LIQUEFACTION OF SANDS UNDER MULTI-AXIAL LOADING

by

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ABSTRACT

A fundamental study of the undrained behaviour of sands under multi-axial loading is presented. The study was performed by using the hollow cylinder torsional (HCT) device. The HCT is the only device that permits a soil specimen to be subjected to multi-axial loading with controlled variations in the magnitudes of the three principal stresses and the direction of the major principal stress with the vertical deposition direction.

The main objective of the study was to assess the effects of principal stress magnitude, directions and their rotation on sand liquefaction. This is achieved by a systematic study of static and cyclic undrained behaviour of reconstituted loose sand. Shear loading is carried out under strain control. Only such loading permits the needed capture of post peak strain softening characteristics of loose sands. Undesirable runaway strains are inevitable in stress controlled loading modes.

In addition to the investigations in the hollow cylinder torsional device, sand behaviour in simple shear as well as under the triaxial conditions was also assessed as reference for comparisons with that under multi-axial stresses. The investigations were carried out using two sands - Fraser River sand and Syncrude sand. Sand specimens were reconstituted by water pluviation, which is considered to duplicate the fabric of in-situ fluvial and hydraulic fill deposits.

Independence of the effective stress path and stress-strain characteristics from the total stress path under fixed principal stress directions and constant value of intermediate principal stress parameter is illustrated. The undrained response of loose sand is highly dependent on the loading direction, implying inherent anisotropy. The friction angle mobilized at phase transformation or steady state is a unique material property, independent of the mode of loading,
static or cyclic, direction of principal stresses, intermediate principal stress level, consolidation history and the stress and void ratio state prior to undrained shear. There is no unique relationship between steady state or phase transformation strength and void ratio that is independent of the stress path, implying that a unique steady state line does not exist for a sand.

The influence of intermediate principal stress, on undrained response is small when the intermediate principal stress parameter, that reflects value of this stress relative to the major and the minor values, is less than about 0.5. At constant values of other parameters increasing confining stress and decreasing relative density under multi-axial loading promote a higher degree of contractive response.

The history of principal stress directions during principal stress rotation does not seem to have any appreciable effect on the peak and steady state or phase transformation strength. These strengths are apparently controlled by the peak value of major principal stress inclination experienced during shearing with respect to vertical direction.

Principal stresses undergo continuous rotation from 0 to about ±45° in simple shear deformation. A simultaneous change in intermediate principal stress occurs as the major principal stress rotates. The maximum shear stress and maximum shear strain in conventional simple shear deformation approximately equals the horizontal shear stress and shear strain respectively.

For a given initial stress and void ratio state, the number of cycles to liquefaction is smaller under cyclic triaxial than under similar 90° jump rotation that do not invoke the weakest triaxial extension loading mode during shear. For a given direction of principal stresses, if the sand is contractive under static loading, it would also be contractive under cyclic loading,
provided that the cyclic deviator stress amplitude is higher than the steady state or phase transformation strength in static loading.
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<tr>
<td>DPT</td>
<td>Differential pressure transducer</td>
</tr>
<tr>
<td>DVPC</td>
<td>Digital volume pressure controller</td>
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<tr>
<td>$D_r$</td>
<td>Relative density</td>
</tr>
<tr>
<td>$D_r_e$</td>
<td>Relative density at the end of consolidation</td>
</tr>
<tr>
<td>$e_c$</td>
<td>Void ratio at the end of consolidation</td>
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<tr>
<td>FRS</td>
<td>Fraser River sand</td>
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<tr>
<td>$F_z$</td>
<td>Vertical force</td>
</tr>
<tr>
<td>H</td>
<td>Specimen height</td>
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<tr>
<td>HCT</td>
<td>Hollow cylinder torsional shear device</td>
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<tr>
<td>$I_B$</td>
<td>Brittleness index</td>
</tr>
<tr>
<td>LVDT</td>
<td>Linear variable differential transformer</td>
</tr>
<tr>
<td>$P_e$</td>
<td>External cell pressure</td>
</tr>
<tr>
<td>$P_i$</td>
<td>Internal cell pressure</td>
</tr>
<tr>
<td>PT</td>
<td>Phase transformation state</td>
</tr>
<tr>
<td>$R_e$</td>
<td>External specimen radius</td>
</tr>
<tr>
<td>$R_i$</td>
<td>Internal specimen radius</td>
</tr>
<tr>
<td>SS</td>
<td>Steady state</td>
</tr>
<tr>
<td>SYN</td>
<td>Syncrude sand</td>
</tr>
<tr>
<td>$T_h$</td>
<td>Torque in the horizontal plane</td>
</tr>
<tr>
<td>$\gamma \theta$</td>
<td>Horizontal shear strain</td>
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</tbody>
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\( \gamma_{\text{max}} \) Maximum shear strain
\( \Delta \) Incremental change
\( \Delta \theta \) Angular displacement of specimen
\( \varepsilon_1 \) Major principal strain
\( \varepsilon_2 \) Intermediate principal strain
\( \varepsilon_3 \) Minor principal strain
\( \varepsilon_a \) Axial strain
\( \varepsilon_x \) Axial strain or vertical strain
\( \varepsilon_r \) Radial strain
\( \varepsilon_{\text{vol}} \) Volumetric strain
\( \varepsilon_\theta \) Tangential strain
\( \sigma, \sigma' \) Total and effective normal stresses
\( \sigma_m, \sigma'_m \) Total and effective mean normal stress
\( \sigma'_{\text{mc}} \) Effective mean normal stress at the end of consolidation = \((\sigma'_1+\sigma'_2+\sigma'_3)/3\)
\( \sigma_1, \sigma_2, \sigma_3 \) Total major, intermediate and minor principal stresses
\( \sigma'_1, \sigma'_2, \sigma'_3 \) Effective major, intermediate and minor principal stresses
\( b \) Intermediate principal stress parameters \((\sigma_2-\sigma_3)/(\sigma_1-\sigma_3)\)
\( \sigma_{\text{dcy}} \) Cyclic deviator stress
\( \sigma_c \) Cell pressure
\( \sigma_v \) Vertical stress
\( \sigma_r \) Total radial stress
<table>
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<tr>
<td>$\sigma_0$</td>
<td>Total tangential stress</td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>Deviator stress ($\sigma_1-\sigma_3$)</td>
</tr>
<tr>
<td>$\tau_{\theta}$</td>
<td>Horizontal shear stress</td>
</tr>
<tr>
<td>$\alpha_\sigma$</td>
<td>Major principal stress direction with respect to vertical</td>
</tr>
<tr>
<td>$\alpha_{\Delta\sigma}$</td>
<td>Major principal stress increment direction with respect to vertical</td>
</tr>
<tr>
<td>$\alpha_{\Delta e}$</td>
<td>Major principal strain increment direction with respect to vertical</td>
</tr>
<tr>
<td>$\phi_{\text{CSR}}$</td>
<td>Friction angle mobilised at peak</td>
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<td>$\phi_{\text{cv}}$</td>
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<td>$\phi_f$</td>
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Rapid loading due to an earthquake or static loads may induce large pore pressures in loose, saturated sandy deposits. The associated decrease in effective stress could cause sand to liquefy. The term liquefaction encompasses all phenomena involving excessive deformation of saturated cohesionless soils (National, 1985). Under static loading, liquefaction is associated with the sand responding in a strain softening manner. The field consequences of this type of sand behaviour could be trigger of a flow slide. Liquefaction under cyclic loading can be either due to strain softening, in a manner similar to that in static loading, or on account of cyclic mobility or a combination of the two (Vaid and Chern, 1985). The term cyclic mobility, in triaxial tests, refers to excursions of the stresses in sand through transient states of zero effective stress (Castro, 1969; Seed, 1979; Vaid and Chern, 1985). Its consequences are unacceptable deformations in the sand structure.

The nature of stresses acting on soil elements, both during construction and performance, involve principal stress rotation. In addition, principal stress direction may vary from element to element in the soil mass. Embankment loading and cutting of slopes are two examples where gradual rotation of principal stresses occurs. Figure 1.1 shows two cases where the direction of major principal stress varies from element to element along the potential failure surface. Repetitive loading due to an earthquake can result in continuous cyclic rotation of principal stresses. Cyclic rotation of principal stresses also occurs in soil elements in the ocean floor due to wave loading, and under pavements due to traffic loading.
(a). Limiting equilibrium stress states beneath an embankment

(b). Limiting equilibrium stress states beneath a gravity platform

Figure 1.1. Examples of principal stress direction variation along potential failure surfaces
Most natural sands have anisotropic strength and deformation characteristics (Saada, 1988). This anisotropy is of two types; one is inherent anisotropy, which occurs due to preferred particle arrangement during gravitational sedimentation through either air or water. From such sand deposits if identical specimens are sheared with identical principal stresses, but with directions inclined at different angles to the vertical, their response may be different. The second type is the stress induced anisotropy. This is caused by straining leading to a continuous evolution of particle arrangement during shear loading.

Despite the large number of studies carried out on the undrained response of saturated sands using triaxial or simple shear devices, only a few have dealt specifically with the anisotropic nature of sands. These few studies have established that the response of sands is stress path dependent. For example, the behaviour at identical initial states under triaxial compression loading is dramatically different from that under triaxial extension loading (Saada and Ou, 1973; Hanazawa, 1980; Miura and Toki, 1982; Kuerbis, 1989; Vaid et al., 1990a; Pillai and Stewart, 1994; Vaid and Thomas, 1995). Dilative (strain hardening) response may occur even for the loosest deposition state in compression, while in extension it could be contractive (strain softening). In triaxial compression loading the major principal stress $\sigma_1$ acts in the vertical direction (or in most cases in the deposition direction). In extension test however, the stress in vertical direction is the minor principal stress, while the horizontal stress becomes the major principal stress. Therefore, a systematic variation in the undrained response could be expected, if the direction of major principal stress varies, from the direction of deposition to the direction of bedding plane.
Steady state concepts used to assess potential for flow failures (Castro, 1969, 1987) assume that the steady state strength of the soil is only a function of its void ratio, regardless of the stress path or initial condition. This may not be valid for sands which are inherently anisotropic. Although the stress path dependence of undrained behaviour and, in particular the shear strength of clays has been routinely considered in practice (Bjerrum, 1972; Ladd and Foott, 1977; Eide and Anderson, 1984; Lacasse et al., 1988), it is surprising that similar concerns have not been expressed for sands.

Commonly, the steady state line for a sand is determined from triaxial compression tests on samples reconstituted by the moist tamping technique. Moist tamped sands are invariably contractive in compression over a large range of void ratios (Castro et al., 1985). Due to capillary tensions, there is a resistance to particle packing during moist tamping of sands. This results in the development of a fabric which has been described as metastable honeycomb (Casagrande, 1976). Loose moist tamped specimens of fine grained soils have been found to experience very large strains during the saturation process (Sladen et al., 1985a). This collapsing characteristic on mere removal of capillary tensions suggests a metastable fabric that is likely to promote contractive deformation during shear. Tamping may also inhibit the development of a strong anisotropic fabric, thus giving rise to a material that responds more isotropically to hydrostatic stress increments (Saada, 1988).

The research undertaken herein addresses the question of undrained anisotropy in sand. The primary focus is to investigate the effects of principal stress directions and their rotation on sand liquefaction. This objective is achieved by a study of static and cyclic undrained behaviour of loose sand under multi-axial stresses. Shear loading is carried out using the strain controlled
loading mode. Only the strain controlled loading permits the needed capture of post peak strain
softening characteristics of loose sands by overcoming undesirable runaway strains, inevitable in
stress controlled loading modes.

Undrained behaviour of sands was investigated utilizing a Hollow Cylinder Torsional
(HCT) device. Behaviour in simple shear as well as under the triaxial conditions was also
assessed as the reference for comparisons with that under multi-axial stresses. This investigation
was carried out using Fraser River sand. The specimens were reconstituted by water pluviation.
Water pluviation, as opposed to the moist tamping technique, is considered to simulate the fabric
of natural and artificial fluvial and hydraulic fill sands (Oda et al., 1978; Saada, 1988; Vaid et al.,
1990a), and hence provides a convenient means for a systematic study of their undrained
behaviour. In addition to the comprehensive investigation of the response of Fraser River sand,
undrained static behaviour of another sand, (Syncrude sand - CANLEX) was also investigated,
though on a limited scale.

In chapter two, a review is presented on the current state of understanding regarding the
undrained response of sands. Previous research in this area, and the stress conditions associated
with the testing devices used in those studies, are reviewed. This is intended to provide a
framework which forms the basis for the investigation presented in this thesis. In chapter three,
the University of British Columbia (UBC) HCT apparatus together with the data acquisition and
control system utilised in this study are described. Chapter four presents a description of testing
procedures and an outline of the testing program designed for this investigation. Chapter five
presents test results and their interpretations followed by conclusions in chapter six.
CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

In the conventional approach, the sand response at a given void ratio is considered to depend only on the levels of initial shear stress (or deviator stress $\sigma_d = \sigma_1 - \sigma_3$) and confining pressure $\sigma'_3$ (or effective mean normal stress $\sigma'_m = (\sigma'_1 + \sigma'_2 + \sigma'_3)/3$). Specifically, the effects of intermediate principal stress and of principal stress directions are disregarded. This approach originated not from a lack of recognition of the anisotropic nature of sand deposits, and of the possible significance of $\sigma_2$ (usually accounted for by means of the normalised parameter $b = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$) on sand behaviour, but stemmed from the limitations of commonly used laboratory testing devices (i.e., the conventional triaxial device) to simulate more realistic field stress paths.

The hollow cylinder torsional (HCT) device is the only type of equipment that enables control of simple as well as generalized stress paths. Specifically, it has the unique capability of allowing independent control of the magnitudes of the three principal stresses together with the direction $\alpha_\sigma$ of $\sigma_1$ with respect to the vertical (deposition direction). HCT device is thus suitable for a systematic investigation of the isolated effects of each of the four stress parameter ($\sigma'_m$, $b$, $\sigma_d$ and $\alpha_\sigma$) on the deformation response of sands. The directional shear cell (Arthur, 1981; Wong and Arthur, 1986) can also simulate generalised stress paths, but they are limited to plane strain conditions only. Hence no independent control can be exercised on $\sigma_m$ and $b$ levels.

In the following sections, a brief overview of three laboratory testing devices (triaxial, simple shear and HCT) is presented. This is intended to highlight the different regions of stress
space that can be investigated with each of these devices. In addition, a discussion of the non-uniformities of stress and strain, which are inherently associated with all soil testing devices, is given, and the means adopted to minimize them are discussed. Advances in fundamental experimental soil mechanics research have been achieved only by accepting, but minimizing these non-uniformities. The review clearly brings out the versatility of the HCT device over other devices in conducting fundamental research on soil behaviour under generalized stress paths frequently encountered in practical problems.

In the second part of this chapter, previous investigations into the undrained response of sand, under both static as well as cyclic loading are reviewed. Specifically, findings related to the nature of inherent anisotropy are examined. Findings on the effects of principal stress rotation as well as the magnitude of intermediate principal stress on sand response are also summarized.

2.2 Laboratory Shear Testing Devices

In this section triaxial, simple shear and hollow cylinder torsion devices are described. Their merits and limitations in characterizing sand behaviour are also discussed.

2.2.1 Triaxial Device

In the triaxial device only normal stresses can be applied to the specimen boundaries. As a consequence, principal stresses have their directions fixed with respect to the specimen axis, and only 90° jump rotations of principal stresses can be simulated. The most common example is the standard triaxial test on cylindrical specimens. The two independent stress components (σ₁ and σ₃) can be controlled during shear.
The standard triaxial device has been used extensively in the past and it has become a standard piece of equipment in every soil testing laboratory. Occasionally, plane strain and true triaxial device have also been employed in research, but in a very limited number of investigations. Like the standard triaxial device, these devices are also not suitable for controlling either the directions or the rotations of principal stresses.

Interpretation of the standard triaxial test data is normally based on the assumption that the radial stress $\sigma_r$ equals the tangential stress $\sigma_\theta$, and the radial strain $\varepsilon_r$ equals the tangential strain $\varepsilon_\theta$. However, experimental observations (Casbarian and Jamal, 1963; Frydman et al., 1971) have indicated that this assumption may be inaccurate at large strain levels due to the end restraint effects. This aspect has been discussed in detail by Saada and Townsend (1981).

Recognition of the fact that many geotechnical engineering problems can be better approximated by a plane strain deformation condition has led to the development of plane strain testing techniques (Cornforth, 1964; Campanella and Vaid, 1973). Rectangular prismatic specimens are normally used and a condition of zero longitudinal strain is imposed by a pair of fixed rigid plates. Consequently, no control on the magnitude of $\sigma_m$ or $\sigma_2$ can be exercised. Minimization of frictional forces at the rigid boundary enforcing the zero strain component must be attempted in order to keep $\sigma_2$ in the longitudinal direction.

Improvement on the stress path testing capabilities of sands can be further achieved with true triaxial devices, wherein the three principal stress magnitudes can be independently controlled. In particular, the influence of the parameter $b$ on the stress-strain characteristics can be properly assessed, but neither arbitrary inclination of the principal stresses nor their rotation with respect to the specimen axes is possible.
Several true triaxial devices have been described in the literature. Depending on the nature of the loading on the specimen, three basic types can be pointed out;

1. flexible boundaries, where ideally uniform principal stresses are applied to all six prismatic specimen faces through pressurized bags or membranes (Ko and Scott, 1967; Arthur and Menzies, 1968),

2. rigid boundaries, where uniform strains are imposed by rigid boundaries on all faces (Pearce, 1971; Hosseini and Cousens, 1988), and

3. mixed boundaries, where a combination of rigid plates and flexible membranes are used to apply boundary strains or stresses (Green, 1971; Lade 1978)

A simpler alternative for imposing multi-axial stress paths with fixed principal stress direction could also be achieved with hollow cylindrical specimens, by independently controlling external and internal confining pressures together with axial load. When no torque is applied, the vertical stress $\sigma_z$, $\sigma_r$ and $\sigma_\theta$ are the principal stresses. Criticism against this device has often been raised due to the inherent nonuniform distribution of stresses across the specimen's wall, when internal and external pressures are different. More recently, it has been shown that these stress non-uniformities can be greatly minimized by a suitable choice of specimen dimensions and by avoiding exploration of certain stress space for delineating sand behaviour (Hight et al., 1983; Vaid et al., 1990b).

In all triaxial (and plane strain) devices mentioned so far, principal stresses are fixed in the vertical and horizontal directions. Thus, stress paths with $\alpha_\sigma$ different from 0 or 90° can only be simulated by using tilted specimens. This has been frequently reported in investigations on inherent anisotropy (Arthur and Menzies, 1972; Oda et al., 1978; Negussey and Islam, 1994).
Results from tests on tilted specimens of anisotropic material have been the subject of serious criticism (Saada and Townsend, 1981; Saada, 1988). When the specimen's axis of symmetry does not coincide with the loading axis, highly nonuniform distortions may result due to end restraint. Preference should thus be given to test methods where the principal stresses rather than the specimen axis is inclined.

2.2.2 Simple Shear Device

One of the merits of simple shear device is that it enables rotation (though uncontrolled) of principal stresses while the sample is being sheared under plane strain. Since principal stress rotation takes place in most geotechnical engineering problems, simple shear device has been used in the past to simulate such rotations in the laboratory.

Two types of simple shear devices are commonly used:

1. **NGI type**: It uses a circular disk like specimen laterally confined by a reinforced rubber membrane.

2. **Cambridge type**: It uses soil specimens of square cross section confined by six rigid boundaries.

Undrained response of sands has been investigated by means of constant volume simple shear tests (Pickering, 1973; Moussa, 1975; Finn et al., 1977, 1982). In these tests, total volume of the soil specimen is held constant by restricting the vertical deformation together with the apparatus imposed zero lateral deformations. This test is considered to be equivalent to an undrained test. During shear, the vertical stress on the soil specimen will change so as to hold the vertical strain zero at zero pore pressure. It is assumed that the change in vertical stress is equal
to the excess pore pressure, that would develop in a truly undrained test with constant total vertical stress. Experimental evidence in support of this assumption has been provided by Dyvik et al. (1987).

The limitations of simple shear devices have been the subject of extensive discussions (Saada and Townsend, 1981; Lacasse and Vucetic, 1981; Budhu, 1984). Complimentary shear stresses are inherently absent on the lateral boundaries of the specimen. Hence, considerations of equilibrium and boundary conditions require the distribution of both shear and normal stresses on the specimen's surfaces to be necessarily nonuniform. Saada and Townsend (1981) criticize the use of simple shear device for its inability to yield either reliable stress-strain relations or failure parameters. In addition, principal stresses and their directions are neither known nor controlled. For these reasons, the simple shear device is considered not suitable for investigations of the effects of principal stress directions or rotations on soil behaviour. Nevertheless, its ability to closely model some of the field loading situations has made the simple shear device attractive in practical applications.

2.2.3 Hollow Cylinder Torsion (HCT) Device

In the HCT device a thin, long, hollow cylindrical specimen is subjected to a combination of axial and torsional stresses, in addition to fluid pressures on inside and outside cylindrical surfaces. It is a valuable tool to investigate the response of cross anisotropic materials. Applied stresses in the HCT device are symmetric with respect to its axis of symmetry, and unlike the tilted sample in a triaxial device, no unwanted bending moments develop at the ends on loading. The three principal stresses can be controlled independently, and the major and minor principal
stresses can be subjected to either, continuous or jump rotations within the plane of specimen wall.

Like other laboratory testing devices, HCT device also deficient from a theoretical standpoint. When shear stress about the vertical axis or different internal and external cell pressures are applied, stress gradients develop across the wall of the specimen. Radial frictional restraint at the specimen boundaries, if present, also causes stress non-uniformities within the specimen. These are inherent limitations of the HCT device, which cannot be eliminated, but could be minimised by a careful selection of the geometry of the specimen. Thin walled specimens with large mean radius, suffer from lesser stress non-uniformities. It has also been found that when the height to external diameter ratio of the specimen is within the range of 1.8 to 2, non-uniformities caused by end restraint will be minimal (Saada and Townsend, 1981; Lade, 1981; Vaid et al., 1990b).

2.3 Anisotropy in Sand

It has long been recognized that the mechanical behaviour of granular materials is anisotropic. One of the earliest experimental observations of anisotropic response of sand was reported by Kjellman (1936). He noted significant difference in the three principal strains during hydrostatic compression of a cubical specimen.

A convenient distinction between the two types of anisotropy in soils was first suggested by Casagrande and Carrillo (1944). Inherent anisotropy was considered as a physical characteristic, inherently present in the material, before the straining process is initiated. Induced
anisotropy, on the other hand, was defined as due exclusively to the strains associated with the applied stresses.

2.3.1 Inherent Anisotropy

In nearly all natural sand deposits, the gravitational mode of deposition and the shape of individual grains introduce some form of inherently anisotropic fabric. Yet, in practice, soils are frequently modelled as isotropic, in the interest of simplicity. Alternatively, some models consider soil as possessing inherent cross-anisotropy. This is characterized by a vertical axis of symmetry and, consequently, a horizontal plane of isotropy. Cross-anisotropic fabric could result naturally from vertical sedimentation in approximately horizontal layers. Experimental evidence in support of inherent anisotropy in undisturbed sand samples, obtained by the in-situ freezing technique, has been presented by Ladd et al. (1977). Conventional hydrostatic compression in the triaxial cell indicated the radial strains in loose Niigata sand to be about 2.4 times larger than the vertical strains.

During deposition, individual particles assume positions in the sand mass depending on their shape and the mode of deposition. The term fabric has been used to describe the spatial arrangement of solid particles, and the associated voids in a granular mass. Oda (1972) investigated the grain structure of sand by examining thin sections of both natural and reconstituted sand samples. Thin sections were obtained after infiltrating a resin binder into the voids. The results indicated that during deposition under gravity, non-spherical sand grains preferred to rest with their long axis oriented in a nearly horizontal direction. Oda (1972) also found that the majority of particle contact normals were oriented in a direction parallel to the
deposition direction. These observations revealed that the preferred alignment of non-spherical particles and the orientation of particle contacts created an anisotropic fabric. Experimental studies by Lade and Wasif (1988) confirm these findings. Lade and Wasif (1988) have further shown that the deposition of elongated particles produces a cross-anisotropic fabric.

Several investigations have shown that samples of spherical particles deposited under gravity also exhibit anisotropic behaviour (Haruyama, 1981; Oda, 1981; Shibuya and Hight, 1987). Since there is no longitudinal axis in the case of spheres, this anisotropic behaviour has been attributed solely to the preferred orientation of particle contact normals in the deposition direction.

Oda et al. (1978) have shown that sand specimen reconstitution by pluviation under gravity closely simulates the field sedimentation process, and thus yields the fabric of an in-situ soil mass. They also demonstrated the anisotropic behaviour of pluviated Toyoura sand by testing tilted specimens in a plane strain device. The most important conclusion from Oda et al. seems to be that any granular soil, when subjected to vertical gravitational deposition, tends to develop a significantly anisotropic fabric. As a result, inherently anisotropic stress-strain-strength characteristics are to be expected, with the stiffest response occurring for loading with $\alpha_c = 0$.

It is important to note, however, that the specimen reconstitution methods other than air or water deposition may result in a less marked degree of inherent anisotropy as a consequence of a more random orientation of contact normals.
2.3.2 Induced Anisotropy

The mechanism of fabric changes associated with shearing of a granular mass is significantly dependent on the initial (inherent) geometric fabric (Oda, 1976). Induced anisotropy is more conveniently studied starting from an initially isotropic fabric. When subjected to shearing stresses, the spatial arrangement of solid particles and the associated voids of a granular mass progressively change (Oda et al., 1985). As a result, new geometric fabric gradually evolves and the sand becomes increasingly anisotropic. Elongated grains tend to become aligned in a direction perpendicular to the major principal stress direction. Simultaneously, increasing concentration of contact normals along the direction of $\sigma_1$ is produced (Oda et al., 1985). Numerical simulation of tests on plane granular assemblies by Rothenburg and Bathurst (1989) also support the findings of Oda et al. (1985).

2.4 Undrained Response of Sands

Undrained response and liquefaction of sands has been a subject of active research for the past several decades. This is mainly because of the concern that arose from the flow slides and the damage that occur following liquefaction of loose sand deposits during earthquakes. The term *spontaneous liquefaction*, was first used by Terzaghi and Peck (1948) to describe the sudden change in the response of loose sands, to that of a viscous fluid. The 1964 Alaska and Niigata earthquakes, and flow failures led to the recognition of the gravity of the liquefaction phenomena associated with undrained loading of sands. Thus, static as well as cyclic undrained loading behaviour of saturated sands has been investigated extensively, using mostly triaxial and simple shear devices.
2.4.1 Static Loading Response in Triaxial Compression

The typical range of undrained response of sands in triaxial compression loading is shown in Fig. 2.1. The change in response from type 1 to type 3 is associated with increase in relative density at a fixed initial confining stress level. This type of response has been reported by several investigators (Castro, 1969; Chern, 1985; Thomas, 1992). Type 1 and 2 are the characteristic strain softening responses, exhibited by loose sand and sand at high effective confining stress levels. Sand exhibiting such a response has been termed contractive (Castro, 1969). Type 1 response is generally associated with flow failures. The type 3 response is the characteristic dilative type, exhibited by dense sand and possibly by sand at low effective confining stress levels.

Loss of shear resistance after the occurrence of a peak followed by unlimited unidirectional strain at constant stresses is a feature of type 1 response. This type of deformation has been termed steady state deformation and the soil is said to have reached a steady state. The shear stress mobilized at the steady state is termed the steady state strength or residual strength. The basic concept of steady state used for sand is essentially the same as the critical state, defined for clay (Ishihara 1993). Poulos (1981) defined steady state deformation as: the state in which the sand mass is continuously deforming at constant volume, constant normal effective stress, constant shear stress and constant velocity. Castro (1969, 1987) and Seed (1979) called steady state deformation as true liquefaction. In a three dimensional space of void ratio, shear stress and effective confining stress, steady states reached following undrained deformation from several initial void ratio and confining stress states plot as a line called the steady state line (Castro et al., 1982; Chern, 1985).

Type 2 response represents strain softening with limited unidirectional strain and has been called limited liquefaction response (Castro, 1969). In limited liquefaction, strain hardening
Figure 2.1. Characteristic undrained response of sands under monotonic triaxial compression loading.
follows strain softening after a minimum in undrained strength. Strain hardening, that is accompanied by the increase in effective confining stress and decrease in excess pore pressure, limits the amount of strain developed on further straining. Type 3 response represents the strain hardening behaviour, reflecting no loss of shear resistance. Sand showing such a behaviour is called dilative.

**Critical Stress Ratio**

The peak of deviator stress in Fig. 2.1a marks the initiation of strain softening behaviour. On the effective stress path diagram, such as shown in Fig. 2.1b, the effective stress ratio corresponding to the peak deviator stress has been called Critical Stress Ratio (CSR) by Vaid and Chern (1983) and Kuerbis (1989). CSR has been shown to be unique for a sand if its response is strain softening in triaxial compression (Chern, 1985; Chung, 1985; Kuerbis, 1989; Thomas, 1992). Sladen et al. (1985a), on the other hand, show that CSR varies with void ratio and confining stress level in moist tamped specimens. This difference can be attributed to the difference in specimen preparation techniques. Vaid and Chern (1985) and Chung (1985) adopted water pluviation technique which guarantees a uniform specimen while specimen uniformity and fabric are difficult to control by the moist tamping method.

Although unique in triaxial compression, CSR has been shown to be dependent upon the deposition void ratio (Chung 1985), soil gradation and sample preparation technique in triaxial extension (Kuerbis 1989). Chung (1985), Kuerbis (1989) and Thomas (1992) show that the triaxial extension CSR values are considerably lower than the compression values, implying stress path dependent trigger of contractive deformation.
Phase Transformation State

One of the characteristic features of type 2 response is that after the termination of strain softening, shear resistance of the sand increases and excess pore pressure decreases with further straining. Termination of strain softening can be seen by a sharp turnaround in effective stress path diagram (see Fig. 2.1b, type 2 response). The state at which the pore pressure is a maximum was termed as the phase transformation state by Ishihara et al. (1975). Such a state can also be defined for the dilative response of type 3. The friction angle \( \phi_{\text{PT}} \) mobilized at the phase transformation state has been shown to be unique for a given sand (Chern, 1985; Thomas, 1992). Furthermore, under undrained loading, \( \phi_{\text{PT}} \) equals the friction angle mobilized at steady state, and under drained loading, it equals the constant volume friction angle \( \phi_{\text{cv}} \) (Chern, 1985; Negussey et al., 1988). For the type 1 response, the stress state remains on the phase transformation/steady state line while deformation continues indefinitely. In type 2 and 3 response, after the phase transformation state, the effective stress path moves up and follows the undrained failure envelope that represents the line of maximum obliquity.

Steady State and its Application

It has been suggested by Castro (1969), that if sufficient undrained unidirectional shear strain is imposed on a soil specimen, regardless of the stress path or initial consolidation stress condition, a unique stress state (the steady state) will be reached, which depends only on the initial void ratio. In other words, the undrained steady state strength, \( S_{\text{uu}} \), of a sand is dependent only on its void ratio and not on initial stress state or the type of undrained loading (static or...
cyclic) or initial structure (Castro 1987). Ishihara (1993) also states that the stress conditions at steady state deformation are determined by the void ratio alone.

Steady state concepts imply that a sand is either dilative or contractive depending on its state relative to the steady state line. If the density is high, or the initial confining stress is low, the sand tends to show dilative characteristics. Castro (1987) and Ishihara (1993) suggest that for such a sand, steady state is achieved only after all soil dilation is complete, which may occur at very large shear resistance. At this state, the shear stress reaches its maximum, and this value can be taken as the undrained strength $S_u$. However, data from Ishihara (1993) show that the shear resistance is increasing even at axial strain levels of 25% (see Fig. 2.2). Steady state, which corresponds to deformation with constant shear stress, may never be reached for such a dilative sand. Also, measurements of stresses corresponding to such large levels of strain may not be reliable, due to non-uniform deformations in the soil specimens. If the sand is dilative, Poulos et al. (1985) state that drained strength parameters should be used for the assessment of flow slide potential in a given problem.

Type 2 response in Fig. 2.1, shows that a sand may be both strain softening and strain hardening, depending upon the strain level. One of the important aspect of type 2 response is that the shear stress mobilized at phase transformation state is markedly smaller than the strength mobilized at the steady state, if it does exists at larger strain level. The phase transformation state at which the shear resistance is a minimum has also been called a quasi steady state (Alarcon-Guzman et al., 1988; Been et al., 1991; Ishihara, 1993). Selecting a residual strength, for practical problems, may become a serious issue, since the same sand shows two different strengths both, under large strain levels. It should be remembered, that the drop in shear resistance from the peak until phase transformation state will be overcome by further straining.
Effective confining stress, \( p' = \frac{(\sigma_1' + 2\sigma_3')}{3} \) (MPa)

Figure 2.2. Undrained response of loose Toyoura sand samples prepared by moist placement method (Ishihara 1993).
However, the amount of strain needed to mobilize it may not be acceptable in some practical problems.

2.4.2 Undrained Anisotropy

Investigations using triaxial tests have revealed that at identical initial states, the undrained response in compression is dramatically different from that in extension. (Saada and Ou, 1973; Hanazawa, 1980; Miura and Toki, 1982; Kuerbis, 1989; Vaid et al., 1990a; Pillai and Stewart, 1994; Vaid and Thomas, 1995). Dilative behaviour may ensue even for the loosest sand in compression, while the response may be contractive in extension. Note, that both $\alpha_{\sigma}$ and $b$ are zero in triaxial compression, while they are $90^\circ$ and 1 respectively in extension. A study of triaxial and simple shear test data from Vaid and Sivathayalan (1995) revealed that the loosest deposited sand is much less contractive in simple shear when compared to triaxial extension, while the behaviour in triaxial compression is dilative.

Difference between compression and extension undrained behaviour in the triaxial tests is a reflection of undrained anisotropy in sand. This difference in the behaviour is a combined effect of different $\alpha_{\sigma}$ and $b$ in the two loading modes. The effect of $\alpha_{\sigma}$ alone (directional dependence) on the undrained response can only be assessed by HCT tests in which $b$ is held constant during shear loading. Such experimental studies on the inherent undrained anisotropy of pluviated Ham River sand have been reported by Symes et al. (1984, 1985). Several tests at identical initial void ratio and stress state, but with different $\alpha_{\sigma}$, were performed. The deviator stress was increased in a stress controlled mode by holding $\alpha_{\sigma}$ constant. During shearing, total mean normal stress and intermediate principal stress parameter were also held constant at 600 kPa and 0.5 respectively.
Figure 2.3 shows the effective stress path and the corresponding deviator stress-strain response (Symes et al., 1984). An increase in $\alpha_\sigma$ is seen to promote contractive behaviour. Similar data is presented by Symes et al. (1985).

A contractive phase may be seen for each $\alpha_\sigma$ in Fig. 2.3. Further straining halted contractive strain softening response, and for $\alpha_\sigma = 22.5^\circ$ and $45^\circ$, and eventually the stress paths became tangent to the failure envelope, relevant to each $\alpha_\sigma$. Failure envelope for $\alpha_\sigma = 0$ appears uncertain, because the test was terminated at a small strain level.

Although, it is evident from Symes et al. (1984, 1985) that the undrained response of Ham river sand is anisotropic, the past data may be questionable. A stress controlled loading system induces runaway strains once peak deviator stress has been reached in a contractive sand (Chern, 1985). Stresses and strains measured during this runaway state without suitable corrections for inertia forces of the moving masses will be subject to serious errors. A sharp increase in $(\sigma_1 - \sigma_3)/2$, after about 6% strain in test A4 appears to be a consequence of dynamic impact, at the commencement of strain hardening behaviour.

Experimental evidence from triaxial compression and extension tests shows that $\phi_{PT}$ and the friction angle $\phi_f$ mobilized at maximum obliquity are unique for a sand (Chern 1985, Vaid and Thomas 1995). These findings are supported by tests conducted on several sands. The three undrained test results from Symes et al. (1984), at $\alpha_\sigma = 0$, $22.5^\circ$ and $\alpha_\sigma = 45^\circ$, however, show that $\phi_{PT}$ and $\phi_f$ are not unique. As noted earlier, these stress states and friction angles, evaluated for the contractant phase of a stress controlled shear test, may be erroneous due to unaccounted for inertia effects.
Figure 2.3. Undrained response of Ham river sand (Symes et al., 1984).
Shibuya and Hight (1987) also performed stress controlled undrained HCT tests on glass ballotini using stress controlled. Testing conditions and initial stress states were identical to those of Symes et al. (1984,1985). Test results with $\alpha_c$ held constant at 0, 45° and 90°, respectively, during shear are shown in Fig. 2.4. There appear to be two phase transformation states for tests B1 and B2. After the stress path turn around following the first phase transformation state, a sudden drop in deviator stress occurs leading to the second phase transformation state. Two phase transformation states, for a single monotonic stress path, is in contrast with the observed behaviour of granular materials, in general. As noted previously, a stress controlled loading system may not be suitable for investigating undrained response of contractive sands. In particular the measured magnitude of the phase transformation/steady state strength and the friction angles mobilized at these states would be subject to an unknown amount of error.

**Principal Stress Rotation**

Although sand deposits are known to be inherently anisotropic, they are treated as isotropic in most deformation or stability analyses. Consequently, the results of such analyses are independent of the directions of the principal stresses. As correctly pointed out by Arthur et al. (1980), principal stress rotations are a major feature of nearly all field stress paths, and yet little attention is paid to the possible effects of these rotations on deformation response of sands.

Under undrained conditions, a progressive increase in pore water pressure of a medium loose Ham River sand ($D_{rc} = 44\%$) has been shown to occur by a mere rotation of principal stress directions, while their magnitudes are held constant (Symes et al., 1984,1985). This may cause a medium loose sand to liquefy with no additional increment in shear stresses.
150, Test a* b.
B1 0 0 (↓) peak
B2 +45 0.5  (→) phase transformation
B3 +90

Figure 2.4. Undrained response of glass ballotini (Shibuya and Hight 1987).
A simple framework for a qualitative understanding of the effects of both principal stress rotation and initial anisotropy was proposed by Symes et al. (1984). This concept of bounding surface (BS), originally introduced by Roscoe et al. (1958), was extended to include $\alpha_\sigma$ as an additional variable. The bounding surface could be visualized in a 3-D space, by keeping $b$ constant in undrained HCT tests.

Although attractive because of its simplicity, the concept of bounding surface should be applied with caution. As indicated by Sayao (1989), induced anisotropy and previous stress history effects are intrinsically neglected when the BS is considered unique.

**Intermediate Principal Stress**

The intermediate principal stress has been shown to have an important effect on the drained behaviour of sands (Symes et al., 1984; Yamada and Ishihara, 1979; Sayao, 1989). However, its effect has not been studied on the undrained response. Almost all the previous work on undrained response, using HCT device, has been performed by holding the intermediate principal stress parameter $b$ at a single constant value of 0.5. It would be of interest to examine the effect of intermediate principal stress on undrained response if any, from a practical standpoint. Undrained shear at the same initial relative density, $\sigma_m$ and $\alpha_\sigma$ but at different constant values of $b$, will enable isolation of the effects of $b$ alone.

**Steady State Strength**

Although Castro (1969,1987) and Ishihara (1993) state that the steady state strength is only a function of the initial void ratio, ample experimental evidence exists on the contrary.
Laboratory tests on specimens reconstituted by pluviation (Hanazawa, 1980; Miura and Toki, 1982; Vaid et al., 1990a; Vaid and Thomas, 1995), and undisturbed sand specimens sampled by ground freezing technique (Pillai and Stewart, 1995) show the steady state strength of a sand is stress path dependent. These studies demonstrate that the response in triaxial extension is contractive over a wide range of relative density, while compression loading may not even result in contractive response. Thus, for a given initial void ratio and stress state, a systematic variation in steady state strength could be expected as $\alpha_e$ varies from 0 to 90°.

In addition, simple shear tests on undisturbed sand specimens from the foundation of Duncan dam in British Columbia, show that the steady state strength is not only a function of void ratio, but also of effective confining stress level (Pillai and Stewart, 1995). Tests on specimens with identical void ratio but with different confining stress levels resulted in different steady state strengths. Triaxial extension test data by Vaid and Thomas (1995) also show that the steady state strength depends on the effective confining stress level. These studies reveal that the steady state strength of a sand depend not only its initial void ratio but also on several factors such as stress path, confining stress level, direction of principal stress, intermediate principal stress level etc. A comprehensive investigation is thus necessary to assess the influence of each parameter on the steady state strength of a sand.

2.4.3 Cyclic Loading Response

Liquefaction potential under cyclic loading is frequently assessed by cyclic undrained triaxial tests on account of its simplicity and easy availability. In these tests, cyclic deviator stress $\pm \sigma_{cyc}$, is applied to the specimen following consolidation under isotropic or anisotropic stresses.
In a specimen that is isotropically consolidated, application of $+\sigma_{dcy}$ imposes triaxial compression ($\alpha_\sigma = 0$ and $b = 0$) in one half of the loading cycle (see Fig. 2.5a). In the other half of the cycle, application of $-\sigma_{dcy}$ causes deformation under triaxial extension ($\alpha_\sigma = 90^\circ$ and $b = 1$). In other words, cyclic triaxial loading imposes a stress path, which induces jump rotation in $\alpha_\sigma$, between 0 and 90°, and b parameter between 0 and 1 concurrently. In addition, total mean normal stress fluctuate between $\sigma_c+(\sigma_{dcy})/3$ and $\sigma_c-(\sigma_{dcy})/3$, in which $\sigma_c$ represent cell pressure.

In an anisotropically consolidated specimen, if the static deviator stress ($= \sigma_{vc-\sigma_{hc}}$, where the subscript c refers at the end of consolidation) is higher than the cyclic deviator stress amplitude, no stress reversal occur during cyclic loading (see Fig. 2.5b). Thus, $\alpha_\sigma$ and b remain zero at all times. In contrast, if the amplitude of cyclic deviator stress is higher than the static deviator stress, shear stress reversal occurs in each cycle. Simultaneously, a jump rotation occurs in $\alpha_\sigma$ form 0 to 90° and in b from 0 to 1 (see Fig.2.5c). The bulk of cyclic triaxial test data reported in the literature are from tests on isotropically consolidated specimens.

An element of soil below a level ground, under a vertical effective overburden pressure $\sigma'_v$, is shown in Fig. 2.6. During earthquake shaking, horizontal shear stresses are induced due to the upward propagation of shear waves. This causes the soil element to deform horizontally with the stress system shown in Fig. 2.6b. The direction of major principal stress $\alpha_\sigma$ undergoes a continuous rotation on either side of the vertical direction. The magnitude and the sign of $\alpha_\sigma$ depends on the magnitude and the sense of cyclic shear stress $\tau$, but the maximum value generally does not exceed 45°. If the initial stress state is hydrostatic, the cyclic shear stresses induces jump rotations of $\alpha_\sigma$, between $+45^\circ$ and $-45^\circ$, and the b parameter stays at 0.5 during shear loading.
Figure 2.5. Variation of cyclic deviator stress with time in cyclic triaxial tests.
Figure 2.6. States of stress (a) prior to, and (b) during cyclic loading.
Several studies on monotonic (static) behaviour of pluviated sands, both reconstituted and naturally occurring, have revealed that undrained response of sands is anisotropic (Symes, 1984; Shibuya and Hight, 1987; Pillai and Stewart, 1994; Vaid and Thomas, 1994). Specifically, the response depends upon the directions of principal stresses, even though their magnitude stays constant. Therefore, cyclic triaxial test which imposes jump rotations of principal stresses between 0 and 90° may not necessarily represent many field loading situations. In view of the available evidence (though limited in scope) that triaxial extension mode may represent the weakest undrained response, such weakest loading modes in each half cycle of cyclic loading may underestimate liquefaction resistance of sand. HCT and simple shear devices can simulate field stress paths during earthquake shaking more realistically. The unknown lateral stresses, and often serious nonuniformities in stresses and strains, which are inherent features of the commonly available simple shear devices, make them less than desirable for fundamental studies of undrained behaviour of sands.

Cyclic shear loading of sands can lead to a sudden or gradual development of large strains. Triaxial tests have shown that the development of strain during cyclic loading, may be due to true liquefaction, limited liquefaction or cyclic mobility (Castro, 1969; Seed, 1979). As a consequence, results of cyclic loading tests are generally assessed in terms of the strain criterion. Cyclic strength or resistance to liquefaction is thus defined as the cyclic shear stress required to cause a specified level of strain (e.g., 2.5 % axial strain in triaxial test or 3.75 % shear strain) in a fixed number of cycles.

Castro (1969) and Vaid and Chern (1985) have shown cases in which true liquefaction developed during cyclic loading, much in the same manner as that observed under monotonic loading (see Fig. 2.7a). Vaid and Chern (1985) have also shown cases of cyclic loading of sand,
Figure 2.7. Cyclic loading behaviour of contractive sand - true liquefaction and limited liquefaction (Vaid and Chern, 1985).
wherein limited liquefaction developed in the same way as that observed in type 2 (Fig. 2.1) response under static loading (see Fig. 2.7b). There are many similarities between cyclic and monotonic loading behaviour of sands. Castro et al. (1982) and Chern (1985) have shown that the steady state line in void ratio - effective stress space, for liquefaction is unique under static and cyclic conditions, as is the friction angle mobilized at that state.

The observations reported in preceding paragraphs were derived from cyclic triaxial tests. For a full understanding of the cyclic response of anisotropic sands, behaviour under jump or continuous rotations of principal stresses along various stress paths would be needed. The loading scheme should be such that principal stress direction could be rotated between any specified angles under controlled conditions. Studies of sand liquefaction in simple shear device, intended to simulate this principal stress rotation during earthquake shaking, have been performed in the past (Silver and Seed, 1971; Finn et al., 1977, 1982; Lacasse et al., 1981). But, $\alpha$, $\sigma_m$ and $b$ change simultaneously during cyclic shear application. Therefore, fundamental studies aimed at isolating the effects of independent parameters $\alpha$, $\sigma_m$ and $b$ on undrained behaviour cannot be achieved from simple shear test data.

2.5 Summary

It is apparent from the review presented in this chapter that sands possess undrained anisotropy. Triaxial and simple shear devices are suitable for investigations of only specific regions of the general stress space with regard to the assessment of undrained behaviour of sand. Of all the testing devices currently available, HCT device is the only one that offers the possibility of independent control of three principal stresses and rotation of two principal stresses in one
plane. Consequently, studies of fundamental behaviour of initially anisotropic materials and, in particular, isolation of the effect of principal stress direction and rotation and the effect of intermediate principal stress can only be performed with a HCT device. Although some attempts have been made in assessing the drained response of sands as influenced by \(\alpha\) and b, few investigations on the undrained response have been made.

Although several studies of undrained behaviour have been made using the triaxial and simple shear devices, little effort has been made to correlate response with and without principal stress rotations. Symes et al. (1984,1985) do report some studies on undrained response using the HCT device, but the use of stress controlled loading system inhibited investigations of the post peak behaviour. This crucial part of the response involving residual/steady state strengths is required for the assessment of potential for a flow slide. A comprehensive and systematic fundamental investigation on the effects of \(\alpha\) and b parameter on the behaviour of loose sands is, therefore, necessary under strain controlled loading. The research program in this thesis is designed to investigate the undrained response of sands under general stress paths. Particular emphasis is placed on delineating the effects of principal stress directions and rotations on undrained behaviour with other parameters (\(\sigma_m\) and b) held constant. Both static and cyclic loading behaviour of sand are investigated.
CHAPTER 3
HOLLOW CYLINDER TORSIONAL DEVICE

3.1 Introduction

The UBC Hollow Cylinder Torsional (HCT) device used in this research was fabricated in the civil engineering workshop in 1986, and a detailed description is given by Sayao (1989) and Vaid et al. (1990b). In order to carry out complex stress path tests, a fully automated test control and data acquisition system was added (Wijewickreme et al., 1994). As a part of this research program, the HCT device was somewhat modified to enable tests under stress as well as strain controlled loading.

3.2 General Description

A schematic diagram of the UBC HCT device, together with the data acquisition system is shown in Fig. 3.1, together with some details in Fig. 3.2. This test apparatus is capable of applying axial load and torque about the vertical axis and independent internal and external pressures to a hollow cylindrical soil specimen. Independent control of these four tractions enables the specimen to be loaded along a prescribed stress path in the four dimensional stress space - $\sigma'_m$, $b$, $\sigma_d$ and $\alpha$.

The HCT specimen is approximately 30 cm high and possesses internal and external diameters of 10.2 cm and 15.2 cm, respectively. Vaid et al. (1990b) describe the basis for the selection of these dimensions in detail. These dimensions are arrived at from the considerations of minimizing stress non-uniformities across the specimen wall. The specimen is fixed at the top and
Figure 3.1: Schematic diagram of the HCT device.
Figure 3.2. Details of the HCT device (Vaid et al. 1990b).
laterally confined by internal and external water pressures, acting on flexible, 0.3 mm thick rubber membranes.

A double-acting water saturated frictionless Bellofram piston, mounted at the bottom of the supporting table, applies vertical normal stress to the sample, in either, compression or extension. The load is transmitted through a 25 mm diameter, polished stainless steel shaft that has its vertical alignment guaranteed by two frictionless, Thompson combination bush bearings. These bearings permit linear and rotary motions of the loading shaft and are located at the base of the cell chamber.

Torsional shear stresses are applied by two pairs of identical, single-acting water saturated Bellofram pistons together with a system of cables and pulleys. Diametrically opposite pistons are inter-connected to a common regulated pressure supply. This configuration is necessary to apply torque in either direction, and to eliminate horizontal side forces on the loading shaft.

Transfer of torsional shear to the specimen and minimization of radial shear at the boundaries is achieved by using polished, anodized aluminum platens having 12 thin radial ribs (1 mm thick and 2.3 mm deep), as shown in Fig. 3.3. Drainage from the specimen is provided by six 12.8 mm diameter porous discs set 60° apart, flush with each platen surface.

Internal and external cell pressures and back pressure are applied through air-water interfaces, as illustrated in Fig. 3.4. Air diffusion into the specimen or into the cell's de-aired water is prevented by 1.5 m long, 4 mm I.D. saran tubings diffusion loops. Control of the air pressure is obtained by precision regulators. Pressure monitoring is done on the water side by pressure transducers placed close to the cell's base platen. The measurement resolution of all pressures was in the order of 0.25 kPa.
Figure 3.3. Polished end platen with radial ribs (Vaid et al 1990b).
Figure 3.4. Fluid pressure and volume change measuring systems (Sayao 1989).
3.3 Stress Nonuniformities in HCT Specimens

Laboratory tests on soil specimens assume that the entire specimen represents a soil element. The validity of this assumption depends on the uniformity of stress and strain distributions within the specimen. Like other soil testing devices, the HCT device also suffers from the fact that the stress distribution within the specimen is not strictly uniform. The nonuniformity of stresses arises on account of the curvature of the specimen wall, in addition to that due to end restraint. The latter is common to most soil testing devices, and is usually minimized by selecting specimens with height/diameter ratio in the range of 1.8-2.2, together with loading platens that reduce radial friction at the ends (Saada and Townsend, 1981; Hight et al., 1983; Vaid et al., 1990b). The ratio of specimen height/diameter used in the UBC-HCT device is approximately 2, and the radial friction at the end platens is minimized as discussed in the previous section.

Stress nonuniformities due to the curvature of the HCT specimen occur even when the boundary stresses are uniformly applied. Specifically, unequal internal and external cell pressures result in radial and tangential stress gradients across the wall of the specimen. Also, the application of torque, \( T_h \) causes shear stress \( \tau_{z\theta} \) gradients across the wall.

Comprehensive investigations of the stress nonuniformities in HCT specimens have been reported by Hight et al. (1983), Sayao (1989), Wijewickreme (1990), and Vaid et al. (1990b). Hight et al. (1983) examined these nonuniformities in terms of the distribution of individual stress components (radial, tangential and axial) within the specimen, assuming the soil to be an elastic or strain hardening elasto-plastic material. Sayao (1989) and Vaid et al. (1990b) argued that instead of the individual stress components, the assessment of stress nonuniformities in a frictional
material should be based on the variation of effective stress ratio across the wall of the specimen. Assuming sand behaviour to be linear elastic, Sayao (1989) delineated the domain of stress space that could be explored with the HCT device without incurring unacceptable nonuniformities. Using a finite element analysis, Wijewickreme (1990) showed that a much larger domain of stress space than that proposed by Sayao (1989) could be explored if the soil was more realistically modelled as nonlinear elastic Coulomb type frictional material. These studies reveal that the levels of stress and strain nonuniformity depend predominantly on the specimen geometry, and on the level of the effective stress ratio mobilized. Nonuniformities progressively decrease as higher levels of stress ratio are mobilized (Wijewickreme, 1990). The study reported herein recognizes that the problem of stress nonuniformity cannot be completely eliminated, but it can be minimized by a careful selection of specimen geometry and avoiding exploration of certain extreme regions of the stress space, that may contribute unacceptable stress nonuniformities.

3.4 Definition of Stresses and Strains

The stresses in the hollow cylindrical specimen are generated by the four surface tractions. They are: vertical load ($F_z$), torque ($T_h$), external and internal pressures ($P_e, P_i$). Figure 3.5 shows these tractions together with the stress state in an element in the wall of the specimen. The four stress components $\sigma_z, \sigma_r, \sigma_\theta$ and $\tau_{z\theta}$ induce the four strain components $\varepsilon_z, \varepsilon_r, \varepsilon_\theta$, and $\varepsilon_{z\theta}$ in the soil element.
Figure 3.5. Surface tractions and stress state in an element in the hollow cylindrical specimen.
3.4.1 Average Stresses

Interpretation of the results of hollow cylinder torsion tests is made by considering the entire specimen as a single element, deforming as a right circular cylinder. Since the stresses may vary across the wall of the cylinder for a variety of loading conditions, it becomes necessary to work in terms of average stresses and strains. The following expressions are used for calculating average stresses (Vaid et al., 1990b).

\[ \sigma_z = \frac{F_z + \pi (P_e R_e^2 - P_i R_i^2)}{\pi (R_e^2 - R_i^2)} \]  
\[ \sigma_r = \frac{P_e R_e^2 - P_i R_i^2}{R_e^2 - R_i^2} - \frac{2(P_e - P_i)R_e^2 R_i^2 \ln(R_e / R_i)}{(R_e^2 - R_i^2)^2} \]  
\[ \sigma_\theta = \frac{P_e R_e^2 - P_i R_i^2}{R_e^2 - R_i^2} + \frac{2(P_e - P_i)R_e^2 R_i^2 \ln(R_e / R_i)}{(R_e^2 - R_i^2)^2} \]  
\[ \tau_{\varphi \theta} = \frac{4 T_h (R_e^3 - R_i^3)}{3 \pi (R_e^4 - R_i^4)(R_e^2 - R_i^2)} \]

where \( R_e \) and \( R_i \) are external and internal radii respectively of the hollow cylindrical specimen.

The radial stress \( \sigma_r \) is usually the intermediate principal stress \( \sigma_2 \). The major and minor principal stresses \( \sigma_1 \) and \( \sigma_3 \) in the plane of the specimen wall can be computed from \( \sigma_z \), \( \sigma_\theta \) and \( \tau_{\varphi \theta} \).

The vertical stress \( \sigma_z \) is assumed to be distributed uniformly across the cross-section, and thus, obtained using equilibrium considerations only. Stresses \( \sigma_r \), \( \sigma_\theta \) and \( \tau_{\varphi \theta} \) are obtained by averaging them over the volume of the specimen and assuming the soil to be linear elastic.
Hight et al. (1983), Miura et al. (1986) and Pradel et al. (1990) used slightly different expressions for \( \sigma_0 \), \( \sigma_\theta \) and \( \tau_{xy} \). The differences arise on account of:

1. averaging across the wall instead of over the volume of specimen, and
2. assuming linear elastic behaviour for evaluating \( \sigma_r \) and \( \sigma_\theta \) and perfectly plastic behaviour for \( \tau_{xy} \).

These differences, however, are minor and for the UBC-HCT device and do not exceed 2\% among different expressions proposed (Sayao, 1989; Wijewickreme, 1990; Vaid et al., 1990b). However, for the sake of consistency, as Vaid et al. (1990b) correctly pointed out, all stress components should be computed by assuming a single constitutive law and not a hybrid combination of elastic for some and plastic for other components.

### 3.4.2 Average Strains

Average strain components are evaluated using the following expressions,

\[
\varepsilon_x = -\frac{\Delta H}{H} \tag{3.5}
\]

\[
\varepsilon_r = -\frac{\Delta R_s - \Delta R_i}{R_s - R_i} \tag{3.6}
\]

\[
\varepsilon_\theta = -\frac{\Delta R_s + \Delta R_i}{R_s + R_i} \tag{3.7}
\]
\[ \gamma_{t\theta} = \frac{2\Delta \theta (R_e^2 - R_i^2)}{3H(R_e^2 - R_i^2)} \]  

(3.8)

where \( H \), \( \Delta H \) are the height and height change and \( \Delta \theta \) angular displacement of the specimen.

These definitions stem from considerations of compatibility of displacements together with the assumption of a linear variation of displacement across the specimen wall. The change in the inner radius \( \Delta R_i \) is obtained from the measured values of \( \Delta H \) and volume change of the inner chamber. The external radius change \( \Delta R_e \) can then be computed from the measured values of \( \Delta H \), volume change of the sample and \( \Delta R_i \).

Likewise for stresses, major and minor principal strains can be computed from the known average strain components \( \varepsilon_z \), \( \varepsilon_\theta \) and \( \gamma_{z\theta} \). The radial strain \( \varepsilon_r \) corresponds to \( \varepsilon_2 \).

### 3.5 Measurement of Strains

Four deformation measurements are needed for computing the four components of strain in the hollow cylinder specimens. Two displacement transducers (linear variable differential transformers LVDT) monitor vertical and angular displacements of the specimen's base pedestal (see Fig. 3.1). From these, average axial and shear strains (\( \varepsilon_z \) and \( \gamma_{z\theta} \)) can be obtained. Two differential pressure transducers register volume changes of the saturated soil specimen and of the inner pressure chamber. These are required for the evaluation of radial and tangential normal strains.

The vertical LVDT is mounted on the loading frame's top cross beam, as shown in Fig. 3.1. The transducer's core reacts against the top of a thin vertical steel rod (central rod), rigidly
attached to the center of the specimen's pedestal. In this manner, vertical movement of the specimen's base is transmitted to the transducer's core. An O-ring placed in the top cap seals the inner chamber pressure around the central rod (see Fig. 3.2).

Improvement in the accuracy of vertical or axial displacement measurements is obtained by minimization of bedding and seating errors. This has been achieved by adopting several improvements in the testing technique;

1. use of small diameter porous stones flush with the platen surface,
2. careful levelling of the specimen's top surface prior to placement of the top platen and
densification of the specimen after placement of the top platen.

Items 2 and 3 are described in detail in chapter 4.

Rotation of the specimen's base platen is converted to linear tangential displacement by using the system schematically shown in Fig. 3.6. The LVDT is placed at a distance large enough to avoid the need for corrections in the measured tangential displacement due to vertical movements of the rotation arm when the specimen deforms vertically.

Both displacement transducers (axial and tangential) were calibrated against a precision micrometer. They can reliably detect movements in the order of $10^{-3}$ mm. This results in a resolution of about $5 \times 10^{-6}\%$ in both $e_z$ and $\gamma_{\theta\theta}$.

Volume changes of the inner chamber and soil specimen are obtained by monitoring the height of water column in graduated pipettes, as illustrated in Figs. 3.4.(a) and 3.4.(c) respectively. Sensitive differential pressure transducers (DPT) are used to electronically register the water levels in the pipettes. The DPT used for measuring sample volume change can detect volume changes in the order of 3 mm$^3$, resulting in a resolution of about $5 \times 10^{-4}\%$ in volumetric
Figure 3.6. Rotational displacement measuring system (Vaid et al 1990b).
strain. The DPT for the inner chamber can detect volume changes in the order of 10 mm$^3$, which results in a resolution of about $2 \times 10^{-4}$%.

3.6 Measurement of Surface Traction

Four surface tractions - vertical load, axial torque, internal and external chamber pressures are monitored. These tractions are applied by means of four motor set (stepper motor) precision regulators. Pore water pressure and chamber pressures are measured by using sensitive pressure transducers having a resolution in the order of 0.25 kPa.

Vertical load is measured using a load cell placed outside the cell chamber. Friction on the loading shaft due to the U-cup seal causes a negligible error in vertical stress $\sigma_z$ ($\sim 0.1$ kPa). Vertical stress as low as 0.2 kPa can be accurately measured using this load cell.

Torque measurements are made with a torque transducer placed just above the central torque pulley (see Fig. 3.1). As low as 0.05 Nm of torque can be measured with this transducer. This corresponds to a resolution of shear stress $\tau_{xz}$ in the order of 0.1 kPa. The torque cell has a negligible amount of cross talk between the vertical load and torque.

3.7 Instrumentation and Data Acquisition System

The system consists of nine transducers that monitor various loads, pressures, volume changes and displacements through a multi-channel scanner and an A/D converter, together with a personal computer. All transducers are exited with a stable D.C. power supply of 6 V. The LVDT's being high output devices, do not need any signal amplification. Other transducers are of
bonded strain gauge type with full scale outputs of about 3 mV/V. All transducer signals are scaled to 2 V full scale by variable gain amplifiers.

Data scanning is triggered simultaneously on all channels at specified time intervals, depending on the stress path being followed. The signal from each transducer over a 50 ms period is integrated by an analog circuit and the average value is obtained. This average value is then digitized by the A/D converter (one for each channel) and temporarily stored. The digital output from the A/D conversion yields a decimal number ranging from 1 to 40000 (≈ 16 bits in binary) corresponding to an analog range of 0 to 4 V. This translates to a resolution of the transducer outputs in the order of 0.1 mV. At the end of conversion, the digital voltages are retrieved by the computer in a sequential manner (see Fig. 3.7).

3.8 Stress/Strain Path Control

The stress path control is achieved by controlling the four surface tractions ($F_z$, $T$, $P_e$ and $P_i$) applied by the stepper motors. In response to an input to the computer, a series of square pulses are sent to the stepper motors. The input contains information about the number of pulses and their directions for a prescribed increment in traction. A single pulse advances the motor set regulator by one step, and approximately 12 to 13 pulses are needed to alter the pressure output by 1 kPa, thus enabling pressure change as low as 0.1 kPa. The regulators can be set to alter pressures at a rate of approximately 3 to 14 kPa/s.

A computer code, which enable application of a generalised stress path under either undrained or drained conditions has been written. Details about the stress path, the number of steps needed to apply stresses in incremental form from one stress state to another, and the rate of
START CHANNEL SCAN
PULSE FROM PC

\[ t = t_0 \]

SEQUENTIAL RETRIEVAL OF CONVERTED INFORMATION BY PC

\[ t > t_0 + 50 \text{ ms} \]

END OF A/D CONVERSION PULSE TO PC

A/D = A/D CONVERTER
M = MEMORY CHIP
INT = ANALOG INTEGRATOR

Figure 3.7. Data acquisition process (Wijewikreme 1990).
loading are specified in a data file. The program allows incremental tractions to be applied simultaneously in each step.

Stresses and strains are calculated using the scanned data and these current stresses are compared with the target stresses for each step. Each surface traction is then adjusted until the applied stresses are within a specified tolerance from the target values. Stresses, strains and deformations are then stored as acquired data before the next load increment is applied. This type of feedback permits the system to follow any specified stress path accurately. The flow chart used for the execution of stress path control is presented in Fig. 3.8.

Strain control in axial ($e_z$) or torsional ($\gamma_{zb}$) mode is achieved by digital volume pressure controllers (DVPC), connected to axial and torsional loading pistons (Fig. 3.1). A DVPC consists, essentially, of a water cylinder, ball screw and a piston. The piston is attached to the ball screw which is advanced by a stepper motor. Depending on the amount of motor rotation, diameter of the cylinder and pitch of the ball screw, specified amounts of water volumes can be either pumped into or extracted from the saturated torque pistons. This in turn gets translated into a constant rate of vertical or torsional strain. All tests reported in this thesis were performed at constant rates of strain.

It is possible to carry out stress path tests under mixed stress and strain control. For example, undrained static loading at constant $\alpha_\infty$, $\sigma_m$ and $b$ can be carried out by imposing a constant rate of $\gamma_{zb}$ and simultaneously controlling axial load, internal and external cell pressures. To accomplish this, water saturated torsional and axial loading piston are connected to the DVPC and the Axial Loading Stepper Motor Regulator (ALSMR) respectively (see Fig. 3.9a). Air pressure from the ALSMR is transferred to the axial loading piston through an air-water interface.
Figure 3.8. Flow chart for stress or strain controlled loading.
Figure 3.9. Schematic diagram for strain control mechanism in (a) vertical (axial) mode, and (b) torsional mode.
in the water reservoir \( R2 \) connected in series to the piston and the valve \( V2 \). Stepper motor which control the pressure input to the torsional loading pistons is de-activated and valves \( V1 \) and \( V4 \) are closed.

Alternatively, constant rate of axial strain can be imposed and torque, and internal and external cell pressures are controlled simultaneously. This is achieved by connecting the DVPC to the axial loading piston by opening valve \( V4 \) and closing \( V2 \) and \( V3 \) (see Fig. 3.9b). Air pressure from torque stepper motor regulator is transferred to the torque piston through the water reservoir \( R1 \) and valve \( V1 \).
4.1 Test Materials

Laboratory tests were performed on two sands: Fraser River sand, and Syncrude sand. The main focus was on the undrained response of Fraser River sand, which was dredged from the Fraser River in British Columbia. This sand was selected, since it underlies the populated Fraser delta, which is located in the region of high seismicity. The sand as dredged was found to have about 1% of clay fraction. For testing purposes, it was decided to clean the sand of this clay fraction.

Fraser River sand is uniform, grey coloured, medium grained with sub-angular to sub-rounded particles. The average mineral composition is 40% quartz, 11% feldspar, 45% unaltered rock fragments and 4% other minerals (Garrison et al 1969). The average grain size $D_{50}$ is 0.3 mm and the coefficient of uniformity $C_u$ is 2.4. Maximum and minimum void ratio in accordance with ASTM (D4353-91 and D4254-91) are 1.0 and 0.68 respectively, and the specific gravity of solids $= 2.72$.

Syncrude tailings sand is a waste material left after the processing of oil sand at the open-pit mine near Fort McMurray, Alberta, operated by Syncrude Canada Ltd. This is also a fine, uniform, angular to sub-angular, predominantly quartz sand with a trace of silt and clay. Its mineral composition is 95% quartz, 2% feldspar, 1-2% pyrite and 1% amphibole, $D_{50} = 0.2$ mm and $C_u = 2.4$. Maximum and minimum void ratios are 0.965 and 0.522 respectively (ASTM
D4353-91 and D4254-91), and $G_v = 2.62$. Grain size distribution curve for the sands are shown in Fig. 4.1. Photomicrographs of the sands are shown in Fig. 4.2a and 4.2b, respectively.

4.2 Specimen Reconstitution

Ideally the undrained behaviour of a given sand should be assessed using undisturbed samples for tests in the laboratory. But the conventional undisturbed sampling techniques significantly alter the mechanical properties of sands (Seed et al., 1982). Loose sands tend to get densified and dense sands get loosened during sampling. The ideal method to obtain undisturbed samples is by *in-situ* freezing and coring the frozen ground using suitable drilling equipment. This sampling method was used recently in connection with seismic upgrade studies of Duncan Dam (Pillai and Stewart, 1994). Ground freezing technique has also been used in similar studies by Japanese researchers (Yoshimi et al., 1978, 1984, 1989; Hatanaka et al., 1985; Goto et al., 1992). This technique, however, is very expensive and therefore not economically feasible, except in major critical structures. In general, the fundamental property characterization of sands is carried out using reconstituted specimens, which enables tests on several repeatable samples. This is not generally possible using undisturbed frozen samples of natural or man made sand formations, as they are likely to be inherently more variable.

Several techniques for reconstituting sand specimens in the laboratory have been developed in the past, the most common being pluviation (Kolbuszewski, 1948; Chaney and Mulilis, 1978; Miura and Toki, 1982) and moist tamping (Lambe, 1951; Ladd, 1978). Comparative reviews of the methods of sand specimen preparation have been presented by Mulilis et al. (1977), Mortensen (1982) and Vaid and Negussey (1988).
Figure 4.1. Grain size distribution curves for Fraser River and Syncrude sands.
Figure 4.2. Photographs of Fraser River and Syncrude sands (magnification = 100).
Water pluviation followed by densification by vibrations - if necessary - was selected in this study, as the most suitable technique for reconstituting soil specimens. This technique enables preparation of repeatable homogeneous saturated specimens of uniform sands with controlled density (Vaid and Negussey, 1988). This method has been used at the University of British Columbia for almost thirty years. Pluviation is considered to duplicate the sedimentation process and, hence, the fabric of many natural or artificial sand deposits (fluvial and hydraulic fills). Laboratory studies on water pluviated sands should, therefore, give a close indication of the behaviour of these deposits (Oda et al., 1978; Miura and Toki, 1984).

The main limitation of water pluviation is the segregation of particles during sedimentation of well-graded and silty sands. For these materials, an alternative _slurry deposition technique_ has been developed by Kuerbis and Vaid (1988) for preparing homogeneous saturated specimens. For this research, uniform sands were used and specimen reconstitution was done by water pluviation.

### 4.2.1 Preliminary Steps

A known dry weight of sand (about 5 kg) was boiled for about 10 minutes in several long necked 500 ml flasks and allowed to cool at room temperature, followed by application of about 70 kPa vacuum to each flask. Porous stones were also boiled and allowed to cool in water at room temperature. The drainage lines in the HCT device were flushed through by boiled and de-aired water and saturated.

Prior to sample preparation, a dial gauge was mounted on a removable stand to obtain a reference reading on the top platen. This was used later for determining the height of the sand
specimen after deposition. First the inner rubber membrane was positioned and sealed to the inner surface of the base platen. De-aired water was flushed through the base drainage line, and saturated porous stones placed in position. The four piece inner split mould was then assembled and the inner membrane stretched around it. The four pieces of the inner mould are held together by two internal metallic discs, the annular base platen and one O ring at the top. The outer membrane was then positioned and sealed to the outer surface of the base platen, and the outer split mould assembled around. The outer mould has its inner surface lined with porous plastic through which vacuum was applied for stretching the membrane, against the inner surface of the mould.

4.2.2 Specimen Preparation Steps

The flasks with boiled sand were completely filled with de-aired water up to the top. The annular cavity formed by the two mould was then filled with de-aired water. Once the sand flask was inverted and had its mouth submerged in the specimen water cavity, sedimentation of the sand proceeds under gravitational influence and mutual displacement with water as shown in Fig. 4.3.

During the pluviation process, flasks are slowly traversed over the annular area to deposit sand resulting in an approximately level surface at all times. Water pluviation results in loose density close to the ASTM minimum regardless of the height of particle drop (Vaid and Negussey, 1988).

Sedimentation was continued until an excess of sand over that required for the final grade had been deposited. The upper surface was then carefully levelled by siphoning off excess sand
Figure 4.3. Specimen preparation by water pluviation (Vaid et al 1990b).
using a suction of about 2 kPa as shown in Fig. 4.4. This causes minimal disturbance of sand grains below the surface. The excess siphoned sand was oven-dried to allow determination of the dry weight of sand used in the specimen.

The top platen containing saturated porous stones was then carefully seated on the levelled sand surface. Since the pluviated specimen was loose, no further loosening of the top layers will occur because of the penetration of the platen's ribs into sand (Vaid et al., 1990).

De-aired water was then percolated upwards through the specimen under a very small gradient. This was done to remove entrapped air bubbles, if any, between the rubber membranes and the vertical face of the top platen. After sealing both membranes to the top platen with O rings, the top drainage line was closed and a vacuum of about 20 kPa applied to the bottom drainage line. This provides an effective confinement to the specimen prior to dismantling the forming moulds. At this stage, the dimensions of the specimen were recorded. The measured diameters were appropriately corrected for membrane thickness. The top loading cap was then carefully installed, thus completing the specimen preparation process.

4.3 Specimen Assembly

The cell chamber was placed in position, and de-aired water allowed to simultaneously fill the inner and outer chambers slowly. Care was taken to ensure full saturation of the inner chamber. The top cross beam was then swivelled in position and firmly bolted to the reaction frame. The central rod used for monitoring vertical displacement was then installed thus sealing the inner chamber. The specimen was then lifted upwards by pressurizing the bottom chamber of the vertical loading piston until the top cap contacted the cross beam of the reaction frame. The
Figure 4.4. Levelling the specimen's upper surface (Vaid et al 1990b).
top cap was secured against the cross beam by a bolt. This ensures coincidence of the specimen's vertical axis and the frame's center axis. A locating pin, protruding from the cross beam at 30 mm from the specimen axis as shown in Fig. 3.2 was then inserted into the loading cap. This pin serves to arrest any rotational movement of the loading cap in tests requiring application of torque. De-aired water was allowed to simultaneously fill the inner and outer chambers of the specimen slowly. Care was taken to ensure full saturation of the inner chamber. The central rod used for monitoring vertical displacement of the specimen was then installed, thus sealing the inner chamber.

Measurement of Skempton's B value for checking specimen's saturation then proceeded in several increments of hydrostatic confining pressure under undrained conditions. Full saturation of the soil specimen was ensured by insisting on a B value greater than 0.98. Consolidation, isotropically or anisotropically, was then performed in small increments, until the desired stress level prior to undrained shear was reached. Monotonic or cyclic shearing, along the specified stress paths was then carried out.

4.4 Performance and Control

Simultaneous and independent control of the four stress parameters is required for the stress controlled multi-axial loading tests with the HCT device. For the mixed stress/strain controlled tests, either (1) torsional strain, axial stress, internal and external cell pressures or (2) axial strain, torque, internal and external cell pressures need to be controlled.

In order to follow the prescribed stress path precisely, smooth changes in the control pressures must be made, either continuously or in small increments. Since the control system
operates in a feed-back loop, the rate of loading should be slow enough so as to make appropriate adjustments in each stress parameters such that the desired stress path is followed accurately. In addition, mixed stress/strain controlled tests require careful selection of the rate of strain. To determine the appropriate rate of strain in these tests, several trial tests were performed. It was found that the required stress path will be followed accurately at an axial or torsional strain rate of about 0.1% per minute. This rate was, therefore, adopted in all tests.

A typical example of stress path control is shown in Fig. 4.5. In this undrained test $\sigma_m$, $\alpha$, and $b$ were held constant while the specimen was sheared at 0.1% per minute torsional strain rate. The remarkable stress path control capability of the UBC-HCT device may be noted. The maximum excursion in any of the stress parameters $\sigma_m$, $\alpha$, and $b$ from the prescribed values may be noted to be less than 2%.

4.5 Repeatability of Test Results

Repeatability of test results on identical specimens is an important requirement for ensuring the consistency of test results and the conclusions to be derived therefrom. Strict adherence to identical specimen preparation technique and test control routines is central to achieving repeatable test results.

Undrained shear test data on two identical samples under identical stress paths are shown in Fig. 4.6. The specimens were hydrostatically consolidated under 200 kPa. During shearing, $\sigma_m$, $\alpha$, and $b$ were held constant at 300 kPa, 30° and 0, respectively. Excellent repeatability of test results may be noted, both in stress-strain and effective stress path response.
Figure 4.5. Experimental control in undrained multi-axial loading tests.
Figure 4.6. Repeatability of HCT test results.
4.6 Membrane Penetration

Intrusion of the flexible rubber membrane into the peripheral voids of granular specimens due to a change in confining stress is termed as membrane penetration. The effect of membrane penetration on the undrained behaviour of granular materials has, in recent years, been the subject of a large number of analytical and experimental studies. When a sand specimen is hydrostatically loaded under drained conditions, the degree of membrane intrusion into the peripheral voids increases with the level of effective confining stress. The decrease in effective confining stress that is accompanied by an increase in pore pressure during undrained shearing on the other hand will cause the membrane, which had penetrated into the peripheral voids during consolidation, to rebound. This creates a *partially drained state* within the specimen, thus inhibiting truly undrained conditions. Membrane rebound leads to a decrease in void ratio of the specimen and hence, the excess pore pressure developed is smaller than that would have developed under truly undrained conditions. An increase in effective confining stress on the other hand, would lead to pore pressures that would be larger than those under truly undrained shear. In cyclic loading, membrane penetration would result in over estimation of the number of cycles required for liquefaction (Martin et al. 1978, Tokimatsu and Nakamura 1986, Nicholson et al. 1993b).

Factors affecting membrane penetration are mean grain size, effective confining stress level and to a lesser extent relative density, angularity of grains and membrane thickness (Ramana and Raju 1982, Evans and Seed 1987, Seed et al. 1989, Nicholson et al. 1993a). From a comprehensive study on membrane penetration effects, Nicholson et al. (1993a) concluded that in comparison to the effects of mean grain size and gradation characteristics, all other factors affecting membrane penetration are of relatively secondary importance.
Some analytical procedures have been proposed to correct for the effects of membrane penetration on the undrained behaviour of granular material (Kramer and Sivaneswaran 1989, Tokimatsu 1990, Byrne 1995). This requires modelling soil behaviour within a suitable constitutive framework. The corrections therefore, are a mere reflection of the suitability of the constitutive model used. The effect of membrane penetration should alternatively be directly assessed by performing truly undrained tests and comparing the results with conventional tests. A truly undrained test can be carried out using the water injection technique (Raju and Venkataramana 1980, Tokimatsu and Nakamura 1986, Nicholson et al. 1993b, Uchida and Vaid 1994). Recent advancements in computer controlled testing have made this technique relatively simple and easy to adopt. By monitoring the changes in pore pressure during undrained loading, corrections can be made by feeding in or extracting volumes of water from the specimen equivalent to membrane penetration volume over the corresponding effective confining pressure change. The volume correction is determined from an independently established membrane penetration-effective confining stress relationship. This relationship developed for Fraser River sand is presented in Appendix A.

Membrane penetration effects on the undrained response of Fraser River sand were directly assessed by comparing behaviour in conventional and truly undrained triaxial tests. Standard triaxial extension tests, utilizing solid cylindrical specimen, 6.35 cm diameter × 12.7 cm high were used. Fig. 4.7 compares behaviour of two specimens at identical initial density and void ratio. Little difference in stress/strain and pore pressure/strain response is noted. This direct evidence correspond to the shearing mode in which very large pore pressures developed, and hence the effect of membrane compliance, if any, would be most severe. In other loading modes,
Figure 4.7. Comparison of truly undrained and conventional undrained triaxial extension behaviour of Fraser River sand.

\[ \sigma'_m = 200 \text{ kPa}, \ b = 1, \ \alpha_\sigma = 90^\circ \]
lesser pore pressure would develop, and consequently, the effect of membrane penetration on measured pore pressure in conventional undrained tests would be negligible.

The results shown in Fig. 4.7 are in agreement with those reported for other sands of similar mean grain size. Frydman et al. (1973) show that the effect of membrane penetration on sand behaviour is negligible for sands with mean grain size 0.1 - 0.3 mm. Lade and Hernandez (1977) report that this effect is not significant even for a sand of $D_{50} = 1.2$, tested with an initial confining pressure of 100 kPa, although this claim may be questionable in light of the data by Nicholson et al. (1993b) and Evans et al. (1992). In cyclic triaxial tests on Toyoura sand ($D_{50} = 0.17$ mm), Tokimatsu and Nakamura (1986) also report little difference between the behaviour of sand in truly undrained and conventional undrained shear. Other studies, such as Martin et al. (1978), Evans et al. (1992) further confirm that for sands with $D_{50}$ less than about 0.3 mm ($D_{50}$ of the Fraser River sand is 0.3 mm) membrane penetration effects are negligible on the undrained behaviour (see Fig. 4.8).

Clearly, membrane penetration effects on undrained behaviour of a given sand will be greater in specimens with larger ratio of the surface area covered by the membrane to its volume (Wong et al. 1975, Martin et al. 1978, Nicholson et al. 1990, Nicholson et al. 1993b). The area/volume ratio of the UBC-HCT specimens (0.79 cm$^{-1}$) and the standard triaxial specimen (63 mm diameter) used in this study (0.63 cm$^{-1}$) are not too different. Nevertheless, the possible effect of this larger area/volume ratio on undrained behaviour was examined directly by performing comparative triaxial tests using the UBC-HCT and the standard triaxial devices (see Fig. 4.9). Identical initial stress states and void ratio states were used and no compensation for membrane
Figure 4.8 Error in cyclic resistance ratio due to membrane compliance as a function of $D_{50}$ (Evans et al., 1992).
compliance was allowed. The essentially identical stress-strain and pore pressure response implies that the membrane penetration effects are negligible for this sand.

4.7 Experimental Program

The main objective of this study was to investigate the undrained response of sands under general stress path loading. Emphasis was placed on assessing the nature of undrained anisotropy in loose sands and its consequences in practice, which often utilizes undrained behaviour measured in conventional axisymmetric triaxial compression tests - the test that shears specimens under fixed principal stress directions. This was accomplished by a study of the undrained behaviour in tests where principal stresses applied were controlled in magnitude and direction, as well as rotated during shear. In addition, the possible effects of intermediate principal stress on undrained response were assessed. Both monotonic and cyclic undrained loading responses were considered. Accordingly, an experimental program, described in the following sections, was developed. All tests (static and cyclic) were performed under strain controlled conditions at constant rate of 0.1%/min shear or axial strain. This prevented the occurrence of post-peak runaway strains inevitable in stress controlled loading of strain softening materials.

4.7.1 Static Tests

The static testing program comprised of the following groups of tests:

1. Effect of total stress paths on undrained response.

Effect of total stress paths for a fixed direction of major principal stress was investigated by imposing different total stress paths on specimens with identical initial conditions.
Figure 4.9. Response of triaxial tests performed using UBC-HCT and standard triaxial devices.
(stress state and relative density). During shear loading only total mean normal stress varied along selected stress paths, while $\sigma_d$ increased while $b$ and $\alpha_o$ were held constant.

2. Undrained anisotropy ($\sigma_d$ tests).

The sand was sheared by holding total mean normal stress $\sigma_m$, $\alpha_o$ and $b$ parameter constant at pre-specified values. Several tests at identical relative density, $\sigma_m$ and $b$, but different values of $\alpha_o$, enabled isolation of the effect of $\alpha_o$ alone on undrained response (i.e.; anisotropy or directional dependence). The degree of this undrained anisotropy was assessed with respect to possible influence of initial $b$ level, confining stress level and relative density.

3. Principal stress rotation tests ($\sigma_d - \alpha_o$ tests)

Sand with identical initial conditions (density and stress state) was sheared under constant $\sigma_d/\alpha_o$, $b$ and $\sigma_m$. The major principal stress direction, which coincides initially with the deposition direction ($\alpha_o = 0$), undergoes continuous rotation as $\sigma_d$ increases. These series of tests shed light on the influence of principal stress rotation on undrained behaviour.

4. Torsional shear (TS) tests.

Torsional shear strain $\gamma_{\theta\theta}$ was applied to initially anisotropically consolidated specimens ($\sigma_z > \sigma_r = \sigma_\theta$), while total normal stresses $\sigma_z$, $\sigma_r$ and $\sigma_\theta$ were held constant. This represented continuous rotation of major principal stress at constant $\sigma_m$ with increasing deviator stress $\sigma_d$.

5. Torsional simple shear (TSS) tests.

Simple shear tests on anisotropically consolidated specimens were performed to investigate the effect of principal stress rotation and that of the variation of intermediate
principal stress during simple shear deformation. Simple shear conditions were imposed
by arresting vertical deformation and maintaining constant sample volume and internal
cavity volume. Such simulated closely the constant volume simple shear conditions. This
simulation of simple shear response in the HCT device will not be subject to the severe
degree of stress non-uniformity inherent in the NGI or Roscoe type simple shear tests.

4.7.2 Cyclic Tests

A series of cyclic shear tests were performed on the loosest deposited sand at the initial
state represented by $\sigma'_{mc} = 200$ kPa and density $D_{re} = 30\%$. These included,

1. Conventional cyclic triaxial tests: These impose jump rotations in $\alpha_o$ between 0 and 90°
in each cycle, with simultaneous jumps in b value between 0 and 1.

2. Cyclic torsional shear tests, which imposed jump rotations in $\alpha_o$ between $-45^\circ$ and $45^\circ$,
$-60^\circ$ and $30^\circ$, $-75^\circ$ and $15^\circ$ Total mean normal stress $\sigma_m$ and b were held constant at 300
kPa and 0.5, respectively, during cyclic shearing.

Details of tests with initial states (relative density and confining stress) and the undrained
stress paths imposed are given in Tables 4.1 - 4.7. The subscript c in these tables refers to the end
of consolidation. FRS and SYN refer to Fraser River and Syncrude sands respectively.
Table 4.1. Effect of Total Stress Path.

\(\sigma_m\), \(b\) and \(\alpha_\sigma\) held constant during undrained shear

<table>
<thead>
<tr>
<th>Consolidation state</th>
<th>Undrained stress path</th>
<th>Remarks</th>
<th>Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dr_c (%)</td>
<td>(\sigma_{mc}^\prime) (kPa)</td>
<td>(b_c)</td>
<td>(\alpha_{\sigma c}) (deg)</td>
</tr>
<tr>
<td>30</td>
<td>200</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>200</td>
<td>0</td>
<td>0</td>
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</table>
Table 4.2. Undrained Anisotropy $\sigma_d$ Tests.

$\sigma_m$, $b$ and $\alpha_\sigma$ held constant during undrained shear

<table>
<thead>
<tr>
<th>Consolidation state</th>
<th>Undrained stress path</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_r$ (%)</td>
<td>$\sigma_{mc}$ (kPa)</td>
<td>$b_c$ (deg)</td>
</tr>
<tr>
<td>30</td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>200</td>
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<tr>
<td>30</td>
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<td>0</td>
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<tr>
<td>35</td>
<td>200</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td></td>
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</tr>
</tbody>
</table>

Sand
Table 4.3. Principal Stress Rotation Tests $\sigma_d$-$\alpha_\sigma$ Tests.

$\sigma_m$, $b$ and $\sigma_d / \alpha_\sigma$ held constant during undrained shear

<table>
<thead>
<tr>
<th>Consolidation state</th>
<th>Undrained stress path</th>
<th>Remarks</th>
<th>Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_c$ (%)</td>
<td>$\sigma'_mc$ (kPa)</td>
<td>$b$</td>
<td>$\sigma_d / \alpha_\sigma$ (kPa/deg)</td>
</tr>
<tr>
<td>30</td>
<td>200</td>
<td>0</td>
<td>1.5 2.5 3.0 5.0</td>
</tr>
</tbody>
</table>

Table 4.4. Torsional Shear Tests.

$\sigma_m$ held constant during shear

<table>
<thead>
<tr>
<th>Consolidation state</th>
<th>Undrained stress path</th>
<th>Remarks</th>
<th>Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_c$ (%)</td>
<td>$\sigma'_mc$ (kPa)</td>
<td>$b$</td>
<td>$\sigma_d / \alpha_\sigma$ (kPa/deg)</td>
</tr>
<tr>
<td>30</td>
<td>200</td>
<td>0</td>
<td>1.5 2.0</td>
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Normal stresses $\sigma_z$, $\sigma_r$, $\sigma_\theta$ held constant

<table>
<thead>
<tr>
<th>Remarks</th>
<th>Sand</th>
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<tbody>
<tr>
<td>Normal stresses $\sigma_z$, $\sigma_r$, $\sigma_\theta$ held constant</td>
<td>FRS</td>
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</table>

81
Table 4.5. Simple Shear Tests.

<table>
<thead>
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<th>Consolidation state</th>
<th>Remarks</th>
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</tr>
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<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>$D_r$ (%)</td>
<td>$\sigma'_{mc}$ (kPa)</td>
<td>$b_c$</td>
</tr>
<tr>
<td>30</td>
<td>200</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.6. Cyclic Triaxial Tests.

<table>
<thead>
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<th>Consolidation state</th>
<th>Cyclic stress ratio</th>
<th>Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_d / (2 \sigma_{3c}')$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>FRS</td>
</tr>
<tr>
<td>$D_r$ (%)</td>
<td>$\sigma'_{mc}$ (kPa)</td>
<td>$b_c$</td>
</tr>
<tr>
<td>30</td>
<td>200</td>
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</table>
Table 4.7. Cyclic Torsional Shear Tests.

\( \sigma_{m}, b \) and held constant during shear. \( \sigma_{1} \) subjected to jump rotation

<table>
<thead>
<tr>
<th>Consolidation state</th>
<th>Stress path</th>
<th>Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_r ) (%)</td>
<td>( \sigma'_{mc} ) (kPa)</td>
<td>( b_{c} )</td>
</tr>
<tr>
<td>30</td>
<td>200</td>
<td>0</td>
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<td>30</td>
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<td>0</td>
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<td>30</td>
<td>200</td>
<td>0</td>
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</table>

\( \sigma_{m} \) held constant during shear. \( \sigma_{1} \) subjected to continuous rotation

<table>
<thead>
<tr>
<th>Consolidation state</th>
<th>Stress path</th>
<th>Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_r ) (%)</td>
<td>( \sigma'_{mc} ) (kPa)</td>
<td>( b_{c} )</td>
</tr>
<tr>
<td>30</td>
<td>200</td>
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</table>
CHAPTER 5
RESULTS AND DISCUSSION

5.1 Introduction

Response of loose Fraser River and Syncrude sands under static and cyclic undrained loading conditions is presented in this chapter. The behaviour under general stress path loadings is presented in terms of effective stress path diagrams and deviator stress - maximum shear strain responses. This enables assessment of the effects of principal stress directions and their rotations on undrained behaviour. Deviations in the observed direction of major principal strain increment, in relation to the directions of major principal stress and stress increment during shear, are presented and discussed in a systematic manner. Unless otherwise stated, loosest deposited sand specimens were hydrostatically consolidated prior to shearing under a mean effective stress of 200 kPa. The relative density at the conclusion of consolidation was found to be about 30% (e_c = 0.905) for Fraser River sand. For Syncrude sand, consolidation to a mean hydrostatic effective stress of 400 kPa resulted in a relative density of about 40% (e_c = 0.790).

The discussion on static undrained response is divided into two sections. In the first section, test results aimed at investigating inherent undrained anisotropy (σ_d series) are presented. The effects of total stress path, confining stress level, relative density, inclination of the major principal stress direction with sand deposition direction and intermediate principal stress level are systematically assessed. Behaviour under similar stress paths is examined for the two sands.

In the second section, static undrained behaviour under stress paths involving rotation of principal stresses is presented. Behaviour in torsional simple shear simulated by the HCT device
and in conventional simple shear are also described and compared. Thus the possible effects of principal stress rotation on undrained behaviour are assessed.

Cyclic undrained response is assessed both under continuous and 90° jump rotation of principal stresses. The 90° jump rotation is imposed not only between $\alpha_0 = 0$ and 90°, that is characteristic of the conventional triaxial test, but also between $\alpha_0 = \alpha$ and $(90-\alpha)^\circ$, about the deposition direction. This was intended to inhibit invoking the weakest loading mode typical of the triaxial extension test. Cyclic resistance curves from different types of cyclic tests are also presented and compared. A number of specimens were loaded post cyclic monotonically in axial compression ($\alpha_0 = 0$) in order to define the post cyclic stress-strain response, as influenced by the nature of prior cyclic loading.

The entire study of undrained behaviour undertaken constitutes exploration of the regions of stress space that will not induce serious levels of non-uniformity in stress distribution. The only exception is the stress path with $b = 0$ and $\alpha_0 = 90^\circ$, which may induce non-uniformities larger than acceptable. Its possible effects on sand behaviour will be discussed later in sub-section 5.3.2.

As demonstrated in chapter 4, the effect of membrane penetration is negligible on the undrained behaviour of Fraser River sand. This effect may be even smaller for Syncrude sand which has a smaller mean grain size ($D_{50}$ of 0.2 mm) than that of Fraser River sand. No attempt was, therefore, made to insist on truly undrained tests using the water injection technique.
5.2 Drained Hydrostatic Loading

The strain response during hydrostatic compression of Fraser River sand commencing from an initial hydrostatic stress state of 50 kPa and proceeding up to 200 kPa is illustrated in Fig. 5.1. The strain paths at two relative densities are shown in the $\varepsilon_r$ (radial strain) - $\varepsilon_z$ (axial strain) space. Volumetric strains were suitably corrected for membrane penetration effects as recommended by Vaid and Negussey (1984). The measured relationship between the confining stress and unit membrane penetration is included in Appendix A.

Higher deformability in the horizontal than in the vertical (deposition) direction may be noted. Radial strain $\varepsilon_r$ and tangential strain $\varepsilon_\theta$ were found to be identical, suggesting that the material response is cross-anisotropic. With increasing relative density, the strain ratio $\varepsilon_r/\varepsilon_z$ decreases, implying that the degree of inherent anisotropy decreases accordingly. Similar observations have been reported by others (Wijewickreme, 1990 - using the HCT device; El-Sohby and Andrawes, 1972, Rowe, 1971, Negussey, 1984 - using the triaxial device). Considering that the strain paths are linear, it may be pointed out that for a given relative density, the degree of inherent anisotropy is not altered by hydrostatic compression, at moderate stress levels (Rowe, 1971). Identical strains $\varepsilon_z = \varepsilon_r = \varepsilon_\theta$ or $\varepsilon_{vol} = 3\varepsilon_1$ would have resulted, if the sand was isotropic.

Studies by Oda (1972) show that during vertical deposition through air or water, non-spherical grains prefer to rest with their long axis oriented in the horizontal direction, and therefore particle contact normals are preferentially oriented in the vertical deposition direction. This creates an anisotropic fabric. Consequently, for Fraser River sand, which is comprised of non-spherical particles, strains induced on hydrostatic compression loading are smaller in the direction of contact normals than that in the orthogonal horizontal directions. Densification
Figure 5.1. Strain response under hydrostatic loading.

\[ \Delta \varepsilon_r / \Delta \varepsilon_z = 2.00, \ D_r = 30\% \]

Hydrostatic loading
from \( \sigma'_m = 50 \) to 200 kPa

\[ \Delta \varepsilon_r = \Delta \varepsilon_\theta \]
clearly tends to alter the orientation of particle contact normals, causing a reduction in the degree of inherent anisotropy, as reflected by a decrease in the strain ratio $e_1/e_2$ (Fig. 5.1).

5.3 Undrained Static Behaviour with Fixed Principal Stress Directions

5.3.1 Effect of Total Stress Path

Conventional triaxial compression (constant cell pressure) behaviour of Fraser River sand using the HCT device is shown in Fig. 5.2 by solid lines. Changes in the total mean normal stress in this test are represented by $\sigma_m = \sigma_{mc} + \Delta(\sigma_1-\sigma_3)/3$, where $\sigma_{mc}$ is the total mean normal stress prior to the start of shear loading. In this test $b = 0$ and $\alpha_\sigma = 0$. Undrained triaxial compression behaviour on an identical specimen under a different total stress path with $\sigma_m =$ constant ($= \sigma_{mc}$) but identical, $b = 0$ and $\alpha_\sigma = 0$ is also shown in Fig. 5.2 by dashed lines. The effective stress path and stress-strain response may be noted to be identical. Similar behaviour has been demonstrated in several earlier studies (Henkel, 1960; Vaid and Campanella, 1974; Bishop and Wesley, 1975; Vaid et al., 1988; Kuerbis, 1989). This is an affirmation of the statement that for a given strain path (or fixed principal stress directions) and constant $b$ parameter, undrained behaviour of sand is independent of the total stress path.

Results similar to those shown in Fig. 5.2, but with $\alpha_\sigma = 30^\circ$ and $b = 0$ are illustrated in Fig. 5.3. Again independence of undrained behaviour with respect to the total stress path may be noted, even though this time the major principal stress direction is not coincident with the deposition direction, as is the case with triaxial compression. Uniqueness of the effective stress path requires that the pore pressure induced during undrained loading to be dependent on the total stress path. That this would be the case at constant $b$ and $\alpha_\sigma$ is explained in the following.
Figure 5.2. Effective stress path and deviator stress-strain response at $\alpha_\sigma$ and $b = 0$. 

During shear, $b$ and $\alpha_\sigma$ held constant

$\sigma_0' = 200$ kPa, $b = 0$, $\alpha_\sigma = 0^\circ$, $D_\varepsilon = 30\%$
Figure 5.3. Effective stress path and deviator stress - strain response at $\alpha_\sigma = 30^\circ$ and $b = 0$. 

During shear, $b$ and $\alpha_\sigma$ held constant

$\sigma_{mc} = 200$ kPa, $b = 0$, $\alpha_\sigma = 30^\circ$, $Dr_e = 30\%$
Excess pore water pressure generated in a saturated sand could be divided into that due to hydrostatic and shear induced components. Hydrostatic undrained loading of a saturated soil induces an equal amount of pore pressure. A given total stress increment tensor can be divided into a hydrostatic component and a deviatoric component as shown below. Note, that the division represented is unlike used in mechanics where the hydrostatic tensor component represents $\Delta \sigma_m$ and not $\Delta \sigma_3$.

\[
\begin{bmatrix}
\Delta \sigma_1 & 0 & 0 \\
0 & \Delta \sigma_2 & 0 \\
0 & 0 & \Delta \sigma_3
\end{bmatrix}
= \begin{bmatrix}
\Delta \sigma_1 & 0 & 0 \\
0 & b(\Delta \sigma_1 - \Delta \sigma_3) + \Delta \sigma_3 & 0 \\
0 & 0 & \Delta \sigma_3
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\Delta \sigma_3 & 0 & 0 \\
0 & \Delta \sigma_3 & 0 \\
0 & 0 & \Delta \sigma_3
\end{bmatrix}
= \begin{bmatrix}
\Delta \sigma_1 - \Delta \sigma_3 & 0 & 0 \\
0 & b(\Delta \sigma_1 - \Delta \sigma_3) & 0 \\
0 & 0 & 0
\end{bmatrix}
\] (5.1)

It can be seen from the above equation that for a given $b$ value at a given deviator stress level $\sigma_1 - \sigma_3$, the total stress increments $\Delta \sigma_1, \Delta \sigma_2$ and $\Delta \sigma_3$ can be divided into an identical deviator stress tensor and a hydrostatic stress tensor. The magnitude of the hydrostatic stress increment equals the increment in total minor principal stress. This manner of dividing the total stress increment tensor implies that for a given $b$ value, deviator stress increment tensor is independent of the total stress path, and the differences in induced pore pressure for different total stress paths are a mere manifestation of the difference in the hydrostatic stress increment tensor. Since the undrained response of sands depend only on the deviator stress tensor, effective stress path and the stress-strain response should be independent of the total stress path, and this independence of
undrained response with total stress path would be valid for any other constant value of b, and not merely confined to the usually quoted axisymmetric stress conditions of the triaxial compression test (b = 0 and $\alpha_\sigma = 0$).

The above analysis of pore pressures developed in response to total stress increments would render incorrect the usual assumption that pore pressure development can be related to the increments of mean normal stress and shear stresses. It will be correctly related to the increments in $\Delta \sigma_3$ and $(\Delta \sigma_1 - \Delta \sigma_3)$ together with the associated b value. The data presented in Fig. 5.3 under multi-axial loading conditions are evidence in support of this contention. In all tests reported in this study, the total mean normal stress during shear was held constant, for convenience.

5.3.2 Initial Undrained Anisotropy ($\sigma_d$ Tests)

Figures 5.4 and 5.5 present the effective stress paths and deviator stress-strain response of Fraser River and Syncrude sands respectively, sheared at several constant values of $\alpha_\sigma$. Prior to shearing, the stress states of each specimen was $\sigma_{\text{me}} = 200$ kPa and $D_r = 30\%$ ($e_c = 0.905$) for Fraser River sand and 400 kPa and 40$\%$ ($e_c = 0.790$) respectively, for Syncrude sand. Shearing was carried out by holding total mean normal stress $\sigma_m$ constant. The b parameter was also held constant at 0 for Fraser River sand and 0.5 for Syncrude sand. A very profound undrained anisotropy is apparent in both sands. As the direction of $\sigma_1$ deviates from the deposition direction ($\alpha_\sigma = 0$) towards that of the bedding plane ($\alpha_\sigma = 90^\circ$), undrained response changes from dilative to contractive for Fraser River sand and less contractive to more contractive for Syncrude sand.

As pointed out earlier, the level of stress non-uniformity in the HCT specimen for the stress path $\alpha_\sigma = 90^\circ$ and $b = 0$ would be slightly higher than the allowable levels suggested by
Figure 5.4. Effective stress path and deviator stress - strain response at $b = 0$ for Fraser River sand.
Figure 5.5. Effective stress path and deviator stress - strain response for Syncrude sand.
Wijewickreme (1990). This is likely to result in the deviator stress strain response somewhat weaker than if non-uniformities were within the acceptable level.

The dramatic change in undrained response with $\alpha_\sigma$ may be attributed to the fabric anisotropy formed by the parallel alignments of sand particles during the deposition process. Higher deformability in the horizontal than in the vertical direction during hydrostatic drained compression (see Fig. 5.1) would support the above conclusion, as do the observations on volumetric deformations which, for a given $\sigma_d$, increase with increase in $\alpha_\sigma$ (Sayao, 1989; Wijewickreme, 1990).

Figures 5.4 and 5.5 show that both sands undergo steady state (SS) type response when major principal stress is directed along the bedding planes ($\alpha_\sigma = 90^\circ$). Syncrude sand shows steady state response even at $\alpha_\sigma = 45$ and $60^\circ$. In contrast, Fraser River sand suffers limited liquefaction type of response at $\alpha_\sigma = 30^\circ$ to $60^\circ$, that eventually changes into dilative response at $\alpha_\sigma = 0$.

The maximum shear stress $(\sigma_1-\sigma_3)/2$ at phase transformation (PT) state or steady state (SS) for initially identical void ratio and confining stress level depends on the direction of major principal stress $\alpha_\sigma$ (Figs. 5.4 and 5.5). This is in variance to the claim by Castro (1969) and Ishihara (1993) who associate the maximum shear stress $(\sigma_1-\sigma_3)/2$ at steady state to void ratio alone, based only on triaxial compression loading. It is clear that under multi-axial loading conditions, the phase transformation state or steady state strength of a sand is not a function of void ratio alone.
The degree of undrained anisotropy in loose Fraser River sand under non-axisymmetric stress states at b values other than zero (b = 0.5 and 0.4) is illustrated in Figs. 5.6 and 5.7. The results show behaviour similar to that observed under b = 0 (see Fig. 5.4).

The proponents of the steady state concepts have used test results from moist tamped triaxial specimens in formulating these concepts (Castro, 1969; Sladen et al., 1985; Ishihara, 1993). This specimen reconstituting technique is used solely for the purpose that contractive deformation be triggered because of the very loose density states, that can be simulated by moist tamping. Such loose density states may not even be accessible to the sand on water deposition. Contractive deformation of moist tamped specimen over a wide range of relative density may thus be due to the collapse of its unstable honeycomb fabric (Casagrande, 1976). Studies on the effect of different specimen reconstitution techniques on undrained behaviour of Syncrude sand have shown that at identical initial void ratio and effective stress state, moist tamped sand is highly contractive, but water pluviated sand is dilative even in the loosest deposited state (Vaid et al., 1995). As noted in chapter 1, moist tamping may also inhibit a strong anisotropic fabric that ensues on water pluviation. A single steady state strength, which is a function of void ratio alone may be an attractive concept, but hydraulically deposited sands may not conform to such a concept. If laboratory results obtained using reconstituted specimens are to be meaningful in practice, care should be exercised when selecting the appropriate specimen reconstitution technique that simulates closely the deposition process of the in-situ sand. In addition, stress path dependence of undrained behaviour typical of the anticipated field loading paths must be recognized.
Figure 5.6. Effective stress path and deviator stress - strain response at $b = 0.5$ for Fraser River sand.
Figure 5.7. Effective stress path and deviator stress - strain response at $b = 0.4$ for Fraser River sand.
Radical difference between triaxial compression ($\alpha = 0$ and $b = 0$) and extension ($\alpha = 90^\circ$ and $b = 1$) response of sands - including Fraser River sand - have been reported in a number of previous studies (Bishop, 1971; Hanazawa, 1980; Miura and Toki, 1982; Kuerbis et al., 1988; Vaid et al., 1990a; Vaid and Thomas, 1995). These differences are a combined effect of different $\alpha$ and $b$ parameter in the two tests, and thus the effect of $\alpha$ alone cannot be isolated. Many sands water deposited in the loosest state show dilative response in compression but are strongly contractive in extension. Similarly, drained tests on water pluviated sands show that larger volumetric deformation ensue as $\alpha$ deviates more and more from the deposition direction (Yamada and Ishihara, 1979; Symes et al., 1982; Sayao, 1989; Wijewickreme, 1990).

**Initiation of Contractive Deformation**

The degree of strain softening of a contractive sand can be conveniently characterized by the brittleness index $I_B$ (Bishop, 1971),

$$ I_B = \frac{\sigma_{d\text{peak}} - \sigma_{d\text{min}}}{\sigma_{d\text{peak}}} $$

in which $\sigma_{d\text{peak}}$ and $\sigma_{d\text{min}}$ are the peak and minimum deviator stresses, respectively.

Brittleness index is considered indicative of the flow potential of a contractive sand. The dependence of brittleness index on $\alpha$, for the data in Figs. 5.4 and 5.5 is shown in Fig. 5.8. A dramatic increase in $I_B$ with $\alpha$ may be noted. Syncrude sand, which was consolidated to a mean effective stress of 400 kPa prior to undrained shear, shows significantly higher values of $I_B$ than
For each test $\sigma_m$, $b$ and $\alpha_\sigma$ held constant during shear.

Syncrude sand, $D_r = 40\%$
$\sigma'_{mc}=400$ kPa, $b=0.5$

Fraser River sand
$\sigma'_{mc}=200$ kPa, $D_r=30\%$

Figure 5.8. Variation of brittleness index with $\alpha_\sigma$. 
Fraser River sand at a lower mean effective stress (200 kPa). Additional data on I_B for Fraser River sand at b = 0.5, but otherwise identical stress conditions show a somewhat increase in I_B with increase in b parameter. The effect of b alone on undrained behaviour is further discussed in section 5.3.3.

Contractive deformation in Fraser River sand, at the stress and void ratio considered prior to undrained shear, occurs when \( \alpha_e \) is higher than 30° for the b = 0 path (Fig. 5.4). The friction angle \( \phi_{CSR} \) mobilized at the peaks of deviator stress for the states that showed contractive response is shown in Fig. 5.9a by open circles, as a function of \( \alpha_e \). A systematic decrease in \( \phi_{CSR} \), and hence trigger of contractive deformation at progressively smaller mobilized effective stress ratio, may be observed as \( \alpha_e \) increase from 30° to 90°. Similar variation of \( \phi_{CSR} \) was observed during shear at other b values. A decrease in \( \phi_{CSR} \) with increasing \( \alpha_e \) has important implications on the undrained cyclic loading response, as will be shown later in section 5.5.

Triaxial test data from Vaid and Thomas (1995) show that \( \phi_{CSR} \) in triaxial compression (\( \alpha_e = 0 \) and b = 0) for Fraser River sand is about 26° and independent of the relative density and confining stress. Vaid and Thomas (1995) also show that under triaxial extension (\( \alpha_e = 90° \) and b = 1), \( \phi_{CSR} \) is much smaller than its value in compression and tends to increase with increase in deposition density. Chern (1985), Chung (1985), and Kuerbis (1989) have also shown that \( \phi_{CSR} \) of other Ottawa and Brenda mine tailings sands in triaxial extension is much lower than the compression values.

The dependence of peak shear strength \( S_{u_{peak}} \) (\( S_u = \sigma_d/2 \)) on \( \alpha_e \) for contractive Fraser River sand at b = 0 is shown in Fig. 5.9b by open circles. A systematic decrease in \( S_{u_{peak}} \) may be observed as \( \alpha_e \) increases. Shibuya and Hight (1987) also report such decreases in peak undrained
Figure 5.9. Variation of (a) $\phi_{CSR}$ and (b) peak shear strength with $\alpha_\sigma$. 
shear strength of Ham River sand and glass ballotini as $\alpha_\sigma$ increases from 0 to 90°. Their observations, however, correspond to stress controlled loading in the HCT device, in which the peak condition cannot be captured precisely.

$\sigma_d$ tests with $\alpha_\sigma = 45°$ and $b = 0$ were also performed on anisotropically consolidated Fraser River sand. The states prior to undrained shear were $K_c (= \sigma'_{1o}/\sigma'_{3o}) = 1.5$ and 2.0, $\sigma'_mc = 200$ kPa, $b = 0$, $\alpha_\sigma = 45°$ and $Dr_c = 30\%$. Solid circles in Figs. 5.9a and 5.9b show $\phi_{CSR}$ and $Su_{peak}$ respectively for $K_c = 1.5$. $\phi_{CSR}$ and $Su_{peak}$ appear unique for a given $\alpha_\sigma$ and $b$, and do not depend on the consolidation history, isotropic or anisotropic.

Steady State and Phase Transformation State

Effective stress conditions at phase transformation state (open circles) and steady state (solid circles) are shown in Fig. 5.10 for Fraser River sand. For clarity in presentation, data is shown separately for shear under $b = 0$, 0.4 and 0.5. Figure 5.11 show similar data for Syncrude sand at $b = 0.5$. Stress conditions at phase transformation state for the two anisotropically consolidated Fraser River sand are also shown in Fig. 5.10a.

Each symbol in Figs. 5.10 and 5.11 corresponds to a single $\sigma_d$ test at different, constant $\alpha_\sigma$. The phase transformation state is defined, in the triaxial test, as the instance corresponding to the peak of excess pore pressure, which occurs when the undrained response changes from contractive (positive pore pressure generation) to dilative (negative pore pressure generation) (Ishihara et al., 1975). The same definition under multi-axial loading, is used herein for both dilative and limited liquefaction types of response. If steady state (liquefaction type response) was reached, the effective stress conditions at the steady state are shown by solid circles.
Figure 5.10. Effective stress conditions at phase transformation state for Fraser River sand at (a) $b = 0$, (b) $b = 0.4$ and (c) $b = 0.5$. 

$\sigma_{mc} = 200$ kPa, $b = 0$, $Dr_c = 30\%$

$\sigma_m$, $b$ and $\alpha_g$ held constant during shear.
Figure 5.11. Effective stress conditions at phase transformation state for Syncrude sand.

$\sigma'_{mc} = 400$ kPa, $b = 0.5$

$D_{r_c} = 40\%$

$\sigma_m$, $b$ and $\alpha_\sigma$ held constant during shear

$\alpha_\sigma = 0$

$\sigma'(1 - \sigma'_{3})/2$ (kPa)

$(\sigma'_1 + \sigma'_3)/2$ (kPa)

Solid circle - S.State

30, 45

60, 90

105
The effective stress conditions at phase transformation (or steady state for tests which realized that state) may be seen to lie on unique straight lines, which pass through the origin, regardless of the direction of major principal stress $\sigma_\alpha$ or the consolidation history. The slope of these lines is also independent of the level of intermediate principal stress parameter $b$. For Fraser River sand, the friction angle mobilized at phase transformation state is about $33^\circ$ and for Syncrude sand about $35^\circ$. Clearly, the friction angle $\phi_{\text{ss}}$ mobilized at steady state for the sands tested, equals that at phase transformation state. Triaxial compression and extension tests on Fraser River sand (Vaid and Thomas, 1995) and a tailings sand (Vaid and Chern, 1985) also confirm that $\phi_{\text{PT}} = \phi_{\text{ss}}$.

Independence of $\phi_{\text{PT}}$ with respect to stress path, relative density, consolidation history or mode of loading has been reported by Bishop (1966, 1971), Ishihara et al. (1975), Vaid and Chern (1985), Kuerbis et al. (1988), and Vaid et al. (1990a) for other sands, based on sand behaviour in the triaxial test. In an attempt to seek a broader significance of this friction angle mobilized at phase transformation state, Vaid and Chern (1985) have shown that $\phi_{\text{PT}}$ equals the mobilized friction angle at steady state, and Negussey et al. (1988) have shown that $\phi_{\text{PT}}$ equals constant-volume friction angle $\phi_{\text{cv}}$ of the sand under drained conditions.

Continued straining beyond the phase transformation state causes little additional change in the mobilized friction angle (a maximum of 2 to 3$^\circ$). The effective stress paths then become tangent to a line of maximum obliquity. This angle of maximum obliquity is unique for a given sand; $36^\circ$ for Fraser River and $37^\circ$ for Syncrude sand, and is not influenced like $\phi_{\text{PT}}$ by the initial state or the stress path during shear.
The relationship between undrained shear strength ($S_{uPT}$) at phase transformation or steady state for contractive deformation, and $\alpha_\sigma$ is shown in Fig. 5.12 for both sands. At a given $b$ value, a systematic decrease in $S_{uPT}$ with $\alpha_\sigma$ may be noted, and for a given $\alpha_\sigma$, $S_{uPT}$ decreases somewhat with increase in the $b$ parameter. $S_{uPT}$ at $\alpha_\sigma = 90^\circ$ is only 1/3 of that at $\alpha_\sigma = 0$ for Fraser River sand. For this sand, it appears that the influence of intermediate principal stress parameter on $S_{uPT}$ at a given $\alpha_\sigma$ become smaller as the major principal stress direction orients towards the bedding planes. $S_{uPT}$ somewhat increase with the level of $K_c$ at $\alpha_\sigma = 45^\circ$ and $b = 0$.

Further discussion on the response of anisotropically consolidated sands is presented in section 5.4.2. As pointed out earlier, for the stress path $b = 0$ and $\alpha_\sigma = 90^\circ$, the nonuniformity in stress distribution is likely to be higher than the acceptable level, and hence, $S_{uPT}$ for $\alpha_\sigma = 90^\circ$ shown in Fig 5.12a is most likely an underestimate.

Principal stress directions in soil elements vary along potential failure surfaces commonly encountered in the field. In a slope susceptible to flow slide, for example, $\alpha_\sigma$ may range approximately from 0 at the crest to 90° at the toe. Thus, in hydraulically placed granular embankments, undrained shear strength at steady phase transformation state may decrease as $\alpha_\sigma$ increases from zero at the crest to 90° at the toe. Use of a single undrained shear strength for a given void ratio estimated from triaxial compression test ($\alpha_\sigma = 0, b = 0$) alone may therefore overestimate the factor of safety against a flow failure.

If variation in undrained steady state strength along the potential failure surface was taken into account in the analysis of the failed zone of the Lower San Fernando dam, it may not have been necessary to apply a reduction factor of 20 to the measured triaxial compression strength. Castro et al. (1985) used this large reduction factor in order to obtain the back-analyzed strength.
Figure 5.12. Variation of phase transformation strength with $\alpha_\sigma$. 

- (a) Fraser River sand
  - $\sigma'_m = 200$ kPa, $D_r = 30\%$
  - $\sigma_m$, $b$, and $\alpha_\sigma$ held constant during shear
  - $K_e = 2$, $b = 0.0$, $b = 0.5$

- (b) Syncrude sand
  - $\sigma'_m = 400$ kPa, $D_r = 40\%$
  - $b = 0.5$
  - $\sigma_m$, $b$, and $\alpha_\sigma$ held constant during shear
  - $\alpha_\sigma$ (deg) range from 0 to 100.
at failure. A similar study of offshore liquefaction failure (Nerlerk berm slides) was performed by Sladen et al. (1985b). Back analysis of the failed slope, which inherently assumed a single undrained strength throughout the failure surface, shows the relative density of the sand in the failed region should have been about 15-20% less than that measured in-situ. Parameters used for the back analysis were obtained from laboratory triaxial compression tests, and in-situ relative density was derived from cone penetration data (Sladen et al., 1985a, 1985b). Sladen et al. blamed cone penetration test data for this difference in the measured and back calculated relative densities, whereas the real reason might rest in the undrained strengths substantially less than the apparently maximum ($\alpha_o = 0$) observed in the triaxial compression mode.

**Strain Increment Directions**

Changes in directions of major principal strain increment $\alpha_{ae}$ for undrained shearing at $\alpha_o = 15, 30, 45^\circ$ and $60^\circ$ are shown in Figs. 5.13a and 5.13b for Fraser River and Syncrude sands respectively. In these $\sigma_d$ tests where $\alpha_o$ is fixed, principal stress increment direction $\alpha_{ae}$ coincides with the principal stress direction (i.e.; $\alpha_{ae} = \alpha_o$). Except at small strain levels (the pre-peak domain), the direction of strain increment for a given $\alpha_o$ is close to that of the stress direction. Wijewickreme (1990) and Sayao (1989) reported similar findings in drained shear of Ottawa sand. Arthur et al. (1981), however, found no difference between stress and strain increment directions in studies on drained behaviour of sand in the directional shear cell. At the other end of the spectrum, Symes et al. (1984) report large deviations in principal strain increment and stress directions during undrained shear of Ham River sand.
Figure 5.13. Direction of principal strain increments and stress for (a) Fraser River, and (b) Syncrude sands.
For deformation under $\alpha_\sigma = 0$ and $90^\circ$, principal stress and stress increment directions coincide with the principal axes of anisotropy, (i.e.; axial and radial). Hence the directions of principal stress and strain increments coincide in triaxial compression and extension modes of deformation.

During shear deformation granular materials experience a continuous evolution of inter-particle contacts and their orientations and hence anisotropy (Oda, 1985). Numerical simulations by Rothenburg (1992) show that the particles change their orientation such that the contact normals along the major principal stress direction stay always in excess of those in any other direction. It appears therefore that contact normals which predominate in the deposition direction at the beginning, would tend to change their orientation gradually along the imposed major principal stress direction $\alpha_\sigma$.

Figure 5.13 suggests that the contact orientations have reached an essentially stationary state by the time the effective stress ratio corresponding to the phase transformation/steady state is mobilized. Little change in $\alpha_{\Delta e}$ between the phase transformation and maximum obliquity states is apparently because of a very small increase in the mobilized friction angle between the two states.

Figure 5.14 shows strain paths in $\varepsilon_2/\varepsilon_3$ space in $\sigma_d$ shear of Fraser River sand at $b = 0$ (Fig. 5.4). The equality of the two strain increments (slope of $45^\circ$) beyond the phase transformation/steady states is an indication that the evolved anisotropy may also be of the cross anisotropic type about the new axis of symmetry $\alpha_\sigma$ for each $\alpha_\sigma$ loading.
Figure 5.14. Strain paths at b = 0.

\[ \sigma'_m = 200 \text{ kPa}, \ \beta = 0, \ \Delta r_e = 30 \% \]

\( \sigma_m, \ \beta \) and \( \alpha_\sigma \) held constant during shear.
Effect of Intermediate Principal Stress Parameter on Undrained Anisotropy

Undrained response of Fraser River sand at constant values of $b = 0, 0.25, 0.4, 0.5$ and $1$ and a fixed value of $\alpha_o = 45^\circ$ is shown in Fig. 5.15. Similar response at $\alpha_o = 90^\circ$ and $b = 0, 0.5$ and $1$ is presented in Fig. 5.16. The conditions prior to undrained shear in each case correspond to $D_r = 30\%$ and $\sigma_{mc} = 200$ kPa. $\sigma_m$, $\alpha_o$ and $b$ were held constant during shear. At $\alpha_o = 45^\circ$ a small increase in the degree of contractiveness may be observed with increase in $b$ parameter in the region $b = 0$ to $0.5$. But for $b = 1$, PT/SS strengths are substantially smaller than those corresponding to other $b$ values. Peak undrained shear strength ($\sigma_{dpeak}/2$) for $b = 1$ is also substantially smaller (about 25%) compared to the value at $b = 0$.

The dependence of $\phi_{CSR}$ and peak undrained shear strength on $b$ parameter for the direction $\alpha_o = 45^\circ$ is shown in Figs. 5.17a and 5.17b respectively. These values were obtained from the data shown in Fig. 5.15. The effect of $b$ parameter on both $\phi_{CSR}$ and peak shear strength may be noted to be small when $b$ is less than $0.5$, but $\phi_{CSR}$, in particular, suffers a dramatic degradation when $\alpha_o = 90^\circ$.

The dependence of brittleness index and phase transformation/steady state strength on $b$ parameter are shown in Figs. 5.18a and 5.18b respectively. An increase in brittleness index and decrease in phase transformation/steady state strength with increase in $b$ parameter may be noted. The friction angle $\phi_{PT}$ mobilized at phase transformation/steady state, however is essentially constant and equals the value at $b = 0$ (Fig. 5.19a). Changes in $I_B$ and phase transformation state or steady state strength however, do not appear to be very significant for the range of $b$ between $0$ and $0.5$. On the other hand, for $b = 1$, significantly higher $I_B$ and lower phase transformation/steady state strength values are noted compared to those at $b = 0$. It
Figure 5.15. Effective stress path and deviator stress - strain response at $\alpha_\sigma = 45^\circ$. 

\begin{align*}
\sigma'_m &= 200 \text{ kPa} \\
\alpha_\sigma &= 45, \quad D_r = 30\% \\
b &= 0
\end{align*}
Figure 5.16. Effective stress path and deviator stress-strain response at $\alpha_g = 90^\circ$.
Figure 5.17. Variation of (a) $\phi_{\text{CSR}}$ and (b) peak undrained shear strength with $b$ parameter.
Figure 5.18. Variation of (a) brittleness index and (b) phase transformation strength with b parameter.
Figure 5.19. (a) Effective stress conditions at PT state (b) Variation of intermediate principal strain with maximum shear strain.
appears that at a given \( \sigma_\alpha \), the effect of intermediate principal stress level may not be significant when \( b \) is less than about 0.5. Drained torsion shear tests on Ottawa, sand in a study of shear modulus and damping in sand, also confirm such a conclusion (Uthayakumar, 1992).

Figure 5.19b shows the relationship between intermediate principal strain \( \varepsilon_2 \) and maximum shear strain \( \varepsilon_1 - \varepsilon_3 \), during shear at various \( b \) values and \( \alpha_\sigma = 45^\circ \). Since the strain increment direction in \( \sigma_d \) tests is essentially constant, the strain path in this strain space is linear. The intermediate principal strain during shearing is very close to zero throughout the stress path corresponding to \( b = 0.4 \). This implies, that virtually plane strain conditions prevailed along this undrained \( b = 0.4 \) path. Previous studies on drained behaviour of sands also support such a conclusion (i.e.; plane strain deformation for \( b = 0.4 \) path, Sayao 1989).

The triaxial extension deformation mode (\( \alpha_\sigma = 90^\circ \) and \( b = 1 \)) yields much smaller undrained steady state/phase transformation strengths than the plane strain mode (\( b = 0.4 \) to 0.5) at \( \alpha_\sigma = 90^\circ \) (see Figs. 5.16 and 5.18b). At the toe of an embankment where \( \alpha_\sigma = 90^\circ \), the soil elements would be closer to the plane strain extension rather than the axisymmetric triaxial extension state. The use of triaxial extension steady state/phase transformation strength for such elements in the analytical process, would therefore be on the conservative side.

The relative directions of stress and strain and stress increments are similar to those observed under \( b = 0 \) and \( \alpha_\sigma = 0 \) (Fig. 5.20). This would be expected, since under any \( b \) loading the direction of principal stresses remain fixed.

**Effect of Initial Confining Stress Level on Undrained Anisotropy**

Figure 5.21 presents effective stress paths and deviator stress-strain response of loose Fraser River sand under initial hydrostatic confining stress \( \sigma_{hc}' = 100, 200 \) and 400 kPa. The
Figure 5.20. Directions of principal stress and strain and stress increments.
Figure 5.21. Effective stress path and deviator stress - strain response.

\[\sigma_1 - \sigma_3 \quad (\text{kPa})\] 

\[\left(\frac{\sigma_1 + \sigma_2 + \sigma_3}{3}\right) \quad (\text{kPa})\] 

\[\sigma_m = 400 \text{ kPa}\]
relative density, at the conclusion of consolidation, was targeted at 30% for all specimens, and during shearing $\sigma_m$, $\alpha = 45^\circ$, $b = 0.5$ were held constant. An increase in $\sigma_{mc}'$ may be seen to enhance contractive tendency (larger $I_B$). Previous studies using the triaxial compression tests have also shown similar trend (Castro, 1987; Been et al., 1991; Ishihara, 1993; Vaid and Thomas, 1995).

The contractive deformation is triggered at a mobilized friction angle $\phi_{CSR}$ of approximately $22^\circ$, which is identical to that noted under $\sigma_{mc}'$ of 200 kPa under other conditions, i.e.; $b = 0$ and $\alpha = 45^\circ$ (Fig. 5.22).

The influence of initial confining stress level under multi-axial drained loading conditions has been investigated by Sayao (1989) and Wijewickreme (1990). They report that Ottawa sand when sheared at a fixed $\alpha$ undergoes much larger volumetric and shear strains at higher confining stress levels than that at low levels, at any mobilized stress ratio $\sigma_1'/\sigma_3'$. The similar effects of decreasing relative density at constant confining stress, and increasing confining stress at constant relative density, is a well recognised characteristic of granular materials. In contrast, the triaxial extension behaviour of Fraser River sand shows that its degree of contractiveness decreases instead of increasing with increase in confining stress level (Vaid and Thomas, 1995).

It is important to point out that for a given relative density (or void ratio) the shear strength at phase transformation or steady state increases with confining stress level (see Fig. 5.23). This is contrary to the steady state concepts which assume that this shear strength is a function only of the void ratio prior to undrained shear. Although the data acquired is not too extensive, the ratio $(S_{SS/PT})/(\sigma_{mc}')$ may be noted to be essentially constant at 0.2. These normalised residual strength concepts have been utilised in practice in the seismic evaluation of Duncan Dam in British Columbia (Pillai and Stewart, 1995). Pillai and Stewart (1995) also report
Figure 5.22. Variation of $\phi_{CSR}$ with $\sigma'_{mc}$.
Figure 5.23. Relationship between steady state/phase transformation strength and $\sigma_{\text{mc}}$. 

$\sigma_m$, $b$ and $\alpha_\sigma$ held constant during shear. 
$\alpha_\sigma = 45^\circ$, $b = 0.5$, $D_{\text{S}} = 30\%$
the ratio \((S_{SS/PT})/(\sigma'_{mc})\) for Duncan Dam foundation sand (unit 3c) to be about 0.2. Physical characteristics and gradation curves of Duncan Dam unit 3c sand are very similar to those of Fraser River sand.

Even though, the steady state/phase transformation strength of Fraser River sand depends on the level of \(\sigma'_{mc}\), the friction angle mobilized \((\phi_{PT})\) at this state is essentially constant and equals approximately to the value noted from the \(\sigma_d\) tests at \(\sigma'_{mc} = 200\text{ kPa}\) (Fig. 5.24a).

Steady state concepts were developed based on undrained triaxial compression test data only. The experimental evidence presented so far shows that such concepts cannot be extended to multi-axial loading. Test results on Fraser River sand by Vaid and Thomas (1995) also show, that even under triaxial loading, the concept of residual strength, that is dependent only on the void ratio is not valid.

The higher levels of \(\sigma'_{mc}\) do not alter the essential coincidence of principal stress direction and strain increment direction noted at the lower \(\sigma'_{mc} = 200\text{ kPa}\) level (Fig. 5.24b).

**Effect of Relative Density on Undrained Anisotropy**

Undrained behaviour of Fraser River sand at three relative densities of 30%, 35% and 50%, is shown in Fig. 5.25. The initial confining stress \(\sigma'_{mc}\) was identical in each case at 200 kPa. During undrained shearing, \(\sigma_o\) and \(b\) parameter were held constant at 60° and 0 respectively. Contractive response of the steady state type at \(Dr_c = 30\%\) may be noted to turn into a limited liquefaction type and finally dilative as \(Dr_c\) increases. Several studies in triaxial and simple shear also reveal that a contractive sand in loose state becomes dilative after densification (Bishop,
Figure 5.24. (a) Effective stress conditions at PT state, and (b) Variation of principal strain increment, stress increment and stress directions with maximum shear strain.
Figure 5.25. Effective stress path and deviator stress response - effect of relative density.
1971; Vaid et al., 1990a; Been et al., 1991; Vaid and Thomas, 1995; Vaid and Sivathayalan, 1995).

It may be noted from Fig. 5.26 that the contractive deformation is triggered at a $\phi_{CSR}$ of approximately 20° which equals the mobilized friction angle $\phi_{CSR}$ observed in $\sigma_d$ tests at $\alpha_d = 60°$. The friction at phase transformation/steady state ($\phi_{PT} = 33°$) is independent of relative density, and is essentially identical to that observed in other $\sigma_d$ tests. Also, increase in relative density does not appear to alter the coincidence of principal strain increment direction and stress direction (Fig. 5.27b, 5.7c and 5.27c).

5.3.3 Summary

Based on the experimental evidence presented it may be concluded that the undrained response of sands, under fixed principal stress direction and constant $b$, is independent of the total stress path.

Undrained response of loose sand is highly dependent on the loading direction. A sand which is dilative when the major principal stress coincides with the deposition direction may show limited liquefaction type response, which transforms eventually to the steady state type as the direction of major principal stress changes from the deposition direction to that of the bedding plane.

Steady state/phase transformation strength of a sand is not uniquely related to its void ratio alone, as suggested by Castro (1969) and Ishihara (1993). For a given void ratio, steady state/phase transformation strength depends, in addition, on the direction of principal stresses, and to some extent on the magnitude of $\sigma_2$. Thus the use of conventional triaxial compression test to estimate the steady state/phase transformation strength of sands may not be justifiable.
Figure 5.26. Variation of $\phi_{CSR}$ with $Dr_c$. 

$\sigma'_m=200$ kPa, $\alpha_\sigma=60^\circ$, $b=0$

$\sigma_m$, $b$ and $\alpha_\sigma$ held constant during shear.
Figure 5.27. (a) Effective stress conditions at PT state (b) Variation of principal strain increment, stress increment and stress directions with maximum shear strain.
Laboratory characterization of sand for solving practical problems should consider the anticipated range of principal stress directions and intermediate principal stress level in the field as being important variables that influence undrained behaviour.

Mobilised friction angle at peak deviator stress ($\phi_{CSR}$) for contractive response is a function of the major principal stress direction and intermediate principal stress parameter. For Fraser River sand $\phi_{CSR}$ decreases from about $26^\circ$ to $16^\circ$ as $\alpha_\sigma$ increase from $30^\circ$ to $90^\circ$. $\phi_{CSR}$ remain essentially constant as $b$ increases from 0 to 0.5. The friction angle at phase transformation/steady state is a unique material property, independent of the direction of principal stress, intermediate principal stress level and the stress and void ratio state prior to undrained shear.

The influence of intermediate principal stress, on undrained behaviour is small when $b$ parameter is less than about 0.5. Approximately plane strain conditions prevail ($\varepsilon_2 = 0$), when shearing occurs along a $b = 0.4$ path.

At constant values of other parameters, increasing confining stress and decreasing relative density under multi-axial loading promote a higher degree of contractive response.

5.4 Undrained Static Shear Loading With Principal Stress Rotation

5.4.1 Controlled Rotation of Principal Stresses ($\sigma_d-\alpha_\sigma$ Tests)

A series of undrained tests in which $\alpha_\sigma$ was increased in a controlled manner is described in this section. Prior to shearing, all specimens were isotropically consolidated to an effective stress of 200 kPa. The targeted relative density was 30%. During shear loading $\sigma_m$, $b$ and $\sigma_d/\alpha_\sigma$ were held constant. This represents simultaneous increase in $\sigma_d$ and $\alpha_\sigma$. Figure 5.28(a) and
Figure 5.28. (a) Effective stress path, (b) deviator stress, and (c) major principal stress direction variation with maximum shear strain.
5.28(b) show effective stress paths and deviator stress - strain response for several $\sigma_d/\alpha_o$ values. The degree of contractiveness may be noted to decrease with increase in $\sigma_d/\alpha_o$, and the response become almost dilative at the largest $\sigma_d/\alpha_o (= 5.0)$ used.

It was evident from the $\sigma_d$ series tests that the degree of contractiveness increases with $\alpha_o$. In the $\sigma_d - \alpha_o$ test series, shear loading with a smaller value of $\sigma_d/\alpha_o$ imposes larger rotation of principal stresses at a given $\sigma_d$. Therefore, it appears (Fig. 5.28) that larger the rotation of principal stresses for a given $\sigma_d$, the larger is the degree of contractive deformation. Limited studies on the effect of principal stress rotation by Symes et al. (1984,1985) and Shibuya and Hight (1987), on Ham River sand also show that principal stress rotation alone, under constant deviator stress, $\sigma_m$ and b, causes increase in pore pressures.

The rotation of major principal stress direction with maximum shear strain under controlled principal stress rotation is shown in Fig. 5.28(c). Largest rotations may be seen for the lowest $\sigma_d/\alpha_o (= 1.5)$. Since $\sigma_d/\alpha_o$ was held constant during shear, decrease in deviator stress in the post peak region accompanies corresponding decrease in $\alpha_o$.

The dependence of $\phi_{CSR}$ and $S_u$ with peak $\alpha_o$ experienced in $\sigma_d - \alpha_o$ shear are shown in Figs. 5.29(a) and 5.29(b) respectively by open circles. The solid lines correspond to these relationships noted in $\sigma_d$ tests (see Fig. 5.9). In $\sigma_d$ tests, $\alpha_o$ was fixed and not allowed to change during shear loading. Also, for both series of tests, the initial stress states and relative densities were identical. It may be observed that the relationship between $\phi_{CSR}$ and $S_u$ with $\alpha_o$ in $\sigma_d$ tests and $\sigma_d - \alpha_o$ tests are not too different. This essential identity in the relationships implies that the effect of principal stress rotation or the history of $\alpha_o$ imposed may not be important, and only
\[ \sigma_d / \alpha_d = 5.0 \]

\[ \sigma_{mc} = 200 \text{ kPa}, \ b = 0, \ \text{Dr} = 30 \%
\]

\( \sigma_m, \ b \) and \( \sigma_d / \alpha_d \) held constant during shear.

Solid line from \( \sigma_d \) tests.

Figure 5.29. Variation of (a) \( \phi_{CSR} \) and (b) peak shear strength with peak \( \alpha_d \).
the peak value of $\alpha_\sigma$ experienced governs $\phi_{CSR}$ and $S_{peak}$. Further evidence in support of this conclusion will be presented in the following sections.

Effective stress conditions at phase transformation state in $\sigma_d - \alpha_\sigma$ loading are shown in Fig. 5.30(a) by data points. The phase transformation line found from the $\sigma_d$ series tests is also shown. Clearly the friction angle mobilized at the phase transformation state is independent of whether principal stress directions stayed fixed or were rotated.

Figure 5.30(b) presents the relationship between phase transformation steady state strength and maximum $\alpha_\sigma = (\text{open circles})$. The solid line represents this relationship in $\sigma_d$ shear. The essential identity of the relationship under principal stress rotations and no rotations implies that only the peak value of $\alpha_\sigma$ experienced during shearing governs the peak and post peak undrained response of sand.

Variations in the major principal strain increment direction, stress increment direction and stress direction with maximum shear strain in the $\sigma_d - \alpha_\sigma$ series tests are shown in Fig. 5.31 and 5.32. The small strain region is presented on an expanded scale in the inset. Close agreement between strain increment direction and stress direction may be noted over the entire strain range, pre-peak and post-peak. This coincidence of strain increment direction and stress direction implies that deformations are predominantly non-recoverable.

Gradual evolution of anisotropy over and above the inherent on straining, like in $\sigma_d$ shear, seems to be essentially complete by the time phase transformation/steady state conditions are reached. Further straining tends to indicate that this new anisotropy for the case of $b = 0$ is of the cross anisotropy type about the new direction of $\alpha_\sigma$. This is apparent from the equality of $\Delta e_2$ and
Figure 5.30. (a) Effective stress conditions at phase transformation state and (b) variation of phase transformation or steady state strength with peak $\alpha_\sigma$. 

$\sigma'_m = 200$ kPa, $b = 0$, $Dr_c = 30\%$

$\sigma'_d / \sigma'_m = 5.0$

$\sigma'_m = 200$ kPa, $b = 0$, $Dr_c = 30\%$

$\sigma_m$, $b$ and $\sigma'_d / \sigma'_m$ held constant during shear.
Figure 5.31. Variation of principal strain increment, stress increment and stress directions with maximum shear strain for (a) $\sigma_d/\alpha_\sigma = 5.0$ and (b) $\sigma_d/\alpha_\sigma = 3.0$. 

100
$\sigma''_{mc}=200$ kPa, Dr$_c=30\%$
$b=0, \; \sigma_d/\alpha_\sigma=5.0$

$\sigma_m$, $b$ and $\sigma_d/\alpha_\sigma$ held constant during shear

(a)

100
$\sigma''_{mc}=200$ kPa, Dr$_c=30\%$
$b=0, \; \sigma_d/\alpha_\sigma=3.0$

$\sigma_m$, $b$ and $\sigma_d/\alpha_\sigma$ held constant during shear

(b)

$\varepsilon_1 - \varepsilon_3$ (%)
Figure 5.32. Variation of principal strain increment, stress increment and stress directions with maximum shear strain for (a) $\sigma_d/\alpha_\sigma = 2.5$ and (b) $\sigma_d/\alpha_\sigma = 1.5$. 

\[ \sigma'_m = 200 \text{ kPa}, \ Dr_c = 30 \% \]
\[ b = 0, \ \sigma_d/\alpha_\sigma \text{ held constant during shear} \]
\(\Delta \varepsilon_3\) (Fig. 5.33) in the post phase transformation/steady state region (strain path is sloped at approximately 45° regardless of the level of \(\sigma_d/\alpha_o\).

### 5.4.2 Arbitrary Rotation of Principal Stresses (Torsional Shear Tests)

In this section, principal stress rotation test results on initially anisotropically consolidated sand are described. Test specimens were consolidated to an effective mean normal stress of 200 kPa, with \(K_c = 1.5\) and 2.0. Relative density at the conclusion of consolidation was targeted at 30%. Following consolidation, specimens were sheared by holding the boundary normal stresses \(\sigma_z, \sigma_\theta\) and \(\sigma_r\) constant, while shear strain \(\gamma_{\theta}\) was increased monotonically. Figure 5.34 presents the effective stress path diagram and deviator stress response. In these tests, rotation of principal stresses occurred simultaneously with increase in \(\sigma_d\) and \(b\) parameter (see Figs. 5.35(a) and 5.35(b)). Unlike, in an undrained or constant volume simple shear test, specimens in this series tests were free to deform in axial, radial and tangential directions.

The direction of major principal stress in these tests coincides with the deposition direction at zero strain level (\(\alpha_o = 0\)). For both \(K_c\) levels, the rotation of principal stress directions reached their respective peaks at about 0.5% maximum shear strain. Also, the peak \(\sigma_d\) and peak \(\alpha_o\) occur at nearly the same strain level. For \(K_c = 1.5\), peak \(\alpha_o\) is about 25°. On further straining, \(\alpha_o\) dropped by about 2°, and then increased to about 30° at 10% maximum shear strain. For \(K_c = 2.0\), on the other hand, \(\alpha_o\) in the post peak region increased from 15° to about 25° on account of larger dilation in the post phase transformation state. For both, \(K_c = 1.5\) and 2.0 the highest value of \(b\) parameter is less than 0.2. It was apparent in \(\sigma_d\) test series that the effect of \(b\) parameter is
Figure 5.33. Strain paths in $\sigma_d-\alpha_0$ shear.
Figure 5.34. Effective stress path and deviator stress - strain response.
Figure 5.35. Variation of $\alpha_\sigma$ and b.
not very significant when \( b \) is less than about 0.25. Therefore, it is reasonable to assume that a change in \( b \) from 0 to 0.2 in these tests may not influence the deformation response significantly.

The dependence of \( \phi_{CSR} \) and \( S_{upeak} \) with peak \( \alpha_\circ \) experienced during torsional shear loading is shown in Figs. 5.36(a) and 5.36(b) by solid circles. Both \( \phi_{CSR} \) and \( S_{upeak} \) decrease with a decrease of \( K_c \). For comparison, similar relationship in \( \sigma_d \) shear are shown by solid lines. These tests were performed on isotropically consolidated specimens. A unique relationship between \( \phi_{CSR} \) and \( \alpha_\circ \) seems to exist irrespective of the consolidation history. However, peak undrained shear strength increase as \( K_c \) increase. In triaxial compression loading Chern (1985) and Thomas (1992) also report increase in undrained peak shear strength with increase in \( K_c \).

Effective stress conditions at phase transformation states for anisotropically consolidated initial states are shown in Fig. 5.37(a). Regardless of the consolidation history and rotation of principal stresses during undrained shearing, phase transformation states may be seen to lie on the same effective stress ratio line observed for this sand in previous sections (i.e.; \( \phi_{PT} = 33^\circ \)). This observation further confirms that \( \phi_{PT} \) is a unique material property that is independent of the principal stress directions, principal stress rotations, intermediate principal stress, and consolidation history. In triaxial compression, Chern (1985) also reported that \( \phi_{PT} \) is independent of the consolidation history.

Strain increment directions on torsional shear loading seem to deviate from stress directions at small strain levels (see Figs. 5.38a and 5.38b). At the start of shear, \( \alpha_{\Delta e} \) is about 45° and the stress direction \( \alpha_\circ \) is zero. However, with increase of strain level, the deviation becomes small and coincidence of the two directions can be observed at shear strain levels in excess of about 0.5%. It can be shown, that the direction of principal strain increment is 45° for constant
Figure 5.36. Variation of (a) $\phi_{CSR}$ and (b) peak shear strength with peak $\alpha_\sigma$. 

\[
\sigma'_{mc} = 200 \text{ kPa}, \ b = 0, \ Dr_c = 30\%
\]

- Solid circles — $\sigma_z$, $\sigma_s$, and $\sigma_r$ held constant during shear.
- Solid line — from $\sigma_d$ tests.

1. Figure 5.36. Variation of (a) $\phi_{CSR}$ and (b) peak shear strength with peak $\alpha_\sigma$. 

2. The diagram shows the variation of $\phi_{CSR}$ with respect to $\alpha_\sigma$ under specified conditions ($\sigma'_{mc} = 200 \text{ kPa}$, $b = 0$, $Dr_c = 30\%$). 

3. The figure indicates that $\phi_{CSR}$ decreases as $\alpha_\sigma$ increases. 

4. The solid line represents data from $\sigma_d$ tests, whereas solid circles denote values of $\sigma_z$, $\sigma_s$, and $\sigma_r$ held constant during shear. 

5. The peak shear strength decreases linearly with increasing $\alpha_\sigma$. 

6. The diagram is used to illustrate the relationship between $\phi_{CSR}$ and $\alpha_\sigma$ under controlled conditions, providing insights into the material's shear behavior. 

7. Further analysis and discussion are necessary to fully understand the implications of these findings in the context of soil mechanics and geotechnical engineering.
Figure 5.37. (a) Effective stress conditions at phase transformation state, and (b) variation of undrained shear strength.
Figure 5.38. Variation of principal strain increment, stress increment and stress direction with maximum shear strain for (a) $K_c = 1.5$, and (b) $K_c = 2.0$. 

$\sigma'_m = 200$ kPa, $D_{re} = 30\%$

$K_c = 1.5$

$\sigma_m$ held constant but $b$ and $\alpha_\sigma$ were not controlled during shear

$K_c = 2.0$
volume simple shear deformation conditions, in which all normal strains in the axial and in the plane of shear are equal to zero. Since normal strains were not controlled in this series of tests, \( \Delta \varepsilon_n \) decreased from about 45° at the start finally coinciding with \( \alpha_n \) as the strain level increased. The principal stress increment direction, \( \alpha_{\Delta \sigma} \) remain unchanged at 45° throughout shear. This is attributed to the fact that the horizontal shear stress was increased while the three boundary total normal stresses \( \sigma_z, \sigma_0 \) and \( \sigma_r \) were held constant.

5.4.3 Behaviour in Torsional Simple Shear

Results from constant volume - undrained torsional simple shear tests performed on anisotropically consolidated specimens are presented in this section. Constant volume (undrained) simple shear deformation conditions were achieved indirectly by restricting the axial deformation, and volume changes of the internal chamber and of the soil specimen to zero. This resulted in zero normal strains in the axial, radial and tangential directions. Shearing was carried out at a constant rate of shear strain \( \gamma_{z0} \) in the horizontal direction.

The ability of the UBC-HCT device in simulating constant volume simple shear deformation is illustrated in Fig. 5.39. Zero normal strains \( \varepsilon_n, \varepsilon_0 \) and \( \varepsilon_z \) may be noted, as would be expected for the undrained simple shear mode. It may also be noted that the maximum shear strain \( \varepsilon_1 - \varepsilon_3 \) equals the horizontal shear strain \( \gamma_{z0} \).

In Fig. 5.40, effective stress path diagram and shear stress-strain response is shown by solid lines. Dashed lines represent response in \( \sigma_d \) shear with \( \alpha_0 = 45° \) and \( b = 0.4 \) and 0.5, respectively. The simple shear test specimen was consolidated to an effective mean normal stress of 200 kPa along \( K_c = 2.0 \), while \( \sigma_d \) test specimens were isotropically consolidated to identical effective mean normal stress level. Close agreement in shear stress-strain response between \( \sigma_d \)
Figure 5.39. Observed strain paths due to torsional simple shear.
Figure 5.40. Effective stress path and shear stress-strain response in torsional simple shear tests.
tests and torsional simple shear test may be noted. The phase transformation/steady state strength in the three shearing modes are essentially the same (about 40 kPa). In addition, the peak horizontal shear stress is also the same. It may be recalled that deformation in \( \sigma_d \) tests with \( \alpha_\sigma = 45^\circ \) and \( b = 0.4 \) represents approximately plane strain conditions, and simple shear happens to be a special case of plane strain deformation (zero lateral normal strain).

The major principal stress direction, \( \alpha_\sigma \), undergoes a continuous rotation from 0 to about 45\(^\circ\) during constant volume simple shear. A simultaneous increase in \( b \) parameter occurs as it changes from 0 to about 0.4 (see Figs. 5.41(a) and 5.41(b)). Both, \( \alpha_\sigma \) and \( b \) seem to have reached their constant steady values at 45\(^\circ\) and 0.4 respectively, at about 3\% shear strain, and continued deformation does not seem to alter these constant steady values. In drained simple shear tests on Leighton Buzzard sand, Roscoe (1970) reports similar changes in \( \alpha_\sigma \). Experimental results reported by Roscoe were obtained from tests on the Cambridge type simple shear (Mk 6) apparatus.

As a consequence of the imposed zero normal strains in the axial, radial and tangential directions, the major principal strain increment direction remain constant at 45\(^\circ\) right from the start of shear deformation. The coincidence of principal stress and strain increment direction after about 3\% shear strain implies that the deformations are predominantly non-recoverable.

The major principal stress direction \( \alpha_\sigma = 45^\circ \) implies that the horizontal shear stress is the maximum shear stress. The lateral stresses in conventional simple shear tests are not known and, it is often assumed that the horizontal shear stress equals the maximum shear stress. This assumption is utilized to compute friction angle mobilized and the principal stresses. Figure 5.41 shows that this assumption in the interpretation of conventional simple shear tests has its support in the evidence presented. An increase in \( b \) parameter from its initial value zero to 0.4 also
Figure 5.41. (a) Variation of principal strain increment, stress increment and stress directions and (b) $b$ parameter with maximum shear strain in torsional simple shear.
confirms the findings in the previous sections that \( b = 0.4 \) represent approximately plane strain conditions. The effective stress conditions at phase transformation state show that a \( \phi_{PT} \approx 33^\circ \) was mobilized at the PT state, implying that the angle \( \phi_{PT} \) is independent of the stress or strain path (Fig. 5.42a). The relationship between phase transformation/steady state strengths and \( \alpha_\sigma \) in torsional simple shear also conforms essentially to that observed in \( \sigma_d \) tests (Fig. 5.42b). This again implies that the effect of the history of rotation of principal stresses during simple shear deformation may not be very significant, and only the maximum value of \( \alpha_\sigma \) (= 45° in this case) governs the phase transformation/steady state strength.

A comparison of the response of Fraser River sand in constant volume torsional simple shear and conventional constant volume simple shear is illustrated in Fig. 5.43. The latter were performed by Sivathayalan (1994) using the NGI type simple shear apparatus (Bjerrum and Landva, 1966). Results from four conventional simple shear tests (dashed lines) and two torsional simple shear tests (solid lines) over a range of vertical effective stresses are shown. Although it is not possible to measure lateral stresses and therefore, \( K_c \) level in conventional simple shear tests, it is estimated that \( K_c \) would be about 2.0 to 2.4 (based on \( K_o = 1 - \sin \phi, = 30-33^\circ \) for loose sand). Torsional simple shear test specimen was anisotropically consolidated along \( K_c = 2.0 \) and 2.4 paths. The targeted relative density of all specimens at the end of consolidation was about 30%.

The stress paths in Fig. 5.43(a) show excellent similarity in shape between conventional and torsional simple shear. However, the shear stress-strain relationships demonstrate considerable differences (see Fig. 5.43b). In the pre-peak region, conventional simple shear response is not as stiff as under torsional simple shear. The absence of complimentary shear stresses along the lateral boundary causing non-uniform distribution of vertical normal stress and
Figure 5.42. (a) Effective stress conditions at phase transformation state (b) variation of undrained shear strength in constant volume simple shear.
$\sigma_m = 200$ kPa, $D_r = 30\%$

Dashed lines – Conventional simple shear
Solid lines – Torsional simple shear

Figure 5.43. Comparison of torsional simple shear and conventional simple shear deformation.
shear stresses may be the reason for a softer stress-strain response under conventional simple shear.

The demonstrated agreement in effective stress paths and experimental verification of the assumption that the horizontal shear stress approximates the maximum shear stress tend to lend support for the use of simple shear device for an estimation of the undrained strength. The relative ease of setting up undisturbed specimens and in carrying out this test compared to the HCT test thus makes the simple shear device a valuable testing device in practice. This device may not however, be appropriate for basic research, as it may not yield reliable stress-strain data.

5.4.4 Summary

Based on the principal stress rotation test results presented in this section, it can be concluded that the effect of the history of principal stress rotation may not be very significant. Only the peak $\alpha_\eta$ experienced during shearing seems to govern pre-peak and post-peak response of the sand. It should be emphasized that the type of principal stress rotations investigated were such that $\alpha_\eta$ increases from its initial value of zero. Therefore, the term history of $\alpha_\eta$ refers to values of $\alpha_\eta$ lower than the current value. Many field loading situations involve increasing $\alpha_\eta$ with loading, and therefore, the findings of this research are most relevant to real field problems.

Under multi-axial loading, the mobilised friction angle at phase transformation/steady state is independent of the consolidation history and the direction or rotation of principal stresses, stress or strain paths during shear.

Principal stresses undergo a continuous rotation, form 0 to about 45° in simple shear deformation. A simultaneous change in intermediate principal stress occurs as $\alpha_\eta$ increases. Both
the major principal stress direction and the intermediate principal stress parameter reach their maxima (45° and 0.4), at about 3% shear strain, whereafter they remain unchanged with further straining.

The maximum shear stress and maximum shear strain in simple shear deformation approximately equal the horizontal shear stress and strain respectively. Comparative study of the conventional and torsional constant volume simple shear behaviour shows that the assumption of horizontal shear stress equal to the maximum shear stress in conventional simple shear testing is reasonably valid. The conventional simple shear device may however yield unreliable stress-strain relationships on account of the absence of complementary shear stresses on the lateral boundary, and non-uniform vertical stress distribution on top of the soil specimen.

5.5 Undrained Cyclic Loading Response

Undrained cyclic loading behaviour of Fraser River sand under constant deviator stress amplitude involving cyclic principal stress rotation is presented in this section. The behaviour in conventional triaxial tests carried out in the UBC-HCT device is presented first. In this test, \( \alpha_\sigma = 0^\circ \) and \( 90^\circ \) in each half cycle, and thus a \( 90^\circ \) jump rotation of \( \sigma_1 \) is involved. Simultaneously, the \( b \) parameter experiences similar jump variation between 0.0 (\( \alpha_\sigma = 0^\circ \)) and 1.0 (\( \alpha_\sigma = 90^\circ \)). Thus the conventional cyclic triaxial loading invokes the strongest and the weakest deformation modes in each half cycle (see Figs. 5.4 and 5.16).

Cyclic loading behaviour involving jump rotations of \( 90^\circ \) between -45° and 45° (\( \alpha \approx 0.5 \) tests), -60° and 30° (\( \alpha \approx 0.5 \) tests) and -75° and 15° (\( \alpha \approx 0.5 \) tests) was also studied. In these modes, neither the strongest nor the weakest loading mode is activated in each half cycle.
Furthermore, the b parameter was held constant at 0.5 in order to eliminate its possible influence on cyclic response.

Continuous principal stress rotation during cyclic loading in which both \( \alpha_a \) and \( b \) were allowed to vary simultaneously was also investigated. In all tests, except the conventional triaxial, the magnitude of \( \sigma_m \) was held constant during cyclic loading.

The cyclic resistance curve for the loose Fraser River sand was generated for a specific jump rotation deformation mode \( \alpha_o = \pm 45^\circ \) and compared to that obtained under the conventional triaxial mode (\( 90^\circ \) jump rotation between \( \alpha_o = 0 \) and \( 90^\circ \)).

Except for cyclic loading under continuous principal stress rotation, test specimens were hydrostatically consolidated to an effective mean normal stress of 200 kPa. The targeted relative density of all specimens prior to cyclic shearing was about 30%. Cyclic shear loading was applied under strain rather than stress controlled conditions. The constant strain rate used was \( \varepsilon_a = 0.1%/\text{min} \) for the conventional triaxial, and \( \gamma_{st} = 0.1%/\text{min} \) for other cyclic tests. Strain controlled loading was chosen in order to capture confidently the stress-strain behaviour during the loading cycle in which the sand might develop strain softening. The specimen-apparatus interaction, when such a response ensues, precludes confident assessment of stress-strain behaviour by stress controlled loading (Chern 1985). The strain controlled loading also permits termination of the cyclic shear loading precisely at the specified strain level. This has the advantage of enabling a systematic study of post cyclic behaviour of sand, which is profoundly affected by the maximum strain during cyclic loading. Stress controlled cyclic loading does not enable such a control on strain development.
The term *liquefaction*, as emphasized before, refers herein to all phenomena involving excessive deformation. It is defined as the development of a maximum shear strain \((\varepsilon_1 - \varepsilon_3)\) in excess of 3.75%, single amplitude. This is the usual definition adopted in the literature (NRC, 1985). This strain can accumulate with or without the occurrence of contractive deformation during some stage of the cyclic loading.

5.5.1 Conventional Cyclic Triaxial Response

The conventional cyclic triaxial response using specifically the HCT device was assessed primarily to ensure that the results were not influenced by the geometry of the specimen. Little difference was noted in cyclic response as assessed using the conventional triaxial specimens of 63 mm diameter and the HCT specimens. All conventional cyclic triaxial tests reported herein were carried out using the HCT device.

Typical response at a cyclic deviator stress amplitude of 50 kPa is shown in Fig. 5.44. As is well known, the effective stress path during cyclic loading is nonsymmetrical about the effective mean normal stress axis. In each cycle, positive increments of excess pore pressure in the first and fourth quarter cycles and negative increments in the second and third quarters may be observed (see Fig. 5.44b). These are closely related to the constant in compression and decrease and then increase in extension phase of the total minor principal stress \(\sigma_3\). The cumulative excess pore pressure developed at any given time is nevertheless positive, and increases with cycles of loading. The effective stress path is inclined to the right, which signifies larger effective stress ratio mobilized in extension than compression mode of each cycle. This results in the axial strain accumulation that is biased on the extension side (Fig. 5.44c). Strain softening response in the last cycle in extension triggered liquefaction. It is interesting to note that steady state strength of
Figure 5.44. Undrained cyclic triaxial loading response.
about 20 kPa in Fig. 5.44 equals approximately that obtained in static extension loading. Strain softening was expected (Vaid and Chern, 1985) because the cyclic shear stress of 25 kPa exceeded the steady state strength, and Fraser River sand is strain softening under static extension loading (Fig. 5.16).

Very small strains are induced with loading cycles until the last (26th), in which strain softening developed and the sand liquefied. The residual excess pore pressure on the other hand, increased with each loading cycle, and the pore pressure ratio $\Delta U/\sigma'_m$ reached about 90% at the conclusion of the last cycle.

5.5.2 Cyclic Loading with Other 90° Jump Rotations

These tests were carried out at a constant $b = 0.5$ during cyclic loading. The results for the $\alpha45b0.5$ cyclic test under a deviator stress amplitude of 50 kPa (identical to that for the conventional triaxial) are shown in Fig. 5.45. Unlike the conventional cyclic triaxial behaviour (Fig. 5.44), the effective stress paths are now symmetrical about the effective mean normal stress axis. This is apparently due to the fact that the rotation of $\sigma_1$ is symmetrical about the axis of cross-anisotropy and both $\sigma_m$ and $b$ are held constant during cycles. Incremental pore pressure development within each half cycle is positive, regardless of the sign of $\alpha_\varphi$. The excess pore pressure with number of cycles shows a smooth increase and does not reflect oscillations within typical cycles of the conventional cyclic triaxial loading (Fig. 5.45b). The excess pore pressure ratio at the conclusion of cyclic loading was about 0.90.

No strain softening occurred in either positive or negative $\alpha_\varphi$ mode (see the sign convention for $\alpha_\varphi$ in Fig. 5.45a). Since Fraser River sand at identical initial relative density and
Figure 5.45. Cyclic loading response due to jump rotation of $\alpha_\circ$ between $+45^\circ$ and $-45^\circ$. 

\begin{align*}
\sigma_{d(cy)} &= 2 \tau_{zo} \\
\sigma_m &= 300 \text{ kPa}, \ b(=0.5) \ \text{held constant during shear} \\
\sigma_{mc} &= 200 \text{ kPa}, \ b=0.5, \ D_{r_c}=30\% \\
\gamma_{z0} &= \gamma_{zo} \\
\gamma_{max} &= \gamma_{zo} \\
\sigma_{d(cy)}/(2\sigma_{mc}) &= 0.125
\end{align*}
stress state is barely strain softening under static shear loading at \( \alpha_o = 45^\circ \) and \( b = 0.5 \) path (Fig. 5.6), strain softening response in cyclic loading would not be expected.

The shear strain induced with cycles of loading is very small until the 28\(^{th}\) cycle in which the maximum shear strain defined as liquefaction occurs. This happens when the stress path gets close to the phase transformation state noted in static tests. The shear strain response is symmetrical in both positive and negative modes of \( \alpha_o \), which may again be attributed to the fact that the jump rotation of \( \sigma_1 \) is symmetrical about the axis of cross-anisotropy and \( b \) parameter is held constant throughout.

The response in another \( \alpha_{45}b_{0.5} \) cyclic loading, but with a larger deviator stress amplitude of 70 kPa is illustrated in Fig. 5.46. Like the \( \alpha_{45}b_{0.5} \) cyclic loading at deviator stress amplitude of 50 kPa, liquefaction occurred when the effective stress path crossed the phase transformation line noted in static tests. This occurred in the seventh stress cycle in the \( \alpha_o = +45^\circ \) mode. Since the rotation of principal stresses is symmetrical about the axis of cross-anisotropy (i.e., is identical in both positive and negative modes of \( \alpha_o \)), liquefaction could occur in either \( \alpha_o \) mode. Liquefaction in the \( \alpha_o = -45^\circ \) mode may be noted in the results of test at \( \sigma_{dev} = 50 \) kPa (Fig. 5.45).

The response under cyclic jump rotation of principal stress between \( +30^\circ \) and \( -60^\circ \) (\( \alpha_{30}b_{0.5} \) loading) with a deviator stress amplitude of 70 kPa is shown in Fig. 5.47. Liquefaction occurred in the seventh cycle through contractive deformation in the \( \alpha_o = -60^\circ \) mode. This was expected since for the same initial state, the static response is contractive (Fig. 5.6) and the amplitude of cyclic shear stress 35 kPa was greater than the phase transformation strength under static loading with \( \alpha_o = 60^\circ \) and \( b = 0.5 \).
Figure 5.46. Cyclic loading response due to jump rotation of $\alpha_\sigma$ between $+45^\circ$ and $-45^\circ$.

\[ (\sigma_d)_{cy} / (2\sigma'_{mc}) = 0.175 \]

$\sigma_m$ (300 kPa), $b(0.5)$ held constant during shear

$\sigma'_{mc} = 200$ kPa, $b = 0.5$, $D_R = 30\%$

$\gamma_{zo} = \gamma_{zo}$

Number of cycles $N$
Figure 5.47. Cyclic loading response due to jump rotation of $\alpha_\sigma$ between $+30^\circ$ and $-60^\circ$. 

- $\sigma_{d(cy)} = 70$ kPa, $\tau_{zo(cy)} = 30$ kPa
- $\sigma_m = 300$ kPa, $b = 0.5$ held constant during shear
- $\sigma_{me} = 200$ kPa, $b = 0.5$, Dr$_c = 30%$
The excess pore pressure increased with cycles of loading, but again no oscillations of pore pressure occurred within loading cycles. A pore pressure ratio of 95% was noted at the conclusion of cyclic loading.

Undrained response of Fraser River sand due to cyclic jump rotation of principal stresses between +15° and -75° (\(\alpha_{1.5b0.5}\) loading) is shown in Fig. 5.48. Deviator stress amplitude of 70 kPa was selected as for the \(\alpha_{45b0.5}\) and \(\alpha_{30b0.5}\) loadings (Fig. 5.46 and 5.47). Again, very small strain and little oscillation of pore pressure with loading cycles occurred, until liquefaction took place by strain softening in the 8th stress cycle.

5.5.3 Comparison of Cyclic Behaviour with Different 90° Jump Rotations

Figure 5.49 illustrates shear stress-strain relationships under various 90° jump rotations but identical cyclic stress amplitude. Strain development during cycles prior to the one in which liquefaction developed is small regardless of the type of 90° jump rotation. Accordingly, the area of the hysteresis loops is small, implying low values of damping. Except for the \(\alpha_{45b0.5}\) loading, occurrence of liquefaction was attributable to contractive deformation. As pointed out earlier, this was expected, since the sand was statically contractive in these \(\alpha_\theta\) loading modes and the cyclic shear stress amplitude exceeded the steady state/phase transformation strength in each case.

Data similar to Fig. 5.49 comparing response in the conventional cyclic triaxial and \(\alpha_{45b0.5}\) loading at a smaller but identical cyclic shear stress is illustrated in Fig. 5.50. Again a difference in the mechanism of strain development, strain softening and no strain softening type is present, for reasons explained earlier.
Figure 5.48. Cyclic loading response due to jump rotation of $\alpha_e$ between $+15^\circ$ and $-75^\circ$. 

(a) 

(b) 

(c) 

Strain magnified 10 times
Figure 5.49. Cyclic shear stress-maximum shear strain response.
Figure 5.50. Cyclic shear stress - maximum shear strain response.
5.5.4 Cyclic Loading Resistance of Loose Fraser River Sand

The relationships between cyclic stress ratio and number of cycles to liquefaction at $D_{rc} = 30\%$ is shown in Fig. 5.51. Cyclic stress ratio is defined as the ratio $\sigma_{dc}/(2\sigma_{mc}')$, which is identical to $\sigma_{dc}/(2\sigma_{3c}')$ because of the initial hydrostatic consolidation state. These results correspond to the $90^\circ$ cyclic jump rotation loading of the $\alpha 45b0.5$ type. For comparison the resistance curve obtained under conventional cyclic triaxial loading in the HCT device is also shown. The cyclic resistance corresponding to 10 cycles to liquefaction on the curve is identical to that presented earlier by Thomas (1992) using the conventional triaxial test.

For a given cyclic stress ratio, the number of cycles required to cause liquefaction (i.e.; exceedance of 3.75% maximum shear strain) seems to be higher under $\alpha 45b0.5$ cyclic shear than that under the conventional cyclic triaxial shear. However, with decreasing cyclic stress ratios this difference in the number of cycles tends to decrease. This difference in cyclic loading in the domain of cycles typical of earthquake shaking may be attributed to the most damaging extension loading ($\alpha_o = 90^\circ$) mode being activated together with high value of $b = 1$ in each half loading cycle. In the $\alpha 45b0.5$ loading, only the $\alpha_o = 45^\circ$ modes are activated. The triaxial compression ($\alpha_o = 0$) mode in each half cycle is not excessively stronger than the $\alpha_o = \pm45^\circ$ mode, but the triaxial extension mode ($\alpha_o = 90^\circ$) in the other half cycle is extremely weaker than the $\alpha_o = \pm45^\circ$ mode (Fig. 5.16). In addition, conventional cyclic triaxial loading activates high $b = 1$, whereas $b$ was held to 0.5 in the other comparable mode. It would then appear that the conventional cyclic triaxial test might underestimate the cyclic resistance in problem, where such extreme principal stress rotations with respect to the deposition axis do not occur.
(1) - Cyclic torsional shear
Jump rotation of $\alpha_\sigma$ between $+45^\circ$, $-45^\circ$
while $\sigma_m(300 \text{ kPa})$, $b(0.5)$ held constant

$\sigma'_m = 200 \text{ kPa}$, $D_{r_c} = 30\%$

(2) - Conventional cyclic triaxial shear
Jump rotation of $\alpha_\sigma$ between 0, $90^\circ$
together with change in $b$ between 0 and 1

Figure 5.51. Cyclic resistance curve of loose Fraser River sand.
5.5.5 Cyclic Loading with Continuous Principal Stress Rotation

This type of loading can only be applied provided the initial state of stress is nonhydrostatic. Thus, the soil is initially under a finite value of static $\sigma_1 - \sigma_3$ and the cyclic deviator stress is applied about this ambient value. Prior to cyclic loading, anisotropic consolidation was done along the $K_c = \sigma'_1/\sigma'_3 = 1.5$ path to an effective mean normal stress of 200 kPa. This corresponds to $\sigma'_2c = 257$ kPa and $\sigma'_re = \sigma'_oe = 171$ kPa and thus a static deviator stress of 86 kPa. Following consolidation, horizontal cyclic shear stress $\tau_{eh}$ was applied while the total boundary stresses $\sigma_z$, $\sigma_r$ and $\sigma_\theta$ were held constant. The amplitude of the horizontal shear stress was 25 kPa, identical to that in the $\alpha 45B0.5$ cyclic loading data reported in Fig. 5.45. This cyclic $\tau_{eh}$, however resulted in a cyclic deviator stress amplitude of only 13 kPa. The deviator stress oscillated between 89 kPa and 102 kPa during each half cycle.

Since the vertical effective stress was higher than the radial and tangential stresses, major principal stress direction coincided with vertical (i.e.; $\alpha_\sigma = 0$) prior to shear loading. The application of shear stress induced continuous principal stress rotation. The rotation $\alpha_\sigma$ of the major principal stress direction and $b$ parameter with number of cycles is shown in Figs. 5.52. The principal stress rotation $\alpha_\sigma$ is symmetrical with respect to the vertical direction (axis of cross-anisotropy). The amplitude of $\alpha_\sigma$ remain constant at about 15°. A larger amplitude of horizontal cyclic shear stress, with other parameters held constant, will increase the amplitude of $\alpha_\sigma$. Similarly, at higher $K_c$, with other parameters held constant, will reduce the amplitude of $\alpha_\sigma$. The change in $b$ parameter is small, less than 0.07, whose effect can be considered as insignificant.

Cyclic loading response for this continuous cyclic principal stress rotation test is shown in Fig. 5.53. Symmetry of the stress path with respect to a line parallel to the effective mean normal
Continuous cyclic principal stress rotation

Total normal stresses $\sigma_z$, $\sigma_r$ and $\sigma_\phi$ held constant during shear

$\sigma_{mc} = 200$ kPa, $K_c = 1.5$, $Dr_c = 30\%$

$\sigma_{d(\phi)}/(2\sigma_{mc}) = 0.033$

Figure 5.52. Variation of (a) $\alpha_\sigma$ and (b) B with number of cycles due to continuous cyclic principal stress rotation.
\[ \sigma_{d(cy)} = 13 \text{kPa}, \ \tau_{zo(cy)} = 25 \text{kPa} \]

PT line from \( \sigma_d \) tests

\[ \sigma_d/2 \ (\text{kPa}) \]

\[ 0 \ 25 \ 50 \ 75 \ 100 \]

\[ \sigma'_m \ (\text{kPa}) \]

\[ 0 \ 50 \ 100 \ 150 \ 200 \]

(a)

\[ \sigma_{d(cy)}/(2\sigma'_m) = 0.033, \ \tau_{zo(cy)}/(\sigma'_m) = 0.125 \]

Continuous principal stress rotation

\[ \sigma'_m = 200 \text{kPa}, \ Dr_c = 30 \% \]

(b) \( K_s = 1.5, b_s = 0, \ \alpha_{se} = 0 \)

\[ \gamma_{zo} \ (\text{\%}) \]

Strain magnified 10 times

\[ -5 \ 0 \ 5 \]

Number of cycles \( N \)

\[ 0 \ 5 \ 10 \ 15 \ 20 \ 25 \ 30 \]

(c)

Figure 5.53. Undrained cyclic loading response due to continuous cyclic rotation of \( \alpha_s \).
stress axis is apparent because of equal rotation about the axis of cross anisotropy. The excess pore pressure (Fig. 5.53b) response is very similar to that for the case of 90° jump rotation $\alpha_{45b0.5}$.

Like the $\alpha_{45b0.5}$ cyclic response reported in Fig. 5.45c, shear strain developed symmetrically about the zero strain axis (see Fig. 5.53c). However, unlike the $\alpha_{45b0.5}$ and conventional triaxial responses, strains increased progressively, and liquefaction ensued in the 24th cycle and the residual pore pressure ratio at the end of cyclic loading was only about 0.75.

The number of cycles to liquefaction (24) for this continuous cyclic principal stress rotation under $\sigma_{dcy}/2\sigma_{mc}=0.033$ approximate the number (26) under conventional triaxial and (28) under $\alpha_{45b0.5}$ loading at a much higher $\sigma_{dcy}/2\sigma_{mc}=0.125$ (Figs. 5.44, 5.45 and 5.53). This greatly reduced cyclic resistance may be attributed to the fact that continuous cyclic principal rotations of even smaller amplitude of rotation angles are much more damaging than that the cyclic jump rotations of even 90°. Results presented by Symes et al. (1984) show that liquefaction can be induced by a mere rotation of principal stress at $\Delta \sigma_{dcy}=0$.

5.5.6 Summary

Under identical initial state and cyclic stresses, the number of cycles required to exceed a specified level of maximum shear strain $\varepsilon_1-\varepsilon_3$ (occurrence of liquefaction) is less under cyclic triaxial than under $\alpha_{45b0.5}$ loading. Although both loadings impose a 90° jump rotation of principal stresses, in $\alpha_{45b0.5}$ test horizontal shear stress is cycled while holding total normal stresses $\sigma_z$, $\sigma$, and $\sigma_0$ constant. Therefore, the stress system in a $\alpha_{45b0.5}$ test may be considered
to duplicate the stresses on a soil element below horizontal ground when subjected to vertically propagating shear waves.

The strain development is extremely small prior to the cycle in which liquefaction occurred. The number of cycles required to exceed a certain level of strain development do not seem to depend on the manner in which a 90° jump rotation is imposed. However, the mechanism of strain development may change from limited liquefaction to liquefaction type as the major principal stress direction approach the bedding plane in one half of the cycle. Contractive deformation (liquefaction type) and strain accumulation always occurred in the half cycle, when $\alpha_\sigma$ exceeded 45°. For a given direction of major principal stress ($\alpha_\sigma$), if the sand manifests contractive deformation under static loading, it would be contractive under cyclic loading conditions, provided the maximum shear stress (static + cyclic) amplitude exceeds the steady state/phase transformation strength in static loading.

The friction angle mobilised at phase transformation state (33°) during cyclic shear loading is essentially identical to that under static loading. For a sand which undergoes contractive deformation, $\phi_{PT}$ equals the friction angle mobilised at steady state.

### 5.6 Post-Cyclic Loading Behaviour

Following cyclic shear loading specimens were subjected to post cyclic static undrained shear in the triaxial compression mode ($\alpha_\sigma = 0$, $b = 0$). The state of stress prior to post cyclic loading was hydrostatic and the residual strain, $\varepsilon_x$, for the given $\gamma_{\text{max}} = 3.75\%$ imposed, varied with the type of cyclic loading. Conventional triaxial loading gave rise to extensional $\varepsilon_x$, and torsional loading ($\alpha 45^\circ b 0.5$ loading) compressive $\varepsilon_x$. For representation of post cyclic behaviour,
the zero reference for the strain is taken as the residual state at the conclusion of cyclic loading.

The residual excess pore pressure ratio PP/\(\sigma'_{mc}\) at the conclusion of cyclic loading was less than one for all cyclic tests. For post cyclic static shearing an axial strain rate of 0.1%/min was used.

Typical post cyclic undrained static response of loose Fraser River sand under \(\sigma'_{mc} = 200\) kPa (prior to cyclic loading) is shown in Fig. 5.54. Cyclic loading was of the \(\alpha45\beta0.5\) type. For comparison, pre-cyclic static response (loading on virgin specimen) in triaxial compression mode is also shown in the same figure. For the specimen which developed a residual excess pore pressure ratio of 0.95, the stiffness may be noted to increase with strain. This type of stress-strain response in which the modulus increases with strain is contrary to the usual response of soils. The unusual response of the liquefied sand results from the fact that, upon shearing it dilates, causing the effective stresses to increase. The deformation progresses throughout at a mobilized friction angle that equals the angle of maximum obliquity in static pre-cyclic shear.

The post cyclic static undrained response with a residual excess pore pressure ratio of 0.87 is similar to the pre-cyclic response, wherein the modulus first decreases before it starts increasing with strain as dilation commences. Thus the post cyclic behaviour of a sand depends profoundly on the magnitude of residual excess pore pressure ratio.

The post cyclic response following conventional cyclic triaxial liquefaction is shown in Fig.5.55. The response for the three cases shown is such that the modulus starts to increase from the start of post cyclic loading. The modulus at a given strain level now depends on the excess pore pressure ratio at the conclusion of cyclic loading. The higher the excess pore pressure ratio the lower is the modulus. The sand which developed an excess pore pressure ratio of 0.98, deforms with essentially zero stiffness from the start of post cyclic loading. Similar observation of post cyclic static response in triaxial compression following conventional cyclic triaxial
Figure 5.54. Effective stress path and deviator stress-strain response in post cyclic undrained static loading.
Figure 5.55. Effective stress path and deviator stress-strain in post cyclic static undrained response in triaxial compression mode following a conventional cyclic triaxial test.
liquefaction has been reported by Vaid and Thomas (1995). They also report that the deviator stress-strain response become as a straight line at large strains and no apparent approach to a residual strength occurs even after about 30% axial strain. Similar trends may be noted in the data shown in Fig. 5.55.

The full spectrum of post cyclic static response following several \( \alpha = 45\degree \pm 0.5 \) (solid lines) and conventional cyclic triaxial loading (dashed lines) is shown in Fig. 5.56. For a given excess pore pressure ratio and initial void ratio, the stress-strain response seems to be independent of the type of the cyclic loading the sand has undergone. In other words, the post cyclic static response of sand after liquefaction may not depend on the type of principal stress rotation imposed during cyclic loading.
Figure 5.56. Effective stress path and deviator stress-strain in post cyclic static undrained response in triaxial compression mode.
CHAPTER 6
SUMMARY AND CONCLUSIONS

The undrained behaviour of sands under multi-axial loading conditions has been investigated utilising the Hollow Cylinder Torsion (HCT), triaxial and simple shear devices. The HCT is the only device that enables independent control of four stress parameters; three principal stresses and the direction of the major principal stress with the vertical deposition direction.

Little research has been done in the past on the independent effects of principal stress direction and rotation on the undrained behaviour of sands. In devices other than the HCT, principal stress rotation is either not possible or is accompanied by uncontrolled changes in other stress parameters. A study of undrained anisotropy and the associated effects of principal stress directions and their rotation on sand liquefaction are addressed in this research. This objective is achieved by a study of static and cyclic undrained behaviour of loose sands under multi-axial stresses. Shear loading was carried out in the strain controlled loading mode. Only such loading permits the needed capture of post peak strain softening characteristics of loose sands as undesirable runaway strains, inevitable in stress controlled loading modes, cannot occur.

The investigation was carried out on Fraser River and Syncrude sands. The specimens were reconstituted by water pluviation. Water pluviation, as opposed to moist tamping technique, is considered to simulate the fabric of natural and artificial fluvial and hydraulic fill sands, and hence provides a convenient means for a systematic study of their undrained behaviour.
Based on the experimental evidence presented, it is concluded that the undrained response of sands under fixed principal stress direction and constant b is independent of the total stress path.

Undrained response of loose water deposited sand is highly dependent on the loading direction implying inherent anisotropy. A sand which is dilative when the major principal stress coincides with the deposition direction may show limited liquefaction type response which transforms eventually to the steady state type of contractive response as the direction of major principal stress changes from the deposition direction to the bedding plane direction.

Steady state/phase transformation strength of a sand is not uniquely related to its void ratio alone. For a given void ratio, steady state/phase transformation strength depends in addition, on the direction of principal stresses, and to some extent on the magnitude of $\sigma_2$. Thus the use of conventional triaxial compression test alone for estimating the steady state/phase transformation strength of sands is not appropriate. Laboratory characterization of sand for solving practical problems should consider the anticipated range of principal stress directions and intermediate principal stress level in the field as being important variables that influence undrained behaviour.

Mobilised friction angle at the initiation of contractive deformations ($\phi_{CSR}$) is a function of the major principal stress direction together with the magnitude of intermediate principal stress. For Fraser River sand $\phi_{CSR}$ decreases from about $26^\circ$ to $16^\circ$ as $\alpha_\theta$ increase from $30^\circ$ to $90^\circ$.

The influence of intermediate principal stress on undrained response is small when b parameter is less than about 0.5. Approximately plane strain conditions prevail ($\varepsilon_2 = 0$), when undrained shearing occurs along a b = 0.4 path.
At constant values of other parameters, increasing confining stress and decreasing relative density under multi-axial loading promote a higher degree of contractive response.

Based on the test results presented that involve principal stress rotation, it is concluded that the effect of principal stress rotation may not be very significant. The undrained response tends to depend only on the direction of principal stress. In other words, it is not the history of \( \alpha_o \) but the peak \( \alpha_o \) experienced during shearing that governs the peak and post-peak response of the sand. These findings are limited to the type of principal stress rotations in which \( \alpha_o \) increase from its initial value of zero. Many field loading situations involve increasing \( \alpha_o \), and therefore, the investigation carried out in this study regarding the effect of principal stress rotation are very relevant to real field problems.

The friction angle \( \phi_{PT} \) mobilized at phase transformation or steady state is a unique material property, independent of the direction of principal stresses, intermediate principal stress level and the stress and void ratio state prior to undrained shear. \( \phi_{PT} \) is also unaffected by the rotation of principal stresses. Furthermore, under multi-axial loading conditions, \( \phi_{PT} \) is independent of the consolidation history, hydrostatic or nonhydrostatic.

Principal stresses undergo a continuous rotation, form 0 to about 45° in simple shear deformation simulated in the HCT device. A simultaneous change in intermediate principal stress occurs as \( \alpha_o \) increases. Both principal stress direction and intermediate principal stress parameter reach their maxima (about 45° and 0.4), at about 3% shear strain, whereafter they remain unchanged with further straining.

Maximum shear stress and maximum shear strain in simple shear deformation equal approximately the horizontal shear stress and strain respectively. Comparative study of the
conventional and torsional constant volume simple shear behaviour shows that the assumption of
horizontal shear stress equal to the maximum shear stress in conventional simple shear testing is
justified. Conventional simple shear device may however yield unsatisfactory stress-strain
behaviour on account of the absence of complementary shear stress on lateral boundaries, and the
non-uniform vertical stress distribution on top and bottom of the soil specimen.

Number of cycles to liquefaction at a given amplitude of maximum shear stress is smaller
for cyclic triaxial than for $\alpha_{45b0.5}$ loading. Although both tests impose a $90^\circ$ jump rotation of
principal stresses, in $\alpha_{45b0.5}$ loading horizontal shear stress is cycled at constant $\sigma_z$, $\sigma_r$ and $\sigma_0$
thus, the stress system in $\alpha_{45b0.5}$ loading may be considered as duplicating the stress system of a
soil element in the field, subjected to vertically propagating shear waves.

The mechanism of strain development during cyclic loading varies from limited
liquefaction to liquefaction type as the major principal stress direction approach the bedding plane
in each half cycle. Contractive deformation (liquefaction type) and strain accumulation always
occurred in the half cycle, when $\alpha_\sigma$ was higher than $45^\circ$. For a given direction of principal stress,
the sand that is contractive under static loading conditions is also contractive under cyclic loading
conditions, provided the maximum shear stress amplitude exceeds the relevant steady state or
phase transformation strength for that direction.

Friction angle mobilised at phase transformation state during cyclic shear loading equals
that under static shear loading. For sand which suffers contractive deformation $\phi_{PT}$ equals the
friction angle mobilised at steady state.

Pore pressure ratio at end of cyclic loading, when maximum shear strain exceeded 3.75%,
is also independent of the manner in which a $90^\circ$ jump rotation of principal stress is given.
Suggestions for Further Research

Undrained behaviour of loose sands under multi-axial loading has been investigated using the HCT device. This allowed a systematic assessment of the effects of principal stress magnitudes, directions and rotations on static and cyclic liquefaction. Experiments were carried out on water deposited specimens of two medium uniform sands.

Most of the work on static liquefaction was focused on loose sand at a single hydrostatic confining stress and relative density. Nevertheless, a limited work was aimed at delineating the effect of initial $\sigma_{mc}$ on liquefaction potential but only at $\alpha_\sigma = 45^\circ$. Similarly the effect of relative density on static liquefaction was also carried out on a limited scale.

Undrained behaviour under cyclic loading focused on 90° jump rotations, both symmetrically and nonsymmetrically about the axis of deposition. Only isotropically consolidated loose initial density states were considered. The effect of continuous cyclic principal stress rotation during cyclic loading of anisotropically consolidated ($K_c > 1$) loose sand was also studied, but on a limited scale. Post cyclic static response assessed was of the conventional triaxial compression type, regardless of the direction of major principal stress during cyclic loading.

Clearly several important aspects of static and cyclic liquefaction have not been explored comprehensively. A list of topics that would supplement the study reported is given below as suggestions for further research.

1. Extension of similar study to other sands with emphasis on the influence gradation and angularity, and fines content on undrained anisotropy.

2. The influence of initial state (stress and void ratio), on static and cyclic liquefaction.
3. The effect of simultaneous changes in $b$ parameter and total mean normal stress on undrained behaviour as opposed to maintaining these stress variables constant during shear. This will be the normal scenario in field problems.

4. Detailed exploration of the relationships between static and cyclic undrained behaviour, and cyclic loading resistance response in which cyclic mobility contributes to strain development as opposed to strain softening behaviour investigated so far.

5. Post cyclic monotonic testing should be directed in modes other than triaxial compression (ie; $\alpha_0 > 0$) and its relationship to the strain direction during cyclic loading.

6. The response of moist tamped sand under multi-axial loading, to further confirm the potentially collapsible fabric that ensues by this technique of specimen reconstitution, and hence their inability to model behaviour of water deposited sands in practice.

7. Design guidelines suggesting incorporation of the findings from these studies as to the method of, for example, specifying residual strength and cyclic loading resistance in a given field problem.
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APPENDIX
MEMBRANE PENETRATION CORRECTION

Experimental studies using the UBC-HCT device require measurement of four strain components ($\varepsilon_z$, $\gamma_{2\theta}$, $\varepsilon_r$ and $\varepsilon_\theta$). The two strain components, $\varepsilon_z$ and $\gamma_{2\theta}$ are measured directly from axial and angular deformation of the soil specimen. However, strain components $\varepsilon_r$ and $\varepsilon_\theta$ are derived from measured volume changes of the soil specimen and that of the internal chamber (see chapter 3 for details). An accurate determination of these volume changes is therefore essential.

Both inside and outside vertical surfaces of the HCT specimen are covered by 0.3 mm thick rubber membranes. Under drained loading paths requiring changes in confining pressure, volume changes of coarse grained soils occur not only in response to soil deformations but also because of the penetration or withdrawal of the membrane into or out of the interstices of the granular soil specimen. The measured volume changes of the soil specimen and the internal chamber must therefore be corrected for membrane penetration effects in order to compute the volumetric deformation of the soil skeleton alone.

As a part of this study, membrane penetration curves for Fraser River sand has been developed using the method proposed by Vaid and Negussey (1984). The unit membrane penetration $\varepsilon_M$ as a function of effective confining pressure is shown in Fig. A.1. Strains in all drained tests were corrected for the membrane penetration effects according to Fig. A.1.

The measured volume change of the soil specimen was corrected as follows;

$$
\Delta V_s = \Delta V_{s_m} - \Delta \varepsilon_M A_i - \Delta \varepsilon_M A_e
$$

(A.1)
Figure A.1. Unit membrane penetration with effective confining stress for loose Fraser River sand.
where

\[ \Delta V_S = \text{corrected volume change of the soil specimen} \]

\[ \Delta V_{Sm} = \text{measured volume change of the soil specimen} \]

\[ \varepsilon_{Mi} = \text{from Figure A.1 (corresponding to the current and increment of internal effective chamber pressure)} \]

\[ \varepsilon_{Me} = \text{from Figure A.1 (corresponding to the current and increment of external effective chamber pressure)} \]

\[ A_i = \text{surface area of the internal membrane covering the specimen} \]

\[ A_e = \text{surface area of the external membrane covering the specimen} \]

Volume change of the internal chamber was corrected as follows;

\[ \Delta V_I = \Delta V_{Im} + \Delta \varepsilon_{Mi} A_i \]  \hspace{1cm} (A.2)

where,

\[ \Delta V_I = \text{corrected volume change of the inner chamber} \]

\[ \Delta V_{Im} = \text{measured volume change of the inner chamber} \]