability is found to be critical in an ultimate limit state design context, it is likely that the failure will correspond to a collapse situation. In a more generalized formulation, the performance function is described as follows:

$$G(X) = g(x_1, x_2, \ldots, x_n)$$  \hspace{1cm} (5.7)

Where $X$ is a vector descriptor of the variables $(x_1, x_2, \ldots, x_n)$. The probability of failure of this system is the chance that the point $X = (x_1, x_2, \ldots, x_n)$ lies in the failure region defined by $G < 0$. This can be expressed as follows:

$$P_f = \int_{x \in (G<0)} G(X)dx$$  \hspace{1cm} (5.8)
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The solution of equation has been completed by others, see Foschi et al. (1989, 1993) and Blokley (1980). The solutions are First and Second Order Methods (FORM/SORM) which involve the normal distribution function $\Phi$. Using these methods, an estimate of the probability of failure is obtained if the probability density function $G(X)$ is normal and give us:

$$P_f = \Phi(-\beta) \quad (5.9)$$

Where $\beta$ is the reliability or safety index. The reliability index is graphically defined in Figures 5.2 and 5.3. The reliability index $\beta$ represents the minimum distance between the origin: mean value of $X$, and the failure surface: $G(X) = 0$.

If $G$ has a linear form and all variables are independent and normally distributed, the probability of failure computed with FORM is exact. Otherwise, the FORM method executes a linear or plane approximation of the $G$ function at the tangent point, which corresponds to the design point ($G = 0$ and $\beta$ minimum: see Figure 5.3). The SORM iteration provides the closest approximation for a non-linear system, by developing a quadratic surface.

Ang et al. (1975) and Benjamin et al. (1970) presented methods to approximate the expected value and the variance of a multivariate situation. If $G(X)$ is a function of several random variables, that is, $G(X) = g(x_1, x_2, ..., x_n)$, then the second-order approximation of the expected value is:
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Figure 5.2. Definition of the safety index $\beta$

Figure 5.3. Geometric representation of $\beta$ and FORM/SORM approximation. [Source: Foschi et al.: 1989]
CHAPTER 5: Reliability analysis

\[ E[G(X)] \approx g(\mu_x_1, \mu_x_2, \ldots, \mu_x_n) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left( \frac{\partial^2 g}{\partial x_i \partial x_j} \right) \text{Cov}(x_i, x_j) \] (5.10)

and the first-order approximation of the variance is:

\[ \text{Var}[G(X)] \approx \sum_{i=1}^{n} C_i^2 \text{Var}(x_i) + \sum_{i \neq j}^{n} C_i C_j \text{Cov}(x_i, x_j) \] (5.11)

where \( C_i \) and \( C_j \) are the values of the partial derivatives \( \frac{\partial g}{\partial x_i} \) and \( \frac{\partial g}{\partial x_j} \) evaluated at \( \mu_x_1, \mu_x_2, \ldots, \mu_x_n \).

The reliability index \( \beta \) is then defined as:

\[ \beta = \frac{E[G(X)]}{\sqrt{\text{Var}[G(X)]}} \] (5.12)

Since \( G(X) \) is a function of the demand and the supply of the system, \( \beta \) can be written in terms of the Supply \( S(x) \) and the Demand \( D(x) \), as follows:

\[ \beta = \frac{E[S(x)] - E[D(x)]}{\sqrt{\text{Var}[S(x)] + \text{Var}[D(x)]}} \] (5.13)

where: \( \beta \) = reliability or safety index, \( E[S], E[D] \) = expected value of the supply, and the demand, \( \text{Var}[] \) = respective variance of the supply and the demand.
5.2 Reliability Analysis, Based on Skidding Criterion, for PTEC Experiment

In contrast to the structural elements for which failure occurs when an extreme load of sufficient magnitude physically ruptures the element (ultimate failure), the failure mode for a geometric design element is more ambiguous. To have an ultimate failure in the case of a horizontal curve, the $G < 0$ condition should result in a non-compliance with the design of the C/D system. In fact, due to the nature of the factors involved in the road system, an accident in a horizontal curve can occur even if $G > 0$. The probabilities computed in the following, refer to the probabilities of non-compliance ($P_{nc}$) of a highway design element (Navin: 1992). The procedure to compute $P_{nc}$ is similar to the one described above to compute $P_f$.

Figure 3.8 and 3.9 are the best graphical descriptors for the margin of safety of the subject drivers. In the following, the coefficient of friction is the variable used to provide an objective level of safety against sliding off the curves.

Figure 5.4 shows three variable distributions related to the coefficient of friction for treatment T5 (R3-V1). The bold distribution represents the coefficient of friction of the regular drivers when driving at a comfortable speed. The second distribution represents the coefficient of friction on wet pavement of the expert drivers, when driving at their maximum speed before sliding. The third distribution represents the maximum lateral acceleration that the cars can handle when cornering in a 150 ft. (46 m) radius skidpad (Car/driver road test: 1995).
CHAPTER 5: Reliability analysis

Figure 5.4 Distribution of the coefficient of friction for three variables, and respective safety margin for PTEC radius R3 = 60 m.

These three variables are either representing D or S.

5.2.1 Reliability Analysis Using RELAN

RELAN software (RELiability Analysis), (Foschi: 1993) computes the probability of non-performance and the reliability index $\beta$ by using FORM or SORM (First and Second Order Methods) analysis for a given function $G$. RELAN can be used for any engineering system. This software is structured around the Rackwitz algorithm and programmed in FORTRAN. The capacity of the program is of 50 variables per system. The variables can follow ten different distribution types. RELAN transforms those distributions to the normal distribution. Even if
FORM/SORM analysis required independent variables, the software takes into consideration the eventual correlation between the variables.

5.2.1.1 Performance function $G$

The performance function involves only one variable for the demand and one variable for the supply. Both variables are related to the side friction experienced at PTEC, and they are both random and normal. Referring to Equation 5.6, the performance function $G$ is written in its simplest form:

$$ G = f_y S - f_y D $$

(5.14)

And the probability of noncompliance $P_{nc}$ is the probability that $G < 0$ or $S < D$.

5.2.1.2 Supply (variable $X(1)$)

The supplied side friction is estimated by the driving result of the expert drivers. During the experiment, expert drivers drove in the four curves at the maximum cornering traction capacity of the vehicle. Because subject drivers did not significantly reduce speed on wet pavement, the expert distribution on wet pavement is used as the safety threshold, or as the supply friction of the system.

5.2.1.3 Demand (variable $X(2)$)

The demand is the distribution of the lateral acceleration required by the subject’s drivers rides for the two scenarios V1 and V2. The data collected on wet and dry pavement is regrouped since the subjects did not significantly reduce their speed on wet pavement.
CHAPTER 5: Reliability analysis

5.2.1.4 Variables

With the lateral acceleration being a function of the speed, and the speed in the curves being represented by the normal distribution, then the lateral acceleration follows a normal distribution. The two variables $S$ and $D$ are assumed to be uncorrelated. Table 5.2 is the statistical information on the variables for the four radii and for the two scenarios V1 and V2.

<table>
<thead>
<tr>
<th>Rad.-Scen.</th>
<th>Variable</th>
<th>Mean (in g)</th>
<th>Stdev. (in g)</th>
<th>Rad.-Scen.</th>
<th>Variable</th>
<th>Mean (in g)</th>
<th>Stdev. (in g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1-V1</td>
<td>X(1)</td>
<td>0.60</td>
<td>0.06</td>
<td>R3-V1</td>
<td>X(1)</td>
<td>0.53</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>X(2)</td>
<td>0.32</td>
<td>0.08</td>
<td></td>
<td>X(2)</td>
<td>0.21</td>
<td>0.06</td>
</tr>
<tr>
<td>R1-V2</td>
<td>X(1)</td>
<td>0.60</td>
<td>0.06</td>
<td>R3-V2</td>
<td>X(1)</td>
<td>0.53</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>X(2)</td>
<td>0.48</td>
<td>0.07</td>
<td></td>
<td>X(2)</td>
<td>0.37</td>
<td>0.06</td>
</tr>
<tr>
<td>R2-V1</td>
<td>X(1)</td>
<td>0.60</td>
<td>0.04</td>
<td>R4-V1</td>
<td>X(1)</td>
<td>0.46</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>X(2)</td>
<td>0.29</td>
<td>0.07</td>
<td></td>
<td>X(2)</td>
<td>0.20</td>
<td>0.04</td>
</tr>
<tr>
<td>R2-V2</td>
<td>X(1)</td>
<td>0.60</td>
<td>0.04</td>
<td>R4-V2</td>
<td>X(1)</td>
<td>0.46</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>X(2)</td>
<td>0.46</td>
<td>0.06</td>
<td></td>
<td>X(2)</td>
<td>0.28</td>
<td>0.06</td>
</tr>
</tbody>
</table>

5.2.1.5 Results

Example of safety value

The first safety value is the central factor of safety (CFS) of Equation 5.3 which is the ratio of the expected mean of $S$ and the expected mean of $D$. The second value of safety is more conservative than the first one and is the factor of safety (FS) of Equation 5.4 calculated as the expected value of $S$ minus $k$ times its standard deviation, over the expected value of $D$ plus $k$ times its standard deviation. The safety margin ($M$), in Equation 5.5 is the difference between $S$.
and $D$. The PTEC data collected in the sharper curve R1 and in the flatter curve R4 for both scenarios V1 and V2, are used as an example to compute the preceding values of safety:

<table>
<thead>
<tr>
<th>Treatment</th>
<th>$CFS$</th>
<th>$FS (\kappa=1)$</th>
<th>$M (g)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1-V2</td>
<td>1.27</td>
<td>0.99</td>
<td>0.13</td>
</tr>
<tr>
<td>R1-V1</td>
<td>1.92</td>
<td>1.39</td>
<td>0.29</td>
</tr>
<tr>
<td>R4-V1</td>
<td>2.34</td>
<td>1.65</td>
<td>0.26</td>
</tr>
<tr>
<td>R4-V2</td>
<td>1.63</td>
<td>1.14</td>
<td>0.18</td>
</tr>
</tbody>
</table>

**Probability of non-compliance**

With the use of the software RELAN (Foschi *et al.*: 1993), the probability of non-compliance and the reliability index $\beta$ are computed, based on the following performance function:

$$G = X(1) - X(2)$$

Since the performance function is of linear form, the FORM calculation provides a precise value of $\beta$ and the probability of non-compliance. The results are summarized in Table 5.3 and are graphically presented in Figures 5.5 and 5.6.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>$\beta$</th>
<th>Probability of noncompliance, $P_{nc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1-V2</td>
<td>1.302</td>
<td>0.09653</td>
</tr>
<tr>
<td>R1-V1</td>
<td>2.800</td>
<td>0.00255</td>
</tr>
<tr>
<td>R2-V2</td>
<td>1.941</td>
<td>0.00261</td>
</tr>
<tr>
<td>R2-V1</td>
<td>3.845</td>
<td>0.00006</td>
</tr>
<tr>
<td>R3-V2</td>
<td>1.886</td>
<td>0.02967</td>
</tr>
<tr>
<td>R3-V1</td>
<td>3.771</td>
<td>0.00008</td>
</tr>
<tr>
<td>R4-V2</td>
<td>1.952</td>
<td>0.02545</td>
</tr>
<tr>
<td>R4-V1</td>
<td>3.225</td>
<td>0.00063</td>
</tr>
</tbody>
</table>
CHAPTER 5: Reliability analysis

Figure 5.5 is a representation of the distribution of the demand and the supply for the two radii R1 and R4 and for the two scenarios V1 and V2.

Figure 5.5. Demand and Supply with respective Safety margin for R1 and R4 under scenarios V1 and V2
Figure 5.6. Probability of noncompliance, based on skidding criterion, versus the radius of PTEC curves.

Because the variables are independent, the coefficient of correlation is zero and the variance of the performance function distribution is the sum of the variances of $S$ and $D$ distributions. The expected value of $G$ is the difference between the expected value of $S$ and the expected value of $D$. Figure 5.7 is a representation of the $G$ distribution for treatment T1 (R1-V2). On the same Figure, the safety index $\beta$ and the probability of non-compliance are graphically represented.
The reliability function $G$ compared the supply and the demand only on the basis of the lateral acceleration. Strictly based on this criterion, the probability of non-compliance at a comfortable speed is $26/10,000$ for the smaller radius $R_1$, and $6/10,000$ for the bigger radius $R_4$. At the maximum safe speed of the subjects, the probability of non-compliance is $10\%$ for $R_1$ and $2.5\%$ for $R_4$. The level of safety, based on side friction when cornering, decreases with the decrease of speed and radius from $R_1=16m$ to $R_2=25m$. The probability of non-compliance is, thereafter, constant or slightly decreasing to $R_4=100m$ for scenario V2. However, for scenario V1, the probability increases to ten-fold between $R_2=25m$ and $R_4=100m$. This information tends to contradict the regression presented in Figure 3.8 and 3.9, which show an increase of the safety margin with an increase of the radius for both scenarios. Even if, as presented in Figure 5.6, this
increase represents very small values of non-compliance, it exhibits the influence of the dispersions of the variables on the level of safety of a specific curve.

Comparison with AASHTO’s design speed of 30 mph (48 km/h)

Within the scope of their analysis, Harwood et al. (1994) concluded that "the existing [High-Speed Horizontal Curves Design Criteria] provides adequate margins of safety against skidding and rollover by both passenger cars and trucks ..." However, for the Low-Speed Horizontal Curves Design Criteria, "the minimum radius ... may not provide an adequate margin of safety for trucks [in a poor environment] or with low rollover thresholds." They supported their analysis with the help of the AASHTO (1990) curve formula. The result of non-compliance of Treatments T7 and T8 are compared in Table 5.4 with the Harwood et al. data, for a design speed of 30 mph (48 km/h).

<table>
<thead>
<tr>
<th>Source</th>
<th>superelevation (e)</th>
<th>$f$ demand</th>
<th>mini. Radius (ft/m)</th>
<th>$f$ supply</th>
<th>Marg. Safety wet</th>
<th>Marg. Safety dry</th>
<th>Risk of skidding</th>
</tr>
</thead>
<tbody>
<tr>
<td>AASHTO</td>
<td>0.04</td>
<td>0.16</td>
<td>302 (92)</td>
<td>0.51</td>
<td>0.35</td>
<td>0.78</td>
<td>low</td>
</tr>
<tr>
<td>PTEC T7</td>
<td>0</td>
<td>0.20</td>
<td>328 (100)</td>
<td>0.46</td>
<td>0.26</td>
<td>X</td>
<td>0.06 %</td>
</tr>
<tr>
<td>PTEC T8</td>
<td>0</td>
<td>0.28</td>
<td>328 (100)</td>
<td>0.46</td>
<td>0.18</td>
<td>X</td>
<td>2.5 %</td>
</tr>
</tbody>
</table>

In their study, Harwood et al. (1994) could only provide information on the margin of safety, which compares two nominal values of lateral acceleration. In their analysis, the variation
among drivers and among vehicle capacity was not taken into consideration. The last calculation of non-compliance considered the variation among subject drivers. For treatment T7, which corresponds to a radius of 100 m driven at scenario V1, the probability of non-compliance is 0.06% and for treatment T8, the same radius driven at scenario V2, the probability of non-compliance is 2.5%. If it is assumed that people always drive at a comfortable speed, then 0.06% might be associated with a low risk of sliding off the road. If one assumed that people might be late for an important interview, then it is doubtful that 2.5% of non-compliance would be associated to a low risk of sliding off the curve.

5.3 Reliability for Highway 99

5.3.1 Analysis based on skidding criterion

Using the coefficient of friction as the main variable in the performance function $G$, the same procedure as described in Section 5.2.1 can be followed, to compute a value of non-compliance for the four Highway 99 curves observed.

5.3.1.1 Performance function $G$

The performance function for this exercise is the same as Equation 5.14:

$$G = f_y S - f_y D \tag{5.16}$$

And the probability of noncompliance $P_{nc}$ is the probability that $G < 0$ or $S < D$. 

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5.3.1.2 Supply (variable X(1))

The supply of the system is associated with the maximum lateral acceleration that an expert would develop in the four curves with a given car. Assuming a decrease of lateral acceleration with an increase of speed in the horizontal curves, the exponential regression of the expert drivers' lateral acceleration, presented in Figure 4.9, is extended to $R= 450$ m. Assuming the Mazda Protégé, driven at PTEC, is a representation of the mean car capacity for all passenger cars, the exponential regression represents the distribution of the mean, and maximum lateral acceleration in a horizontal curve without superelevation of a car driven by an expert. The standard deviation is the same for all radii, and refers to the variation of the holding capacities in a horizontal curve among cars (see Figure 2.1). For the four radii, the values of the statistical characteristics of these maximum lateral accelerations are presented in Table 5.5 and resumed in Table 5.6.

Table 5.5. Definition of the Supply for the four radii.

<table>
<thead>
<tr>
<th>Radius</th>
<th>Average**</th>
<th>Superelevation</th>
<th>Total</th>
<th>Stdev.*</th>
<th>Distrib.</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>0.45</td>
<td>0.07</td>
<td>0.52</td>
<td>0.07</td>
<td>Normal</td>
</tr>
<tr>
<td>#2</td>
<td>0.31</td>
<td>0.04</td>
<td>0.35</td>
<td>0.07</td>
<td>Normal</td>
</tr>
<tr>
<td>#3</td>
<td>0.41</td>
<td>0.03</td>
<td>0.44</td>
<td>0.07</td>
<td>Normal</td>
</tr>
<tr>
<td>#4</td>
<td>0.47</td>
<td>0.04</td>
<td>0.51</td>
<td>0.07</td>
<td>Normal</td>
</tr>
</tbody>
</table>

** refer to Figure 4.9.; *refer to Figure 2.1

5.3.1.3 Demand (variable X(2))

The demand is represented by the lateral acceleration experienced by the driver population going through the inner and outer lanes of the four Highway 99 curves. The statistical characteristics are presented in Table 5.6.
CHAPTER 5: Reliability analysis

5.3.1.4 Variables

All the lateral accelerations are assumed to be normally distributed, and assumed to be independent variables. Table 5.5 is the statistical information on the variables involved in the $G$ function for the four radii.

Table 5.6. Variables: $X(1)$: Supply and $X(2)$: Demand.

<table>
<thead>
<tr>
<th>Radius</th>
<th>Variables</th>
<th>Mean</th>
<th>Stddev</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>$X(1)$</td>
<td>0.52</td>
<td>0.07</td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>$X(2)$</td>
<td>0.19</td>
<td>0.03</td>
<td>Normal</td>
</tr>
<tr>
<td>#2</td>
<td>$X(1)$</td>
<td>0.35</td>
<td>0.07</td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>$X(2)$</td>
<td>0.06</td>
<td>0.02</td>
<td>Normal</td>
</tr>
<tr>
<td>#3</td>
<td>$X(1)$</td>
<td>0.44</td>
<td>0.07</td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>$X(2)$</td>
<td>0.19</td>
<td>0.06</td>
<td>Normal</td>
</tr>
<tr>
<td>#4</td>
<td>$X(1)$</td>
<td>0.51</td>
<td>0.07</td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>$X(2)$</td>
<td>0.27</td>
<td>0.07</td>
<td>Normal</td>
</tr>
</tbody>
</table>

5.3.1.5 Result

The results of the reliability index and the probability of non-compliance are summarized in Table 5.7. They are computed with RELAN and refer to the same performance function $G$ of Equation 5.15.

Table 5.7. RELAN (FORM) results for $G$ Equation 5.15 or 5.16.

<table>
<thead>
<tr>
<th>Radius (m)</th>
<th>Curve</th>
<th>$\beta$</th>
<th>$P_{nc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>#4</td>
<td>2.424</td>
<td>0.767 E (-02)</td>
</tr>
<tr>
<td>120</td>
<td>#1</td>
<td>3.721</td>
<td>0.992 E (-04)</td>
</tr>
<tr>
<td>180</td>
<td>#3</td>
<td>2.712</td>
<td>0.335 E (-02)</td>
</tr>
<tr>
<td>450</td>
<td>#2</td>
<td>3.983</td>
<td>0.340 E (-04)</td>
</tr>
</tbody>
</table>
As expected, the probability of non-compliance is increasing with the decrease of radius, except between curves #1 and curve #3. The probability of non-compliance is higher in curve #3 than in curve #1. The superelevation, which is higher in curve #1, can explain this offset.

5.3.1.6 Total probability of non-compliance

The variable X(2) in Table 5.6 is the average of the acceleration experienced by the cars in the inner lane and the outer lane. In Chapter 4, it was shown that the path selected in the curves was different among the drivers and between the inner and the outer lanes. The variation of the path selected, influences the variation of the experienced lateral acceleration. Table 5.8 shows that lateral acceleration experienced in the inner lane is higher than the one experienced in the outer lane.

<table>
<thead>
<tr>
<th>Curve</th>
<th>Inner lane</th>
<th></th>
<th>Outer lane</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Stdev.</td>
<td>Mean</td>
</tr>
<tr>
<td>#1</td>
<td>0.23</td>
<td>0.04</td>
<td>0.24</td>
</tr>
<tr>
<td>#2</td>
<td>0.08</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>#3</td>
<td>0.21</td>
<td>0.05</td>
<td>0.16</td>
</tr>
<tr>
<td>#4</td>
<td>0.30</td>
<td>0.06</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 5.9 is the probability of non-compliance computed by using the same G function of Equation 5.15 or 5.16 for the inner lanes and the outer lanes. The supply values are X(1) from Table 5.6, and the demand values are from Table 5.8. It is assumed that the supply is the same for inner and outer lanes.
Table 5.9. RELAN (FORM) results for Equation G 5.15 or 5.16 for inner and outer lane.

<table>
<thead>
<tr>
<th>Curve</th>
<th>Inner lane</th>
<th>Outer lane</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta$</td>
<td>$P_{nc}$</td>
</tr>
<tr>
<td>#1</td>
<td>3.597</td>
<td>0.161E(-03)</td>
</tr>
<tr>
<td>#2</td>
<td>3.818</td>
<td>0.672E(-04)</td>
</tr>
<tr>
<td>#3</td>
<td>2.674</td>
<td>0.375E(-02)</td>
</tr>
<tr>
<td>#4</td>
<td>2.278</td>
<td>0.114E(-01)</td>
</tr>
</tbody>
</table>

The probability of non-compliance based on the lateral acceleration, is higher for the people driving in the inner lane than the people driving in the outer lane for all radii. The outer lane supplies drivers with flatter curves than the inner lane. The difference of path radius selected by the drivers between the two lanes, is the main factor that influences the differences of probability of non-compliance.

The performance of the system is a function of the performance of the elements which compose the system. The curve system is in reality to a serial system. A serial system will not perform if one of the elements does not perform. The total probability of non-compliance of a serial system is computed as follows:

$$P_{12} = P_1 + P_2 - P_{12} = \Phi(\beta_1) + \Phi(\beta_2) - P_{12}$$  \hspace{1cm} (5.18)

where: $P_1$ = Probability of inner lane,
$P_2$ = Probability of outer lane,
$P_{12}$ = Joint probability between events 1 and 2.
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If the events $P_1$ and $P_2$ are independent, the joint probability $P_{12}$ is the product of the individual probability. If the events $P_1$ and $P_2$ are correlated, the joint probability is a function of the correlation coefficient between the two events.

The total probability of non-compliance based on skidding criterion for the four curves is presented in the following table:

<table>
<thead>
<tr>
<th>curve</th>
<th>$\beta$</th>
<th>$P_{nc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>3.591</td>
<td>0.165E-03</td>
</tr>
<tr>
<td>#2</td>
<td>3.818</td>
<td>0.673E-04</td>
</tr>
<tr>
<td>#3</td>
<td>2.598</td>
<td>0.468E-02</td>
</tr>
<tr>
<td>#4</td>
<td>2.278</td>
<td>0.114E-01</td>
</tr>
</tbody>
</table>

Figure 5.8 summarizes the three precedent computations, based on the lateral acceleration experienced on the four curves of Highway 99.

To measure the level of safety of a highway curve, the reliability function should include, not only the friction factor, but all the known and unknown variables which play a role in the process of a car driving through a curve. In the following analysis, only the known and basic variables are used to compute the level of non-compliance, based on the radii of the four Highway 99 curves.
CHAPTER 5: Reliability analysis

5.3.2 Reliability analysis, based on the radii, for Highway 99 curves

Navin (1990) suggested the radii as the measures to be compared, because radius is related to the speed and threshold of lateral acceleration. The radius is also an easy physical parameter to control, and therefore, comparisons and studies are easier.

In the following, the probability of non-compliance based on the radius supply $R_s$ and the radius demand $R_D$, is computed using the data collected at PTEC and on the four Highway 99 curves.
CHAPTER 5: Reliability analysis

5.3.2.1 Performance function $G$

To apply the description of Section 5.1 to a horizontal curves, the non-compliance condition has to be defined. The non-compliance for the horizontal curve is: A horizontal curve will not perform if its designed radius ($R_S$) is inferior to the radius demanded by the vehicle-driver system ($R_D$). The performance function is described as follows:

$$G = R_S - R_D$$

The non-compliance mode is stated as: $R_S < R_D$

From the observations, it was found that the behavior of the C/D system differs between the inner and the outer lanes of a particular curve. Therefore, the horizontal curve has to be regarded as a serial system where the inner lane and the outer lane are the two elements. In that condition, the reliability analysis for a horizontal curves has first to cover the probability of noncompliance for each individual lane, before computing the total probability.

5.3.2.2 Supply

The basic curve equation 5.20 provides a curve with a minimum radius which is usually associated to the centre line of a two lane highway.

$$R_H = \frac{V_H^2}{\alpha_y^H} = \frac{V_H^2}{(e^H + f^H_y)g}$$

(5.20)

where $R_H$ = minimum radius of curve, m;
$V_H$ = highway design speed, m/s;
$g$ = acceleration of gravity (9.81 m/s$^2$);
$\alpha_y^H$ = curve design lateral acceleration, m/s$^2$;
$e^H$ = curve superelevation, m/m, and
\[ f_y^H = \text{curve side friction factor,} \]

The supply in the \( G \) function could also represent a single value \( R_H \). However, the resulting \( R_H \), as observed on Highway 99, does not provide a single possible radius to drivers, but supplies a range of radii. For the same curve, the C/D systems going in the same direction have different lateral placements about the centre lane and each corresponding path is associated with a single radius at the centre point of the bend. The designed radius, referred to as \( R_s \), provides a radii population rather than a single radius.

The supply should not be a single value of \( R_H \), but rather a random variable which is a function of the design radius centre lane. The radius selected by the C/D system is assumed to follow a normal distribution.

### 5.3.2.3 Demand

The demand refers to the behavioral characteristics of the C/D system when driving in a horizontal curve.

As presented in Equation 5.20, the speed and the lateral acceleration developed by the C/D system are a function of the radius. Therefore, these two variables together, cannot be compared to the design radius in the performance function. Such use would bias the performance function which would then compare three variables that are highly correlated. The comfortable speed \( V_{C/D} \) alone, is taken as the variable to represent the behavior of the driver in the performance
function. The speed selected by the C/D system in a horizontal curve, is a random variable which follows a normal distribution.

The second element needed to compute $R_D$ is a measure of lateral acceleration. In a previous section, supply was associated to the maximum lateral acceleration $a_y$ that an expert driver could reach in the Highway 99 curves with a Mazda Protégé. The same correlation, between the radius and the maximum lateral acceleration on wet pavement developed by the car when driven by experts at PTEC, is now associated with the average acceleration capacity for all the cars $a_{y}^{car}$, and for a given horizontal curve. It is assumed that the Mazda Protégé used at PTEC, represents the average vehicle. Based on the information provided by Car/Driver Road Test (1995), the Mazda cornering performance is 0.76g and the average cornering performance for all the cars is 0.8g (see Figure 1.2). From $V_{C/D}$ and $a_{y}^{car}$ a basic $R_D$ can be computed as:

$$R_D = \frac{V^2}{g(a_{y}^{car})}$$

(5.21)

As presented in Chapter 1, the superelevation has a safety aspect. The superelevation takes some of the lateral forces and allows a C/D system to develop higher speeds in the curve. This geometric characteristic of the horizontal curve is assumed to have very low variability and will be considered as a constant. However, this variable is part of the performance function, and $R_D$ becomes:
\[ R_D = \frac{V_{C/D}^2}{g(\alpha_{car} + e)} \] (5.22)

5.3.2.4 Variables

The demand and the supply are presented in terms of their respective variables, the \( G \) function is as following:

\[ G(X) = R_S \frac{V_{C/D}^2}{g(\alpha_{car} + e)} \] (5.23)

The variables are summarized in Table 5.11. They represent the elements needed to compute the probability of non-compliance. The coefficient of friction \( f_y \) is absent from the performance function. The value of the coefficient of friction \( f_y \) is implicitly represented by the value of the maximum lateral acceleration \( \alpha_{car} \) for all the cars. To reflect the variability of the coefficient of friction supplied by the pavement, due to the presence of elements such as rain, snow and ice, or due to the long-term polishing effect of traffic, a safety factor \( \alpha_{fy} \) would intervene in the design equation.

<table>
<thead>
<tr>
<th>VARIABLES ((X))</th>
<th>DEMAND</th>
<th>SUPPLY</th>
<th>DISTRIBUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Designed Radius</td>
<td>– ( R_S, \sigma_{Rs} )</td>
<td>( R_S, \sigma_{Rs} )</td>
<td>NORMAL</td>
</tr>
<tr>
<td>C/D speed for the design radius</td>
<td>( \bar{V}_{C/D}, \sigma_V )</td>
<td>–</td>
<td>NORMAL</td>
</tr>
<tr>
<td>Lateral acceleration car for a given curve</td>
<td>( \bar{\alpha}<em>{car}, \sigma</em>\alpha )</td>
<td>–</td>
<td>NORMAL</td>
</tr>
<tr>
<td>Superelevation</td>
<td>( e )</td>
<td>–</td>
<td>UNIFORM</td>
</tr>
</tbody>
</table>

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Since only few variables are involved, an algebraic presentation of the analysis is possible. The approximation of the expected $R_D$ and its variance are derived from the Equations 5.10 and 5.11 respectively. Table 5.12 presents the derivatives and the variations between variables to facilitate the computation.

**Table 5.12. Derivatives for the horizontal curves reliability.**

<table>
<thead>
<tr>
<th>$X_i$</th>
<th>$X_j$</th>
<th>$\frac{\partial (R_D)}{\partial X_i}$</th>
<th>$\frac{\partial^2 (R_D)}{\partial X_i \partial X_j}$</th>
<th>$\text{Cov}(V, \alpha_y)$</th>
<th>$\sigma_y^2$</th>
<th>$\text{Cov}(\alpha_y, V)$</th>
<th>$\sigma_{\alpha_y}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>$\alpha_y$</td>
<td>$\frac{2V}{\alpha_y}$</td>
<td>$-\frac{2V}{\alpha_y^2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V$</td>
<td>$V$</td>
<td>$\frac{2V}{\alpha_y}$</td>
<td>$\frac{2}{\alpha_y}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>$V$</td>
<td>$-\frac{V^2}{\alpha_y}$</td>
<td>$-\frac{2V}{\alpha_y^2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>$\alpha_y$</td>
<td>$-\frac{V^2}{\alpha_y^2}$</td>
<td>$\frac{2V^2 \alpha_y}{\alpha_y^4}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The expected radius is:

$$E[R_D] = \frac{V^2}{\alpha_y} + \sigma_y^2 - \frac{V}{\alpha_y} \left[ \text{Cov}(V, \alpha_y) + \text{Cov}(\alpha_y, V) \right] + \frac{V^2 \sigma_{\alpha_y}^2}{\alpha_y^3}$$  \hspace{1cm} (5.24)$$

and the variance is:

$$Var[R_D] = \left( \frac{4V^2}{\alpha_y^2} \right) \sigma_y^2 + \left( \frac{V^4}{\alpha_y^4} \right) \sigma_{\alpha_y}^2 - \left( \frac{2V^3}{\alpha_y^3} \right) \left[ \text{Cov}(V, \alpha_y) + \text{Cov}(\alpha_y, V) \right]$$ \hspace{1cm} (5.25)$$

where $V$ = Car-Driver system speed ($V_{CD}$),
$\alpha_y$ = Car maximum lateral acceleration ($\alpha_{y,\text{cor}}$).
$R_D$ = Radius demanded
For simplification, the superelevation was not presented in the derivation. However, for the formulation of a reliability model, this variable is represented in the computation. From Equation 5.5, the margin of safety is as follows:

\[ M(R) = E[R_s] - E[R_D] \]  

(5.26)

The probability of non-compliance is the probability that \( G(X) \) is negative: \( P_{nc} = P[G(X) < 0] \) and from Equation 5.13 the safety index becomes:

\[ \beta(R) = \frac{E[R_s] - E[R_D]}{\sqrt{Var(R_s) + Var(R_D)}} \]  

(5.27)

where \( E[R_s] \) is the variable to be defined for designing the curve with a good level of compliance. The expected value and the reliability index are functions of the highway’s basic design elements \( (e, f, R \) and \( V) \), the driver, and the vehicle, which are the three major variables for design and analysis (Navin, 1990).

5.3.2.5 Numerical analysis

From data collected on Highway 99, the variables for the four curves and for the two lanes are defined in the Table 5.13.

The probability of non-compliance of each lane is individually computed using FORM procedure through \( G \) function of equation 5.23. The total probabilities of non-compliance of the curves are computed by using FORM for a series system with two failure modes. All variables are assumed uncorrelated. Table 5.14 and Figure 5.9 summarize the RELAN results.
Table 5.13. Definition of the variables for performance function $G$ of equation 5.23.

<table>
<thead>
<tr>
<th>Curve #</th>
<th>Lane</th>
<th>$R_s$ Mean</th>
<th>Stdev.</th>
<th>$V_{CD}$ Mean</th>
<th>Stdev.</th>
<th>$a_y$ Mean</th>
<th>Stdev.</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Inner</td>
<td>121.6</td>
<td>11.7</td>
<td>70.0</td>
<td>6.4</td>
<td>0.45</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>Outer</td>
<td>136.7</td>
<td>10.7</td>
<td>73.5</td>
<td>7.0</td>
<td>0.45</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>2</td>
<td>Inner</td>
<td>440.9</td>
<td>58.1</td>
<td>84.5</td>
<td>6.9</td>
<td>0.31</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>Outer</td>
<td>678.4</td>
<td>10.3</td>
<td>83.8</td>
<td>5.2</td>
<td>0.31</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>3</td>
<td>Inner</td>
<td>195.0</td>
<td>19.4</td>
<td>77.5</td>
<td>8.2</td>
<td>0.41</td>
<td>0.07</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>Outer</td>
<td>232.9</td>
<td>29.0</td>
<td>75.2</td>
<td>8.4</td>
<td>0.41</td>
<td>0.07</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>Inner</td>
<td>102.2</td>
<td>24.8</td>
<td>65.0</td>
<td>5.5</td>
<td>0.47</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>Outer</td>
<td>135.6</td>
<td>15.0</td>
<td>70.0</td>
<td>6.2</td>
<td>0.47</td>
<td>0.07</td>
<td>0.04</td>
</tr>
</tbody>
</table>

As expected, an increase of non-compliance occurs with a decrease of radius. For Highway 99 horizontal curves, Figure 5.9 shows a high increase of non-compliance for radii under 200 meters.

Table 5.14. Results FORM for non-compliance of Highway 99 curves.

<table>
<thead>
<tr>
<th>Curves</th>
<th>Lane</th>
<th>Reliability index $\beta$</th>
<th>$P_{nc}$ Reliability index $\beta$</th>
<th>Total $P_{nc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Inner</td>
<td>1.997</td>
<td>0.2292E(-01)</td>
<td>1.771</td>
</tr>
<tr>
<td></td>
<td>Outer</td>
<td>2.096</td>
<td>0.1803E(-01)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Inner</td>
<td>2.913</td>
<td>0.1790E(-02)</td>
<td>2.909</td>
</tr>
<tr>
<td></td>
<td>Outer</td>
<td>3.712</td>
<td>0.1029E(-03)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Inner</td>
<td>2.084</td>
<td>0.1857E(-01)</td>
<td>2.030</td>
</tr>
<tr>
<td></td>
<td>Outer</td>
<td>2.710</td>
<td>0.3365E(-02)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Inner</td>
<td>1.271</td>
<td>0.1019</td>
<td>1.226</td>
</tr>
<tr>
<td></td>
<td>Outer</td>
<td>2.293</td>
<td>0.1094E(-01)</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.9 Probabilities of non-compliance versus radius for Highway 99 curves.

Figure 5.9 is a characteristic of the alignment of Highway 99. Each road alignment should have a typical $P_{nc}$ and radius relationship since the car/driver systems adopt different behavior for different highway alignments. McLean (1995) presented the desired speed values as a function of overall alignment standard and terrain type: flat, undulating, hilly and mountainous. The same could be realized by combining a standardized reliability index for each highway class as presented in figure 5.10.

Figure 5.10 could be adopted to set consistency alignment criteria. For example, two successive curves should not differ more than a reliability index value of 0.5. The use of reliability analysis
is not constraint to the analysis of an individual highway element, but could also be used for the analysis of the overall alignment of a road section.

Figure 5.10. Reliability index curves for different highway alignment type.

5.4 Reliability-Based Design for Highway Horizontal Curves

Equation 5.23 is now presented in the conditional form:

\[ R_S > \frac{v_D^2}{g(a_{car}^e + e)} \]  

(5.28)
There are two possibilities to satisfy this condition. The first possibility is to increase the radius supplied and the second is to decrease the radius demanded. The main difficulty is that radius demanded is a function of the radius supplied. The speed and the path selected by the driver in a particular horizontal curve is a function of what radius centre line is supplied to the driver. However, the superelevation, the maximum lateral acceleration for a given car, and the side friction factor supplied by the pavement, are elements which are assumed not to intervene in the speed selected by the driver $V_D$. These elements can be adjusted by the designer to satisfy a desirable probability of non-compliance or a desirable value of $\beta$.

5.4.1 Influence of Superelevation ($e$) on Reliability Index ($\beta$)

The superelevation is a geometric characteristic which could be associated to a safety factor. The superelevation increases the maximum lateral acceleration available in a horizontal curve for a given car. The following figures show the influence of the superelevation on the value of the Reliability Index $\beta$ for the four Highway 99 curves, and Inner and Outer lanes respectively.

The values of $\beta$ are computed with RELAN by changing the superelevation in equation 5.23. As expected, the Reliability Index increases with an increase of the superelevation. Curve #1 develops a linear relationship between $e$ and $\beta$. The other curves experience a high increase of $\beta$ between superelevations 0.04 and 0.07, and leveling off thereafter. Except for curve #1, it seems that after 0.07, increased superelevation does not significantly improve, the level of compliance of the curves. With this type of analysis, an optimum superelevation might be derived. This
information could be an excellent tool for designers making decisions, and could be relevant for the economic analysis of a highway.

5.4.2 Influence of Friction Coefficient ($f_y$) on Reliability Index ($\beta$)

The friction coefficient supplied by the road is one element which has not been investigated so far. The grip the pavement supplies at the tires' point of contact, is very important for the safety of the user. In Equation 5.28, the lateral acceleration parameter corresponds to the maximum performance for a given vehicle and a given radius. This value might not be attainable if the pavement does not provide sufficient side friction. Furthermore, drivers do not have the expertise to accurately estimate the available side friction of the road. Nicholson et al. (1992) cited a the McLean (1983) study which stated that drivers do not select their speed around a curve in accordance with an actual or subjectively predicted side friction. The side friction
available is presented in the following equation as a road friction factor $\alpha_{f_y}$ applied on the optimum lateral acceleration for a given vehicle.

$$R_s > \frac{V_D^2}{g(\alpha_{f_y}a_{yar} + e)}$$

(5.29)

The side friction available in a horizontal curve can vary considerably for different reasons:

- poor pavement designs, which does not provide a good micro and macro texture,
- poor construction or too rich bitumen asphalt composition,
- long-term polishing effect of traffic,
- effects of seasonal variations,
- effects of weather variations.

Four values of $\alpha_{f_y}$ could be assumed as follow:

<table>
<thead>
<tr>
<th>safety factor $\alpha_{f_y}$</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Good construction, regular maintenance.</td>
</tr>
<tr>
<td>0.9</td>
<td>Good construction, low polishing.</td>
</tr>
<tr>
<td>0.7</td>
<td>Poor construction.</td>
</tr>
<tr>
<td>0.4</td>
<td>High risk of weather change and presence of black ice</td>
</tr>
</tbody>
</table>

The next figure is the influence of these $\alpha_{f_y}$ on the safety index $\beta$ for the four curves and the inner and outer lanes.
 CHAPTER 5: Reliability analysis

Figure 5.12. Influence of road friction factor $\alpha_{f_r}$ on Safety index $\beta$.

Poor coefficient of friction of the pavement implies negative value for $\beta$ for curves 1, 3 and 4. If such events are expected to occur during the lifespan of the horizontal curve, the designer could compensate the design by providing higher superelevation or by flattening the curve with higher radius. Assuming a desired value for $\beta$ of 2.0, one can see that only the outer lane of curve 2 satisfies a poor coefficient of friction.

Safety or performance factors can be related to all the random variables involved in the design system. The last two examples demonstrate how a road designer could control design element based on a limit state design equation.
One important product of RELAN is the sensitivity factor. It indicates which variable is the most influential in the system. A sensitivity factor is provided for each variable involved in the performance function $G(X)$. Table 5.15 are the sensitivity factors for the three variables involved in the two lanes of the four curves.

Table 5.15: Sensitivity factors for Highway 99 curves.

<table>
<thead>
<tr>
<th>Curve</th>
<th>Lane</th>
<th>$R_s$</th>
<th>$V_D$</th>
<th>$a_{v,car}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>Inner</td>
<td>0.413</td>
<td>0.646</td>
<td>0.642</td>
</tr>
<tr>
<td></td>
<td>Outer</td>
<td>0.331</td>
<td>0.672</td>
<td>0.662</td>
</tr>
<tr>
<td>#2</td>
<td>Inner</td>
<td>0.322</td>
<td>0.325</td>
<td>0.889</td>
</tr>
<tr>
<td></td>
<td>Outer</td>
<td>0.245</td>
<td>0.166</td>
<td>0.955</td>
</tr>
<tr>
<td>#3</td>
<td>Inner</td>
<td>0.361</td>
<td>0.624</td>
<td>0.693</td>
</tr>
<tr>
<td></td>
<td>Outer</td>
<td>0.445</td>
<td>0.577</td>
<td>0.685</td>
</tr>
<tr>
<td>#4</td>
<td>Inner</td>
<td>0.830</td>
<td>0.413</td>
<td>0.374</td>
</tr>
<tr>
<td></td>
<td>Outer</td>
<td>0.473</td>
<td>0.595</td>
<td>0.649</td>
</tr>
</tbody>
</table>

The higher the value of the sensitivity factor, the more the corresponding variable influences the probability of non-compliance. If the sensitivity factors are studied, it can be seen that the distribution of the maximum lateral acceleration of the cars influences the most the reliability for almost all the curves. In this case only few variables are considered, however, such analysis would be of interest for reliability design of more complicate highway systems.

5.4.3 Design Equation

Navin (1992) presented a design parameter equation, of the same form of the basic ultimate limit state equation of structural engineering, as:

$$\phi P = \gamma \tau \psi \xi \eta P_{D/M}$$

(5.30)
where $P$ is the engineering design parameter; $\phi$ is the highway construction and maintenance quality factor; $\gamma$ is the importance of the route factor, $i$ is the terrain factor; $\psi$ is the driver exposure factor, $\eta$ is the vehicle and driver mix factor, $\xi$ is the environmental factor (Navin: 1992). Based on the same model the design equation for an isolated horizontal curve would be:

$$
\phi R = \gamma \psi \eta \xi (\alpha_y, V_{C/D}, \alpha_y e, \alpha_y \alpha_{vehicle})
$$

(5.31)

For all variables involved in the system there is a certain level of uncertainty. The following factors can be added: $\alpha$ is the design characteristic factors related to $V_{C/D}$ which stands for the actual speed of the car-driver system, $e$ for the superelevation, and $\alpha_{vehicle}$ for the maximum attainable lateral acceleration for a given vehicle.

Figures 5.9, 5.10 and 5.11 are three reliability analyses, based on the radius supplied by the highway and based on the radius demanded by the car-driver system, on four existing curves of Highway 99. These analyses were based on the performance function of Equation 5.28. From this performance function, a simple design equation can be written as:

$$
\phi R_s \geq \frac{\alpha_{vd} V^2}{g(\alpha_y \alpha_{vehicle} + e)}
$$

(5.32)

where: $\phi = \text{performance or construction factor},$  
$R_s = \text{effective radius, a function of the nominal radius } R_H \text{ under design and of the inner lane or outer lane},$
\[ \alpha_{VD} = \text{speed factor}, \]
\[ V_D = \text{desired speed or estimated speed for } R_H, \]
\[ \alpha_{f_y} = \text{road friction factor}, \]
\[ \alpha_{y, \text{vehicle}} = \text{maximum lateral acceleration of a given vehicle for } R_H, \]
\[ e = \text{superelevation of the curve}, \]
\[ g = \text{gravity acceleration (9.81 m/s}^2) \]

\( \Phi, \alpha_{VD} \) and \( \alpha_{f_y} \) are factors which have to be further defined. \( R_S \) is a random variable which represents the variation of path selected among drivers, and therefore, which represents the radii population related to the nominal centre line radius \( R_H \) under design. \( V_D \) is the desired or estimated speed of the driver population in the curve of radius \( R_H \). \( \alpha_{y, \text{vehicle}} \) is the maximum lateral acceleration that the average vehicle can develop in the curve of radius \( R_H \) if driven by an expert. The value of \( \alpha_{y, \text{vehicle}} \) can be associated with passenger cars or with good vehicles. In one case, the analysis would be based on skidding criterion whereas in the other case, the rollover criterion would be considered. \( R_S, V_D \) and \( \alpha_{y, \text{vehicle}} \) are random variables which take into consideration the variation of behavior among vehicles and drivers.

Figures 4.2, 4.4 and 4.9 show the three variables \( R_S, V_D \) and \( \alpha_{y, \text{vehicle}} \) as a function of the design radius centre line \( R_H \). The reliability-based design for horizontal curves could, for example, be used in an iterative method. For a specific type of highway, see Figure 5.10, the designer select a first centre line radius. From the other characteristic of the road which influence the path and the speed of the C/D system (width of the lanes, Average Annual Daily Traffic, grades, etc...) the values \( R_H, V_D \) and \( \alpha_{y, \text{vehicle}} \) are defined. In a first check, the designer computes the reliability
index. If $\beta$ does not satisfy the standard, the reliability level is increased by changing the superelevation or applying a performance factor $\phi$. Another trial will check if the new values $R_s$, $V_D$ and $\alpha_y^{\text{vehicle}}$ provide an acceptable $\beta$. The logical sequence of the suggested and simplified design framework is presented in Figure 5.13.

5.5 Conclusion

There are similarities only in appearance between Equation 5.20 and Equation 5.28. In fact, in the last design equation, the highway engineer has a representation of the performance of the driver population through the value of $V_D$ and the value of $R_s$, a representation of the performance of any vehicle with the value of $\alpha_y^{\text{vehicle}}$, a representation of the performance of the road pavement through the value of $\alpha_{f_p}$, and a representation of the performance of the radius $R_H$ through the value of $R_s$. In the simplified curve formula equation, the sole parameter the designer can really control is the design speed which may not correspond to the actual speed of the car-driver system in a given highway horizontal curve.

Not only can reliability analysis be used for designing horizontal curves based on a targeted $\beta$ value, but it could also become a powerful tool in making decisions in the presence of uncertainty, as approached in Section 5.4.1.
CHAPTER 5: Reliability analysis

Figure 5.13. Logical flow diagram for highway design using reliability analysis.
CHAPTER 6

Conclusion

The actual North American highway geometric standards give little attention to the performances of the driver and vehicle in the design of horizontal curves. Highway designers, when using the current procedures to design a horizontal curve, do not necessarily need to know what would be the actual demand, in terms of performance, on any particular curve. A good geometric design method should incorporate a close description of tire-car-driver systems' behavior for each particular situation on the road. Through reliability calculations, highway designers are provided with a powerful tool, which can assist them in understanding the behavior of the road-tire-vehicle-driver system.

In order to conduct reliability-based design calculations, basic information on each random variable, such as the probability distribution and estimates of the mean and standard deviation, is needed. Each variable involved in the system (the performance function) belongs either to the demand, to the supply or both. A great number of studies, on driver behaviors at horizontal curves, provides good information on what drivers do and on the influence of the geometric characteristics of the road. From these studies, statistical information on drivers' speeds in a horizontal curve can be derived. However, very few information on the thresholds speed and lateral acceleration for a vehicle-driver population on a particular road section is available.
CHAPTER 6: Conclusion

A driving experiment on a airport runway (PTEC), and observations of four existing curves of Highway 99 were carried out in order to collect statistical information on speed, lateral acceleration and path radius. The data were collected from regular drivers who drove at 'comfortable' and 'high speeds' scenarios. The maximum speed obtained in the curves by two expert drivers was associated with the threshold performance of the tire-car-pavement system. The level of comfort information provided by the passengers, during the experiment, have helped to define the level of comfort adopted by the drivers going through the four Highway 99's curves.

On highways, people drive at a comfortable speed. To fulfill this level of comfort, unconsciously drivers "trade-off" speed and lateral acceleration. In small radius curves, drivers tend to adopt a speed that corresponds to an "acceleration tolerance" which ranges from 0.35 g to 0.40 g. On a curve with large radius, drivers tend to adopt an "aspiration velocity" which is a function of the overall alignment and the characteristics of the highway. In addition, from the PTEC experiments its was found that drivers do not reduced their speed on wet pavement (water depth 0.5 to 1 mm) and that the difference of speed between gender is not statistically significant. However, the last information may not be valid since the number of subjects was small.

From the PTEC data, RELAN (RELiability ANalysis software) did not show a decrease of probability of skidding between all radii for scenario 'comfortable speed', although the margin of safety against skidding is increasing with increase of radius in Figure 3.19. Even if the probabilities are very small, this shows the impact of the dispersion of the individual values around the mean on the reliability calculation. The probabilities of skidding at comfortable speed
were 0.26% for the curve with 16 m radius, and 0.06% for the curve with radius of 100 m. For ‘high speed’ scenario the risk were 9.7% and 2.5% for the smaller and bigger radii respectively.

In the performance function of a horizontal curve, the basic variables: driver speed, driver path radius, vehicle maximum lateral acceleration, and superelevation where represented either in the radius supplied $R_S$ or the radius demanded $R_D$. From Highway 99’s data, the probabilities of non-compliance for the four observed curves were calculated. The results show an increase in the probability of non-compliance with a decrease of the designed radius centre line. This particular relationship between design radius and probability of non-compliance is thought to be unique to Highway 99. Each highway type is thought to have its own non-compliance or reliability function since drivers have a specific aspiration velocity for each road’s environment.

A reliability index $\beta$ could be assigned to each type of highway in the same way that McLean (1981) assigned a desire speed to different highway environments. From these standardized $\beta$ values, transportation engineers could use an iterative method, as presented in figure 5.13, to design a ‘reliable’ horizontal curve.

With a reliability-based design method, the designer has a representation of the main variables involved in the process of driving in a horizontal curve. The engineer can adjust the design to fulfill a desirable probability of non-compliance in a $\beta$ range allowed by the standards. With, this $\beta$ value the transportation engineering is provided with a highway design method which better responds to the actual driving demand on the road.
CHAPTER 7

Recommendations

Reliability analysis for transportation engineering has been shown to be possible by Navin et al. (1993) and Navin (1992). This thesis demonstrated how to use this concept for the design of highway horizontal curves. However, it did not provide a calibrate reliability model which could be presented in a generalized formulation for design purpose, as proposed by Navin et al. (1993) for stopping sight distance. To provide highway horizontal curve design procedure with a calibrated reliability model the following has to be considered.

1. **Detailing a design procedure for highway horizontal curves using reliability concept:**
   This implies describing the 'Limit State Design' equation for horizontal curves with all the variables and parameters acting in the equation. Its formulation could simply be as equation 5.32. Each element that composes the design equation: road friction factor, speed factor, performance factor, and the variables such as the speed, path radius and vehicle cornering capacity has to be defined. For example, each quality of road friction should be represented by a friction factor.

2. **Designing a road classification:** Such classification could take into consideration the geometric and environmental elements which influence the driving process on the road: rate of curvature, number of lanes, width of the lanes, type and volume of traffic, and scenery. For
each highway sub-group, the statistical information, such as the probabilistic distribution, the mean, the variance and the correlation coefficients, are needed for each variable involved in the 'Limit State Design' equation. The statistical information for the cornering capacity for car and for truck are the same for all type of highways. However, the statistical characteristics of the other variables are expected to differ from one class of highway to another since the behavior of the vehicle/driver system differ from one environment to another.


Bibliography


Bibliography


Bibliography


APPENDIX A1

Latin Square Algorithm

In this appendix, an introduction to the principle of the Latin square design is presented. This short explanation, do not cover all the alternatives and combinations that are possible when using the Latin square model for the analysis of the variance. A complete description is presented in Myers (1972) and in Winer (1971). The Latin square model used for PTEC experimental design is layout. A detail of the calculation is provided.

In the following 3 x 3 arrangement each letter-digit combination occurs just once in each row and just once in each column:

\[
\begin{array}{ccc}
A1 & A2 & A3 \\
A2 & A3 & A1 \\
A3 & A1 & A2 \\
\end{array}
\]

The above Latin square is referred to as a standard Latin square. There are 3 possible standard squares and from each of them 3!2!-1 nonstandard of 3 x 3 Latin square are obtainable by permuting rows and columns. To select a random square from the population of 3 x 3 squares, one can first permute the second row with the third row and, then, permute the first column with the third column of the standard square:
Appendix 1: *Latin Square*

This last square represents a random rearrangement of the rows and columns of the original 3 x 3 standard form. The Latin square model originated from agricultural experimentation for which a x a Latin square defines a^2 experimental units. Usually, the Latin square design represents a single-factor experiment, A - for which 1,2 and 3 refer to three levels of fertilizer type or fertilizer concentration -, randomly sprayed on three fields' crop (columns) at three locations on each of the field (rows). The variability analysis of the level A is then simplified by knowing the variability among fields and the variability within a same fields. In this design it is assumed that treatment effects do not interact with the row and column effects. One of the advantages over other designs, is the potential to investigate several variables with less expenditure of time and subjects than a complete factorial design would involve. Only a^2 of the possible a^3 treatment combinations are investigated by the Latin square.

The Latin square used at PTEC followed the modified Latin square design model presented by Myers (1972).

<table>
<thead>
<tr>
<th></th>
<th>C_1</th>
<th>C_2</th>
<th>C_3</th>
<th>C_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>B_1</td>
<td>A1D1</td>
<td>A1D2</td>
<td>A2D1</td>
<td>A2D2</td>
</tr>
<tr>
<td>B_2</td>
<td>A1D2</td>
<td>A2D1</td>
<td>A2D2</td>
<td>A1D1</td>
</tr>
<tr>
<td>B_3</td>
<td>A2D1</td>
<td>A2D2</td>
<td>A1D1</td>
<td>A1D2</td>
</tr>
<tr>
<td>B_4</td>
<td>A2D2</td>
<td>A1D1</td>
<td>A1D2</td>
<td>A2D1</td>
</tr>
</tbody>
</table>
Appendix 1: *Latin Square*

The total number of 4 x 4 Latin squares from one standard square is $4!3!-1$ or 143. Experiment at PTEC was designed from one random selection of the above standard form.

We have a 4 x 4 Latin square because each of the four treatment combinations appears exactly once in each level C factor and at each level B factor. Such design is incomplete; the complete design would require 64 cells and each B level would require enough drivers and passengers to provide 16 combinations of A, C, and D. There are $ad$ levels of B, C, and AD with $n$ (4) subjects within each of the $a^2d^2$ cells. If the four variables and the variable combination of the Latin square do not interact with each other, the appropriate model is

$$Y_{ijkmp} = \mu + \alpha_j + \beta_k + \gamma_m + \delta_p + (\alpha\delta)_{jp} + \varepsilon_{ijkmp}$$ (1)

where:

- $Y_{ijkmp}$ = Score of the i Th. subject in the jkmp Th. treatment level.
- $\mu$ = The expected value over all ABCD treatment populations.
- $\alpha_j$ = Main effect of treatment Aj.
- $\beta_k$ = Main effect of treatment Bk.
- $\gamma_m$ = Main effect of treatment Cm.
- $\delta_p$ = Main effect of treatment Dp.
- $(\alpha\delta)_{jp}$ = The interaction effect of treatment Aj and Dp.
- $\varepsilon_{ijkmp}$ = The error component.
The first element needed for the analysis of the variance is the correction term:

\[ C = \frac{\text{sum of all scores in the data matrix}}{\text{total number of scores}} \]

Then

\[ C = \frac{(2,588.56)^2}{64} = 104,697.54 \]

The sum of the squares for the two level B (Group) and C (Radius) are:

\[ SS_B = \frac{(644.72)^2 + A + (673.71)^2}{16} - C = 110.56 \]
Appendix 1: Latin Square

\[
SS_c = \frac{(449.56)^2 + \Lambda + (847.53)^2}{16} - C = 6110.17
\]

Table A1.2. Observed data* and subtotal.

<table>
<thead>
<tr>
<th>Level B</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
<th>Sum of rows</th>
</tr>
</thead>
<tbody>
<tr>
<td>B Sum</td>
<td>127.1</td>
<td>122.3</td>
<td>160.09</td>
<td>235.23</td>
<td>644.72</td>
</tr>
<tr>
<td>B1</td>
<td>30.78</td>
<td>25.84</td>
<td>41.39</td>
<td>54.39</td>
<td>112.35</td>
</tr>
<tr>
<td></td>
<td>29.78</td>
<td>28.91</td>
<td>36.06</td>
<td>64.41</td>
<td>169.35</td>
</tr>
<tr>
<td></td>
<td>33.71</td>
<td>35.47</td>
<td>42.14</td>
<td>61.41</td>
<td>172.73</td>
</tr>
<tr>
<td></td>
<td>32.83</td>
<td>32.08</td>
<td>40.5</td>
<td>55.02</td>
<td>160.45</td>
</tr>
<tr>
<td>B1 Stdev</td>
<td>1.81</td>
<td>4.14</td>
<td>2.73</td>
<td>4.90</td>
<td>12.58</td>
</tr>
<tr>
<td>B1 Mean</td>
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<td>30.58</td>
<td>40.02</td>
<td>56.81</td>
<td>158.20</td>
</tr>
<tr>
<td>B2 Sum</td>
<td>96.21</td>
<td>151.06</td>
<td>212.34</td>
<td>195.02</td>
<td>654.63</td>
</tr>
<tr>
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<td>34.73</td>
<td>54.66</td>
<td>42.77</td>
<td>155.30</td>
</tr>
<tr>
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<td>37.52</td>
<td>53.82</td>
<td>46.21</td>
<td>168.44</td>
</tr>
<tr>
<td></td>
<td>24.92</td>
<td>38.76</td>
<td>51.83</td>
<td>53.29</td>
<td>178.88</td>
</tr>
<tr>
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<td>26.02</td>
<td>40.05</td>
<td>52.03</td>
<td>52.75</td>
<td>180.84</td>
</tr>
<tr>
<td>B2 Stdev</td>
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<td>2.27</td>
<td>1.38</td>
<td>5.13</td>
<td>9.05</td>
</tr>
<tr>
<td>B2 Mean</td>
<td>24.05</td>
<td>37.77</td>
<td>53.09</td>
<td>48.76</td>
<td>157.67</td>
</tr>
<tr>
<td>B3 Sum</td>
<td>122.79</td>
<td>155.98</td>
<td>159.17</td>
<td>177.56</td>
<td>615.50</td>
</tr>
<tr>
<td>B3</td>
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<td>39.77</td>
<td>32.8</td>
<td>42</td>
<td>105.35</td>
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<td>39.4</td>
<td>34.52</td>
<td>46</td>
<td>118.17</td>
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<td>31.08</td>
<td>36.16</td>
<td>46.42</td>
<td>45.52</td>
<td>168.22</td>
</tr>
<tr>
<td></td>
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<td>40.65</td>
<td>45.43</td>
<td>44.04</td>
<td>160.96</td>
</tr>
<tr>
<td>B3 Stdev</td>
<td>0.31</td>
<td>1.96</td>
<td>7.13</td>
<td>1.80</td>
<td>10.89</td>
</tr>
<tr>
<td>B3 Mean</td>
<td>30.7</td>
<td>39.00</td>
<td>39.79</td>
<td>44.39</td>
<td>123.98</td>
</tr>
<tr>
<td>B4 Sum</td>
<td>103.46</td>
<td>120.05</td>
<td>210.48</td>
<td>239.72</td>
<td>673.71</td>
</tr>
<tr>
<td>B4</td>
<td>23.67</td>
<td>26.63</td>
<td>48.91</td>
<td>55</td>
<td>114.11</td>
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<td>26.86</td>
<td>51.88</td>
<td>65</td>
<td>148.98</td>
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<tr>
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<td>29.34</td>
<td>33.81</td>
<td>58.56</td>
<td>66.6</td>
<td>188.32</td>
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<tr>
<td></td>
<td>27.17</td>
<td>32.75</td>
<td>51.13</td>
<td>53.12</td>
<td>164.76</td>
</tr>
<tr>
<td>B4 Stdev</td>
<td>2.90</td>
<td>3.80</td>
<td>4.16</td>
<td>6.85</td>
<td>13.63</td>
</tr>
<tr>
<td>B4 Mean</td>
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</tr>
<tr>
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<td>449.56</td>
<td>549.39</td>
<td>742.08</td>
<td>847.53</td>
<td>2588.56</td>
</tr>
<tr>
<td>Level AD</td>
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<td>719.56</td>
<td>553.91</td>
<td>735.14</td>
<td>2588.56</td>
</tr>
<tr>
<td>Subtotal</td>
<td>V1W</td>
<td>V2W</td>
<td>V1D</td>
<td>V2D</td>
<td></td>
</tr>
</tbody>
</table>

* mean speed at the middle of the curve between marks B and D
Appendix 1: Latin Square

For the \( ad \) treatment combinations:

\[
SS_{T.C.} = \frac{579.95^2 + A^2 + 735.14^2}{16} - C = 1637.19
\]

For \( A \) (Scenario) and \( D \) (Pavement):

\[
SS_A = \frac{(719.56 + 735.14 - 579.95 - 553.910)^2}{64} = 1,608.41
\]

\[
SS_D = \frac{(579.95 + 719.56 - 553.91 - 735.14)^2}{64} = 1.71
\]

\[
SS_{AD} = \frac{(553.91 + 719.56 - 579.95 - 735.14)^2}{64} = 27.07
\]

Next we obtain sum of the squares Between cells residual and Total:

\[
SS_{B.Cellsres} = \frac{127.10^2 + 122.30^2 + A^2 + 239.72^2}{4} - C - SS_B - SS_C - SS_{T.C.} = 50.41
\]

\[
SS_{TOT} = 30.78^2 + 29.78^2 + A^2 + 66.60^2 + 53.12^2 - C = 8,613.97
\]

Now the sum of the squares for the error term is:

\[
SS_{SS/cells} = SS_{TOT} - SS_B - SS_C - SS_{T.C.} - SS_{B.cellsres} = 705.64
\]

Table A1.3. Analyze of the variance of the Latin Square design using F. distribution.

<table>
<thead>
<tr>
<th>SV</th>
<th>df</th>
<th>df</th>
<th>SS</th>
<th>EMS</th>
<th>F</th>
<th>F.95</th>
<th>F.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>a^2d^2n-1</td>
<td>63</td>
<td>8613.97</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>ad - 1</td>
<td>3</td>
<td>110.56</td>
<td>36.85</td>
<td>2.51</td>
<td>2.80</td>
<td>4.22</td>
</tr>
<tr>
<td>C</td>
<td>ad - 1</td>
<td>3</td>
<td>6110.17</td>
<td>2036.72</td>
<td>138.54</td>
<td>2.80</td>
<td>4.22</td>
</tr>
<tr>
<td>Treat. Comb</td>
<td>ad - 1</td>
<td>3</td>
<td>1637.19</td>
<td>545.73</td>
<td>37.12</td>
<td>2.80</td>
<td>4.22</td>
</tr>
<tr>
<td>A</td>
<td>a - 1</td>
<td>1</td>
<td>1608.41</td>
<td>1608.41</td>
<td>109.41</td>
<td>4.04</td>
<td>7.2</td>
</tr>
<tr>
<td>D</td>
<td>d - 1</td>
<td>1</td>
<td>1.71</td>
<td>1.71</td>
<td>0.12</td>
<td>4.04</td>
<td>7.2</td>
</tr>
<tr>
<td>AD</td>
<td>(a - 1)(d - 1)</td>
<td>1</td>
<td>27.07</td>
<td>27.07</td>
<td>1.84</td>
<td>4.04</td>
<td>7.2</td>
</tr>
<tr>
<td>B.cells res.</td>
<td>(ad - 1)(ad - 2)</td>
<td>6</td>
<td>50.41</td>
<td>8.40</td>
<td>0.57</td>
<td>2.30</td>
<td>3.21</td>
</tr>
<tr>
<td>S/cells</td>
<td>a^2d^2n-1</td>
<td>48</td>
<td>705.64</td>
<td>14.70</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX 2

Chart of the Power Function for Analysis of Variance Tests Derived From the Non-Central F-Distribution. (Myer: 1972: 446)
APPENDIX 3

Passengers Level of Comfort
0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80
Experienced acceleration (g's)

passenger 3 first day

Scale of comfort

Left turn
Right turn

Very Difficult  Uncomfortable  Comfortable  Easy

0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70
Experienced acceleration (g's)

Passenger 4 first day

154
Experienced acceleration (g's)

Passenger 1 second day

Passenger 2 second day
Passenger 3 second day

Experienced acceleration (g's)

Scale of comfort

Passenger 4 second day

Experienced acceleration (g's)