A NEW, NON-LINEAR ANALYSIS OF LAYERED SOIL-PILE-SUPERSTRUCTURE SEISMIC RESPONSE

by

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A non-linear finite element analysis for determining the seismic response of piles in multilayered soil deposit is presented. The analysis incorporates pile material and geometric non-linearities, P-Δ effects, non-linear load deformation behavior of soil-pile interaction, pile separation and the non-linear shear stress-strain relationship in the free-field soil. The soil shear stress-strain relation is expressed based on Hardin and Drenwich's model. The pile material stress-strain relation is based on a elastic-plastic model. Near field soil is replaced with a continuous foundation based on Yan-Byrne or American Petroleum Institute P-y curves. Time integration is performed utilizing Newmark-β integration scheme. Numerical integration of the virtual work equations has been carried out with Gauss quadrature. The resulting set of non-linear equations is solved by using the Newton-Raphson scheme. The method of analysis are incorporated in two programs: QUIVER and PILEUBC. The program QUIVER captures the seismic response of soil deposit and later is used as a module in the main program PILEUBC, in which the response of a single pile is predicted for an earthquake excitation. The program predicts time varying acceleration, velocity, displacement, bending/axial stress and strain, bending moment and shear force in the pile. The program is able to successfully simulate the seismic response of piles. Numerical investigations have been carried out to verify the results and test the capabilities of the program.

A linearized closed form solution based on Laplace transformation for free-field is developed for comparison with the finite element analysis.
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Anuz K Khan
Inspired by

The Earthquake
When the Earth is shaken to her utmost convulsion
And the Earth throws up Her burdens from within,
And man cries distressed: 'What is the matter with her?'
On that Day will she declare her tidings:
For that thy Lord will have given her inspiration
On that Day will men proceed in companies sorted out
To be shown the Deeds that they had done.
Then shall anyone, who has done an atom's weight of good, see it!
And anyone who has done an atom's weight of evil shall see it.

THE HOLY QURAN
(Surah XCIX)

to my ammi and abba
1.1 INTRODUCTION

Pile-supported structures and foundations are man's oldest method of construction for overcoming the difficulties of founding on soft soils. The use of piles dates back to the prehistoric period. However, up until the late nineteenth century the design was entirely based on experience, or even divine providence (Poulos and Davis, 1980).

Significant damage to pile supported structures during major earthquakes such as Niigata and Alaska earthquakes of 1964 and the San Francisco earthquake of 1906 led to an increase in demand to reliably predict the response of piles. Since then, extensive research have been carried out and several analytical and numerical procedures (Matlock and Reese, 1960; Novak and Aboul-Ella, 1978; Kuhlemeyer, 1979; Kagawa and Kraft, 1980; Velez et. al., 1983; Wu, 1994) have been developed to determine the static and dynamic response of piles subjected to horizontal or vertical loads. Also, full scale experimental observations on the pile's behavior (Abe et. al., 1984; Blaney and O'Neill, 1986; Sy and Siu, 1992) and numerous model testing (Finn and Gohl, 1987, Yan and Byrne, 1991) have been carried out.

The problem with which this thesis is concerned is schematically shown in Figure 1.1. It shows a pile, embedded in a soil deposit, and supporting a superstructure above ground level. The source of the earthquake motion is assumed to be at the base or till and this triggers a vibration of the soil deposit above. In turn, this forces the pile to move under the action of the vibrating soil. Although the actual problem is three dimensional,
Fig. 3.1 General Nature of Soil-Pile-Superstructure System During Seismic Excitation
This thesis would only consider a two-dimensional approximation representing the effect of the soil on the pile by p-y curves relating the applied pressure to the relative movement between pile and soil. The region of applicability of this model is called the "near field".

The results from most of the numerical analysis give an insight to the problem and indicate that the key features of the problem are now better understood. However, complexity of analysis, attribution of failure, damage observation difficulties, and the problems in conducting in-situ tests have hampered the routine methods of analysis for pile foundations subjected to earthquake loading. Also, it has long been recognized that nonlinear effects, such as nonlinear soil behavior, slippage, eventual gapping, and material non-linearity play a fundamental role in the response of piles to seismic excitations.

A number of approaches have been formulated for the analysis of dynamic soil-pile interaction in the past years. The research work carried out in the area of seismic soil-pile-foundation-structure interaction could be most generally classified into the determination of kinematic seismic response, determination of pile-head impedance and determination of superstructure seismic response (Gazetas et. al., 1992). A coupled 3-D analysis of pile foundation and superstructure is generally considered not to be a feasible engineering option. As mentioned earlier, a simpler procedure based on substructuring is usually adopted. Soil-pile interaction analysis is conducted on the foundation alone to determine the pile foundation impedances. These impedances are then incorporated into the model of the superstructure and the dynamic response is calculated using ground motions at the ground surface as input (Luco, 1982). It is assumed that these motions are not affected by the pile itself. But this may not be a tenable assumption for earthquake loading, although it may prove sufficient for pile head loading.
There is a significant difference between seismic excitation of ground and foundation, and cyclic pile head excitation. Pile head excitation creates disturbances in the soil adjacent to the pile only, whereas the entire near-field and its interaction plays a vital role in a pile subjected to earthquake loading. The distinguishing characteristic of strong shaking is the far reaching effects of non-linearity. This type of problem, typical of pile response subjected to strong shaking under maximum probable earthquakes requires a nonlinear analysis.

In addition, in most of the procedures for evaluation of pile response a linear or an equivalent linear analysis is usually done. The whole idea of evaluation of impedance functions exemplifies the methodology of analysis as either linear or equivalent linear to handle non-linearity in the problem, if any. For realistic computations of stress-deformation and ultimate behavior of pile-founded structures it becomes necessary to consider factors like non-linear behavior of soil, interaction effects between soil and structure and nature of loading. If a long pile and its foundation experience large displacements it may become necessary to consider geometric non-linearity in addition to material non-linearity.

The first step in any soil-structure interaction seismic analysis is the calculation of the free-field response of the site, that is, the spatial and temporal variation of motion before excavating or rigging the soil and superimposing the structure. Several methods for evaluating the effect of local soil conditions on the ground response during earthquakes are presently available. One such is the widely accepted procedure incorporated in the program SHAKE (Scnabel et al., 1972). This method is based on the assumption that the main response in the soil deposit is caused by the upward propagation of shear waves from the underlying rock formation. The seismic response analysis taking into account the
effects of nonlinear nature of soil deformation can be made either by equivalent linear
method (Seed and Idriss, 1969) or the step-by-step integration scheme. If the earthquake
induced shear strains are large, say to the order of 1 percent, the accuracy of the
equivalent linear solution fails to yield an acceptably good solution (Ishihara, 1982). For
the problem of inducing larger shear strains in the soil deposits a step-by-step time
integration procedure using a variable tangent stiffness must be used, even if it requires a
greater computational effort to obtain the solution in the time domain.

At present, the state of practice in analyzing nonlinear lateral pile response
represents the pile by beam-column elements and the soil by compliance springs. The non­
linear response of the soil is captured by using non-linear springs termed as p–y curves
(API Code, 1987). These curves are specified for soft clay and sandy materials and are
based on a hyperbolic tangent function. Yan and Byrne (1990) have developed parabolic
p–y curves for sandy soils. Instead of using discrete spring system (lumped model) a
nonlinear continuous foundation would provide an improved solution.

It was observed during cyclic loading that pile deformation, vertical heave and soil-
pile separation (gaps) occur near ground surface. As a result of gapping, the lateral
resistance of the soil-pile system is significantly reduced. When piles are closely spaced,
gaps around each pile may overlap and the effect of group interaction may be very
pronounced.

It is difficult to incorporate all the major factors in a solution procedure, even in a
numerical procedure. Numerical procedures such as the finite element method, can be used
to handle many complexities. The resulting solution strategies are difficult from the view
point of mathematical derivation and involve large computational effort.
In general the solution procedure for soil-pile-superstructure interaction should consider the three dimensions of the problem. A two dimensional solution with interface springs connecting the pile to the free-field is considered a feasible engineering option. These solution techniques could be in the frequency domain utilizing an elastic or equivalent elastic approach. It could also be a time domain inelastic solution. At this point, a nonlinear solution using a continuum approach to the problem of lateral response of piles subjected to earthquake loading is deemed to be needed. In this thesis an attempt is made to obtain such a solution, based on a finite element formulation of the nonlinear soil-pile-superstructure interaction, when subjected to seismic excitations. The effect of gapping is also considered.

1.2 RESEARCH OBJECTIVES

This study aimed at the following objectives:

1. Development of a finite element analysis model for a reliable prediction of the seismic response of piles, based on beam on an inelastic continuous foundation approach using a step-by-step time integration procedure. This analysis will include the nonlinear behavior of the near-field soil. The finite element model will take into account the soil-pile interaction using p-y response curves to depict the nonlinear continuous foundation. Also, the analysis will introduce nonlinear material and geometric properties for the pile as well as account for the development of gaps in the pile-soil interface. It will be shown that the continuum based computational nonlinear approach for soil-pile-superstructure interaction is a viable method for estimating the seismic response.

2. Implementation of the analysis into a computer program which would allow flexibility in placing the layers of soil in different configurations.
3. Validation of this computer program by comparing its predictions with developed analytical and numerical results.

1.3 ORGANIZATION OF THESIS

Chapter 2 provides a critical review of the current methods for estimating dynamic response of pile foundations. Emphasis is given to the study of single piles in a layered media. Chapter 3 presents the finite element formulation of the free-field soil and soil-pile-superstructure interaction. The method of analysis to determine the solution to the problem is discussed. The implementation of computer program is described. The assumptions are pointed out and the advantages and limitations are criticized. Chapter 4 presents typical analysis of the study and example problems. Chapter 5 presents the results and validity of the analysis on mathematical basis.

Summary and conclusions from the research are given in Chapter 6.
CHAPTER 2

REVIEW OF SINGLE PILE ANALYSIS

2.1 INTRODUCTION

Like other dynamic analysis of civil engineering systems, those for single piles may be classified into two major categories: continuum models coupling the soil and pile as a unified system, and lumped mass-spring-dashpot models. Soil-pile interaction for dynamic loading conditions have been represented analytically by discrete models such as the sub-grade reaction theory, by continuous models such as the elastic solutions (Tajimi, 1969; Nogami and Novak, 1977), or by the semi continuous models such as the finite element method (Blaney, Kausel, and Rosset, 1976; Kuhlmeyer, 1979; Rosset and Angelides, 1979). Also Wu (1994) has developed solution procedures for three dimensional idealizations of the coupled soil-pile system. However, a coupled dynamic continuous foundation based approach, incorporating all the important features of the soil-pile-superstructure interaction has not been developed yet.

2.2 REVIEW OF SINGLE PILE ANALYSIS

Linear dynamic analysis in which the soil and pile are considered as an elastic continuum was first formulated by Tajimi (1966) for lateral response of an end bearing pile. Significant contributions followed from Novak, Nogami and their co-workers (Novak, 1974; Nogami and Novak, 1977; Novak and Aboul-Ella, 1978). Their formulation was based on an elastic beam vibrating in a homogeneous or non-homogenous elastic isotropic continuum subjected to dynamic pile head loading. The main
assumptions in their model were: the soil was composed of a horizontal layers that are homogenous, isotropic, and linearly viscoelastic with material damping of a frequency-independent hysteretic type. The soil properties were constant within each layer but may be different in individual layers. The pile was vertical, linearly elastic, of circular cross section that may vary stepwise at the interfaces of the layers, and it is bonded to the soil. If pile head lies above the grade or the pile is assumed to be separated from soil, the adjacent layers were modeled as void. The soil reactions acting on a unit length of pile were described by a complex soil stiffness (Novak et al, 1978) associated with vertical, horizontal, anti-symmetrical (rocking), and torsional displacements of the pile. Closed form expressions were developed for this linear coupled response analysis. This solution procedure offered an insight to the soil-pile system. However, it does not contribute towards the development of a solution for the problem of earthquake excitation where the strains in the free and near-field soil and in the pile itself are subjected to significant nonlinearities. In addition, the reduction of soil stiffness and the increase in damping associated with strong shaking are sometimes modeled crudely by making arbitrary changes in complex stiffness of soil. Hence these studies have not proved very useful for evaluating the response of pile foundations to earthquake loading.

Gazetas and his co-workers (Krishnan et al, 1983; Gazetas, 1984; Makris et al, 1992) compiled available literature and analysis procedures to study the influence of kinematic interaction on differences in pile accelerations at the ground surface relative to that of free field. Their work suggested (Gazetas et al, 1992) that soil-foundation-structure interaction analysis under seismic excitation could be organized as follows: (1) Obtain the motion of foundation in the absence of superstructure. (2) Determine the dynamic impedances associated with swaying, rocking and cross swaying-rocking
oscillations of the foundation. (3) Compute the seismic response of the superstructure supported on the foundation impedances and subjected at its base to the foundation input motion that was obtained earlier. In general, the above procedure is referred as uncoupled analysis as it does not solve the soil-pile-superstructure vibration in an amalgamated form. For each step of the analysis several alternative formulations have been developed and published in the literature, including finite element formulations (Blaney et al, 1976; Gazetas, 1984), boundary element (Kaynia et al, 1982), semi-analytical and analytical solutions (Banerjee et al, 1977) and a variety of simplified methods (Nogami, 1985). The influence of kinematic interaction in an uncoupled superstructure analysis appears to be invalid, provided the free-field surface motions are dominated by relatively low amplitude vibrations. This in result could over predict the dynamic response of the system.

Whitman and Bielak (Rosenbleuth, 1980) define soil-structure interaction as

"If the motion at any point on the soil-structure interface differs from the motion that would occur at this point in the free field if the structure were not present, there is soil-structure interaction ".

This is a very general and widely accepted definition. It exemplifies the significance of the behavior of the coupled soil-pile system at the interface or contact point. Hence the solution procedure discussed by Gazetas et al (1992) may not prove adequate as it fails to address this contact or no contact situation in the soil-pile system. In addition the motion of a point in free-field definitely differs from motion of the same point in the near-field. However, they concluded that the effects of kinematic interaction are relatively minor for excitation frequencies of up about 1.5 times that of fundamental frequency of the free-field. These conclusions were again based on a elastic linear dynamic analysis of soil-pile system.
Kuhlmeyer (1979) developed a three dimensional linear finite element analysis to represent the beam bending aspect of the problem. The results were similar to those of Novak, suggesting that for a linear solution the plain strain models can provide a reasonably adequate prediction of pile head behavior.

The analysis procedures discussed up until now are essentially linear and are only adequate to predict minuscule vibrations. This is usually not the case in a strong motion problem. During an earthquake, piles may behave in a materially nonlinear manner, and a long pile in the case of an off-shore foundation may exhibit geometric non-linearities. Also, additional effects of soil non-linearity at high strain levels, pile separation, slippage and friction are predominant. It is relatively difficult to develop a solution procedure incorporating all the above mentioned behavior. Lumped mass-spring-dashpot models seemed to be an obvious choice to be employed in a nonlinear analyses based on an approach in which pile foundation and the superstructure are analyzed as a combined system. The interaction between the pile and the near-field soil are replaced using a series of nonlinear springs derived from full scale (Matlock et al, 1978) or model test measurements (Yan and Byrne, 1990) or non-linear finite element solutions. The stiffness of these springs represent the combined stiffness of the soil in the near-field and exterior free-field whose properties are governed by the intensity of shaking. Difficulties exists in incorporating all the nonlinear features at high strain levels in a continuum model for the coupled soil-pile-superstructure system. It could be that these difficulties stem in the formulation of a mathematical solution procedure and the computational effort required.

In a lumped mass-spring-dashpot model, interaction between the pile and the near-field soil is usually accounted for by the use of linear or non-linear lateral compliances (springs and equivalent viscous dashpots) placed along the length of the pile. These
compliance springs model the forces acting on the pile due to relative movement between the pile and soil. One particularly attractive method of analysis, because of its ability to simulate non-linear soil-pile interaction, is embodied in the computer program SPASM (Matlock et al., 1978a-b). In using SPASM, the pile is modeled as a linearly elastic beam column incorporating the flexural rigidity of the pile, EI, and the effects of spatially varying axial load, P(z), on pile bending (P-Δ effects) during lateral seismic loading. The model used in SPASM is shown in Fig. 2.1.

Fig. 2.1 Pile model used in SPASM analysis. (after Finn and Gohl, 1990)

Interaction between the near-field soil and the pile during shaking is represented using non-linear lateral springs k, placed along the length of the pile. The spring stiffness
ks for a particular pile deflection level is defined from tangent slope of the non-linear p−y curves. A back bone p−y curve f(y) is specified in the positive and negative quadrants of the p−y space. Unloading p−y response from a peak loading point (p_{max}, y_{max}) is simulated by following a path in the p−y space that mimics the initial backbone response specified using a function f(y - y_{max}).

Equivalent viscous dashpots c_r are also placed parallel with the near-field springs to simulate the radiation of P and S waves away from the pile (Fig. 2.1). Time varying free-field displacements, which are varied along the length of the pile, are applied to the free-field end of the spring-dashpot assemblage and represent the input seismic excitation applied to the pile. The input ground displacements could be computed using free-field response analysis, SHAKE (Schnabel et al., 1972). In dynamic analysis of the above system radiation damping coefficients are assumed to remain constant during shaking.

Verification studies of SPASM's method of analysis were done by Gohl (1992) using data from centrifuge tests. They showed that the dynamic response of the structure is sensitive to the time history of free-field displacements. The SPASM program may underpredict pile flexural response. A key difficulty in using SPASM is the accurate determination of free-field input motions to be used along the embedded length of the pile. In realistic terms the so called coupled analysis in SPASM is actually a semi-coupled method. The method only couples the superstructure with piles, but it does not couple piles with the surrounding soil directly. In addition, the program SHAKE, in predicting the free-field ground motion, adopts a equivalent-linear dynamic analysis. If the earthquake induced shear strains are large, say to the order of 1 percent, the accuracy of the equivalent linear solution fails to yield an acceptably good solution (Ishihara, 1982). For
the problem of larger shear strains in the soil deposits a step-by-step time integration procedure must be used.

Kagawa and Kraft (1980a, 1980b) carried out a comprehensive study on the soil-pile stiffness using a closed-form solution for linear soil-pile-superstructure systems and a finite element analysis. They studied the $p-y$ response for homogenous and linear soil conditions as well as for heterogeneous and nonlinear soil conditions. The free-field soil was adjusted to accompany hyperbolic models (Konder, 1963 Hardin and Drnevich, 1972; Ramberg-Osgood model(1943), and Martin-Davidenko model (1975)) with backbone

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**Fig. 2.2 Analytical model for $p-y$ evaluation and mathematical model for soil-pile springs**

*after Kagawa and Kraft, 1980*
type curves using Masing rule (1926). Their analysis procedure is illustrated in Fig 2.2. Based on this analysis they concluded that 70 percent of the pile displacement is concentrated in the soil mass within 2 pile radius distance. And the increase in shear strain due to soil-pile interaction is concentrated in this zone. Although it appears to be an effective solution technique it does not include pile separation, nonlinear material and geometric characteristics of pile and the dynamic P-Δ effects.

The “state of the art” procedure for three dimensional dynamic soil-structure interaction was developed by Wu (1994). It includes a quasi 3D finite element method of analysis to determine the dynamic response of pile foundations in a coupled form subjected to horizontal loading. Both elastic response of piles and non-linear soil-pile-superstructure were performed for single piles and pile groups. In the soil domain, movement in vertical direction and horizontal cross shaking were neglected and hence a quasi 3D situation was established. The free and near-field soil were modeled using brick elements with horizontal displacement as a degree of freedom in each of the nodes. The pile was modeled with beam elements with horizontal translation and its slope as the degrees of freedom in each node. The linear analysis was carried out in the frequency domain and the accuracy of the model was compared with that of Kaynia and Kausel (1982), Novak et al., (1977) and Fan et al., (1991). The computed impedances and kinematic interaction were found to be similar. It may be concluded that for a linear solution the plain strain models can provide reasonably adequate prediction of pile head behavior.

Extensive time domain non-linear finite element analysis were also carried out by Wu to study the soil-pile-superstructure problem and they are incorporated in the program
PILE3D (Wu, 1994). It adopts an equivalent-linear approach in which the stiffness is represented by a complex shear modulus $G^* = G(1 + i. 2\lambda)$. In it, the stiffness is associated with the real part of the complex shear modulus and hysteretic damping with the imaginary part, using equivalent viscous damping for soil (Seed and Idriss, 1970). As equivalent viscous damping is frequency dependent in Raleigh damping (Humar, 1990), frequencies were computed for the degrading stiffness of the system at selected steps and updated.

As mentioned earlier, the equivalent linear approach fails to predict the response of the free-field soil at high strain levels in the order of 1%. Secondly, in long piles the geometric non-linearity becomes significant since the beam-column effect may not be discounted. Thirdly, in an earthquake loading situation, nonlinearity due to yielding in the pile material at specific points where the state of stress is high should be taken into account. Wu's (1994) work does not consider these issues. However, it is one of the best available analysis procedures available to obtain the three dimensional response of a coupled soil-pile-superstructure system.

The preceding section discussed some of the formulation and analysis procedures available for single pile response. On this bases, the study of a coupled approach for dynamic soil-pile-superstructure interaction analysis, accommodating the free and near-field non-linearities, pile-gapping, the material non-linearity and P-\Delta effects for earthquake loading was considered worthwhile. An attempt is made in this thesis to study all the above mentioned effects. A finite element formulation is presented in next Chapter. In Chapter 4 analysis for sample problems are performed. Discussions of the results are presented in Chapter 5.
3.1.0 INTRODUCTION

Piles subjected to earthquake loading are of particular interest in connection with founding structures on soft soils, drilling platforms and other pile-supported industrial and defense installations. Lateral dynamic loads induced from wind, wave and earthquakes are frequently the most critical factor in the design of such structures. Solutions in a general form of this problem could be applicable in a variety of cases on shore, including power poles, pile-supports for earthquakes and pile-supported structures which may be subjected to lateral blast forces.

The problem of earthquake loading in a pile is closely related to the familiar problem of a beam vibrating on a elastic foundation; however, in one respect, it represents a more specialized case. All time varying external forces and moments applied to the soil-pile system are introduced through boundary conditions existing at the top of the pile, while time varying pressures are applied along the beam. Solutions of dynamic soil-pile-superstructure interaction problems require generalization of the beam vibrating on a elastic foundation theory to account for non-linear soil and pile behavior.

Fig 1.1 shows the general nature of soil-pile-superstructure subjected to seismic excitations. If a long pile (for example, as used in an offshore environment) and its
foundation experience large displacements, it may become necessary to consider geometric non-linearity in addition to material non-linearity in the problem.

During an earthquake the soil-pile-superstructure system is subjected to loading, unloading and reloading which in turn induces significant non-linearities in the near-field and free-field soil and in the pile itself. Utilizing a continuum approach the geometry of the pile at any instant, including gaps, if any, could modeled by cushioning or iterating to attain that shape. A finite element analysis procedure and a computer program, PILEUBC, to obtain the soil-pile-superstructure interaction was developed based on a coupled and unified system.

The basic concept underlying the finite element method is that a physical domain can be modeled by subdividing it into a finite number of elements. Within each finite element, a set of functions are assumed to approximate the stresses or displacements in that region. The set of interpolation functions contain unknown parameters and are chosen to ensure continuity throughout the domain. Application of the principle of virtual work yields a system of algebraic equations for the parameters in the shape functions. This Chapter sets out to explain the procedures of finite element formulation of soil-pile-superstructure system.

The conventional 'stiffness' approach or the formulation based on an assumed displacement field is followed here. The virtual work method is then used to set up the mass and stiffness matrices and to determine nodal forces equivalent to body forces.
3.1.1 PROBLEM FORMULATION AND FINITE ELEMENT DISCRETIZATION

In this new formulation a single pile is allowed to experience large deformations due to ground excitation and could behave as an elastic-plastic material. Superstructure effect could be incorporated by introducing a concentrated mass at the top of the pile in the global domain. To model the soil-pile interaction, a continuous non-linear foundation is defined using p-y curves of either API, Yan-Byrne or defined by data points. The time-varying free-field displacements, which vary along the length of pile buried inside the soil are obtained from a continuum based one dimensional finite formulation (Section 3.2). The attenuated ground displacement time-histories are obtained from a finite element program called QUIVER exclusively developed for this purpose. The theory and formulation behind this program considers the response associated with vertical propagation of shear waves, which in turn induces transverse vibration throughout a heterogeneous soil deposit.

A one dimensional element of length $\Delta_p$ with two end nodes, as shown in Fig. 3.2, is used in the finite element formulation of the soil-pile-superstructure interaction. With the element shape functions expressed in local coordinates $\xi(-1 \leq \xi \leq 1)$ and $\eta(-1 \leq \eta \leq 1)$ the coordinates of any point inside the element can be expressed as

$$\xi = \frac{2(z - z_c^e)}{\Delta} \quad \text{and} \quad dz = \frac{\Delta}{2} d\xi$$

(3.1.1.1)

$$\eta = \frac{2y}{d} \quad \text{and} \quad dy = \frac{d}{2} d\eta$$

(3.1.1.2)

where $z_c^e$ is the z-coordinate of the center of the element length, $d$ is the diameter of the pile in a circular section. The width of the cross-section, $b(\eta)$, may be constant or varying with $\eta$, as shown in Fig 3.3. The consistent element mass matrix is evaluated in closed

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Fig. 3.2 Finite Element Discretization for Pile

Degrees of freedom
\[ b(\eta) = b \]

\[ b(\eta) = d \sqrt{1 - \eta^2} \]

Fig. 3.3 Transformation of typical pile sections
form as will be seen later. The pile is represented by an assemblage of one dimensional
finite elements that are connected through nodal points. A finite element of length \( \Delta \)
between nodes i and j is shown in Fig. 3.2. The degrees of freedom at each node of the
element are the bending (lateral) deflection, \( w \), its slope and curvature, the axial
translation, \( u \), and its first derivative. Thus there are five degrees of freedom per node and
are arranged in a vector \( \{a\} \) as follows

\[
a^T = \{ w, w', w'', u, u', w_1, w_2, w'_1, w'_2, u_1, u_2 \}
\]  

(3.1.1.3)

The interpolation function that describes the variation of the unknown lateral
displacements within the element in terms of nodal values is quintic, and it produces a
cubic variation in translational lateral strain within the element. The interpolating function
for the axial displacements \( u \) is cubic. The different pile elements and the associated layers
are numbered from bottom to top.

3.1.2 MATERIAL STRESS-STRAIN RELATIONSHIPS

The pile is assumed to behave as an elastic-perfectly plastic body. By defining the
yield stress of the material and the initial modulus of the stress-strain curve, the stress-
strain behavior is fully defined. Unloading and reloading is assumed to be parallel to the
initial loading curve resulting in hysteresis loop as shown in Fig 3.4.

3.1.3 NEAR FIELD AND PILE RELATIONSHIPS

An earthquake basically has two effects on a pile foundation. It excites the
superstructure supported by the piles so that the inertia forces (lateral shear, overturning
moment and axial load) are applied to the pile at the ground surface. And it excites the
Fig. 3.4 Elastic, perfectly plastic pile material
ground surrounding the pile so that additional soil "interaction" forces are applied to the piles along their embedded depth due to relative movements between the pile and the moving ground. Extensive studies, mentioned earlier, have shown that non-linear soil-pile interaction dramatically influences pile response. In particular it is indicative that a strain softening is predominant whose stiffness and hysteretic damping are load level dependent. This is important from a practical point of view, since the majority of dynamic seismic analysis carried out use equivalent linear characterizations of pile head stiffness and damping (referred to as pile head compliance functions). This is indeed practical for low amplitude vibrations representative of machine vibrations.

The near-field soil in this formulation is modeled as continuous non-linear foundation using $p-y$ curves as shown in Fig 3.5. One of the $p-y$ curves generally used in analysis are those specified in the American Petroleum Institute (API) Code (1982) for design of fixed offshore platforms. These curves are specified for soft clay and sandy materials. The API $p-y$ curve was initially developed by Cox et al., and Reese et al.,(1974) and modified by Murchison and O’Neill (1984). The $p-y$ curves are expressed by a hyperbolic tangent function:

$$p = \eta A p_u \tanh \left( \frac{n h z}{A n p_u} y \right)$$

(3.1.3.1)

in which

$p_u$ is the ultimate soil resistance taken as the lesser value from the following:

$$p_u = (C_1 z + C_2 D) n z$$

(3.1.3.2)

$$p_u = C_3 D n z$$

(3.1.3.3)

in which
Fig. 3.5 Soil-Pile Interaction
$C_1$, $C_2$ and $C_3$ are a function of soil friction angle, $\phi$ 

$\gamma$ is the unit weight of soil 

$\eta$ is a factor used to describe the pile shape effect 

$z$ is the soil depth at consideration 

$D$ is the pile diameter 

and $n_{hi}$ is the coefficient of subgrade reaction modulus and is a function of soil relative density. 

Yan and Byrne (1992) conducted extensive experimental testing to verify the validity of the API $p$-$y$ curves based on hydraulic gradient similitude technique (HGS) for sandy soils. They concluded that the API curves based on the hyperbolic tangent tend to over-estimate the pile stiffness and proposed a parabolic $p$-$y$ curve that better resembles experimental curves. Their average curve is expressed as:

$$\frac{P}{E_{\text{max}} D} = \alpha \left( \frac{y}{D} \right)^\beta$$

(3.1.3.4)

where 

$D$ = diameter of the pile. 

$E_{\text{max}}$ = Initial modulus at that particular depth. 

$\alpha$ is a function of soil density, and $\beta$ has a value of about 0.5. 

The computer program PILEUBC written for non-linear seismic soil-pile-superstructure interaction is capable of handling the API curve, the Yan-Byrne curve or any other user defined curve through data points. In addition the program allows for different type of curves at different depths. The initial modulus in a heterogeneous soil deposit,
\[ E_{max} = 2G_{max}(1 + \nu), \] where \( G_{max} \) is the shear modulus of the soil and \( \nu \) is the Poison’s ratio. \( G_{max} \), in turn, is a function of the effective pressure at the particular depth.

### 3.1.4 PILE SEPARATION (GAPPING)

Soil response to cyclic pile head loading in saturated clay has been summarized by Bea (1980). For example, it has been shown that lateral resistance \( p \) of a soft clay to lateral pile motions \( y \), during a sufficient number of cycles of loading, soil-pile separation (gapping) and soil remolding occurs. The confining stresses in the soil near the pile head are not sufficient to close the gaps that may develop between soil and the pile during the cyclic loading, resulting in dog-boned hysteresis loops of the \( p-y \) response. The development of gaps can be expected to lead to significant increases in pile displacement, shifting into a longer period of the coupled system and reduction or no contact in pile to soil interaction.

Soil gapping in this continuum dynamic formulation is modeled by assuming that the soil is incapable of carrying any tensile forces and monitoring whether there is soil-pile contact or not. Numerical studies on static response were made by Vazhinkhoo (1994) on single pile, incorporated in the program CYCPILE, subjected to cyclic loading. In comparing field tests it has been observed that this assumption gives reliable results. Further work (Vazhinkhoo, 1994) is underway to develop a mathematical model for cyclic \( p-y \) behavior.
3.1.5 KINEMATICAL RELATIONS AND SHAPE FUNCTIONS

For the lateral response of pile the external work comprises the work done by the inertial forces, the soil reaction due to relative movement of soil and the pile and the damping forces. The internal work done corresponds to internal stresses which depend on the geometry of the pile and its constitutive relations. The stresses in the pile are a function of strain as the pile undergoes large geometric deformations and behaves inelastically. Let the stresses $\sigma$ obey the constitutive equation

$$\sigma = \sigma(\varepsilon) \quad (3.1.5.1)$$

where $\varepsilon$ is the strain at any point in the pile. This strain is the sum of axial strain plus bending strain.

Axial strain, $\varepsilon_0$

$$\varepsilon_0 = \frac{du}{dx} + \frac{1}{2} \left( \frac{dw}{dx} \right)^2 \quad (3.1.5.2)$$

Bending strain, $\varepsilon_b$

$$\varepsilon_b = \frac{dw_b}{dx} = -z \left( \frac{d^2w}{dx^2} \right) \quad (3.1.5.3)$$

Therefore total strain, $\varepsilon$

$$\varepsilon = u' - z(w') + \frac{1}{2} (w)^2 \quad (3.1.5.4)$$
Continuity conditions require a displacement field with continuous derivatives of one order less than the highest derivative appearing in the strain-displacement relation. Thus \( w, w', \) and \( u \) must be continuous. Specifying \( w, w' \) as degrees of freedom ensure continuity of displacements and slopes, while \( w'' \) ensures continuity of bending moment in the elastic case. In general the inclusion of \( w'' \) and \( u' \) as degrees of freedom permits the use of fewer elements in obtaining the solution.

Since it is necessary to choose the displacement function such that \( w' \) be continuous at the nodes, \( w \) must be assumed at least a cubic polynomial. However, a complete quintic interpolation is used to approximate \( w \) within each element. A complete quintic polynomial requires 6 parameters to define a function. For \( u \), a linear interpolation would be sufficient, but a cubic polynomial is used instead, requiring 4 parameters. The lateral displacements, their first and second derivatives, and the axial displacement and its first derivative at the two nodes provide sufficient parameters to fully describe the specified polynomial functions. Hence we could express \( w, w', w'', u \) and \( u' \) as

\[
\begin{align*}
  w &= N_1 w_i + N_2 \left( \frac{dw}{dx} \right)_i + N_3 \left( \frac{d^2 w}{dx^2} \right)_i + N_4 u_i + N_5 \left( \frac{du}{dx} \right)_i + \\
  &\quad N_6 w_j + N_7 \left( \frac{dw}{dx} \right)_j + N_8 \left( \frac{d^2 w}{dx^2} \right)_j + N_9 u_j + N_{10} \left( \frac{du}{dx} \right)_j 
\end{align*}
\]  

(3.1.5.5)

where subscripts \( i \) and \( j \) refer to the first and second nodes respectively of each element.

Alternatively, Equation 3.1.6.1 could be written as

\[
\begin{align*}
  w &= M^T \{ a \}, w' = M^T \{ a \}, w'' = M^T \{ a \}, u = N^T \{ a \} \text{ and } u' = N^T \{ a \} 
\end{align*}
\]  

(3.1.5.6)

In which
\[ M_0(\xi) \]

- \[ M_0(1,\xi) = \left( 8 - 15\xi + 10\xi^3 - 3\xi^5 \right) / 16 \]
- \[ M_0(2,\xi) = \left( 5 - 7\xi - 6\xi^2 + 10\xi^3 + \xi^4 - 3\xi^5 \right) / (\Delta^2 / 32) \]
- \[ M_0(3,\xi) = \left( 1 - 2\xi^2 + 2\xi^3 + \xi^4 - \xi^5 \right) / (\Delta^2 / 64) \]
- \[ M_0(4,\xi) = 0 \]
- \[ M_0(5,\xi) = 0 \]
- \[ M_0(6,\xi) = \left( 8 + 15\xi - 10\xi^3 + 3\xi^5 \right) / 16 \]
- \[ M_0(7,\xi) = \left( -5 - 7\xi + 6\xi^2 + 10\xi^3 - 3\xi^4 - 3\xi^5 \right) / (\Delta^2 / 32) \]
- \[ M_0(8,\xi) = \left( 1 + \xi - 2\xi^2 + 2\xi^3 + \xi^4 + \xi^5 \right) / (\Delta^2 / 64) \]
- \[ M_0(9,\xi) = 0 \]
- \[ M_0(10,\xi) = 0 \]

\[ M_1(\xi) \]

- \[ M_1(1,\xi) = \left( -15 + 30\xi^2 - 15\xi^4 \right) (2 / 16\Delta) \]
- \[ M_1(2,\xi) = \left( -7 - 12\xi + 30\xi^2 + 4\xi^3 - 15\xi^4 \right) / 16 \]
- \[ M_1(3,\xi) = \left( -1 + 4\xi - 6\xi^2 + 4\xi^3 - 5\xi^4 \right) / (\Delta^2 / 32) \]
- \[ M_1(4,\xi) = 0 \]
- \[ M_1(5,\xi) = 0 \]
- \[ M_1(6,\xi) = \left( 15 - 30\xi^2 + 15\xi^4 \right) (2 / 16\Delta) \]
- \[ M_1(7,\xi) = \left( -7 + 12\xi - 30\xi^2 - 4\xi^3 - 15\xi^4 \right) / 16 \]
- \[ M_1(8,\xi) = \left( 1 - 4\xi - 6\xi^2 + 4\xi^3 + 5\xi^4 \right) / (\Delta^2 / 32) \]
- \[ M_1(9,\xi) = 0 \]
- \[ M_1(10,\xi) = 0 \]

\[ M_2(\xi) \]

- \[ M_2(1,\xi) = \left( 60\xi - 60\xi^3 \right) (4 / 16\Delta^2) \]
- \[ M_2(2,\xi) = \left( -12 + 60\xi^2 + 12\xi^2 - 60\xi^3 \right) (2 / 16\Delta) \]
- \[ M_2(3,\xi) = \left( -4 + 12\xi^2 - 20\xi^3 \right) / 16 \]
- \[ M_2(4,\xi) = 0 \]
- \[ M_2(5,\xi) = 0 \]
- \[ M_2(6,\xi) = \left( -60\xi + 60\xi^3 \right) (4 / 16\Delta^2) \]
- \[ M_2(7,\xi) = \left( 12 + 60\xi^2 - 12\xi^2 - 60\xi^3 \right) (2 / 16\Delta) \]
- \[ M_2(8,\xi) = \left( -4 - 12\xi + 12\xi^2 + 20\xi^3 \right) / 16 \]
- \[ M_2(9,\xi) = 0 \]
- \[ M_2(10,\xi) = 0 \]

\[ N_0(\xi) \]

- \[ N_0(1,\xi) = 0 \]
- \[ N_0(2,\xi) = 0 \]
- \[ N_0(3,\xi) = 0 \]
- \[ N_0(4,\xi) = \left( 2 - 3\xi + \xi^3 \right) / 4 \]
- \[ N_0(5,\xi) = \left( 1 - \xi - \xi^2 + \xi^3 \right) (\Delta / 8) \]
- \[ N_0(6,\xi) = 0 \]
- \[ N_0(7,\xi) = 0 \]
- \[ N_0(8,\xi) = 0 \]
- \[ N_0(9,\xi) = \left( 2 + 3\xi - \xi^3 \right) / 4 \]
- \[ N_0(10,\xi) = \left( -1 - \xi + \xi^2 + \xi^3 \right) (\Delta / 8) \]

\[ N_1(\xi) \]

- \[ N_1(1,\xi) = 0 \]
- \[ N_1(2,\xi) = 0 \]
- \[ N_1(3,\xi) = 0 \]
- \[ N_1(4,\xi) = \left( -3 + 3\xi^2 \right) / (2\Delta) \]

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\[ N_1(5, \xi) = \frac{-1 - 2\xi + 3\xi^2}{4} \]
\[ N_1(6, \xi) = 0 \]
\[ N_1(7, \xi) = 0 \]
\[ N_1(8, \xi) = 0 \]
\[ N_1(9, \xi) = \frac{3 - 3\xi^2}{2\Delta} \]
\[ N_1(10, \xi) = \frac{-1 + 2\xi + 3\xi^2}{4} \]

where \( \xi \) is the Gaussian coordinate \((-1 < \xi < 1)\) between nodes i and j.

### 3.1.6 VIRTUAL WORK EQUATIONS

The principle of virtual work is used to derive the equations of motion for the soil-pile-superstructure system. This is illustrated in Fig. 3.4.

Let \( \delta a \) be a virtual perturbation of the vector \( a \). This results, by Equation 3.1.5.6 in

\[
\delta w = M \{ \delta a \}, \Delta w = M \{ \Delta a \}, \Delta w'' = M \{ \Delta a \}, \Delta u = N \{ \Delta a \} \text{ and } \delta u = N \{ \delta a \}
\]  \hspace{1cm} (3.1.6.1)

respectively.

The internal virtual work per unit volume done by the stresses is

\[
\sigma(\epsilon) \delta \epsilon
\]  \hspace{1cm} (3.1.6.2)

In this formulation the interaction between soil and pile during shaking could be represented using nonlinear continuous foundation along the length of the pile. The near-field stiffnesses for a particular deflection level are defined from the tangent slope of nonlinear p-y curves defined earlier. Soil gapping could be heeded by keeping track of the hysteresis loop. In other words while unloading the previous displacement could be considered as gap. The soil deposit could be heterogeneous stratum with modulus varying nonlinearly.

Let \( v(x,t) \) is the ground displacement relative to the till, the displacement of which is \( v_g(t) \). Then the relative displacement of the pile to which the soil would react would be
\[ s = w - v(x, t) \]  
(3.1.6.3)

The virtual work done by the soil reaction forces is

\[ \int p(w - v(x, t)) \delta w dx \]  
(3.1.6.4)

Similarly, the work done by the inertial forces in the pile, assuming no rotatory inertia and that there is no vertical accelerations at the till is

\[ \rho A \int \left( \ddot{w} + \ddot{v}_g \right) \delta w + u \delta u dx \]  
(3.1.6.5)

Since it is a two dimensional domain, equating the external work with the total internal work obtained by integrating over the volume of the element, \( V \), we have

\[ \rho A \int \left( \ddot{w} + \ddot{v}_g \right) \delta w + u \delta u dx + \int p(w - v(x, t)) \delta w dx + \int \sigma(e) \delta e dV = 0 \]  
(3.1.6.6)

This must hold good for any \( \delta a \), substituting Equations 6.1.6.2 through 6.1.6.4 in Equation 6.1.6.6 to work in terms of local coordinates and introducing vector notations form, we obtain the equation of motion

\[ \rho A \frac{1}{2} \left[ \left( M_0(\xi) M_0^T(\xi) \ddot{\alpha} + N_0(\xi) N_0^T(\xi) \alpha \right) dx + M_{structure} \{ e \} \right] \]

\[ - \frac{\Delta A}{4} \int \left[ N_1(\xi) - \left( \frac{d}{2} \right) (M_2(\xi) + M_1(\xi)) + M_1(\xi) M_1^T(\xi) \alpha \right] \delta(\eta) dx + \frac{\Delta A}{2} \int p|s| M_0(\xi) \delta(\xi) S = \]

\[ - \frac{\Delta A}{2} \rho A \int M_0 \delta(\xi) \dot{v}_e + M_{structure} \{ e \} \dot{v}_e \]  
(3.1.6.7)

in which \( M_{structure} \) is the mass matrix for the superstructure and \( \{ e \} \) is unit vector.
This equation of motion is integrated using the Newmark-\(\beta\) average acceleration method (Newmark, 1959). Thus we write a function \(\Psi(a)\):

\[
\rho A \frac{\Delta t}{2} \left[ \int_{t_i}^{t_{i+1}} \left( M_0(\xi) \dot{M}_0(\xi) \ddot{a}_{i+1} + N_0(\xi) N_0^T(\xi) \dot{a}_{i+1} \right) d\xi + M_{\text{structure}} \{a_{i+1}\} \right] + \frac{\Delta t^2}{\alpha} + \frac{\Delta t}{2} \int_{t_i}^{t_{i+1}} \frac{\partial}{\partial a} \left( M_0(\xi) \dot{M}_0(\xi) \right) \dot{a}_{i+1} \frac{\Delta t}{2} = 0
\]

\[
\frac{\Delta t}{4} \left[ \int_{t_i}^{t_{i+1}} \sigma(\xi, t_i) \left( \tau_d(\xi) \dot{M}_0(\xi) + M_0(\xi) \dot{M}_0^T(\xi) \dot{a}_{i+1} \right) d\eta + \frac{\Delta t}{2} \int_{t_i}^{t_{i+1}} M_0(\xi) \ddot{a}_{i+1} \right] + \frac{\Delta t}{2} \int_{t_i}^{t_{i+1}} \frac{\partial}{\partial a} \left( M_0(\xi) \dot{M}_0(\xi) \right) \dot{a}_{i+1} \frac{\Delta t}{2} = 0
\]

Solution of the problem then requires finding \(\{a_{i+1}\}\) so that \(\Psi(a_{i+1}) = 0\). However, it is not possible to achieve an exact solution in this nonlinear problem, and an iterative procedure is required. It is necessary to ensure that the total error \(\Psi(a)\) is reduced to a given tolerance. The Newton-Raphson method is a widely used iterative technique to solve non-linear equations. It uses a truncated Taylor series expansion of the function \(\Psi(a)\)

\[
\Psi(a + \Delta a) = \Psi(a) + \frac{\partial \Psi(a)}{\partial a} \Delta a = 0
\]

\[
\Delta a = \left[ \frac{\partial \Psi(a)}{\partial a} \right]^{-1} \Psi(a)
\]

As \(\Psi(a + \Delta a) = 0\),

\[
\Delta a = a_{i+1} - a_i = \left[ \frac{\partial \Psi(a)}{\partial a} \right]^{-1} \Psi(a_i)
\]
or in matrix language

\[ a_{i+1} = a_i - \left[ \frac{\partial \Psi(a)}{\partial a} \right]_i^{-1} \Psi(a_i) \]  \hspace{1cm} (3.1.6.12)

where \( \{\Psi_i\} \) is the vector at previous iteration and \( \{\Psi_i^*\} \) is the solution vector at the next iteration. It should be realized that these iteration are in between time steps \( t_i \) and \( t_{i+1} \). The matrix \( \left[ \frac{\partial \Psi(a)}{\partial a} \right] \) Equation 3.1.6.9 is

\[
\frac{d\Psi}{da} = \rho A \frac{\Delta}{2} \int_{-1}^{1} \left\{ M_0(\xi) M_0^T(\xi) + N_0(\xi) N_0^T(\xi) + 2 \varepsilon \right\} d\xi + M_{\text{structure}} \bigg/ \omega \Delta t^2 \\
+ \frac{\Delta d}{4} \int_{-1}^{1} \int_{-1}^{1} \frac{d\sigma}{d\xi} \left\{ N_1(\xi) - \left( \frac{d}{2} M_2(\xi) + N_1(\xi) M_1^T(\xi) a_{i+1} \right) \right\} b(\eta) d\xi d\eta \\
+ \frac{\Delta d}{4} \int_{-1}^{1} \sigma(\xi) \left\{ N_1(\xi) M_1^T(\xi) \right\} b(\eta) d\xi d\eta + \frac{\Delta}{2} \int_{-1}^{1} \frac{d\Phi(\xi)}{d\varepsilon} M_0(\xi) M_0^T(\xi) d\xi 
\]  \hspace{1cm} (3.1.6.13)

The displacement solution vector will take the following form at the end of every iteration within a time step.

\[ \{a\} = \{a\}^* - [\nabla \Psi]^{-1} \{-\Psi\}^* \]  \hspace{1cm} (3.1.6.14)

The displacement vector obtained through the solution procedure as in Equation 3.1.6.14 is checked for convergence of the error, \( \Psi \), to a tolerance. The tangent stiffness matrix is evaluated numerically using Gauss quadrature and the consistent mass matrix is evaluated in closed form.
3.1.7 CONSISTENT MASS MATRIX FOR PILE ELEMENT

The consistent mass matrix of the pile element was evaluated analytically and is given as follows:

\[
M = \begin{bmatrix}
462 & 4620 & 55440 & 0 & 0 & 4620 & 55440 & 0 & 0 & 0 \\
52pA^3 & 23pA^4 & 18480 & 0 & 0 & 1980 & 13860 & 0 & 0 & 0 \\
3465 & 9240 & 0 & 0 & 55440 & 13860 & 11088 & 0 & 0 & 0 \\
13pA^3 & 210 & 0 & 0 & 0 & 9pA & 13pA^2 & 0 & 0 & 0 \\
35 & 105 & 0 & 0 & 0 & 70 & 420 & -\frac{13pA^2}{420} & -\frac{pA^3}{140} \\
\end{bmatrix}
\]

Symmetrical about the leading diagonal

3.1.8 NUMERICAL INTEGRATION

The near-field continuous foundation, pile material and geometric non-linearity leads to changing stiffness in the system when excited. Hence it is convenient to numerically integrate Equation 3.1.6.14 during every iteration in the Newton-Raphson procedure. The well proved Gauss quadrature is used. In general, an integral I

\[
I = \int_{-1}^{1} f(\xi) \, d\xi \quad \text{then} \quad I = \sum_{-n}^{n} H_i f(\xi) \quad (3.1.8.2)
\]
where $\xi_i$ are fixed abscise and $H_i$ corresponding weights for the $n$ chosen sampling points. If a polynomial expression is to be integrated it is easy to see that for $n$ sampling points, there are $2n$ unknowns ($\xi_i$ and $H_i$) and hence a polynomial of degree $2n-1$ can be constructed and exactly integrated. The error thus is of the order of $O(\Delta^{2n})$.

As indicated earlier, $n$ Gauss points integrate a polynomial of order $(2n-1)$ exactly. In the gradient of Equation 3.1.6.13 in the Newton-Raphson procedure, there is a polynomial of order 8. Hence a 5-point Gauss Quadrature would suffice to integrate exactly. However it is noteworthy to mention that in the Program PILEUBC there is a flexibility in choosing the number of Gauss points from 1 to 9.

It is understood that as numerical integration using Gaussian scheme is employed the pile stresses and strains could only be evaluated at the sampling points. A suitable interpolation should be adopted to evaluate them at other locations.

3.1.9 DISCRETIZATION ERROR AND CONVERGENCE

The assumed shape functions limit the infinite degrees of freedom of the continuous soil-pile-superstructure system, and the true minimum of the energy may never be reached, irrespective of the fineness of the subdivision (Zienkiewicz, 1977). However it is possible to minimize the total error to a tolerated value.

An Euclidean norm criterion has been used to check the convergence at the end of every iteration in the Newton-Raphson procedure. If $\|\sigma\|$ is the solution vector at the end
of $r^{th}$ iteration and $\|\mathbf{a}_{r+1}\|$ is the solution at the end of $(r+1)^{th}$ iteration, then

$$\Delta a = \sqrt{\mathbf{a}_{r+1} - \mathbf{a}_r}$$

represents the difference between the two displacement vectors. In which,

$$\|\mathbf{a}_r\|^2 = \sum_{i=1}^{NDOP} a_r^2(i)$$  \hspace{1cm} (3.1.9.1)$$

$$\|\mathbf{a}_{r+1}\|^2 = \sum_{i=1}^{NDOP} a_{r+1}^2(i)$$  \hspace{1cm} (3.1.9.2)$$

where $a_r(i)$ and $a_{r+1}(i)$ are components of $\mathbf{a}_r$ and $\mathbf{a}_{r+1}$ respectively. Then $\Delta a$ could be written as

$$\Delta a = \sum_{i=1}^{NDOP} \sqrt{(a_{r+1}(i) - a_r(i))^2}$$  \hspace{1cm} (3.1.9.3)$$

The convergence criterion for this norm is given as

$$\frac{\Delta a}{\|\mathbf{a}_r\|} \leq \text{specified tolerance}$$  \hspace{1cm} (3.1.9.4)$$

Secondly, it is verified that the total error in the problem $\{\Psi^r\}$ is zero. The mathematical form are

$$\|\mathbf{a} - \mathbf{a}^*\| \leq TOL1 \quad \text{and} \quad \|\mathbf{a}^*\| \leq TOL2$$  \hspace{1cm} (3.1.9.5)$$

The convergence tolerance for Equation 3.1.6.14 needs special mention. As the unit for $\{\Psi\}$ is that of force, a tolerance $\text{IN}$ might be appropriate.

If the solution obtained has not converged then the whole procedure is repeated by updating the solution vector in Equation 3.1.6.8. If converged, then the corresponding
acceleration and velocity vectors are calculated by Newmark-β method. The entire procedure is repeated for all the time steps.

### 3.1.10 PILEUBC- THE COMPUTER PROGRAM

A computer program, PILEUBC, based on the aforementioned nonlinear finite element procedure was developed. Its salient features include changing $E_{\text{max}}$ for every single element as a function of effective or total stress, multi-materialing of the pile, multilayering in the soil deposit, energy dissipation (hysteretic damping) by following through the elastic-plastic pile material behavior, non-linear $P-y$ curves for near field in every single Gauss sampling point throughout the depth for the entire time history, evaluation of cyclic stress ratio and flexibility in giving the input motion at anywhere in the soil profile. Results of a few runs are enclosed in the next chapter.

### 3.2 FINITE ELEMENT FORMULATION OF FREE-FIELD SOIL DEPOSIT

#### 3.2.0 INTRODUCTION

In the coupled problem of seismic soil-pile interaction, the pile displacement are sensitive to spatial variation of the free-field soil. Various idealized models and analytical techniques available are, in general, equivalent-linear, and their validity is questionable when the earthquake-induced shear strains are high. Whatever procedure is followed, it is necessary to determine the appropriate shear stress-strain and energy absorbing properties in the deposit. During an earthquake a soil deposit is subjected to an irregular loading pattern due to a vertically propagating wave which consists of intervals of loading,
unloading, and reloading. It has different behavior characteristics in each of the loading phases. The finite element method is a versatile tool to handle the complexities of the soil problem.

3.2.1 FINITE ELEMENT DISCRETIZATION OF SOIL DEPOSIT

The soil deposit is modeled as an assemblage of one dimensional finite elements interconnected at a finite number of nodal points. Details of the element arrangement and the finite element discretization is shown in Fig. 3.6. The domain could be subdivided into elements of length $\Delta$. The degrees of freedom at each node of the element are the horizontal translation and its first derivative. The interpolation function that describes the variation of the unknown displacement within the element in terms of nodal displacements is cubic, and it produces a quadratic variation in strain within the element. The different soil layers are numbered from top to bottom.

3.2.2 SOIL SHEAR STRESS-STRAIN RELATIONSHIPS

Vertically propagating seismic waves at a site produces transverse movement throughout the depth of the soil stratum. This movement may induce large shear strains in the soil. The shear stress-strain modeling of soils for drained and undrained conditions is usually done by the hyperbolic model. In addition, seismic loading imposes irregular loading pulses which consist of loading, unloading and reloading. Modeling of this soil behavior in cyclic loading is usually made by first specifying the stress-strain relation in the virgin or initial loading, using a backbone or skeleton curve. Subsequent unloading and reloading characteristics are handled using the Masing criterion (1926) as illustrated in
Fig. 3.6. Soil Deposit
Fig. 3.7. In the program QUIVER, the behavior of soil is treated to be non-linear and hysteretic, exhibiting Masing behavior during loading and unloading.

The tangent shear modulus $G_t$ for any one particular level of strain could be represented by the three parameter hyperbolic model (Duncan and Barkeley) given by

$$G_t = \frac{\partial \tau}{\partial \gamma} = G_{\text{max}} \left[ 1 - \frac{\tau}{\tau_f} R_f \right]^2 \quad (3.2.2.1)$$

in which,

$G_{\text{max}}$ = the maximum shear modulus that occurs at zero strain at the beginning of cycling,

$\tau$ = the shear stress,

$\tau_f$ = the failure shear stress,

$R_f$ = the failure ratio $\tau_f/\tau_{\text{ult}}$ in which $\tau_{\text{ult}}$ is the ultimate strength. $R_f$ may also be considered a factor that defines the strain $\gamma_f$ at which the failure stress occurs,

$$\gamma_f = \frac{\tau_f}{G_{\text{max}}} (1 - R_f)$$

The parameter $R_f$ is used to modify the hyperbola to fit the laboratory data. $R_f = 0$ specifies a linear elastic plastic material with $\gamma_f = \tau_f/G_{\text{max}}$ as shown in Fig. 3.8. $R_f = 1$ specifies a strain hardening material with $\gamma_f$ equal to infinity. For most sands the, $R_f$ lies between 0.5 and 0.9 (Byrne and McIntyre).

The modified shear stress-strain relationship under cyclic loading could be modeled as a hyperbola and the tangent stiffness at any stress level is expressed as

$$G_t = \frac{\partial \tau}{\partial \gamma} = G_{\text{max}} \left[ 1 - \frac{\tau^*}{\tau^*_f} R_f \right]^2 \quad (3.2.2.2)$$

$G_{\text{max}}$ = the shear modulus immediately upon unloading,
\[ G = G_{\text{max}} \left(1 - \frac{\tau}{\tau_R}\right)^2 \]

Fig. 3.7 Loading-Unloading Curve for Shear Stress-Strain Modeling
Fig. 3.8 Modified Hyperbolic Shear Stress-Strain Formulation

(after Byrne and McIntyre, 1994)
\[ \tau^* = (\tau_A + \tau), \]
\[ \tau_f^* = (\tau_A + \tau_f) \text{ and} \]
\[ \tau_A = \text{the shear stress at the reversal point as shown in Fig. 3.7.} \]

Rearranging and integrating Equation 3.2.2.2 gives equations 3.2.2.3 through 3.2.2.5, hence the generalized expression for \( \tau \) for all types of loading could be obtained.

\[ \frac{1}{1 - \frac{\tau^*}{\tau_f^*}} R_f G_{\text{max}} \gamma + C \]

at \( \gamma = 0, \tau = 0 \)

or,

\[ \tau = \frac{R_f G_{\text{max}} \gamma}{1 + \frac{R_f}{\tau_f} G_{\text{max}} \gamma} \]

or,

\[ \tau = \begin{bmatrix} \frac{G_{\text{max}} \gamma}{1 + \frac{R_f}{\tau_f} G_{\text{max}} \gamma} \end{bmatrix} \]

### 3.2.3 COMPUTATION OF HYPERBOLIC MODEL PARAMETERS

The hyperbolic parameters in Equation 3.2.2.5 are influenced by a number of factors. In order to obtain a reasonable calculation it is necessary that the most important factors are accommodated appropriately. The maximum shear modulus \( G_{\text{max}} \) could be obtained generally from in-situ measurements, laboratory experiments or empirical equations (Byrne, 1994). There is no one procedure that is accepted by different
researchers to evaluate the maximum shear modulus. In QUIVER, $G_{\text{max}}$ for sand is calculated using the following equation

$$G_{\text{max}} = k_g P_a \left( \frac{\sigma'_m}{P_a} \right)^m$$  \hspace{1cm} (3.2.3.1)

in which

$k_g$ varies from 500 to 2000 for sands and is a function of normalized SPT blow count,

$\sigma'_m =$ mean normal effective stress,

$P_a =$ atmospheric pressure,

$m =$ constant depending on the type of soil.

The equation for $k_g$ suggested by Byrne (1994) takes the form

$$k_g = 440 (N_1)^{\frac{1}{3}}_{60}$$  \hspace{1cm} (3.2.3.2)

where,

$(N_1)_{60} =$ is the normalized SPT blow count.

The mean normal stress $\sigma'_m$ is calculated from the following expression,

$$\sigma'_m = \frac{\sigma'_v}{3} (1 + 2k_0)$$  \hspace{1cm} (3.2.3.3)

in which,

$\sigma'_v =$ the effective normal stress,

$k_0 =$ 0.5

The failure shear strength, $\tau_f$ for soils is dependent on the stress system by means of which the soil element is brought to failure. Hardin and Drenvich (1972) suggested that the value of $\tau_f$ calculated using Mohr-Coulomb failure envelope (defined by the static strength parameters such as $c'$, effective cohesion, and $\phi'$, internal angle of friction) is

45
adequate for dynamic loading. Modified Coulomb's model for shear strength of a soil might be expressed in the form (Scott, 1980)

\[ \tau_f = c' + \sigma_v \tan \phi' \] (3.2.3.4)

where

\( \tau \) is the shear stress in the soil at failure,
\( \sigma_v \) is the effective normal stress.
\( c', \phi' \) are approximately constant parameters expressing the shear strength in terms of effective stress.

For sandy soils \( c' \) is close to zero.

### 3.2.4 PROBLEM FORMULATION

A one dimensional element of length \( \Delta \) with two end nodes, as shown in Fig. 3.6 is used in the finite element formulation of the free-field. The displacement \( u \) is relative to the till, which itself displaces an amount \( v_{ge} \) with the acceleration \( v_{ge} \). With the element shape functions expressed in local coordinate \( \xi \) (-1 ≤ \( \xi \) ≤ 1) the axial coordinate of any point inside the element can be expressed as

\[ \xi = \frac{2(z - z_c)}{\Delta} \quad \text{and} \quad dz = \frac{\Delta}{2} d\xi \] (3.2.4.1)

where \( z_c \) is the z-coordinate of the center of the element length. Numerical integration has to be performed inside every element and summed over to obtain element stiffness matrix. The consistent element mass matrix is evaluated in closed form as will be seen
later. The displacement vector consists of nodal degrees of freedom $u$ and its first
derivative $u'$. They are of the following form

$$a^T = \{u_1, u_1', u_2, u_2'\}$$  \hspace{1cm} (3.2.4.2)

This sequence gives the flexibility of having different $G_{\text{max}}$ for each element in a layer and
more than one layer in the problem domain. It permits strain discontinuity between
elements, a parabolic shear strain within an element and satisfaction of continuity in the
shear stresses. The grouping of nodal degrees of freedom is worth mentioning. The nodes
at ground surface and at the till have two degrees of freedom, and the other nodes have
three degrees of freedom, $u_i'$, $u_j$, and $u_j'$, in which $i$ and $j$ correspond to two adjacent
elements. Thus, except at the ends, there are three degrees of freedom per node.

3.2.5 SHAPE FUNCTIONS

For the one dimensional soil element the strain-displacement relationship contains
first derivative in the lateral translations. Hence it is necessary to choose the displacement
function such that $u$ is continuous at the nodes. This can best achieved by adopting a linear
displacement for $u$. However, complete cubic interpolations are used to approximate $u$
within each element. A complete cubic polynomial requires 4 parameters to define a
function. The lateral displacements and their first derivatives at the two nodes provide
sufficient parameters to fully describe a cubic polynomial function. Hence we could
express $u$ and $u'$

$$u = N_1 u_i + N_2 \left(\frac{du}{dx}\right)_i + N_3 u_j + N_4 \left(\frac{du}{dx}\right)_j$$  \hspace{1cm} (3.2.5.1)
where subscripts $i$ and $j$ refer to the first and second nodes respectively of each element and

\[
N_1 = \left( \frac{2 - 3\xi + \xi^3}{4} \right) \tag{3.2.5.2}
\]
\[
N_2 = \left( \frac{1 - \xi - \xi^2 + \xi^3}{8} \right) \tag{3.2.5.3}
\]
\[
N_3 = \left( \frac{2 + 3\xi - \xi^3}{4} \right) \tag{3.2.5.4}
\]
\[
N_4 = \left( \frac{-1 - \xi + \xi^2 + \xi^3}{8} \right) \tag{3.2.5.5}
\]

Alternatively, $u$ and $u'$ could be written as

\[
u = M_0^T \{ a \} \quad u' = M_1^T \{ a \} \tag{3.2.5.6}
\]

where $M_0^T = \{ N_1 \ N_2 \ N_3 \ N_4 \}$ and $M_1^T = \{ N_1' \ N_2' \ N_3' \ N_4' \}$

### 3.2.6 VIRTUAL WORK EQUATIONS FOR FREE-FIELD

Principle of virtual work is used in formulating the element-stiffness and mass matrices. This is illustrated in Fig. 3.9. Let such a virtual displacement be $\delta\mathbf{u}'$ at the nodes. This results, by Equation 3.2.5.6 in virtual displacements and strains within the element equal to

\[
\delta u = M_0^T \{ \delta a \} \quad \delta \gamma = M_1^T \{ \delta a \} \tag{3.2.6.1}
\]

respectively.

The internal work per unit volume done by the stresses $\tau(\gamma)$ is

\[
\tau(\gamma) \delta\gamma \tag{3.2.6.2}
\]

Equating the external work done by the inertial forces with the total internal work obtained by integrating over the volume of the element, one obtains

\[
\int_0^A \int \tau(\gamma) \delta\gamma dz = -\int_0^A \rho \left( \dot{u} + v \dot{\phi} \right) \delta u dz \tag{3.2.6.3}
\]
Fig. 3.9 Virtual work for soil deposit
where \( \ddot{\nu}_g \) is the acceleration at the till. This must hold good for any \( \delta \), substituting Equation 6.2.6.1 in 6.2.6.3 to work in terms of global system, we obtain the equation of motion

\[
\frac{1}{2} \rho \int_{-1}^{1} M_0 M_0^T d\xi \begin{bmatrix} \ddot{t} \\
\end{bmatrix} + \frac{1}{2} \int \tau(\gamma) M_1 d\xi = -\frac{1}{2} \rho \int M_0 d\xi \ddot{\nu}_g
\]

(3.2.6.4)

### 3.2.7 CONSISTENT MASS MATRIX FOR SOIL ELEMENT

The consistent mass matrix for the soil element evaluated analytically is of the form,

\[
M = \begin{bmatrix}
13\rho\Delta & 11\rho\Delta^2 & -13\rho\Delta^2 & 9\rho\Delta \\
35 & 210 & -420 & 70 \\
11\rho\Delta^2 & -13\rho\Delta^2 & 13\rho\Delta^2 & 13\rho\Delta \\
210 & 105 & 140 & 420 \\
-13\rho\Delta^2 & -\rho\Delta^3 & -\rho\Delta^3 & -\rho\Delta^3 \\
210 & 105 & 140 & 420 \\
9\rho\Delta & 13\rho\Delta^2 & -11\rho\Delta^2 & 13\rho\Delta \\
70 & 420 & -210 & 35
\end{bmatrix}
\]

(3.2.7.1)

### 3.2.8 TIME AND NUMERICAL INTEGRATION

Newmark-\( \beta \) integration is adopted for solving the time dependent soil vibration problem. Equation 3.2.6.4 is rewritten below

\[
\frac{1}{2} \rho \int_{-1}^{1} M_0 M_0^T d\xi \begin{bmatrix} \ddot{t} \\
\end{bmatrix} + \frac{1}{2} \int \tau(\gamma_{i+1}) M_1 d\xi = -\frac{1}{2} \rho \int M_0 d\xi \ddot{\nu}_{i+1}
\]

(3.2.8.1)

After approximating with Newmark-\( \beta \) procedure (average acceleration method),
\[
\left[ \frac{\Delta}{2} \int_{-1}^{1} \rho M_0 M_0^T \, d\xi \right] \frac{1}{\alpha \Delta t^2} \{ \mathbf{a}_{i+1} \} + \frac{\Delta}{2} \frac{1}{\alpha} \left( \gamma_{i+1} \right) M_1 \, d\xi = -\frac{\Delta}{2} \int_{-1}^{1} \rho M_0 \, d\xi \{ \mathbf{v}_{i+1} \} \\
+ \left[ \frac{\Delta}{2} \int_{-1}^{1} \rho M_0 M_0^T \, d\xi \right] \frac{1}{\alpha \Delta t^2} \{ \mathbf{a}_i \} + \frac{1}{\alpha} \left[ \frac{\gamma}{2 \alpha - 1} \right] \{ \mathbf{a}_i \} 
\]

(3.2.8.2)

where \( \Delta t \) is the time integration step.

Let the right hand side of the above equation be \( \{ \mathbf{R} \} \). Thus, we must find \( \{ \mathbf{a}_{i+1} \} \) such that

\[
\{ \psi \} = \left[ \frac{\Delta}{2} \int_{-1}^{1} \rho M_0 M_0^T \, d\xi \right] \frac{1}{\alpha \Delta t^2} \{ \mathbf{a}_{i+1} \} + \frac{\Delta}{2} \frac{1}{\alpha} \left( \gamma_{i+1} \right) M_1 \, d\xi - \{ \mathbf{R} \} = 0 
\]

(3.2.8.3)

Using the Newton-Raphson approach, \( \mathbf{a}_{i+1} \) can be found starting from an initial \( \mathbf{a}_i^* \). Then

\[
\mathbf{a}_{i+1} = \mathbf{a}_i^* + \left[ \frac{\partial \mathbf{\Psi}}{\partial \mathbf{a}} \right]^{-1} (-\mathbf{\Psi}^*) 
\]

(3.2.8.4)

where \( \mathbf{\Psi}^* \) is the vector \( \mathbf{\Psi} \) at \( \mathbf{a}_i^* \) and the matrix

\[
\frac{\partial \mathbf{\Psi}}{\partial \mathbf{a}} = \frac{1}{\alpha \Delta t^2} \left[ \frac{\Delta}{2} \int_{-1}^{1} \rho M_0 M_0^T \, d\xi \right] + \frac{\Delta}{2} \frac{1}{\alpha} \frac{\partial \gamma}{\partial \gamma} M_1 M_1^T \, d\xi 
\]

(3.2.8.5)

As indicated in section 3.1.10, \( n \) Gauss points integrate a polynomial of order \((2n-1)\) exactly. In the soil stiffness expression Equation 3.2.8.5 in the Newton-Raphson procedure, there is a polynomial of order 4. Hence a 3-point Gauss quadrature would suffice to integrate exactly. However it is noteworthy to mention that in the program QUIVER there is a flexibility in choosing the number of Gauss points from 1 to 9. It is understood that as numerical integration using Gaussian scheme is employed the shear stresses and strains could only be evaluated at the sampling points. A suitable interpolation should be adopted to evaluate them at other locations.
3.2.9 DISCRETIZATION ERROR AND CONVERGENCE

An Euclidean norm criterion has been used to check the convergence at the end of every iteration in the Newton-Raphson procedure. If $\|\vec{a}_r\|$ is the solution vector at the end of $r^{th}$ iteration and $\|\vec{a}_{r+1}\|$ is the solution at the end of $(r+1)^{th}$ iteration, then $\Delta a = \|\vec{a}_{r+1} - \vec{a}_r\|$ represents the difference between the two displacement vectors. In which,

$$\|\vec{a}_r\|^2 = \sum_{i=1}^{NDOP} a_r^2(i) \quad (3.2.9.1)$$

$$\|\vec{a}_{r+1}\|^2 = \sum_{i=1}^{NDOP} a_{r+1}^2(i) \quad (3.2.9.2)$$

where $a_r(i)$ and $a_{r+1}(i)$ are components of $\vec{a}_r$ and $\vec{a}_{r+1}$ respectively. Then $\Delta a$ could be written as

$$\Delta a = \sum_{i=1}^{NDOP} \sqrt{(a_{r+1}(i) - a_r(i))^2} \quad (3.2.9.3)$$

The convergence criterion for this norm is given as

$$\frac{\Delta a}{\|\vec{a}_r\|} \leq \text{specified tolerance} \quad (3.2.9.4)$$

Secondly, it is verified that the total error in the problem $\{\Psi^r\}$ is zero. The mathematical form are

$$\|\{a\} - \{a\}\| \leq TOL1 \quad \text{and} \quad (3.2.9.5)$$

$$\|\{a\}\| \leq \{\Psi^r\} \leq TOL2 \quad (3.2.9.6)$$
The convergence tolerance for Equation 3.2.9.6 needs special mention. As the unit for $\{\Psi\}$ is that of force, a tolerance of say $1N$ or so might be appropriate.

If the solution obtained has not converged then the whole procedure is repeated by updating the solution vector in Equation 3.2.8.2. If converged, then the corresponding acceleration and velocity vectors are calculated by Newmark-β method. The entire procedure is repeated for all the time steps.

3.2.10 QUIVER - THE COMPUTER PROGRAM

A computer program, QUIVER, based on the aforementioned nonlinear finite element procedure was developed. Its salient features include changing $G_{max}$ and $\tau_{ult}$ for every single element as a function of effective or total stress, multilayering in the soil deposit, energy dissipation (hysteretic damping) by following through the Duncan and Barkaley shear stress-strain model in every single Gauss sampling point throughout the depth for the entire time history, evaluation of cyclic stress ratio and flexibility in giving the input motion at anywhere in the soil profile. Results of sample runs are enclosed in the next Chapter.
CHAPTER 4

VERIFICATION AND NUMERICAL RESULTS

4.1 INTRODUCTION

Numerical results obtained through the proposed formulation described in Chapter 3, and incorporated in the finite element programs QUIVER and PILEUBC are now reported. As in any other analytical procedure the validity of the formulation was verified against linearized closed form solutions to check the accuracy of predictions. The results obtained from the programs are presented in two stages. First, the cases for the finite element analysis of the free-field soil deposit are presented. It includes a comparative analysis with a program SHAKE (Scnabel et. al., 1972).

Secondly, the analysis results from the program PILEUBC are shown. It includes comparative solutions using assumed linear systems and two example problems are dealt. The results comprise of the pile's relative displacement time history at specific nodes, relative displacement profile and bending moment profile at selected time intervals. It is understood that a variety of data could also be obtained from the program. For example, the time varying bending moment, shear force, displacement, velocity, acceleration, bending stress, axial and bending strain and gap history. Explanation of input and sample input data file are attached in Appendix II.
4.2 VERIFICATION OF FREE-FIELD MOVEMENT AGAINST DEVELOPED CLOSED FORM SOLUTION

In order to verify the proposed finite element formulation for free-field soil as illustrated in Section 3.2, a closed form solution for a linear uniform stress field system was developed. If for instance the soil deposit is subjected to a harmonic base excitation and it is idealized as a uniform stress field as described in Appendix A, it is possible to obtain a time dependent exact closed form solution. In this situation, whatever the element subdivision, the finite element solution will coincide with the exact one (Zienkiewicz, 1977). This is an obvious corollary of the formulation and it is useful as a first check of the computer program QUIVER. This solution involves both the transient response and the steady state response of the dynamic soil problem. With this as intention, utilizing Laplace transformation a closed form expression is developed for harmonic base excitation of an elastic soil deposit. The soil deposit is idealized as one layer with constant $G_{max}$ throughout the depth with no damping in the system. The shear stress-strain behavior is assumed to be linear in this boundary value problem. It is important that all the assumptions inherent in the closed form are precisely represented in the finite element program.

The soil deposit used for the comparison between the closed form solution and the finite element approach (QUIVER) was: the depth of the deposit ($H$) was 30m, the maximum dynamic shear modulus ($G_{max}$) was 275000 kN/m$^2$, the failure shear stress was 375 kN/m$^2$, and the simulated sinusoidal base acceleration at the till was $0.5g \sin(\omega t)$, where, $g$ is the acceleration due to gravity and $\omega$ is the frequency of excitation.
To conform with the assumptions used in the closed form solution, the maximum
dynamic shear modulus and the failure shear stress were assumed to be constant
throughout the depth. In addition the free-field was assumed to behave as a undamped
linear system. For comparison, the time history of relative displacements and absolute
accelerations on the surface computed from closed form and QUIVER are plotted in Fig.
4.1. The time history for relative displacements throughout the profile are plotted in Fig.
4.2. It can be seen from both the above mentioned figures that the predicted accelerations
and displacements match exactly with the closed form.

4.3 RESULTS FOR VARYING G_{max} AND \tau_f IN A HETEROGENEOUS SOIL

DEPOSIT

The maximum dynamic shear modulus is a function of effective pressure, \( \sigma' \), and
normalized SPT blow count, \( (N_j)_{60} \). And the failure shear is a function of effective
pressure, effective friction angle \( \phi' \), and the cohesion intercept, \( c' \). These properties vary
with depth and type of soil layer. It may not be possible to develop a closed form solution
for this continuous heterogeneous deposit. Results of such a nonlinear finite element
analysis for a simulated sinusoidal base acceleration of \( 0.2g \ Sin(\omega t) \) is enclosed in Fig 4.3.
The pre-earthquake soil properties of the deposit used in this example problem are shown
in Table 4.1.
Fig. 4.1 Comparison of Finite Element and Closed Form Solution at the Surface for Uniform Stress Field

Relative Displacement (mm)

Absolute Acceleration (m/s²)

Finite Element Solution

Closed Form Solution

Base Excitation = 0.5 × 5 sin(ωt) ω = 6.28 rad/sec

γ = 27500 KN/m²

γ = 20 KN/m²

v = 350 KN/m²

H = 30 m
Base Acceleration = 0.5 g sin (wt)

Fig 4.2 Absolute acceleration along the profile using 10 elements for Uniform Stress Field
Fig. 4.3. Acceleration Time History From QUIVER Simulated Sinusoidal Excitation
Table 4.1

Soil Properties Used In The Example Problem 1

<table>
<thead>
<tr>
<th>Soil</th>
<th>Depth (m)</th>
<th>(N₁)₆₀</th>
<th>kₚ</th>
<th>m</th>
<th>φ</th>
<th>Rₜ</th>
<th>γ (kN/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crust</td>
<td>3.00 *</td>
<td>10</td>
<td>440(N₁)₆₀</td>
<td>0.4</td>
<td>33</td>
<td>0.9</td>
<td>20</td>
</tr>
<tr>
<td>Sand</td>
<td>30.00</td>
<td>10</td>
<td>440(N₁)₆₀</td>
<td>0.4</td>
<td>27</td>
<td>0.7</td>
<td>20</td>
</tr>
</tbody>
</table>

* Water table location

Gₘₐₓ = kg Pa (σᵥ/Pa)ᵐ

Time history of absolute accelerations of the same soil deposit of Table 4.1, for an earthquake excitation (San Fernando, 1971) are shown in Fig. 4.4. A comparative cyclic stress ratio plot from QUIVER and SHAKE is shown in Fig. 4.5. Cyclic stress ratio is defined (Seed and Harder, 1992) by the following relation,

\[ CSR = \frac{0.65 \tau_{\text{max}}}{\sigma_v} \]  \hspace{1cm} \text{(4.1)}

In which

\( \tau_{\text{max}} = \) Maximum driving shear stress

\( \sigma_v = \) Effective overburden pressure.

The results obtained from the finite element analysis are in good agreement with SHAKE except near the water table. It could be observed from Equation 3.2.3.1 that the shear modulus varies parabolically with depth and is a function of effective pressure. If a water table is introduced, the rate of change of \( G_{\text{max}} \) changes rapidly near the water table. As a result the shear strain in the soil at the location of water table would be more, resulting in swift changes in cyclic stress ratio profile. However, the accelerations predicted by
**SAND**

\( N_1 \lambda_0 = 10 \)

\( \gamma = 20 \text{ kN/m}^3 \)

**Till**

**STANDARD LAYER**

Fig. 4.4 Acceleration Time History From QUIVER for Earthquake Excitation
Fig. 4.5. Cyclic Stress Ratio Vs Depth between QUIVER and SHAKE

LEGEND

- No. Elements = 50
- H = 30 m
- \( (N_d)_{60} = 10 \)
- \( \gamma_{60} = 20 \text{ kN/m}^3 \)
- Excitation = San Fernando, 1971
- No. of Layers used in SHAKE = 10

\[
\text{C. S. R.} = \frac{0.65 \tau_{\text{max}}}{\sigma_v^1}
\]
SHAKE at relatively deeper space in the soil deposit is higher than the finite element solution. Fig. 4.6 shows the variation of shear stress-strain at one of the Gauss sampling points in the soil deposit obtained from QUIVER for the above mentioned analysis. It could be observed from Fig. 4.6 that irregular loading the soil deposit and hence the response could be captured appropriately.

4.4 RESULTS FROM AN ANALYSIS FOR CANLEX SITE, AT RICHMOND

Another example was chosen at a site in Richmond, British Columbia, Canada from field data available through seismic cone penetration (SCPTU) tests performed by the In Situ Testing Group, The University of British Columbia. The analysis aimed at the study the free-field response subjected to a base acceleration of the San Fernando, 1971 earthquake record. The site reference indicated in the project report, CANLEX, PHASE II, (Campanella, 1995) was KD9033. This site was chosen for the reason that interpreted data on soil properties was made available upto the till at 48.83m. Fig 4.7 shows the variation of normalized SPT blow count \( N_{60} \) with depth for this site. The pre-earthquake soil properties used in the analysis are tabulated in Table 4.2. The input data file for the program is included in Appendix II. The absolute acceleration time history obtained from the finite element analysis and from SHAKE analysis is shown in Fig. 4.8 and 4.9 respectively. The base acceleration for this site was the acceleration record from San Fernando earthquake record, 1971. It is observed that SHAKE analysis predicts relatively a higher acceleration at one layer above the till.
LEGEND

- $H = 30$ m
- $(N_e)_{60} = 10$
- $\gamma = 20$ kN/m$^3$
- Element No. = 1
- Gauss Point No. = 3
- Excitation = San Fernando Earthquake 1971,
  first 300 data points
  time step = 0.02 sec

Fig. 4.6. Typical Shear Stress-Strain Plot from the Finite Element Program
Fig. 4.7  SPT Data for Test No. KD9303
No. 4 Road at River Road, Richmond, BC, Canada
CLAYEY SILT

\( \gamma = 18.86 \text{ kN/m}^3 \)

No. of Finite Elements = 10

SANDY SILT

\( \gamma = 18.86 \text{ kN/m}^3 \)

No. of Finite Elements = 10

FINE SAND

\( \gamma = 19.65 \text{ kN/m}^3 \)

No. of Finite Elements = 20

CLAY

\( \gamma = 18.86 \text{ kN/m}^3 \)

No. of Finite Elements = 20

Earthquake acceleration induced at till.
(San Fernando, 1971)

Fig. 4.8. Acceleration Time History From QUIVER
Fig. 4.9. Acceleration Time History From SHAKE
Table 4.2
Soil Properties for Nonlinear Analysis of CANLEX SITE

<table>
<thead>
<tr>
<th>Soil</th>
<th>Depth (m)</th>
<th>c’ (kN/m$^2$)</th>
<th>$\phi^*$</th>
<th>$(N_1)_{60}$</th>
<th>$\gamma$ (kN/m$^3$)</th>
<th>OCR (ratio)</th>
<th>No. of Finite Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clayey Silt</td>
<td>1.25*</td>
<td>70.0</td>
<td>0</td>
<td>6</td>
<td>18.86</td>
<td>100.0</td>
<td>5</td>
</tr>
<tr>
<td>Clayey Silt</td>
<td>3.25</td>
<td>55.0</td>
<td>0</td>
<td>6</td>
<td>18.86</td>
<td>35.0</td>
<td>5</td>
</tr>
<tr>
<td>Sandy Silt</td>
<td>6.25</td>
<td>0.0</td>
<td>41</td>
<td>11</td>
<td>18.86</td>
<td>0.0</td>
<td>10</td>
</tr>
<tr>
<td>Fine Sand</td>
<td>23.0</td>
<td>0.0</td>
<td>41</td>
<td>24</td>
<td>19.65</td>
<td>0.0</td>
<td>20</td>
</tr>
<tr>
<td>Clay</td>
<td>48.83</td>
<td>65.0</td>
<td>0</td>
<td>6</td>
<td>18.86</td>
<td>1.0</td>
<td>20</td>
</tr>
</tbody>
</table>

* Water table location

It could be observed from Fig. 4.4 and Fig. 4.8 that the time varying relative displacement obtained by the finite element approach continues to vibrate even after the base acceleration has attained very small amplitudes. The reason is that as only hysteretic damping is considered in the soil stress-strain model, on account of minuscule vibrations, the strain levels will be in the linear range and hence energy dissipation is infinitesimal. As a result, the undamped free vibration response is observed.

4.5 RESULTS FROM FINITE ELEMENT ANALYSIS OF SOIL-PILE-SUPERSTRUCTURE INTERACTION

The results for seismic soil-pile-superstructure is presented in this section. Comparison is made with a linear system. Later results from two examples problems are
illustrated. A similar formulation for soil-pile interaction for static cyclic loading was formulated and incorporated in the finite element program CYCPILE at the University of British Columbia (Vazhinkhoo, 1995). A comparison between analysis and experimental for a case study is shown in Fig. 4.10. The analysis was done (Vazhinkhoo, 1995) for a laboratory test on an instrumented aluminum pile embedded in a dense sand subjected to monotonic pile head loading. Stresses in the soil were increased to field stress levels using Hydraulic Gradient Similitude technique (Yan and Byrne, 1992). Comparison was made between CYCPILE predictions and test data. The predicted results were observed to be in excellent agreement with experimental data. As CYCPILE and PILEUBC are based on the same theory, the former only considering static cyclic loads, the agreement shown in Fig. 4.10 can also be seen as a verification of PILEUBC for static loads.

In order to check the dynamic component of PILEUBC, the vibration of a simple cantilever was studied. If the pile is assumed to be a linear cantilever beam of length, \( l \), with lumped mass, \( m \), vibrating at the free end for a harmonic sinusoidal base excitation with no viscous damping (Fig. 4.11), closed form solution is available. The time varying relative displacement at the free end is

\[
    u(x,t) = u_0 \cos \omega t + \frac{v_0}{\omega} \sin \omega t - \frac{ma}{k} \frac{1}{1 - \beta^2} (\sin \Omega t - \beta \sin \omega t)
\]  

(4.2)

in which

\( u(x,t) \) = time varying relative displacement

\( u_0 \) = initial displacement

\( v_0 \) = initial velocity

\( m \) = lumped mass
Fig. 4.10 Comparison between Experimental Results and CYCPILE Predictions
Fig. 4.11. Comparison of Closed Form and Finite Element Solutions for a Cantilever Beam.
\[ k = \text{stiffness of the system}, \quad k = \frac{3EI}{L} \]

\[ I = \text{moment of inertia} \]

\[ \omega = \text{natural frequency of the system}, \quad \omega = \sqrt{\frac{3EI}{3Lm}} \]

\[ \Omega = \text{frequency of excitation} \]

\[ \beta = \text{frequency ratio}, \quad \beta = \frac{\Omega}{\omega} \]

The cantilever beam used in a comparative analysis between the closed form and finite element approach (PILEUBC) had a length \( (L) \) of 12m, the external and internal diameter of 2.0 and 1.8 m respectively. Young's modulus \( (E_{\text{max}}) \) was 210000 MPa, the yield stress was 700 Mpa, and the simulated sinusoidal base acceleration at the till was \( 0.2g \Sin(\beta h) \). This system will have a natural frequency of 13.56 rad/sec. The frequency of excitation, \( \beta \), was chosen to be was 11.51 rad/sec. A flag was introduced in the finite element program such that there was no near-field soil and the yield stress of the pile material was set to be a high number such that there was no hysteretic damping in pile. P-\( \Delta \) effects were neglected. Results of this comparative analysis are shown in Fig. 4.11. The finite element solution coincides with the closed form solution.

The proposed non-linear formulation was applied to predict the pile-superstructure behavior for an example problem shown in Fig. 4.12. The pile was a steel pipe, 32.4 m long and the external and internal diameter were 0.9 m and 0.7 m respectively. The pile was assumed to be embedded upto the till in sandy soil for a depth of 30 m. Weight of the superstructure was assumed to be 500 kN. The average normalized
standard penetration value, \((N_1)_{60}\), for sandy soil was assumed to be 10. The following values were adopted for the soil:

(a) Bulk unit weight = 20 kN/m\(^3\)

(b) Angle of internal friction = 33°

A simulated sinusoidal base acceleration, \(0.5 \, g \sin (6.28t)\), was imposed at the till. The number of finite elements for soil and pile were 30 and 15 respectively. Boundary condition for the pile was such that axial displacement at the pile tip was zero. The relative displacement time history of the structural mass and the pile at the surface is shown in Fig. 4.12. It also includes the relative displacement time history of the soil at ground level. Figs. 4.13 and 4.14 show the profiles of relative displacement and bending moment in the pile at selected intervals.

4.6 RESULTS OF AN EXAMPLE PROBLEM AT CANLEX SITE, RICHMOND

To simulate an actual situation another example was considered at the site in Richmond, British Columbia. The soil properties of the site are tabulated in Table 4.2. The same cross section used in the previous example was tested herein. The embedded length of the pile was 48.83 m and length above the ground level was assumed to be 2.4 m. Structural weight was assumed to be 500 kN. The boundary condition for the pile was such that axial displacement at the pile tip was zero. The relative displacement of the pile is relative to the till and it is not constrained (as if the tip of the pile were placed on rollers). An earthquake acceleration (San Fernando, 1971) was applied at the till, and the boundary condition for the soil was such that the relative displacement at the till was zero.
Base Acceleration = 0.5 g Sin(6.28 t)

Example No. 1

Fig. 4.13. Relative Displacement Profile at Selected Time Steps
Base Acceleration = 0.5 g Sin(6.28 t)

Example No. 1

Fig. 4.14 Bending Moment Profile at Selected Time Steps for Example Problem 1
The input data file is enclosed in Appendix II. Results of relative displacement time history of the structural mass and the pile at ground surface is shown in Fig. 4.15. Profiles of the pile relative displacement and bending moment are shown from Fig. 4.16 and Fig. 4.17.

From Fig. 4.15 it is observed that, when the amplitude of base acceleration is small at the end of the quake, the soil-pile-superstructure system undergoes free vibration. The natural period of vibration may be obtained from this time history plot. It could be observed from Fig. 4.17 that during an earthquake a pile would have a maximum bending moment near the surface, approximately at about 1/7th-1/8th the embedded depth of pile below the ground level. The reason could be that as gapping forms the deformation of the pile could be large as compared to the situation with no gapping at all. This is a crude approximation as it depends on number of factors, such as the cross section and material of pile, embedded depth, type of the soil deposit and frequency and amplitude of earthquake excitation.
Base Acceleration =
San Fernando Earthquake, 1971
Example No. 2

Relative Displacement (m)

Depth (m)

-0.1000 -0.0500 0.0000 0.0500 0.1000 0.1500 0.2000 0.2500 0.3000

Fig. 4.16. Relative Displacement Profile of Pile at Selected Time Steps for Example Problem 2
Base Acceleration = San Fernando Earthquake, 1971
Example No. 2

Fig. 4.17 Bending Moment Profile of Pile at Selected Time Steps for Example Problem 2
A finite element seismic response analysis procedure for soil-pile-superstructure interaction is developed incorporating the geometric and material non-linearities of the pile, non-linear near-field-pile relationship, P-Δ effects and the soil shear stress-strain non-linearities. The formulations for free-field and the soil-pile interaction are based on the principle of virtual work. The formulation has been incorporated in two finite element programs: QUIVER and PILEUBC. Finite element predictions were compared with linearized closed form solutions wherever possible. In particular, a linearized closed form solution was developed for shear wave propagation in a uniform stress field subjected to a base acceleration. This solution was in exact agreement with the finite element analysis.

The significance of the procedure presented in this thesis is that the usual assumptions made in other procedures like, "sub-structuring" of the soil-pile-superstructure and hence a equivalent linear analysis has been avoided. Damping in this dynamic system is obtained purely through hysteretic energy dissipation, by following through the non-linear constitutive relations for the pile and soil. Soil-pile separation was considered, keeping track of whether there was contact or not between the soil and the pile throughout the depth.

In this formulation an assumption was made that there is no rise in pore pressure when a deposit with sandy soil is subjected to an earthquake excitation. This assumption could be invalid, as has been observed that the maximum dynamic shear modulus of sandy
soil degrades in an undrained situation during excitation. However, it could be incorporated in the finite element program QUIVER using shear-volume coupling model developed by Byrne, 1990. By incorporating this model in a subroutine called HYSTER in the QUIVER program, the seismic response of piles on liquefied soil could be studied.

The effect of accounting for uncertainties in the geometry or material properties of the pile and soil could be studied using stochastic finite elements. Uncertainties may exist in, for example, the initial elastic modulus of the pile or the shear modulus in the soil which are usually spatially distributed over the region of the soil-pile system and could be modeled as a spatial random process. Since pile foundations are to be designed with a high level of reliability, it would be worthwhile to study effects of such spatial variability of material properties. The analysis developed in this thesis could thus be coupled with a general reliability analysis for earthquake loading, including the uncertainties in the seismic excitation itself.
REFERENCES


APPENDIX I

ANALYTICAL SOLUTION FOR A SINGLE LAYER SOIL SYSTEM USING LAPLACE TRANSFORMS

The Laplace transform serves as a device for obtaining the solution of ordinary or partial differential equations (Powers, 1989). It associates a function \( f(t) \) with a function of another variable, \( F(s) \), Equation A.1, from which the original function can be recovered.

\[
F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} \, dt
\]  

(A.1)

Fig. 6 shows a soil particle of density \( \rho \), undergoing a horizontal translation \( u(x, t) \). For an equilibrium to exist in this assumed linear undamped system with \( G \) as the maximum dynamic shear modulus, the following one dimensional wave propagation equation has to be satisfied

\[
\rho \left( \frac{\partial^2 u}{\partial t^2} + a(t) \right) = G \frac{\partial^2 u}{\partial x^2}
\]  

(A.1)

or

\[
a(t) + \frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}
\]  

(A.2)

In which,

\( \rho \) = mass density of soil,
\( G \) = shear modulus of the soil,
\( c \) = shear wave velocity of the layer, \( \sqrt{\frac{G}{\rho}} \)
\( a(t) \) = harmonic base acceleration,
Fig. A.1.1  Soil deposit as a one dimensional shear wave propagation medium
\( u \) = relative displacement with respect to till.

We want to find the Laplace transform of the one dimensional wave equation whose appropriate initial and boundary conditions are,

\[
\begin{align*}
\text{u}(x,0) &= 0 \\
\frac{\partial u}{\partial x}(x,0) &= 0 \\
\gamma &= \frac{\partial u}{\partial x}(0,t) = 0 \\
u(H,t) &= 0
\end{align*}
\]

(A.3)

Using the properties of Laplace transformation, we transform both sides of the equation, obtaining

\[ \tilde{a}(s) + s^2 \tilde{u}(s) = \frac{\partial^2}{\partial x^2} \tilde{u}(s) \]  

(A.4)

or

\[ \frac{\partial^2}{\partial x^2} \tilde{u}(s) - \frac{s^2}{c^2} \tilde{u}(s) = \frac{\tilde{a}(s)}{c^2} \]  

(A.5)

Solving Equation A.5

\[ \tilde{u}(s) = C_1 e^{\frac{s}{c}x} + C_2 e^{-\frac{s}{c}x} - \frac{\tilde{a}(s)}{s^2} \]  

and

(A.6)

\[ \frac{\partial \tilde{u}(s)}{\partial x} = C_1 \frac{s}{c} e^{\frac{s}{c}x} - C_2 \frac{s}{c} e^{-\frac{s}{c}x} \]  

(A.7)

where \( C_1 \) and \( C_2 \) are arbitrary constants. Using the boundary conditions that the relative displacement is zero at base and the shear strain at the surface is zero, the constants \( C_1 \) and \( C_2 \) are evaluated:

\[ C_1 = \frac{\tilde{a}(s)}{s^2 (1 + e^{-2sH})} e^{-2sH} \]  

and

\[ C_2 = \frac{\tilde{a}(s)}{s^2 (1 + e^{2sH})} \]  

(A.8)
Therefore the transformed relative displacement response at any point can be evaluated,

\[
\tilde{u}(s) = \frac{a(s)}{s^2} \frac{1}{(1+e^{-2zH/c})} \left\{ e^{-2zH/c} e^{\xi x} + e^{-\xi x} \right\} - \frac{a(s)}{s^2}
\]

(A.9)

or,

\[
\tilde{u}(s) = \frac{a(s)}{s^2} \left\{ \frac{e^{-2zH/c} e^{\xi x} + e^{-\xi x}}{(1+e^{-2zH/c})} - 1 \right\}
\]

(A.10)

If \( z = e^{-2zH/c} \) then \( y^{(n)} = (-1)^{n} n! (1+z)^{(n+1)} \) expansion would be sufficient to numerically obtain the \( n^{th} \) derivative of \( y \). Substituting this in Equation A.10 as

\[
(1+e^{-2zH/c})^{-1} = \sum_{n=0}^{\infty} (-1)^n e^{-2nzH/c}
\]

(A.11)

\[
\tilde{u}(s) = \frac{a(s)}{s^2} \left\{ \sum_{n=0}^{\infty} (-1)^n \left\{ e^{-2nzH/c} e^{\xi x} + e^{-\xi x} \right\} - 1 \right\}
\]

(A.12)

As indicated earlier, it is now possible to recover the original function in the time domain

\[
u(x,t) = \sum_{n=1}^{\infty} \left[ f \left(t - \frac{2tH}{c} + \frac{2H}{c} x \right) + f \left(t - \frac{2tH}{c} - \frac{2H}{c} x \right) + f \left(t - \frac{2tH}{c} + \frac{x}{c} \right) + f \left(t - \frac{2tH}{c} - \frac{x}{c} \right) \right] f(t)
\]

(A.13)

In which \( f(t) \) is the base displacement (excitation). For instance, say, we induce a sinusoidal acceleration \( \ddot{a}(t) = A \sin(\omega t) \), the corresponding displacement would be

\[
f(t) = \frac{At}{\omega} - \frac{A}{\omega^2} \sin(\omega t).
\]

Where \( A \) is amplitude and \( \omega \) is the frequency of excitation.

This will be the function \( f(t) \) to be used in the time domain solution. Equation A.13, as pointed out earlier, would be the closed form expression for the relative displacement of a undamped single layer soil deposit with constant \( G_{\text{max}} \) throughout the depth. This involves both transient and steady state solutions. Also this expression holds
good for obtaining the response history at anywhere in the soil profile. If the surface relative displacement is of interest, then \( x = H \) in Equation A.13, therefore

\[
u(H,t) = \sum_{n=0}^{\infty} (-1)^n \left[ 2f \left( t - \left( \frac{2nH}{c} + \frac{H}{c} \right) \right) \Delta \left( t - \left( \frac{2nH}{c} + \frac{H}{c} \right) \right) \right] - f(t)
\]  

(A.14)

or,

\[
u(H,t) = 2f \left( \frac{t}{c} - \frac{H}{c} \right) \Delta \left( \frac{t}{c} - \frac{H}{c} \right) + \sum_{n=0}^{\infty} 2(-1)^n f \left( t - \left( \frac{2nH}{c} + \frac{H}{c} \right) \right) \Delta \left( t - \left( \frac{2nH}{c} + \frac{H}{c} \right) \right) - f(t)
\]  

(A.15)

This solution procedure is compared with the finite element program and it is observed that the finite element solution coincides with exact solution.
A.2.1 INTRODUCTION

The dynamic finite element computer codes QUIVER and PILEUBC was written in FORTRAN 77 and compiled with Microsoft® Fortran with Microsoft Visual Work Bench® Version 3.2. The machine used for developing the program was an IBM® compatible PC with 486DX33 micro processor. The minimum hardware required to run PILEUBC are:

- a 386 based or higher central processing unit.
- a math co-processor
- at least 12.0 Mb of free hard disk space (depends on the size of the problem)
- 8.0 Mb of random access memory (RAM)

Due to large number of arrays defined in the program, the compiler is capable of making an executable file such that memory allocation for all arrays is done at run time rather than including it in the memory of the executable file.

The program has been tested with a wide range of cases and is believed to be free from serious defects. Troubles are usually found to be caused by user-oriented errors in the input files or misrepresentation of the physical system. This in return, could result in unexpected response of the soil-pile-superstructure system. The program has been designed to work with any consistent system of units by specifying the value of atmospheric pressure. On account of the integration scheme employed and the nodal
degrees of freedom relatively fewer elements may be used to represent the pile and the free-field soil for predicting the time varying response. The general flow of the coupled soil-pile-superstructure seismic response program (PILEUBC and QUIVER) are illustrated in Fig. A.2.1 and A.2.2

A.2.2 PROGRAM STRUCTURE

The program PILEUBC consists of a main routine and the following fifteen subroutines: QUIVER, SHAPES, SHAPES1, SHAPES2, GAUSS, GAUSS1, SOMASS, EMASS, STRESS, PSUP, HYSTER, SOACCVEC, PILACCVEC, DECOMP, and SOLV. The main routine is responsible for all input and output, initialization of arrays, construction of local and global stiffness matrices, construction of global mass matrix, inclusion of boundary conditions for the pile and near-field soil. Subroutine QUIVER computes the free-field soil profile as a function of space and time. Subroutine SHAPES obtains the values of shape functions at each Gaussian sampling point for each pile element in a layer. These are used in construction of local stiffness matrix for the pile element. SHAPES1 is similar to SHAPES, but it calculates the values of shape functions for free-field soil elements. Subroutine GAUSS return appropriate Gaussian coordinates and weights for desired number of Gaussian integration points for pile elements. GAUSS1 does it for free-field soil elements. Subroutine SOMASS computes the local consistent mass matrix for the free-field. EMASS returns back the same for the pile elements. Subroutine STRESS looks up the appropriate axial and bending stresses for a given strain level at a Gaussian point in the pile. PSUP computes the tangent stiffness and pressure for
READ INPUT DATA FOR SOIL & PILE

INITIALIZE ARRAYS AND OBTAIN GAUSSIAN COORDINATES AND WEIGHTS

CALL GAUSS & SHAPES

CALCULATE PILE ELEMENT MASS MATRIX AND ACCELERATION VECTOR AND GLOBALIZE IT

CALL EMASS & PILACCVEC

REPEAT FOR EACH TIME STEP

READ INPUT BASE ACCELERATION FROM FILE

CONSTRUCT RHS f(t)

INVOLVE NEW MARK - Beta PROCEDURE

CONSTRUCT DERIVATIVE MATRIX FOR THE NEWTON RAPHSON PROCEDURE

.construct psi vector

IMPOSE BOUNDARY CONDITIONS

DECOMPOSE AND SOLVE FOR INCREMENTAL DISPLACEMENT

CALL DECOMP & SOLV

CALL STRESS & SUP

EXTRACT FREE-FIELD RELATIVE DISPLACEMENTS

SPECIFIED TOLERANCE ACHIEVED

YES

UPDATE STRESSES IN THE PILE AND SOIL PRESSURES ACTING ON THE PILE FOR RECENTLY OBTAINED DISPLACEMENTS

NO

CALCULATE FORCES, MOMENTS, VELOCITY, ACCELERATION VECTORS

PRINT OUTPUT INTO PRESCRIBED FILES

END

Fig. A.2.1 Generalized Flow Chart for Finite Element Program, PILEUBC
INITIALIZE ARRAYS AND OBTAIN GAUSSIAN COORDINATES AND WEIGHTS

CALL GAUSS1 & SHAPE1

CALCULATE SOIL ELEMENT MASS MATRIX AND ACCELERATION VECTOR AND GLOBALIZE IT

CALL SOMASS & SOACCVEC

CONSTRUCT RHS f(t)

INVOKE NEW MARK - Beta PROCEDURE

CONSTRUCT DERIVATIVE MATRIX FOR THE NEWTON RAPHSON PROCEDURE

CONSTRUCT PSI VECTOR

IMPOSE BOUNDARY CONDITIONS

DECOMPOSE AND SOLVE FOR INCREMENTAL DISPLACEMENT

CALL DECOMP & SOLV

SPECIFIED TOLERANCE ACHIEVED

YES

CALL HYSTER

NO

UPDATE SHEAR STRESSES AND TANGENT MODULUS IN FREE-FIELD SOIL FOR RECENTLY OBTAINED DISPLACEMENTS

CALCULATE, VELOCITY, ACCELERATION VECTORS
PRINT OUTPUT INTO PRESCRIBED FILES

RETURN TO THE MAIN ROUTINE

Fig. A.2.2 Generalized Flow Chart for Finite Element Program, QUIVER
a particular $p-y$ curve depending on the amount of lateral movement of the pile in the near field. Subroutine HYSTER gives the tangent shear modulus and shear stress in the soil at a sampling point in the free-field element, based on the Hardin-Drenvich model. Subroutine SOACCVEC calculates the acceleration vectors for the driving forces in free-field element, locally. PILACCVEC does for the pile element. Subroutine DECOMP decomposes the one dimensional derivative of the matrix obtained from the truncated Taylor series for the Newton Raphson procedure. Subroutine SOLV solves the system of algebraic equations to get the incremental displacement of the system. Both DECOMP and SOLV are used in free-field response and pile response.

A.2.3 CURRENT PROGRAM CAPABILITIES

The program QUIVER and PILEUBC are capable of analyzing any of the combination of the following:

1. Total stress free-field soil response subjected to base acceleration at the till or any point in the soil profile. It incorporates shear stress-strain nonlinearities in the soil. Also includes degradation or upgradation of the dynamic stiffness of free field depending on the loading, unloading or reloading conditions.

2. Different soil layers in the deposit.

3. Soil nonlinearities and yielding in the near-field through $p-y$ curves. Also includes degradation or upgradation of the dynamic stiffness of near field depending on the loading, unloading or reloading conditions.

4. Automatic computation of $p-y$ curves based on either API or Yan-Byrne method.
5. Soil-pile gapping at the interface.

6. Earthquake loading.

7. Varying pile cross section along the length of the pile.

8. Different pile materials along the length of the pile.

9. Pile material nonlinearity, yielding, and geometric nonlinearity.

10. Direct loading at any node along the length of the pile.

11. P-Δ effects from applied axial load.


### A.2.4 DESCRIPTION OF INPUT DATA FILES

The input data files can be given in free format. The example files enclosed (Section A.2.4) contain the data using comma separated variables. As mentioned earlier any system of units may be employed by specifying the atmospheric pressure in the units of choice. However, other data such as specific unit weight, depth of soil layer, Young's modulus and yield stress of pile material must be in consistent units. Automatic node number generation is incorporated. It goes from bottom to top in the finite elements. The following describes the data file for soil deposit:

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Variable Name</th>
<th>Format</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TITLE</td>
<td>A80</td>
<td>TITLE FOR THE SOIL DEPOSIT</td>
</tr>
<tr>
<td>2</td>
<td>NLAYERS</td>
<td>I4</td>
<td>NUMBER OF LAYERS IN THE DEPOSIT</td>
</tr>
<tr>
<td>3</td>
<td>SDEPTH(NLAYERS)</td>
<td>E15.6</td>
<td>DEPTH OF LAYER</td>
</tr>
<tr>
<td>Line</td>
<td>Variable Name</td>
<td>Format</td>
<td>Description</td>
</tr>
<tr>
<td>------</td>
<td>-------------------</td>
<td>--------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>TITLE2</td>
<td>A80</td>
<td>Title for the input acceleration record</td>
</tr>
<tr>
<td>2</td>
<td>TSTEP</td>
<td>F15.6</td>
<td>Time step for the base acceleration record</td>
</tr>
<tr>
<td>3</td>
<td>NPOINTS</td>
<td>I8</td>
<td>Number of data points</td>
</tr>
<tr>
<td>4</td>
<td>GRACC(NPOINTS)</td>
<td>E15.6</td>
<td>Acceleration in consistent units for NPOINTS</td>
</tr>
</tbody>
</table>

The data file for the earthquake acceleration should be made available in following format:

The data file PILEUBC should be created as indicated below:

<table>
<thead>
<tr>
<th>Line No.</th>
<th>Variable Name</th>
<th>Format</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TITLE</td>
<td>A80</td>
<td>Title for the problem</td>
</tr>
<tr>
<td>2</td>
<td>ELEMENTS</td>
<td>I4</td>
<td>Number of elements along the pile</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>2</td>
<td>SOIL LAYERS</td>
<td>I4</td>
<td>NUMBER OF DIFFERENT SOIL LAYERS</td>
</tr>
<tr>
<td>2</td>
<td>PILE MATS</td>
<td>I4</td>
<td>NUMBER OF DIFFERENT PILE MATERIALS</td>
</tr>
<tr>
<td>2</td>
<td>BC NODES</td>
<td>I4</td>
<td>NUMBER OF DIFFERENT NODES WITH BOUNDARY CONDITIONS</td>
</tr>
<tr>
<td>2</td>
<td>INPUT PTS</td>
<td>I4</td>
<td>NUMBER OF LOAD INPUT POINTS, IF ANY, ENTER 0 FOR SEISMIC LOADING</td>
</tr>
</tbody>
</table>
| 2 | IS_CYC | I4 | = 'Y' IF LOADING IS CYCLIC  
               = 'N' OTHERWISE, USED IN CALCULATIONS FOR API CURVES |
| 2 | FREE_FLD NODES | I4 | NUMBER OF NODES ASSOCIATED WITH LATERAL STATIC LOADING, ENTER 0 FOR EARTHQUAKE LOADING |
| 2 | PA | F15.6 | ATMOSPHERIC PRESSURE IN DESIRED UNITS |
| 2 | STATIC_VER_LOAD | F15.6 | WEIGHT OF THE STRUCTURAL MASS |
| 2 | TOLERANCE | F15.6 | TOLERANCE FOR INCREMENTAL DISPLACEMENT AND TOTAL ERROR |
| 3 | NODE | I4 | NODE NUMBER. NODES ARE NUMBERED FROM BOTTOM TO TOP. THE TOTAL NUMBER OF NODES IS ALWAYS ONE PLUS TOTAL NUMBER OF ELEMENTS |
| 3 | X_COORD(NODE) | F15.6 | COORDINATE OF THE NODE IN THE VERTICAL DIRECTION |
| 3 | IS_FF(NODE) | I2 | = '0' FREE FIELD  
               = '1' OTHERWISE  
               ENTER '0' FOR EARTHQUAKE LOADING |
| 4 | ELEM | I4 | ELEMENT NUMBER |
| 4 | XSEC_TYPE(ELEM) | I2 | THE SHAPE OF THE PILE CROSS SECTION, THE CHOICES ARE:  
               = 1 - SOLID CIRCULAR  
               = 2 - HOLLOW CIRCULAR  
               = 3 - RECTANGLE |
| 4 | OUT_DIA(ELEM) | F15.6 | IF XSEC_TYPE() = 1:  
               = DIAMETER OF PILE  
               IF XSEC_TYPE() = 2:  
               = OUTER DIAMETER OF PILE  
               IF XSEC_TYPE() = 3:  
               = DEPTH OF PILE (DIRECTION OF BENDING) |
| 4 | IN_DIA(ELEM) | F15.6 | IF XSEC_TYPE() = 1:  
               = 0.0  
               IF XSEC_TYPE() = 2:  
               = INNER DIAMETER OF PILE  
               IF XSEC_TYPE() = 3:  
               = WIDTH OF PILE |
<p>| 4 | RPMASS(ELEM) | F15.6 | MASS DENSITY OF THE PILE MATERIAL |</p>
<table>
<thead>
<tr>
<th>Layer</th>
<th>Description</th>
<th>Data Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>MAT_NUM(ELEM)</td>
<td>I4</td>
</tr>
<tr>
<td></td>
<td>PILE MATERIAL NUMBER FOR THIS ELEMENT</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>IN_LAYER(ELEM)</td>
<td>I4</td>
</tr>
<tr>
<td></td>
<td>SOIL LAYER ASSOCIATED WITH THIS ELEMENT</td>
<td></td>
</tr>
<tr>
<td></td>
<td>REPEAT THE ABOVE FOR EACH ELEMENT</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>LAYER</td>
<td>I4</td>
</tr>
<tr>
<td></td>
<td>SOIL LAYER NUMBER</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>SOIL_TYPE(LAYER)</td>
<td>A4</td>
</tr>
<tr>
<td></td>
<td>SOIL TYPE FOR THE LAYER:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= CLAY</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= SAND</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= USER</td>
<td></td>
</tr>
<tr>
<td></td>
<td>THIS OPTION DEFINES HOW TO CALCULATE P-Y CURVES FOR THE SOIL LAYER</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>PY_TYPE(LAYER)</td>
<td>A4</td>
</tr>
<tr>
<td></td>
<td>METHOD OF CALCULATING P-Y CURVES:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= APIC, FOR APPROACH USED IN THE API CODE;</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= YANB, FOR YAN-BYRNE APPROACH</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= USER, FOR USER DEFINED P-Y CURVES</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IF PY_TYPE() = APIC, ENTER THE FOLLOWING:</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>GAMMA(LAYER)</td>
<td>F15.6</td>
</tr>
<tr>
<td></td>
<td>THE UNIT WEIGHT OF SOIL FOR THIS LAYER</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>DR(LAYER)</td>
<td>F15.6</td>
</tr>
<tr>
<td></td>
<td>THE RELATIVE DENSITY OF SOIL FOR THIS LAYER</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>ETA(LAYER)</td>
<td>F15.6</td>
</tr>
<tr>
<td></td>
<td>THE FACTOR ETA FROM API CODE FOR THIS SOIL TYPE</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>N_HI(LAYER)</td>
<td>F15.6</td>
</tr>
<tr>
<td></td>
<td>THE FACTOR n_hi FROM API CODE FOR THIS SOIL TYPE (COEFFICIENT OF SUBGRADE REACTION MODULUS)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>C1(LAYER)</td>
<td>F15.6</td>
</tr>
<tr>
<td></td>
<td>FACTOR C1 FROM API CODE</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>C2(LAYER)</td>
<td>F15.6</td>
</tr>
<tr>
<td></td>
<td>FACTOR C2 FROM API CODE</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>C3(LAYER)</td>
<td>F15.6</td>
</tr>
<tr>
<td></td>
<td>FACTOR C3 FROM API CODE</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IF PY_TYPE() = YANB, ENTER THE FOLLOWING</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>GAMMA(LAYER)</td>
<td>F15.6</td>
</tr>
<tr>
<td></td>
<td>THE UNIT WEIGHT OF SOIL FOR THIS LAYER</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>DR(LAYER)</td>
<td>F15.6</td>
</tr>
<tr>
<td></td>
<td>THE RELATIVE DENSITY OF SOIL FOR THIS LAYER</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>EMAX(LAYER)</td>
<td>F15.6</td>
</tr>
<tr>
<td></td>
<td>OPTIONAL, ENTER 0.0 IF AUTOMATIC COMPUTATION IS REQUIRED AS A FUNCTION OF EFFECTIVE PRESSURE</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IF PY_TYPE() = USER, ENTER THE FOLLOWING</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>GAMMA(LAYER)</td>
<td>F15.6</td>
</tr>
<tr>
<td></td>
<td>THE UNIT WEIGHT OF SOIL FOR THIS LAYER</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td><strong>NUM_PY(LAYER)</strong></td>
<td>I4</td>
</tr>
<tr>
<td>----</td>
<td>-------------------</td>
<td>----</td>
</tr>
<tr>
<td>6</td>
<td><strong>NODE_PY(LAYER,NUM_PY)</strong></td>
<td>I4</td>
</tr>
<tr>
<td>6</td>
<td><strong>EMAX(LAYER)</strong></td>
<td>F15.6</td>
</tr>
<tr>
<td>6</td>
<td><strong>Y_PY_IN(NODE,J)</strong></td>
<td>F15.6</td>
</tr>
<tr>
<td>6</td>
<td><strong>P_PY_IN(NODE,J)</strong></td>
<td>F15.6</td>
</tr>
<tr>
<td>7</td>
<td><strong>PILE_MAT</strong></td>
<td>I4</td>
</tr>
</tbody>
</table>
| 7  | **MAT_TYPE(PILE_MAT)**  | A4 | DEFINES THE STRESS-STRAIN BEHAVIOR FOR PILE ELEMENT:  
|    |                   |    | - ELPL, FOR ELASTIC PERFECTLY PLASTIC CURVE  
|    |                   |    | - WOOD, FOR TIMBER PILES |
| 7  | **E(PILE_MAT)**  | F15.6 | YOUNG'S MODULUS FOR THE MATERIAL |
| 7  | **YIELD_STRS(PILE_MAT)**  | F15.6 | YIELD STRESS OF THE MATERIAL FOR THE ELEMENT.  
|    |                   |    | IF MAT_TYPE = WOOD, ENTER FOLLOWING |
| 7  | **E(PILE_MAT)**  | F15.6 | YOUNG'S MODULUS |
| 7  | **YIELD_STRS(PILE_MAT)**  | F15.6 | YIELD STRESS OF THE MATERIAL IN COMPRESSION  
|    |                   |    | IF BC NODES IS NOT ZERO, FOR EACH BC NODES ENTER THE FOLLOWING |
| 8  | **BC_NODE(I)**  | I4 | NODE NUMBER WITH BOUNDARY CONDITION, I = 1, BC NODES |
| 9  | **NUM_BCS(BC_NODE(I))**  | I4 | NUMBER OF DIFFERENT BOUNDARY CONDITIONS TO BE SPECIFIED |
| 9  | **BC(BC_NODE(I), J, J = 1, NUM_BCS(BC_NODE(I)))**  | I4 |  
|    |                   |    | = 1, IF W = 0  
|    |                   |    | = 2, IF W' = 0  
|    |                   |    | = 3, IF W'' = 0  
|    |                   |    | = 4, IF U = 0  
|    |                   |    | = 5, IF U' = 0 |

NOTE: THE NODE NUMBERS SPECIFIED HERE MUST EXACTLY COINCIDE WITH THAT GIVEN IN THE INPUT DATA FILE
A.2.5 SAMPLE INPUT DATA FILES

The input data files for a few examples are given below:

4 LAYER 15 ELEMENTS INPUT

4
0.325E+01,1,0.1886E+02,6.0D0,0.0D0,89.769
0.325E+01,1,0.905E+01,11.0D0,0.0D0,180.00
0.1625E+02,5,0.984E+01,24.0D0,41.0D0,0.00
0.26E+02,8,0.984E+01,23.0D0,40.0D0,0.00
4
1.0E-04
16
150
1
0.00

CANLEX SITE RICHMOND 1995
15,4,1,1,0,0,'Y',0,101.325,-500.0,0.001
1.4875,0
14,0.0,0
15,-1.20,0
16,-2.40,0
1,2,0.900,0.7,7.4,1,4
2,2,0.900,0.7,7.4,1,4
3,2,0.900,0.7,7.4,1,4
4,2,0.900,0.7,7.4,1,4
5,2,0.900,0.7,7.4,1,4
6,2,0.900,0.7,7.4,1,3
7,2,0.900,0.7,7.4,1,3
8,2,0.900,0.7,7.4,1,3
9,2,0.900,0.7,7.4,1,3
10,2,0.900,0.7,7.4,1,3
11,2,0.900,0.7,7.4,1,3
12,2,0.900,0.7,7.4,1,2
13,2,0.900,0.7,7.4,1,1
14,2,0.900,0.7,7.4,1,0
15,2,0.900,0.7,7.4,1,0
1,'SAND','APIC'
0.1886E+02,60.0,0.0
2,'SAND','APIC'
0.905E+01,60.0,0.0
3,'SAND','YANB'
0.984E+01,70.0,0.0
4,'SAND','APIC'
0.984E+01,60.0,0.0
1,'ELPL'
0.210E+09,0.300E+06
1,1,4
1,10
10,100,105,110,115,120,130,140,150,200,210
A.2.6 PROGRAM OUTPUT

There are 22 output files. Three set of five files consists of relative displacement, velocity and acceleration time histories at selected nodes. Five files consists of time varying bending moment at selected elements. A file contains the profile of the pile at selected time steps. The output consists of time step number, lateral deflection, shear force, bending moment and curvature. Also a file consists of absolute maximum bending moment in the pile. With very little modification in the program other desired data could also be obtained. The CONFIG.SYS file should be modified when run in batch mode to accommodate file management.

A.2.7 PROBLEMS ENCOUNTERED IN RUNNING PILEUBC

Although the program has been thoroughly checked and is believed to be free of errors, as it is a nonlinear finite element program, a few unforeseen errors might have been overlooked. In general, however, mostly problems occur in incorrect or incomplete input data files. Numerical inconsistencies depending on the sudden increase in successive earthquake acceleration (spikes), time step and very small tolerance may also create problems in attaining realistic computations.