LIQUEFACTION INDUCED DEFORMATIONS OF EARTH STRUCTURES

by

HENDRA JITNO

Ir. (Civil Engineering.), Bandung Institute of Technology, Indonesia, 1983
M.A.Sc. (Civil Engineering), University of British Columbia, 1990

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES

CIVIL ENGINEERING

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

February 1995

© Hendra Jitno, 1995
In presenting this thesis in partial fulfillment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Civil Engineering

The University of British Columbia

2324 Main Mall

Vancouver, Canada

V6T 1W5

Date: 15 Feb 1995
ABSTRACT

Liquefaction-induced ground displacements resulting from earthquake shaking are a major cause of damage to earth structures comprising of or underlain by loose saturated sands. A number of dams have failed due to liquefaction-induced deformations. Examples of these are the failures of eleven tailings dams in Chile during the March 1965 earthquake and the Mochikoshi tailings dams in Japan due to the 1978 earthquake. A number of other dams have undergone large deformations but have not failed in as much as the impounded water was not released. The classic example of this was the near failure of the Lower San Fernando dam due to the 1971 earthquake. A liquefaction induced flow slide occurred on the upstream side removing the crest of the dam and leaving only about 1.5 m freeboard. Of more interest from the analytical point of view was the behaviour of the Upper San Fernando dam in which the crest of this dam moved about 1.5 m due to earthquake induced liquefaction.

Of equal importance are the ground failures due to liquefaction-induced lateral spreading. It occurs on gently sloping grounds and sometimes on almost flat grounds, but usually occurs over a very wide area. Although this type of earthquake-induced ground movement does not involve a flow failure where the static shear stresses exceed the residual strength of soils, it is potentially damaging and it has caused over one hundred million US dollars worth of damage in United States alone since the 1964 Alaska earthquake.

The prediction of earthquake induced displacements of earth dams involving soils whose properties change markedly during cyclic loading is a difficult problem. The difficulty mainly arises from modeling the stress-strain relations of soils, particularly when pore pressure rise and liquefaction occur. The strains required to trigger liquefaction are generally small (<1%). Once liquefaction is triggered, however, large but limited deformation may occur on soils whose undrained residual strengths are greater than the
driving stresses. Such soils strain harden, and regain stiffness and strength as they deform, so the displacements are limited. For soils whose residual strengths are less than the driving stresses, unlimited deformation leading to catastrophic failures may occur.

Complex effective stress dynamic analyses procedures have been proposed to predict such deformations but they are essentially research tools and not generally appropriate for analysis of most earth structures in geotechnical engineering practice. It is important, therefore, to develop a simple reliable method for predicting such displacements, and this is the objective of this thesis.

The deformation analysis proposed here is essentially an extension of Newmark's method from a rigid-plastic single-degree-of-freedom system to a flexible multi-degree-of-freedom system. It takes into account the effects of inertia forces from the earthquake, the softening of the liquefied soil, and the settlement following liquefaction. The method is based on the concept that the deformations prior to liquefaction will be small and can be neglected compared to those that occur after liquefaction is triggered. A key aspect of the method is the post-liquefaction stress-strain response for which there is now considerable laboratory data available. The proposed method employs a pseudo-dynamic finite element method in which the additional displacements due to liquefaction and inertia forces are accounted for by applying additional forces that satisfy energy principles. The procedure has been validated by applying it to field case histories involving both one-dimensional sloping ground as well as two-dimensional cases. These case histories include the Wildlife and the Heber Road sites, the Lower and Upper San Fernando dams, the Mochikoshi tailings dams, the La Marquesa and La Palma dams in Chile. It was found that the predicted and observed results in those case histories are in reasonable agreement in terms of both the magnitude and pattern of displacements.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xi</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>xiii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td>xix</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>xx</td>
</tr>
<tr>
<td>CHAPTER 1:</td>
<td>1</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>6</td>
</tr>
<tr>
<td>1.1. Research Objective</td>
<td>6</td>
</tr>
<tr>
<td>1.2. Thesis Organization</td>
<td>7</td>
</tr>
<tr>
<td>CHAPTER 2</td>
<td>9</td>
</tr>
<tr>
<td>UNDRAINED BEHAVIOUR OF SATURATED SANDS</td>
<td>9</td>
</tr>
<tr>
<td>2.1. Introduction</td>
<td>9</td>
</tr>
<tr>
<td>2.2. Definition of Liquefaction</td>
<td>11</td>
</tr>
<tr>
<td>2.3. Pre-liquefaction Behaviour</td>
<td>12</td>
</tr>
<tr>
<td>2.3.1. Undrained Monotonic Loading Behaviour</td>
<td>12</td>
</tr>
<tr>
<td>2.3.2. Undrained Cyclic Behaviour</td>
<td>15</td>
</tr>
<tr>
<td>2.4. Post-liquefaction Behaviour</td>
<td>21</td>
</tr>
<tr>
<td>2.4.1. Post-liquefaction Cyclic Stress-Strain Behaviour</td>
<td>22</td>
</tr>
<tr>
<td>2.4.2. Post-liquefaction Monotonic Stress-strain Behaviour</td>
<td>29</td>
</tr>
<tr>
<td>2.4.3. Steady State or Residual Strength of Liquefied Sand</td>
<td>36</td>
</tr>
<tr>
<td>2.4.4. Limiting Shear Strains</td>
<td>42</td>
</tr>
<tr>
<td>2.4.5. Earthquake-induced Settlement</td>
<td>45</td>
</tr>
</tbody>
</table>
CHAPTER 3: CURRENT METHODS FOR PREDICTING LIQUEFACTION-INDUCED DEFORMATIONS OF EARTH STRUCTURES

3.1. Introduction
3.2. One-dimensional Simplified Method
  3.2.1. Newmark's Method
  3.2.2. Makdisi-Seed Simplified Method
3.3. Two-dimensional Simplified Finite Element Approach
  3.3.1. Modified Modulus Approach
  3.3.2. Dynamic Stress Path Method
3.4. Non-linear Effective Stress Dynamic Analysis
3.5. Empirical Method
3.6. Summary

CHAPTER 4: ONE-DIMENSIONAL METHOD USING ENERGY CONCEPT

4.1. Introduction
4.2. Newmark's Model Based on Energy Concept
4.3. Extended Newmark
4.4. Byrne's One-dimensional Method
4.5. Determination of Required Parameters
  4.5.1. Zone of Liquefaction
  4.5.2. Residual Strength
  4.5.3. Limiting Strain or Strain at Residual Strength
  4.5.4. Peak Ground Acceleration and Velocity
CHAPTER 7: LIQUEFACTION-INDUCED DEFORMATIONS AT THE WILDLIFE SITE

7.1. Introduction ......................................................... 135
7.2. Effects of the 1987 Superstition Hill Earthquake ................. 137
7.2. Soil Investigations ................................................ 138
7.4. Deformation Analysis ............................................ 142
7.4.1. Slope Geometry and Soil Parameters Used in the Analysis .... 142
7.4.2. Results of the Analysis ....................................... 147
7.5. Summary ............................................................. 150

CHAPTER 8: LIQUEFACTION-INDUCED LATERAL SPREADS AT HEBER ROAD SITE, IMPERIAL VALLEY, CALIFORNIA

8.1. Introduction ......................................................... 153
8.2. Ground Movements Due to the Earthquake ....................... 154
8.3. Subsurface Soil Conditions ..................................... 157
8.4. Previous Ground Deformation Analysis .......................... 161
8.5. Current Deformation Analyses .................................. 163
8.5.1. Cross Section and Cases Considered ........................ 163
8.5.2. Soil Properties .................................................. 164
8.8. Results of the Analyses ........................................... 166
8.8. Back-calculated Residual Strength ............................... 172
8.9. Summary ............................................................. 176

10.1. Introduction ........................................................................................................... 214
10.2. The January 1978 Izu-Ohshima-Kinkai Earthquake ............................................ 215
10.3. Effects of the Earthquakes on the Tailings Dams ................................................. 215
   10.3.1. Mochikoshi Tailings Dam No. 1.................................................................. 217
   10.3.2. Mochikoshi Tailings Dam No. 2................................................................. 220
10.4. Site Investigations and Laboratory Tests ......................................................... 221
   10.4.1. Tailings Materials ....................................................................................... 221
   10.4.2. Dam Materials ........................................................................................... 223
10.5. Deformation Analyses ....................................................................................... 224
   10.5.1. Soil Properties Used in the Analyses ......................................................... 224
   10.5.2. Cases Considered ....................................................................................... 226
   10.5.3. Results of the Analyses ............................................................................. 230
   10.5.3.1. Mochikoshi Dam No. 1 ....................................................................... 230
   10.5.3.2. Mochikoshi Dam No. 2 ....................................................................... 232
10.6. Summary ............................................................................................................ 238

CHAPTER 11: TWO DAM FAILURES DURING THE 1985 CHILEAN EARTHQUAKE

11.1. Introduction ........................................................................................................... 239
11.2. The 1985 Chilean Earthquake ........................................................................... 240
11.3. Effects of the Earthquake on the Dams ........................................................... 240
   11.3.1. La Marquesa Dam ..................................................................................... 240
   11.3.2. La Palma Dam .......................................................................................... 243
11.4. Soil Investigations ............................................................................................. 244
## LIST OF TABLES

6.1. Pre-earthquake Soil Properties for Finite Element Analyses ........................................ 126

6.2. Post-earthquake Soil Properties of Liquefied Layer for Both Closed-Form and
Finite Element Analyses for Different $N_1$ ................................. 126

7.1. Measured Displacements at the Surface at the Wildlife Site due to the Superstition
Hill Earthquake ............................................................................. 137

7.2. Soil Properties Used in the Analyses ........................................... 146

7.3. Measured and Computed Displacements Using $S_r = 0.14\sigma'_{wo}$ at the Wildlife
Site .................................................................................................... 146

8.1. Summary of Soil Laboratory Data at Heber Road Site (Vucetic and Dobry, 1986
reported by Dobry et al., 1992) ......................................................... 160

8.2. Pre-and Post-earthquake Soil Properties used in the Analyses ..................... 167

8.3 Predicted and Measured Displacements at Several Locations ................ .......... 167

9.1. Measured Displacements of the Upper San Fernando Dam (after Serff et al.,
1976) .................................................................................................. 185

9.2. Soil Properties Used in the Analyses of the Lower San Fernando Dam ........... 200

9.3. Magnitude and Displacements Computed at the Crest and at the Toe of the Lower
San Fernando Dam ........................................................................ 200

9.4. Soil Properties Used in the Analyses of the Upper San Fernando Dam .......... 209

9.5. Cases Considered in the Analyses of the Upper San Fernando Dam .......... .... 211

9.6. Predicted and Measured Displacements at Several Locations in the Upper San
Fernando Dam ............................................................................. 211

10.1. Soil Properties Used in the Analyses of Mochikoshi Tailings Dams ......... 227

10.2. Predicted Displacements at the Dam Crest of Mochikoshi Tailings Dams .... 230

11.1. Soil Properties Used in the Analyses of the La Marquesa Dam .................. 248

11.2. Cases Considered in the Analyses of the La Marquesa Dam ...................... 248

11.3. Measured and Observed Vertical Displacements at Several Locations of La
Marquesa Dam ............................................................................. 250
11.4. Soil Properties Used in the Analyses of the La Palma Dam.........................256
11.5. Cases Considered in the Analyses of the La Palma Dam.............................256
11.6. Measured and Observed Vertical Displacements, La Palma Dam..................250
LIST OF FIGURES

Fig. 2.1. Stress condition in a soil element beneath flat and sloping grounds subjected to an earthquake ................................................................. 10

Fig. 2.2. Characteristic response of saturated sands under undrained monotonic loading (Vaid and Chern, 1985) ................................................. 14

Fig. 2.3. Liquefaction of sands due to cyclic loading (Vaid and Chern, 1985) .................................................. 17

Fig. 2.4. Limited liquefaction due to cyclic loading (Vaid and Chern, 1985) .................................................. 18

Fig. 2.5. Cyclic mobility due to cyclic loading (Vaid and Chern, 1985) .................................................. 20

Fig. 2.6. Post-liquefaction cyclic stress-strain curves; (a). Limited liquefaction; (b). Cyclic mobility response (Kuerbis, 1989) .................................................. 23

Fig. 2.7. Acceleration time histories during Niigata earthquake recorded at the basement of No. 2 apartment building in Kawagishi-cho, Niigata (Seed and Idriss, 1967) .................................................. 25

Fig. 2.8. Surface acceleration versus relative displacements at the Wildlife site during the Superstition Hills earthquake (Byrne and McIntyre, 1994) .................................................. 26

Fig. 2.9. Recorded data from shaking table tests. (a). Time histories of acceleration, pore pressure and displacements; (b). Approximate stress-strain curves (Sasaki et al., 1992) (c) Approximate effective stress path .................................................. 28

Fig. 2.10. Post-liquefaction monotonic behaviour of sands. (a) Loose Sacramento River sand with relative density of 40 percent; (b). dense Mine tailings sand with relative density of 95 percent based on modified AASHO compaction test (Seed, 1979) .................................................. 30

Fig. 2.11. Effects of excess pore pressure ratio on post-liquefaction stress-strain curves (Yasuda et al., 1994) .................................................. 32

Fig. 2.12. Effects of excess pore pressure ratio on stiffness reduction of sands during post-cyclic monotonic loading (data from Yasuda et al., 1991, 1994 and Thomas, 1992) .................................................. 35

Fig. 2.13. Post-liquefaction monotonic behaviour of Duncan dam sand in simple shear tests (Salgado and Pillai, 1993) .................................................. 38
Fig. 2.14. Tentative relationship between residual strength and equivalent clean sand SPT blow counts, \((N_1)_{60-cs}\) (Seed and Harder, 1990). ........................................ 41

Fig. 2.15. Tentative relationship between limiting strains, cyclic stress ratio and \((N_1)_{60-cs}\) (Seed et al., 1984). ................................................................. 44

Fig. 2.16. Proposed relationship between volumetric strain due excess pore pressure dissipation and \((N_1)_{60-cs}\) (Tokimatsu and Seed, 1986). .................................................. 47

Fig. 2.17. Relationship between volumetric strain due to pore pressure dissipation and maximum amplitude cyclic shear strain for different relative densities (Ishihara and Yoshimine, 1992) ........................................................................... 48

Fig. 2.18. Relationship between post-liquefaction volumetric strains and factor of safety against liquefaction (Ishihara and Yoshimine, 1992) ................................................................. 50

Fig. 3.1. Newmark sliding block model (after Newmark, 1965) ..................................................... 56

Fig. 3.2. Schematic illustration of computing displacements using Newmark method .......................... 57

Fig. 3.3. Relationship between displacements and acceleration ratio, N/A. (a). Symmetrical resistance; (b). Unsymmetrical resistance. (Newmark, 1965) ............................................ 59

Fig. 3.4. Makdisi-Seed method. (a). Variation of effective peak acceleration, \(k_{max}\), with depth of base of potential sliding mass. (b). Variation of yield acceleration with normalized permanent displacement (Makdisi and Seed, 1978). ................................................................. 63

Fig. 3.5. Modified modulus approach. (a). Linear stress-strain curve; (b). Non-linear Stress-strain curve ........................................................................................................ 66

Fig. 3.6. Dynamic stress path method. (a). Determination of equivalent stress; (b). Determination of nodal point forces (Serff. et al., 1976). ................................................................. 68

Fig. 4.1 (a). Block on an Inclined Plane and (b). Rigid Plastic Behaviour in Newmark Model. ......................................................................................................................... 78

Fig. 4.2. Idealized pre and post-liquefaction characteristics of loose sand ........................................ 80

Fig. 4.3. Work-energy principles, extended Newmark ....................................................................... 82

Fig. 4.4. Byrne's one dimensional model. (a). Idealized infinite slope, (b). Model ......................... 84

Fig. 4.5. Linear and non-linear stress-strain models (Byrne, 1990) .................................................... 84

xiv
Fig. 4.6. Predicted and measured displacements using a linear stress-strain assumption...91

Fig. 4.7. Predicted and measured displacements using a non-linear stress-strain assumption..................................................................................................................92

Fig. 4.8. Predicted and measured displacements using a linear stress-strain assumption for cases without local boundary effects.................................................................94

Fig. 4.9. Predicted and measured displacements for Japanese case histories. ...............95

Fig. 4.10. Chart to determine limiting strains proposed by Seed et al. (1984). Dash-lines are interpolated from the existing curves.................................................................98

Fig. 4.11. Chart for determining limiting strains based on the factor of safety against liquefaction (F_L) and (N_1)_{60-cs}. Modified from Fig. 4.10 (Seed et al. (1984))...99

Fig. 4.12. Predicted displacements using the modified method in determining the strain at residual strength...........................................................................................................100

Fig. 5.1. Pre-and post-earthquake stress-strain model of liquefied soils..............................105

Fig. 5.2. Decrease in the static driving stress due to changes in geometry during mass movements.....................................................................................................................111

Fig. 5.3. Stress condition of a soil element in the field.............................................................113

Fig. 5.4. Internal work due to shear strains.............................................................................116

Fig. 5.5. Internal work due to volumetric strain.................................................................118

Fig. 5.6. Assumed velocity distribution in the liquefied soils..............................................120

Fig. 6.1. Finite element model for a sloping ground. (a). Original geometry; (b). Deformed geometry....................................................................................................................125

Fig. 6.2. Computed displacements using the finite element procedure and Byrne's closed form solution for 10 percent slope. Energy balance was achieved by applying k_h and combination of k_r and k_h...........................................................................................................128

Fig. 6.3. Computed displacements using the finite element procedure and Byrne's closed form solution for 5 and 10 percent slope. Energy balance was achieved by applying k_h only..........................................................129

Fig. 6.4. Finite element model for level ground. (a). Original geometry; (b). Deformed geometry....................................................................................................................130
Fig. 6.5. Computed displacements using the finite element procedure and Byrne's closed form solution for different assumption of velocity distribution. 131

Fig. 6.6. Comparison between the displacements using the finite element procedure and Byrne's closed solution. All data combined. 132

Fig. 7.1. Location of the Wildlife site, Imperial Valley, California. 136

Fig. 7.2. Sand boils, ground cracks, and lateral spreading due to the Superstition Hill earthquake (Youd and Bartlett, 1988 and Holzer et al., 1989). 139

Fig. 7.3. Soil stratigraphy and map of the instrumentation (Holzer et al., 1989). 140

Fig. 7.4. (a). Geotechnical properties; (b). Liquefaction resistance and induced stress ratio due to the 1987 M6.6 Superstition Hill earthquake (Holzer et al., 1989). 141

Fig. 7.5. Finite element mesh used in the analysis. 143

Fig. 7.6. Typical deformed geometry obtained from the present analysis. 143

Fig. 7.7. Predicted and measured lateral displacements at the Wildlife site. 149

Fig. 7.8. Sensitivity of the computed displacements to the undrained strength ratio. 151

Fig. 8.1. Deformation pattern at Heber Road site due to the earthquake (surveyed by Youd and Bartlett, presented by Dobry et al., 1992). 155

Fig. 8.2. Cross section at Heber Road site where the maximum displacements occurred during the 1979 earthquake. 156

Fig. 8.3. Soil conditions at Heber Road site (Youd and Bennet, 1983). 158

Fig. 8.4. Grain size distributions. 159

Fig. 8.5. Shear moduli as a function of confining stresses (Kuo and Stokoe, 1982). 161

Fig. 8.6. (a). Rigid plastic assumption of liquefied sand; (b). Maximum displacements versus residual strength at Heber Road site (Castro, 1987). Note: 1 psf = 0.0479 kPa. 162

Fig. 8.7. Part of the finite element mesh used in the analyses. Case 1. 165

Fig. 8.8. Typical deformed geometry obtained from the current analyses of Heber Road Site for Case 1. Displacement magnification factor = 1.0. 168

Fig. 8.9. Horizontal ground displacements at several locations of interest. Case 1 and Case 2. 171
Fig. 8.10. Horizontal ground displacements for several locations as a function of residual strengths .................................................................................................................... 173

Fig. 8.11. Horizontal ground displacements at several locations as function of normalized residual strengths ........................................................................................................ 174

Fig. 8.12. Horizontal ground displacements for several residual strength ................................................................................................................................. 175

Fig. 9.1 Cross-section through the Lower and Upper San Fernando Dams .............. 181

Fig. 9.2. Cross-sections through the Lower San Fernando Dam: (a) Conditions after the 1971 Earthquake, and (b) A Schematic Reconstruction of the Failed Section (Seed and Harder, 1990). Note: 1 ft = 0.3048 m ........................................................................... 184

Fig. 9.3. Displacements of the Upper San Fernando Dam due to the 1971 Earthquake (after Serff et al., 1976) ........................................................................................................ 186

Fig. 9.4. Geometry of the dam, water table and estimated zone of liquefaction of the Lower San Fernando Dam (Seed et al., 1973) ..................................................................... 191

Fig. 9.5. Limit Equilibrium Analyses of the Lower San Fernando Dam (Seed, 1979) .... 194

Fig. 9.6. Finite element mesh used in the analyses of the Lower San Fernando Dam .... 199

Fig. 9.7. Typical results: Deformed mesh and displacement vectors of the Lower San Fernando Dam .................................................................................................................. 201

Fig. 9.8. Geometry of the dam, water table and estimated zone of liquefaction of the Upper San Fernando Dam (Seed et al., 1973) ................................................................. 204

Fig. 9.9. Finite element mesh, water table and material types used in the analyses of the Upper San Fernando Dam ......................................................................................... 208

Fig. 9.10. Typical results: Deformed mesh and displacement vectors of the Upper San Fernando Dam .................................................................................................................. 210

Fig. 10.1. (a). Epicenters of the main shock and aftershocks. (b). Locations of the tailings dams and estimated peak ground accelerations. (Ishihara and Nagao, 1983) .......... 216

Fig. 10.2. Plan of the Mochikoshi tailings dams after earthquake. (Ishihara, 1984) ...... 218

Fig. 10.3. Cross sections of the dams before and after the failures. (a). Dam No. 1; (b). Dam No. 2 (Ishihara, 1984) ................................................................. 219

xvii
Fig. 10.4. The soil profiles and penetration resistance at several bore holes (Ishihara, 1984). ...............................................................222

Fig. 10.5. Finite element mesh used in the analyses of Mochikoshi Dam No. 1 (Case 1). .................................................................228

Fig. 10.6. Finite element mesh used in the analyses of Dam No. 2. Phreatic surface a and b indicate Case 2 and Case 3 ........................................229

Fig. 10.7. Results of the analyses of Dam No. 1 (Case 1). (a). Deformed geometry (displacement magnification factor=2); (b). Displacement vectors ........................................231

Fig. 10.8. Results of the analyses of Dam No. 2 (Case 2). (a). Deformed geometry (displacement magnification factor=2); (b). Displacement vectors ........................................233

Fig. 10.9. Results of the analyses of Dam No. 2 (Case 3). (a). Deformed geometry (displacement magnification factor=2); (b). Displacement vectors ........................................234

Fig. 10.10. Results of re-analyses of Case 3 using kg=0.35 for liquefied soils. (a). Deformed geometry (displacement magnification factor=2); (b). Displacement vectors ........................................236

Fig. 11.1. Location of the epicenter and the recorded peak ground accelerations. ....241

Fig. 11.2. Post-earthquake geometry and reconstructed initial geometry of La Marquesa Dam (De Alba et al., 1988) ........................................242

Fig. 11.3. Pre and post-earthquake geometry of La Palma Dam (De Alba et al., 1988). 245

Fig. 11.4. Finite element mesh used in the analyses, material types and approximate water table during the earthquake. La Marquesa Dam ........................................251

Fig. 11.5. Results of the Analyses of La Marquesa Dam. (a). Deformed geometry (displacement magnification factor=1); (b). Displacement vectors ........................................252

Fig. 11.6. Finite element mesh, material types and approximate water table during the earthquake. La Palma Dam ........................................257

Fig. 11.7. Results of the analyses of La Palma Dam. (a). Deformed mesh (displacement magnification factor=1); (b). Displacement vectors ........................................259
ACKNOWLEDGEMENTS

In the Name of Allah, the Most Gracious, the Most Merciful.

The author would like to express his sincere gratitude to his supervisor Prof. P.M. Byrne for his constant guidance and encouragement during the course of the study.

The author also wishes to express his appreciation to the members of the supervisory and examining committee: Profs. D.L. Anderson, W.D.L Finn and Y.P. Vaid from Civil Engineering, Profs. R.L. Chase and K.W. Savigny from Geological Science, University of British Columbia, Prof. Kenji Ishihara from Civil Engineering, University of Tokyo and Dr. U. Atukorala from Golder Associates for reviewing the manuscript and making valuable suggestions.

The author would like to thank his colleagues in the Geotechnical Engineering Group of the University of British Columbia, especially to Guoxi Wu and Srithar, for creating friendly environment in the office, also to Dr. Francisco Salgado for making valuable suggestions in solving some problems encountered during this research.

The fellowship awarded by the University of British Columbia and the Inter-University Center of Bandung Institute of Technology, Indonesia, are greatly acknowledged.

The author would like to extend his deepest gratitude to his father and mother for their constant support and prayer, to his wife, Linda, for her patient and encouragement, and also to his lovely sons, Amza Ibadurrahman and Reza Ibadurrahim, for their love and understanding of the hard life of their father as a graduate student. This work is dedicated to them.

Finally, praise belongs to Allah, the Sustenance of the Universe. Without His help, the author would not be able to accomplish this work.
NOMENCLATURE

$(\Delta \varepsilon_v)^\text{cycle}$ volumetric strain increment per cycle
$(\Delta u)^\text{cycle}$ pore pressure increment per cycle
$(N_1)^\text{cycle}$ SPT blow counts normalized to an overburden stress of 1 tsf (98 kPa).
$(N_1)_{60}$ $(N_1)$ normalized to 60 percent of hammer energy.
$(N_1)_{60-eq}$ equivalent clean sand SPT blow count
A acceleration pulse
B bulk modulus
c cohesion of soil
CPT Cone Penetration Test
CSR Critical Stress Ratio
CT Characteristic Threshold
$\delta$ node displacement
d soil depth
d, D, $\Delta$ displacement
$D_{5015}$ average mean grain size in layers included in $T_{15}$
DL displacement above which the soil resistance is equal to the residual strength
$D_r$ relative density of soil
$D_{r_c}$ relative density after consolidation
$D_{st}$ static displacement
$\Delta \tau_{\text{max}}$ maximum induced shear stress
$\Delta u$ excess pore pressure
$\Delta u/\sigma'_{vo}, \Delta u/\sigma'_c$ excess pore pressure ratio
e void ratio
E Young’s modulus
\( \varepsilon_a \) axial strain
\( \varepsilon_c \) consolidation void ratio
\( \varepsilon_r \) reconsolidation volumetric strain
\( \varepsilon_f \) final axial strain
\( \varepsilon_i \) initial axial strain
\( E_i \) initial Young’s modulus
\( \varepsilon_v \) volumetric strain, post-liquefaction volumetric strain
\( F \) force
\( \phi \) friction angle of soil
\( F_{15} \) average fines content of saturated granular layers included in \( T_{15} \)
\( F_L \) factor of safety against triggering of liquefaction
\( \gamma \) gravity acceleration
\( G \) shear modulus
\( \gamma \) shear strain
\( \gamma_L \) limiting strain or strain at residual strength
\( G_m, G_o \) maximum shear modulus or shear modulus at low strain
\( \gamma_t \) unit weight of soil
\( H \) thickness of soil layer
\( k_b \) bulk modulus number
\( k_g \) shear modulus number
\( K_L \) stiffness of liquefied soil
\( k_{\text{max}} \) effective peak acceleration
\( m \) bulk modulus exponent, mass of the sliding block
\( M \) earthquake magnitude, constrain modulus of soil, mass of the block
\( N \) number of cycles, number of blow counts, yield acceleration
\( n \) shear modulus exponent
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_L$</td>
<td>number of cycles required to achieve liquefaction</td>
</tr>
<tr>
<td>$P_a$</td>
<td>atmospheric pressure</td>
</tr>
<tr>
<td>$PT$</td>
<td>Phase Transformation state</td>
</tr>
<tr>
<td>$\theta$</td>
<td>slope of the ground</td>
</tr>
<tr>
<td>$R$</td>
<td>horizontal distance from the seismic energy source</td>
</tr>
<tr>
<td>$r_d$</td>
<td>reduction factor due soil depth</td>
</tr>
<tr>
<td>$R_f$</td>
<td>failure ratio</td>
</tr>
<tr>
<td>$\sigma'_1$</td>
<td>major principal effective stress</td>
</tr>
<tr>
<td>$\sigma'_2$</td>
<td>second major principal effective stress</td>
</tr>
<tr>
<td>$\sigma'_3$</td>
<td>minor principal effective stress</td>
</tr>
<tr>
<td>$\sigma'_{30}$</td>
<td>effective confining stress</td>
</tr>
<tr>
<td>$\sigma'_c$</td>
<td>consolidation stress</td>
</tr>
<tr>
<td>$\sigma'_h$</td>
<td>horizontal effective stress</td>
</tr>
<tr>
<td>$\sigma'_{vo}$</td>
<td>initial vertical effective stress</td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>deviator stress</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>mean effective stress</td>
</tr>
<tr>
<td>$SPT$</td>
<td>Standard Penetration Test</td>
</tr>
<tr>
<td>$S_r$</td>
<td>steady state strength or residual strength</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$T_{15}$</td>
<td>cumulative thickness of saturated layers with $(N_1)_{60} &lt; 15$</td>
</tr>
<tr>
<td>$\tau_n/\sigma'_o$</td>
<td>stress ratio</td>
</tr>
<tr>
<td>$\tau_{cy}$</td>
<td>cyclic shear stress</td>
</tr>
<tr>
<td>$T_o$</td>
<td>the first natural period of embankment</td>
</tr>
<tr>
<td>$\tau_{st}$</td>
<td>static shear stress</td>
</tr>
<tr>
<td>$\tau_{ult}$</td>
<td>ultimate stress</td>
</tr>
<tr>
<td>$v$, $V$</td>
<td>velocity of the mass</td>
</tr>
</tbody>
</table>
$W_{\text{ext}}$  

external work

$W_{\text{int}}$  

internal work
CHAPTER 1: Introduction

Over the last three decades, many man-made earth-structures have undergone catastrophic failures due to earthquake induced-liquefaction. In Chile, eleven tailings dams failed during and after the March 28, 1965 earthquake. The most devastating were the total failures of two El-Cobre dams which destroyed part of the town of El-Cobre and claimed more than 200 lives (Dobry and Alvarez, 1966). Similar liquefaction-induced failures also occurred in Mochikoshi, Japan, in 1978 (Marcuson et al., 1979; Ishihara et al., 1984). Two tailings dams associated with the Mochikoshi gold mine failed causing a release of a large volume of tailings materials. The first dam failed about 10 seconds after the occurrence of the main shock, the second one failed approximately one day after the earthquake.

Somewhat less catastrophic failures of other dams have also occurred due to earthquake-induced liquefaction. Although some major soil movement took place in these dams, the dams did not fail in as much as the impounded water was not released. In China, two earth dams experienced major slope failures during the Bohai Gulf earthquake of July 18, 1969 (Wang, 1984). Wangwu dam, a clay-core sand-shell earth dam with a maximum height of 26.5 m and a crest length of 761 m, suffered two flow slides in the upstream part of the dam. The core of this dam was compacted by a roller during construction but the shell was only manually dumped, in lifts over one meter in thickness, without any compaction. Yeyuan dam, which was built with the same construction technique as Wangwu dam, also had a flow slide in the upstream slope. Fortunately, in
both cases, no catastrophic failures occurred as the water levels were far below the crest elevations when the earthquake occurred. In the United States, the near-catastrophic failure of the Lower San Fernando Dam due to the February 1971 earthquake is the classic example of a liquefaction-induced flow slide. Major slides occurred in the upstream slope extending back to the upper part of the downstream slope, removing the crest of the dam, and leaving only about 1.5 m freeboard.

The above cases represent examples of earthquake-induced soil movement where the driving stresses exceed the residual strength of soils. At the end of the movement, the driving stresses should be equal to or less than the residual strength of soils. Thus, significant geometry changes occur in order to reduce the average driving stress to conform with the residual strength. When geometry changes can not accommodate the difference in the driving stress and the residual strength, unlimited deformations such as those of the El-Cobre and Mochikoshi tailings dams will occur.

Of equal interest are cases of lateral spreading on sloping ground due to earthquake-induced liquefaction. Although the driving stresses do not exceed the residual strength of soils, large deformations occur as a result of soil softening due to liquefaction and the inertia forces of the earthquake. A good example of this behaviour was the downstream movement of the Upper San Fernando Dam, which resulted in a horizontal crest displacement of about 1.5 m and settlement of about 1 m.

Lateral spreading also occurs on gently sloping ground and sometimes on almost flat ground. But in these cases, it usually occurs over a very wide area. This type of earthquake-induced ground failure has caused damage worth over one hundred million US dollars in the United States since the 1964 Alaska earthquake. It has been classified by the US National Research Council (NRC, 1985) as the most pervasive and damaging type of ground failure due to earthquakes.
Lateral spreads due to soil liquefaction during the 1906 San Francisco earthquake caused serious damage to several buildings, bridges, roads and pipelines (Youd and Hoose, 1978). The breaking of water lines along Valencia Street between 17th and 18th Streets resulted in the loss of water needed by the fire fighters to control the burning in the Mission District of San Francisco (Scawthorn and O'Rourke, 1989). Similar ground movements occurred in the 1964 Alaska earthquake. Lateral spreading destroyed highway and railway bridges within a radius of 130 km around the zone of energy release (Bartlett and Youd, 1992). The resulting damage from this ground movement was estimated to be over $US 127 million (1964 dollars). Liquefaction-induced lateral spreads also damaged the Joseph Jansen Filtration Plant, Juvenile Hall and Sylmar Converter Station near the Van Norman Reservoir Complex during the 1971 San Fernando earthquake. The ground movements also affected bypass pipelines, channels, pump station and filtration facilities near the Complex (O'Rourke et al., 1992).

Similar damage from this type of ground movement has been documented in seismically-prone countries such as Japan and the Philippines. In Japan, liquefaction-induced ground displacement due to the 1923 Kanto earthquake caused non-repairable damage to houses and bridges in the Nakajima area (Hamada et al., 1992a). Ground failures due to liquefaction during the 1948 Fukui earthquake critically damaged many bridges, railways and other structures in the areas along the Kuzuryu River (Hamada et al., 1992b). Lateral spreads also caused widespread damage during the 1964 Niigata earthquake (Hamada, 1992). The liquefaction-induced movement of the Shinano River bank reached as much as 10 m into the river. In the Philippines, the 1990 Luzon earthquake resulted in widespread liquefaction in Dagupan city (Wakamatsu et al., 1992). Thousands of houses, two bridges and 80 percent of the city roads were destroyed due to liquefaction-induced ground deformation. Large scale lateral spreading and subsidence
due to liquefaction in Barrio Narvacan village caused this village to gradually sink under the sea.

If loose saturated sands are subjected to earthquake loading, pore water pressures will develop due to the tendency of the soils to contract. The rate of pore-pressure development depends on several factors including the level of earthquake shaking and the permeability of the sands. In most cases, however, earthquake loading is too fast to allow significant dissipation to occur during the shaking, consequently the loading condition is essentially undrained. If the duration of the shaking is sufficient to raise the pore pressure to a value equal to the initial overburden stress, thus causes the effective stress to drop to zero, liquefaction will occur which in turn may result in large deformations. The deformation, however, depends on several factors such as relative density of sands, earthquake intensity, duration of shaking and initial static shear stress acting on the soils. For a loose saturated sand on flat ground, for instance, liquefaction may only cause sand boils and settlements after pore water has dissipated. This is due to the fact that although significant deformation may take place during the shaking, the residual deformation at the end of the shaking is limited as there is no driving force imbalance. On the other hand, for sloping ground, even with very small driving force imbalance, the resulting displacements can be very large and potentially damaging if liquefaction occurs. Furthermore, higher peak ground accelerations and a longer earthquake duration will cause larger displacements.

Limit equilibrium analysis has long been used by geotechnical engineers to assess the stability of earth structures under static and dynamic loading. This method is quite simple and in most cases is capable of giving reliable information on the stability of earth structures. The results of the analyses are presented in terms of a factor of safety which is a ratio of soil resistance to driving stress. However, in some cases involving liquefied
soils that exhibit a marked reduction in stiffness, the factor of safety resulting from limit equilibrium analysis is not sufficient to judge whether the earth structures are stable or not. For these cases, a direct measure of seismic deformations is required to assess their stability. Accordingly, many researchers have proposed various methods, from the simple to the complicated, to compute ground deformations due to earthquake including those related to liquefaction of soils.

One of the earliest method for predicting earthquake-induced ground deformation is the Newmark method (Newmark, 1965). This method is very simple which is one of the reason why it is so popular among geotechnical engineers. The method has been shown to work reasonably well for soils that have a potential to develop a distinct failure surface such as dense granular materials (Goodman and Seed, 1966) or rockfill (Elgamal et al., 1990). However, the method does not work equally well for estimating ground displacements involving liquefaction of soils (Seed, 1979).

With the advance in the art of testing soils under dynamic loading conditions, much more information about the dynamic behaviour of soils has been gained (e.g. Seed and Lee, 1966; Lee and Seed, 1967; Thiers and Seed, 1969, Towhata and Ishihara, 1985a; Vaid and Chern, 1985). This information has significantly increased the understanding of soil behaviour under dynamic loading. Furthermore, the acceptance of the finite element approach for solving geotechnical engineering problems (Clough and Woodward, 1967), has led to the development of a more sophisticated method for predicting earthquake-induced ground deformations. One such method is the dynamic stress path method proposed by Seed and his colleagues (Seed et al., 1969; Seed et al., 1973; Lee, 1974; Serff et al., 1976). Their method consists of analyses of static and dynamic shear stresses developed within soil elements in the dam by using the finite element method. These analyses are combined with a comprehensive program of...
laboratory testing to determine strains of soil elements within the dam which potentially develop under the application of combined static and cyclic loads. This method has been claimed to successfully predict the deformation behaviour of several dams under earthquake loading conditions (Seed, 1979). However, recent applications of the procedures for seismic stability evaluations of a number of dams (Babbitt et al., 1983; Marcuson and Franklin, 1983; Smart and Von Thun, 1983) reveal that the method sometimes predicts large potential deformations accompanying soil liquefaction which may not develop in the field (Seed et al., 1988). It was also noted that the method does not provide any basis for evaluating the residual strength of the soil in the zones that were predicted to liquefy. Moreover, its complexity associated with large volume of work and the cost involved, makes the method not suitable for small projects.

A more rigorous method is the coupled dynamic effective stress approach in which both generation and dissipation of excess pore pressure is considered during the prescribed earthquake motion (e.g. Prevost, 1981; Finn et al., 1986; Byrne and McIntyre, 1994). This procedure is even more complex than the dynamic stress path method. It is a state-of-the-art rather than a state-of-practice procedure.

1.1. Research Objective

The objective of this research is to present a simple but reliable method for predicting liquefaction-induced ground displacements of earth structures. The method has to be reliable such that it is capable of predicting reasonably well the observed ground displacements due to earthquake induced liquefaction of soils, and simple such that the required parameters for the analyses can be easily obtained from routine site and laboratory tests. When these two criteria are fulfilled, it is hoped that the method can be useful for both small and large projects.
CHAPTER 1: Introduction

There are four key factors that must be included in a simplified method for predicting liquefaction-induced ground deformations. The first factor is the identification of the zone of liquefaction. The second one is a realistic model for the post-earthquake (post-liquefaction) stress-strain behaviour of soils. This factor is the most influential factor in assessing ground displacements associated with soil liquefaction. The third factor is the inclusion of kinetic energy or inertia forces associated with the earthquake. Although this is not as important as the first two factors, failure to consider the kinetic energy from the earthquake will result in underestimation of the computed ground displacements. The fourth factor is the inclusion of post-earthquake settlements of soils. For cases involving a thick liquefied layer, this factor may play a significant role on damage to structures due to differential settlements. These four key factors are taken into account in the analytical method proposed in this thesis.

1.2. Thesis Organization

Chapter 2 discusses the undrained behaviour of soils under earthquake loading conditions. Emphasis is given to the post-liquefaction stress-strain characteristics of granular soils. Moreover, several factors affecting the post-liquefaction behaviour are also discussed. This information is very important to the understanding of the assumptions used in the proposed method.

Chapter 3 presents a critical review of the current methods for estimating liquefaction induced deformation of earth structures. The key assumptions in each method are pointed out. The advantages and limitations of each method are also discussed.

Chapter 4 deals exclusively with the one-dimensional method proposed by Byrne (1990) which represents a basis for the proposed two-dimensional method. The closed-form solution proposed by Byrne are validated against case histories of liquefaction
induced ground displacements and the results are presented. The advantages and limitations of Byrne's method are highlighted and modifications to enhance the performance of his method are proposed.

Chapter 5 discusses the proposed two-dimensional method based on the energy concept in combination with the use of finite element technique to compute earthquake-induced ground displacements. The method of analysis to determine the displacements that satisfy the work-energy principles are discussed. Furthermore, the method to include the external and internal work is presented.

Chapter 6 presents a verification of the two-dimensional method against the closed-form solution discussed in Chapter 4. Two case studies, i.e. infinite slope and flat ground, are carried out and the results are presented.

The proposed two-dimensional method is validated against eight case histories of liquefaction induced ground displacements of earth structures. The case histories studied involve two case histories of lateral spreads and six cases of dam deformations due to soil liquefaction. The results of the validation are presented in Chapters 7 through 11.

Chapter 7 presents the results of the study of lateral spreads at the Wildlife site. Chapter 8 presents the results of liquefaction-induced deformation analyses of Heber Road site. Chapter 9 discusses the deformation behaviour of the Lower and Upper San Fernando Dams during the 1971 earthquake. Chapter 10 deals with the analyses of the Mochikoshi Dams in Japan due to the 1978 earthquake. Chapter 11 presents the results of deformation analyses of the La Marquesa and La Palma Dams in Chile due to the 1985 Chilean earthquake.

Summary and conclusions of the current research are given in Chapter 12. Suggestions for further research are also presented.
2.1. Introduction

Laboratory tests have served as basic tools to facilitate the understanding by geotechnical engineers of the behaviour of sand under dynamic loading. Early systematic studies on the dynamic behaviour of sand were carried out in the laboratory by using cyclic triaxial and simple shear devices by Seed and Lee (1966), Peacock and Seed (1968) and Castro (1969). Despite some limitations of the cyclic devices in simulating the exact behaviour of soil in the field under dynamic loading (e.g. Seed and Peacock, 1971; Casagrande, 1975; Alarcon et al., 1985), knowledge thus obtained significantly augmented understanding of the dynamic behaviour of soils.

During an earthquake, a soil element in the ground experiences a multidirectional shaking due to erratic ground motion. In most earthquakes, however, a soil element is predominantly subjected to shear stresses induced by the upward propagation of shear waves from underlying soil layers. The combined static and dynamic stresses due to an earthquake in a soil element in level ground and sloping ground are illustrated in Fig. 2.1. These stress conditions can be simulated closely in a simple shear device. In the triaxial device, the stress condition in a soil element beneath flat and sloping grounds can be simulated by consolidating the samples isotropically or anisotropically prior to cyclic load applications.
Fig. 2.1. Stress condition in a soil element beneath flat and sloping ground.
This chapter presents a review of general aspects of pre- and post-liquefaction undrained behaviour of saturated sands. This information provides a foundation for the assumptions used in the proposed procedures that will be discussed in Chapter 5. First, the term *liquefaction* used in this thesis is defined. Secondly, the pre-liquefaction undrained behaviour of saturated sands under monotonic loading conditions is briefly reviewed, and then followed by a review of the undrained cyclic loading behaviour. Finally, discussion focuses on the post-liquefaction behaviour of sand including the stress-strain response under monotonic and cyclic loading as well as the volumetric strain response after dissipation of excess pore pressures.

2.2. Definition of Liquefaction.

The term liquefaction has traditionally been used to describe the response of saturated loose sand to strains or shocks that result in flow slides (Casagrande, 1975). However, with the development of cyclic triaxial devices to study the dynamic behaviour of soils under earthquake loading, this term is now also used to refer to some other phenomena observed during cyclic load tests such as initial liquefaction (Seed and Lee, 1966) and limited liquefaction (Castro, 1969). To prevent confusion concerning this term, as it will be used frequently in the rest of this thesis, the term *liquefaction* must first be defined.

The Committee on Soil Dynamics of the Geotechnical Engineering Division of ASCE (1978) has agreed on several terms related to liquefaction. *Liquefaction* itself was defined as a *process of transforming any substance into a liquid*. For granular materials, liquefaction is defined as a transformation of a solid state to a liquid state as a consequence of increased pore pressure and reduced effective stress. This definition is independent of the initiating disturbance that could be static, dynamic, sea wave, shock
loading or a change in ground-water pressure. The definition is also independent of
deformation or ground movement that might occur following the transformation. Thus,
both flow failures and deformation failures are said to be liquefaction failures (NRC,
1985).

For the purpose of this thesis, the term *liquefaction* will be defined as *all
phenomena that cause a loss of shearing resistance or the development of excessive
strains as a result of transient or repeated disturbance of saturated cohesionless soils*
(NRC, 1985). These phenomena include the condition of transient zero effective stress or
the condition of 5 percent double amplitude cyclic strains in a soil element during triaxial
test in the laboratory, and the conditions of sand boils or excessive deformations observed
in the field.

2.3. Pre-liquefaction Behaviour

Liquefaction of granular soils can be initiated by static, dynamic or impact
loadings, or simply by a change in ground water pressures. In this section, the
development of strains and excess pore pressures of saturated sands under both undrained
monotonic and cyclic loading prior to liquefaction will be highlighted.

2.3.1. Undrained Monotonic Loading Behaviour

In general, the pre-liquefaction undrained behaviour of saturated sands in triaxial
test conditions under monotonic loading can be schematically represented by the curves
in Fig. 2.2. Although the characteristic responses shown in this figure are those from
isotropically consolidated samples, similar responses are also observed if the samples are
anisotropically consolidated. Different responses shown in curves 1, 2 and 3 are
associated with increased relative density at the same initial effective confining stress.
Chapter 2 : Undrained Behaviour of Saturated Sands

Curve 1 represents the response of strain softening (contractive) behaviour of saturated sand under monotonic loading which shows large reduction in soil resistance at large strain. The strain and pore pressure developments are slow until a certain effective stress ratio is reached. This effective stress ratio is called the Critical Stress Ratio (CSR) by Vaid and Chern (1983) and is found to be unique for a given sand in compression loading (Chung, 1985; Vaid and Chern, 1985; Dobry et al., 1985). The CSR in extension is smaller than that in compression mode. Moreover, unlike the CSR in compression mode, the CSR in extension also seems to be affected by deposition void ratio (Vaid et al., 1989).

Once the CSR is reached, the soil exhibits a sudden loss in strength accompanied by a rapid increase in strain and pore pressure. Soil resistance eventually reaches a minimum value at very large strain. This minimum soil resistance was termed steady state strength by Castro (1969) or residual strength by Seed (1979). Castro (1969) considered this steady state strength to be uniquely related to initial void ratio of soils. The phenomenon associated with loss in strength was termed liquefaction by Castro (1969), Casagrande (1975), Seed (1979) and Vaid and Chern (1985). In this thesis, however, this phenomenon will be called *liquefaction flow failure* or simply *flow failure* to differentiate it from the term *liquefaction* in general meaning.

Curve 2 represents the response of contractive sand under the monotonic loading condition which exhibits reduction in strength at some stage of loading yet regains its strength with increasing deformation. This type of response will herein be called *limited liquefaction* following Castro (1969) and Vaid and Chern (1985).
Fig. 2.2. Characteristic response of saturated sands under undrained monotonic loading (after Vaid and Chern, 1985).
As in the liquefaction flow failure type of response, strain softening is initiated at the CSR and continues until a minimum undrained resistance is reached (point 'n' in Fig 2.2.a). However, instead of developing unlimited deformation, the soil begins to regain its strength with further straining. The state where the soil starts to increase its shear resistance is reflected in the effective stress path diagram by a sharp turn around of the effective stress path (Fig. 2.2.b). Such a state has been termed Phase Transformation (PT) by Ishihara et al. (1975) and Characteristic Threshold (CT) by Luong (1980). This PT or CT state has been shown to occur at the same effective stress ratio for a given sand regardless of its relative density (Luong, 1980; Dobry et al., 1985; Vaid and Chern, 1985).

Curve 3 represents the strain hardening behaviour of sand with no loss of shear resistance at any stage of loading. The soil resistance keeps increasing with increasing deformation. Unlike the limited liquefaction type of response, no sharp reversal is observed in the effective stress path diagram. However, as shown in Fig. 2.2.b, a more gradual turn around of effective stress path is observed once the PT or CT line is intersected.

2.3.2. Undrained Cyclic Behaviour

An intimate link exists between behaviour of saturated sands under undrained monotonic loading and cyclic loading. Several researchers have shown cases where liquefaction flow failure and limited liquefaction occur in similar way during cyclic loading and monotonic loading (e.g. Castro, 1969; Vaid and Chern, 1981,1985). The behaviour of sands at a given initial effective stress with different relative density is shown in Figs. 2.3-2.5. Similar to the monotonic response shown in Fig. 2.2, these characteristic responses are also obtained from triaxial tests. However, instead of being isotropically
consolidated, the samples here are anisotropically consolidated to simulate the condition of a soil element in the field having an initial static shear stress.

Fig. 2.3 shows the behaviour of loose saturated sand subjected to cyclic loading. As for monotonic loading behaviour, the strains and pore pressures develop slowly as long as the effective stress ratio is less than the CSR. Sudden increases in strain and pore pressure are observed when the effective stress path hits the CSR line. The soil then exhibits strain softening behaviour with further straining and eventually reaches a steady state strength. If the residual strength is larger than the initial static shear stress, the soil movement will stop at some stage of loading.

Limited liquefaction during cyclic loading is shown in Fig. 2.4. As in monotonic loading, a strain softening response is initiated when the stress state reaches the CSR line and a sudden turn around in the effective stress path is observed when the stress path hits the PT line. This process results in a large axial strain in the sample (Figs. 2.4.a and c). Subsequent unloading from the peak amplitude of cyclic deviator stress causes large increases in pore pressure which leads to a transient zero effective stress condition (Fig. 2.4.b). However, the strain recovery during the unloading process is very small (Fig. 2.4.a). During further unloading in the extension region of the stress cycle, the soil sample shows almost zero stiffness causing the sample to undergo large deformation. Then, during subsequent re-loading, the sample exhibits very soft response for a significant range of strain. The sample begins to regain most of its stiffness when it reaches the strain obtained during the first transient zero effective stress state.
Fig. 2.3. Liquefaction of sands due to cyclic loading (after Vaid and Chern, 1985).
Fig. 2.4. Limited liquefaction due to cyclic loading (after Vaid and Chern, 1985).
Fig. 2.5 shows the cyclic mobility type of response during cyclic loading. The strain and pore pressure develop slowly with the increasing number of cycles. No strain softening response is observed at any stage of loading. When the effective stress path reaches the PT line, a sudden turn around in the stress path is observed and significant strain is accumulated. Further unloading from the peak amplitude of deviator stress brings the sample close to the state of zero effective stress (Fig. 2.5.b) due to large increase in pore pressure. Reloading causes the sample to attain a transient zero effective stress state and a consequent large deformation in the next load cycle.

The response of a soil element with an initial static shear stress can show any one or a combination of the above characteristic responses: flow failure, limited liquefaction and cyclic mobility. The type of soil response will depend on factors such as soil fabrics associated with soil deposition methods, grain angularity, initial void ratio, confining stress, initial static stress and cyclic stress ratio. If liquefaction flow failure is anticipated, the design concern is stability since unlimited deformation will occur. On the other hand, if limited liquefaction or cyclic mobility or a combination of the two is expected, the design concern is large deformations and how to prevent them from occurring.

In the field, a flow failure type of response can be expected in a loose saturated sand deposit (low relative density) with high initial static shear stresses, or even in a saturated sand deposit with high relative density at sufficiently high confining stresses (Castro, 1975; Vaid and Chern, 1985). If the residual strength of soils are less than the static shear stresses, large deformations may occur when an earthquake is triggered. During the movement, however, the static shear stresses will decrease due to the changes in geometry of the soil structures. The movement will eventually stop when the average static shear stress is equal to or less than the residual strength, and all kinetic energy of the movement are dissipated. If the changes in geometry of soil structures can not
Fig. 2.5. Cyclic mobility due to cyclic loading (after Vaid and Chern, 1985).
accommodate the imbalance of forces due to the difference between the average static shear stress and residual strength of the soil, unlimited deformation will occur. Examples of flow failure response were observed in cases including the failures of two tailings dams associated with the Mochikoshi gold mining in Japan during the 1978 Izu-Ohshima-Kinkai earthquake (Ishihara, 1984), and the El-Cobre tailings dam in Chile during the 1965 Chilean earthquake (Dobry and Alvarez, 1966).

For a flat or gently sloping ground, the possibility of liquefaction flow failure is much less in comparison to a steep slope. In this case, the initial static shear stress in the soil deposit is most likely to be smaller than the soil residual strength. However, this does not necessarily mean that there is no potential damage if liquefaction occurs at this site. A limited liquefaction or cyclic mobility type of response may occur which may cause undesirable large deformation. In fact, ground movements due to these types of response were found to be the most pervasive and damaging type of ground movements during earthquakes. Examples of these types of ground displacements were observed including those which occurred during the 1964 Niigata (Hamada, 1992) and the 1906 San Francisco earthquakes (O'Rourke et al., 1989).

2.4. Post-liquefaction Behaviour

The post-liquefaction cyclic deformations of soils have been found to be affected by several factors including relative density, cyclic stress level, and duration of cyclic loading (e.g. Kuerbis, 1989; Kawakami et al., 1994). Similarly, the post-liquefaction volumetric strains are affected by several factors, mainly the relative density and maximum cyclic shear strain. Discussion regarding the post-liquefaction stress-strain relationship and the volume change behaviour of granular soils obtained from the laboratory and field studies are presented in the following sections.
2.4.1. Post-liquefaction Cyclic Stress-Strain Behaviour

The post-liquefaction cyclic stress-strain behaviour of sands during earthquake shaking have been investigated in the laboratory by means of triaxial device (e.g. Seed and Lee, 1966; Ishihara and Towhata, 1983; Towhata and Ishihara, 1985b; Vaid and Chern, 1985; Kuerbis, 1989) and a hollow cylinder triaxial device (e.g. Kawakami et al., 1994). The condition in the field is simulated by subjecting the samples with cyclic loads past the liquefaction stage. Typical post-liquefaction stress-strain characteristics of isotropically consolidated sands under cyclic loads in the triaxial device are shown in Fig. 2.6.a and b (Kuerbis, 1989). The figures show the limited liquefaction (Fig. 2.6.a) and the cyclic mobility type of response (Fig. 2.6.b). As shown in Fig. 2.6.a, the axial strains in the loose sample are small prior to liquefaction. Upon liquefaction, large strains developed in only one additional cycle. Similar trend is also observed in the medium dense sample in Fig. 2.6.b. However, the medium dense sand regains its stiffness much faster than the loose sand.

As also can be seen in Fig. 2.6.b, the post-liquefaction strain increment per cycle initially increases but then tends to decrease after reaching a certain number of cycles. There seems to be a maximum cyclic strain in which additional cycles do not significantly increase the strains. This behaviour is consistent with the observation of De Alba et al. (1976) from the results on large simple shear tests and is confirmed by recent test data obtained by Kawakami et al. (1994) from a hollow cylinder torsional shear tests. This behaviour will be discussed in more detail in Section 2.4.4.
Fig. 2.6. Post-liquefaction cyclic stress-strain curves; (a). Limited liquefaction; (b). Cyclic mobility response (after Kuerbis, 1989).
Laboratory data shown in Figs. 2.6.a and b demonstrate that upon liquefaction, the soil stiffness increases progressively with level of strain during cyclic loading. This implies that upon liquefaction, soils are still capable of transmitting earthquake generated shear wave from the underlying soil layers, provided the soils can deform sufficiently. In fact, even a loose liquefied sand with low static shear stresses also shows a dilative response during post-liquefaction cyclic loading. Field evidence in terms of recorded ground accelerations during the 1964 Niigata earthquake (Seed and Idriss, 1967) and also data from the Wildlife site during the 1987 Superstition Hill earthquake (Holzer et al., 1989) confirm this.

Seed and Idriss (1967) presented the record of a strong motion seismograph located in the basement of an apartment building at Kawagishi-cho, Niigata, as shown in Fig. 2.7. As can be seen in this figure, there is a marked increase in the period of ground motion after about 8 seconds of earthquake motion. This presumably indicates the time of the onset of liquefaction of the soil below the building. Laboratory data show that upon liquefaction, zero stiffness response exists over a large range of strain. Consequently, a much larger strain must be developed before the soil regains its resistance to be able to transmit the earthquake accelerations. The recorded peak ground acceleration after liquefaction is about 0.15 g in comparison to the peak acceleration of 0.16 g before liquefaction.

Holzer et al. (1989) presented the field behaviour of saturated sands at the Wildlife site, California, during the 1987 Superstition Hill earthquake. The Wildlife site is a gently sloping area at the edge of the Alamo River which consists of a loose saturated sand layer, with N values of 1-4 at a depth of 1.5-4 m. This site is heavily instrumented to monitor the expected re-occurrence of liquefaction. Holzer et al. (1989) observed a marked increase in the period of motion after the recorded pore pressures in the loose
sand were close to the initial effective overburden stresses. Moreover, they noticed ground acceleration spikes as high as 0.12 g following this stage. Interestingly, the timings of these spikes coincide with the occurrence of negative pore pressures. This is a strong indication that a cyclic mobility type of response occurred in the field upon liquefaction.

The soil response during earthquake loading at the Wildlife site can be more clearly seen from the results of analyses performed by Byrne and McIntyre (1994) and Zeghal and Elgamal (1994). Byrne and McIntyre (1994) and Zeghal and Elgamal (1994) independently analyzed the earthquake accelerations recorded at the surface and at the non-liquefied base at this site. The displacements at the surface and at downhole were computed by integrating the acceleration records in both locations. The relative displacements between the two were then plotted with the surface accelerations. This plot is similar to the plot of shear stress versus shear strain because shear stress is proportional to acceleration and shear strain is proportional to displacement. The results of analyses carried out by Byrne and McIntyre (1994) are shown in Fig. 2.8. This figure clearly
shows that there is a marked decrease in stiffness at some stage of the earthquake shaking which indicates the onset of liquefaction at some layers between the surface and the stiff base.

Fig. 2.8. Surface acceleration versus relative displacements at the Wildlife site during the Superstition Hills earthquake (Byrne and McIntyre, 1994).

Upon liquefaction, the soil deforms significantly but regains its stiffness after large deformation. The plot in Fig. 2.8 is very similar to the plot shown in Fig. 2.6.

Evidence from the shaking table test data regarding the cyclic mobility type of response upon liquefaction has been presented by Sasaki et al. (1992). They carried out large scale shaking table tests to study the deformation mechanism of soil during and after
liquefaction in the field. They plotted the recorded time history of accelerations, pore pressure increases, and displacements as shown in Fig. 2.9.a. Moreover, they plotted acceleration versus displacement time histories as shown in Fig. 2.9.b. This plot can give an approximate stress strain response of a soil element during shaking table tests, similar to Fig. 2.8. Using the conceptual framework of cyclic loading behaviour discussed previously, a schematic effective stress path diagram for a soil element with this stress-strain response has been drawn in Fig. 2.9.c.

As can be seen in Figs. 2.9.a, b and c, small pore pressure increase and small displacement develop in the first half cycle (point 1 in Figs. 2.9.b and c). During unloading, a large pore pressure develops causing the stress path hits the CSR line (point 2) and accordingly some loss in shear resistance is observed. The stress path shows a sudden turn around when it hits the PT line (point 3) and the soil regains its strength until point 4 where the ground acceleration changes in direction. Reloading causes the stress path to hit the CSR line on the other side of the effective stress diagram (point 5) and a subsequent decrease in soil resistance. At this point, however, the soil element has not reached a zero effective stress condition. The soil resistance increases again when the stress path hits the PT line (point 6) with a consequent decrease in pore pressure until the maximum ground acceleration is reached (point 7). Unloading for the second time brings the soil to a transient state of zero effective stress causing the sample to totally liquefy. This deformation mechanism is similar to the soil response shown in Figs. 2.4. and 2.6.b.
Fig. 2.9. Recorded data from shaking table tests. (a). Time histories of acceleration, pore pressure and displacements; (b). Approximate stress-strain curves (after Sasaki et al., 1992) (c) Approximate effective stress path.
General agreement can be observed between post-liquefaction soil response under earthquake shaking obtained from the results of field and the laboratory studies. The results show that the strain developed during cyclic loading in the pre-liquefaction stage is relatively small in comparison to the strain developed once liquefaction is triggered. This observation confirms the contention of Byrne (1990) that the liquefaction-induced deformation of the earth structures is governed by the stress-strain behaviour of liquefied soils during post-liquefaction loading, regardless of the behaviour of soils at pre-liquefaction stage.

2.4.2. Post-liquefaction Monotonic Stress-strain Behaviour

In some cases, large deformations occur after the cessation of the earthquake loading. Field examples of these are the failures of the Lower San Fernando Dam (Seed et al., 1973) and Dam No. 2 of the Mochikoshi tailings dam (Ishihara, 1984). The post-liquefaction soil behaviour can be studied in the laboratory by subjecting the samples to cyclic loading to bring the samples to liquefaction state, and by subsequently shearing the samples monotonically (e.g. Seed, 1979; Yasuda et al., 1991,1994; Thomas, 1992). In this way, the effects of cyclic loading on the stiffness degradation and the residual strength of soil can be investigated conveniently.

An early attempt to study the post-liquefaction monotonic behaviour of soils was carried out by Seed (1979). The samples tested were Sacramento River sand with relative densities of 40 percent, and mine tailings sands with relative density of 95 percent (based on modified AASHO compaction test). The pre- and post-liquefaction stress-strain curves for the soil tested are presented in Figs. 2.10.a and b.

Seed's data show that upon liquefaction, the sand loses almost all of its stiffness and behaves like liquid over a large range of strain, which is in agreement with the field
Fig. 2.10. Post-liquefaction monotonic behaviour of sands. (a) Loose Sacramento River sand with relative density of 40 percent; (b) dense mine tailings sand with relative density of 95 percent based on modified AASHO compaction test (after Seed, 1979).
response at the Wildlife site during the Superstition Hill earthquake (Byrne and McIntyre, 1994; Zeghal and El-gamal, 1994) and in the laboratory shaking table tests (Sasaki et al. 1992). As can be seen in Fig. 2.10.a, the loose Sacramento River sand is quite stiff upon initial loading with a resistance corresponding to a deviator stress of about 7 kg/cm² (687 kPa) at 20 percent axial strain. However, the soil stiffness drops to almost zero over a large range of strain (from zero to about 20 percent axial strain) during post-liquefaction monotonic loading. At approximately 20 percent of axial strain, the deviator stress begins to increase and reaches its original peak at an axial strain of 40 percent. The deviator stress then levels off with increasing deformation.

Similar stress-strain characteristics are also observed in dense mine tailings sand as seen in Fig. 2.10.b. In this case, the deviator stress begins to increase at axial strain of 10 percent in comparison to 20 percent for the loose Sacramento River sand. Moreover, although the soil resistance increases with increasing deformation, it does not reach its original peak even after an axial strain of 30 percent is exceeded. Some loss in soil resistance is observed in this sand.

Yasuda et al. (1991, 1994) presented the results of a study on the post-liquefaction monotonic behaviour of sand using both triaxial and torsional shear devices. They applied a number of cyclic loads on isotropically consolidated samples until a prescribed residual excess pore pressure was achieved. Subsequently the samples were sheared monotonically. The tests were carried out on the samples with relative densities of 30-70 percent and effective confining stresses of 0.25, 0.5 and 1.0 kg/cm² (or about 25, 50 and 100 kPa). Yasuda et al. (1994) also investigated the effects of additional cyclic loading past the liquefaction stage on the post-liquefaction monotonic stress-strain response. The number of cycles applied to the samples was represented by a factor of
Fig. 2.11. Effects of excess pore pressure ratio on post-liquefaction stress-strain curves (Yasuda et al., 1994).
safety against liquefaction which was defined as $F_L=(20/N)^{0.17}$ for Toyoura sand (Tatsuoka et al., 1980), in which $N$ is the number of cycle. Number of cycles of 20 was taken to correspond to $F_L$ equals 1.0 and number of cycles larger than 20 was indicated by $F_L$ less than unity. Some of the results from the torsional and triaxial tests are respectively shown in Figs. 2.11.a and b.

A similar trend is observed from the results of both torsional and triaxial tests that the stress-strain curves tend to be softer with increasing excess pore pressure ratio $(\Delta u/\sigma'_vo)$. At $\Delta u/\sigma'_vo=1.0$ (or $F_L=1.0$), where the sample liquefies, a marked decrease in soil stiffness is observed which is similar to that observed in Seed's data. Almost zero stiffness exists over large range of strain, and the stiffness starts to increase at 10 percent shear strain and keeps increasing with further straining. Fig. 2.11.a also shows that the strains required by the samples to attain their residual strength increase with decreasing $F_L$.

Thomas (1992) carried out a comprehensive study on post-liquefaction monotonic behaviour of Fraser River sand using a triaxial apparatus. The effects of relative density, effective confining stress and pore pressure ratio developed during cyclic loading on post-liquefaction characteristics were investigated.

Thomas showed that the samples which developed 100 percent excess pore pressure during cyclic loading initially deform with essentially zero stiffness during post-liquefaction monotonic loading. The samples show some increase in soil resistance with further straining and the resistance keeps increasing even after axial strain of about 15 percent (or about 30 percent from the cessation of cyclic loading) is reached. These data agree with those of Seed (1979) and Yasuda et al.(1991, 1994).

Thomas further showed that the rate of stiffness build up during post-liquefaction monotonic loading increases with increasing relative density. Moreover, he found that the
range of strain where the samples exhibit zero stiffness decreases with increasing relative density. This trend was in fact observed by Kawakami et al. (1994) and is in agreement with the limiting strain concept proposed by Seed and De Alba (1975) which will be discussed in further detail in Section 2.4.4.

Yasuda et al. (1994) plotted the reduction in initial stiffness as a function of excess pore pressure ratio, $\Delta u/\sigma'_v$ as shown in Fig. 2.12. It is clear from this figure that the samples experience a marked stiffness reduction with increasing $\Delta u/\sigma'_v$. Large reductions, in the order of 1/500 to 1/1000 of the original stiffness, are observed when the sample is fully liquefied ($\Delta u/\sigma'_v=1.0$). The figure also shows that the modulus reduction for a given excess pore pressure ratio increases with decreasing relative density.

Stiffness reduction calculated from Thomas' data ($\sigma'_c = 400$ kPa) was also plotted as a function of excess pore pressure ratio in Fig. 2.12. The soil stiffness was computed at an axial strain of 2.5 percent due to difficulties in obtaining reliable deviator stress magnitudes at small strain from his data. Similar to Toyoura sand, Fraser River sand tested by Thomas shows a marked decrease in stiffness particularly when the liquefaction state is achieved during previous cyclic loading. Scatter of the data in this figure may be attributed to the different confining pressures, sand types (e.g. particle angularity and gradation), testing procedures (e.g. soil fabric) and testing devices.

Recent laboratory data on the post-liquefaction stress-strain curves were presented by Salgado and Pillai (1993) as shown in Fig. 2.13. The soil samples were obtained from the Duncan Dam foundation in British Columbia and sampled using ground freezing method (Sego et al., 1993). The samples were tested in simple shear apparatus with varying effective confining stresses and initial static shear stresses. The samples were
Fig. 2.12. Effects of excess pore pressure ratio on stiffness reduction of sands during post-cyclic monotonic loading (data from Yasuda et al., 1991, 1994 and Thomas, 1992).

subjected to cyclic loading until the samples reached a maximum shear strain of about 4 percent (or 2.5 percent single amplitude cyclic strain in triaxial test). Subsequently the samples were sheared under monotonic loading. The data show that the samples with initial static shear stress are generally stiffer than those without static shear stress.
Although the data do not show stiffness decrease to the extent obtained by previous investigators, stiffness reduction between 2 to 50 times were observed. Stiffness reduction of this magnitude can still be potentially damaging to soil structures.

2.4.3. Steady State or Residual Strength of Liquefied Sand

One of the key parameters to assess the post-liquefaction stability of earth-structures is the steady state or residual strength of liquefied soils. For the soil which exhibits limited liquefaction type of response, the post liquefaction strength is dependent on the strain level. Thus, researchers have to decide at what stage of deformation they will assume the steady state soil strength. Castro et al. (1982) suggests that the steady state strength is only achieved after all soil dilation process is complete. This may lead to a very high steady state strength for a soil that exhibits strong dilative response. Most researchers, however, have assumed the soil strength at phase transformation state as the steady state strength of liquefied soils. This value of steady state strength is considered to be conservative for post-liquefaction stability of soil structures.

In practice, there are two general methods to determine the steady state strength of soils in the field: direct and indirect methods. The direct method employs laboratory tests on representative samples to obtain the steady state strength, while the indirect method employs the correlation between the normalized Standard Penetration Test (SPT) blow counts for clean sand, \((N_1)_{60-cs}\), in the field with the steady state strength obtained from back-analysis of liquefaction-induced failures which have occurred in the past. The indirect method is based on the concept that the steady state strength depends on the state of the sand (relative density and confining stress). The normalized SPT blow count is a measure of the soil state in the field.
Fig. 2.13 Post-liquefaction monotonic behaviour of Duncan Dam sand in simple shear tests (after Salgado and Pillai, 1993).
A direct method for determining steady state strength of soils has traditionally been using cyclic loading followed by monotonic loading tests on the soil samples using triaxial, simple shear or torsional shear apparatus (e.g. Seed, 1979; Yasuda et al., 1991; Salgado and Pillai, 1993). Typical results have been presented in Figs. 2.11 and 2.13.

As shown in Fig. 2.11., the results of torsional shear tests carried out by Yasuda et al. (1994) show that post-cyclic undrained strength of sand decreases with increase in excess pore pressure ratio. Moreover, continued cyclic loading past the liquefaction stage (i.e. $F_L$ is less than unity) causes more reduction in the undrained strength from its pre-cyclic value. The sample with $F_L$ of 0.90 show a residual strength of about 1/4 of that of the sample with $F_L$ equals unity. Thus, the residual strength of soils of a given relative density can be significantly different depending on the factor of safety against liquefaction.

Post-liquefaction simple shear tests on frozen cored samples from the Duncan Dam foundation (Fig. 2.13) suggest the average residual strength of 0.21 $\sigma'_{vo}$ (Salgado and Pillai, 1993). The equivalent clean sand SPT blow counts, $(N_i)_{60-cs}$, for the materials is about 13. It is important to note that $F_L$ for these samples were approximately unity because the cyclic loads were stopped when the samples reached 4 percent cyclic shear strain. Based on the results presented by Yasuda et al. (1994), the residual strength of this sand can be lower for $F_L$ less than unity.
The other direct method uses the steady state approach (e.g. Poulos et al., 1985). This method does not require one to carry out any cyclic loading test before monotonic loading. This method only requires monotonic undrained compression tests on isotropically consolidated samples in the triaxial apparatus. While it would be desirable to obtain the steady state strength using the simple method proposed by Poulos et al., in many cases this approach tends to yield significantly higher steady state strength values than those back-calculated from the case histories of liquefaction induced failures (Seed, 1979; Jong, 1988).

The steady state concept requires a good quality of undisturbed samples and corrections must be made on void ratio change during sampling, handling, and during reconsolidation in the laboratory to the stress condition in the field. This concept is based on the following assumptions: 1). there is a unique relationship between void ratio and steady state or residual strength, 2). the slope of the steady state line for reconstituted samples is the same as that for undisturbed samples, and 3). the slope of the steady state line is independent of the preparation method and is only a function of grain angularity and grain size distributions.

Despite the acceptance of the steady state concept by many geotechnical engineers, some criticisms have been advanced regarding the concept. For some sands, the uniqueness of steady state line in void ratio-stress space may not exist. As shown by Vaid and Chern (1985), Vaid et al. (1990) and Konrad (1990a,b), the undrained steady state lines may be affected by mode of loading and effective confining stress. As also shown by Riemer et al. (1989), the assumption that the steady state line is a straight line in void ratio-stress space is not entirely correct. They found that there is a limiting minimum density below which saturated cohesionless soils will exhibit zero steady state strength under undrained loading condition. Furthermore, the undrained loading response
and consequently undrained steady state strength for some sands have been shown to be affected by sample preparation method (Miura and Toki, 1982; Ishihara, 1993).

One serious limitation of the steady state concept is that the steady state strength is very sensitive to the change in void ratio, particularly for rounded grain sands which exhibit an almost flat steady state line. The change in void ratio due to sampling and handling is very difficult to control and is very often smaller than the field variation of void ratio in the soil deposit. This results in uncertainty in determining steady state strength of soil. Kramer (1989) identified the void ratio changes during sampling and handling and the slope of the steady state lines as uncertain parameters. He evaluated the uncertainty associated with these parameters and concluded that for many liquefiable soils, significant uncertainty exists in the steady strength predicted using the steady state concept. Kramer (1989) suggested that evaluation of liquefaction hazards using this concept should be supplemented by other more reliable methods.

The indirect method proposed by Seed (1979) offers an alternative way to obtain steady state strength of liquefied soils without concern about the soil disturbance during sampling, handling, and testing. The method employs in-situ standard penetration test (SPT) N-values after being corrected for the effects of hammer energy, in-situ effective overburden stress and fines contents, \((N_1)_{60\text{-cs}}\). As mentioned previously, the undrained steady state strength or residual strength, can be estimated from a correlation between equivalent clean sand SPT blow counts, \((N_1)_{60\text{-cs}}\), and residual strength back-calculated from field case histories of liquefaction flow failures. A tentative relationship between residual strength and \((N_1)_{60\text{-cs}}\) is presented by Seed and Harder (1990) and is shown in Fig. 2.14. Some scatter in the data is observed. This is probably due to the fact that these residual strength values were obtained from different case histories in which the confining stresses varied between 50 to 200 kPa and failed under static and seismic loading.
conditions with different failure surfaces. As well, $S_r$ is back-calculated using limit equilibrium analyses with and without considering the inertia forces of the earthquake.

Based on the relationship shown in Fig. 2.14, Byrne (1990) proposed an empirical formula to determine the average residual strength for post-liquefaction stability analyses as follows:

$$S_r = 0.0284 \ Pa \ e^{(0.173(N_{160cs})}$$

(2.1)

with a lower bound value given by:

$$S_r = 0.087 \sigma'_{vo}$$

(2.2)

in which $Pa$ is the atmospheric pressure and $\sigma'_{vo}$ is the effective consolidation pressure.
The residual strength of Duncan dam foundation soil with equivalent clean sand value of about 13 was computed using Eq. 2.1. The formula yields a residual strength of 27 kPa which corresponds to $0.21\sigma'\nu_0$ for a confining stress of about 125 kPa. This value is in reasonable agreement with the residual strength obtained from Eq. 2.1. However, for the stress range on interest in the Duncan dam project (300 to 600 kPa), the residual strengths from the laboratory are almost three times higher than that computed from Eq. 2.1.

Stark and Mesri (1993) normalized the residual strength presented by Seed and Harder (1990) with respect to preliquefaction effective overburden stress. They compared these values with the critical stress ratio curve used for the triggering of liquefaction (Seed et al., 1985). They found that a large number of normalized residual strength, $S_r/\sigma'\nu_0$, plot above this curve. Based on this, they argued that in a large number of case histories, significant drainage occurred during post-liquefaction deformations. However, for post-liquefaction stability analyses, drainage of the slide mass can not be assumed. Thus, the residual strength at fully undrained condition must be used during the stability analyses. They proposed an equation to estimate the residual strength of liquefied soils corresponding to the undrained condition as follow:

$$S_r/\sigma'\nu_0 = 0.0055 (N_1)_{60-\text{cs}}$$  \hspace{1cm} (2.3)

In comparison to the residual strength obtained from simple shear test shown in Fig. 2.14 (Salgado and Pillai, 1993), Stark-Mesri empirical formula (Eq. 2.3) yields a smaller residual strength. The laboratory measured $S_r$ is about 3 times the empirical values at any confining stresses. Thus, the residual strength computed from Stark-Mesri empirical formula tends to be overly conservative and will tend to highly overestimate the liquefaction induced displacements. Their formula will not be used in the proposed method.
2.4.4. Limiting Shear Strains

De Alba et al. (1976) presented results of large scale simple shear tests on reconstituted medium dense to dense Monterey sand. They found that the cyclic strains increased when a state of transient zero effective stress was reached. However, for the samples tested (medium dense to dense sand) the strains were observed to reach a limiting value which depended on the relative density of the samples. This limiting value of strain decreased with the increase in relative density. This behaviour was also observed by Kawakami et al. (1994) in the torsional shear tests.

Based on these results, De Alba et al. (1976) proposed a concept of limiting shear strain which states that there is a limited amount of shear strain for a given relative density that could be developed during cyclic loading regardless of cyclic stress ratio and number of cycles. For loose contractive sand (relative density less than about 45 percent), liquefaction may cause unlimited strain with additional numbers of cyclic loading. For medium dense and dense sand with dilative response, liquefaction may only result in a limited magnitude of shear strains.

Comparison with field data must be made to justify the magnitude of limiting strains obtained from these simple shear tests. Undisturbed samples obtained using in-situ freezing method (Yoshimi et al., 1984) are considered to be the most suitable method for capturing the in-situ behaviour of saturated sand. Extensive cyclic triaxial tests were carried out on these samples (Tokimatsu and Yoshimi, 1984) and the results were superimposed with the results of De Alba et al. in Fig. 2.15. The generally good agreement between the results from both tests suggests that the limiting shear strain shown in this figure may well be of the correct order of magnitude. Based on these, Seed et al. (1984) proposed a tentative relationship between the cyclic stress ratio, corrected
Chapter 2: Undrained Behaviour of Saturated Sands

Fig. 2.15. Tentative relationship between limiting strains, cyclic stress ratio and $(N_1)_{60-cs}$ (after Seed et al., 1984).
SPT N values \((N_1)_{60-CS}\) and the limiting shear strains for natural deposits as shown Fig. 2.15. In this thesis, these limiting strains will be assumed to be the strain at which the residual strength of the liquefied soils are attained and will be called strains at residual strength.

Based on the data of Seed and Harder (1990), Byrne proposed an empirical formula to estimate the magnitude of limiting shear strains as follows:

\[
\gamma_L = 10^{(2.2-0.05(N_1)_{60-CS}}
\]  
(2.4)

2.4.5. Earthquake-induced Settlement

Although liquefaction induced lateral ground deformations are one of the most potentially damaging factors for civil structures and lifeline facilities (water pipes, sewer, telephone cables, etc.), the potential effects of post-liquefaction settlement due to dissipation of excess pore pressures should not be neglected. The magnitude of earthquake-induced settlement observed in the field range from several centimeters to as much as 380 cm (Yoshimi and Tokimatsu, 1977). Settlement of 380 cm can certainly cause serious damage to a building. However, even ground settlement in the order of tens of centimeters can be potentially damaging for lifeline facilities, particularly those on the transition zone between liquefied and non-liquefied zones. Due to these potential effects, several investigators have studied the volumetric change behaviour of sand during and after earthquake loading (e.g. Seed and Silver, 1972; Lee and Albaisa, 1974; Pyke et al., 1975; Yoshimi and Tokimatsu, 1977; Tatsuoka et al., 1984; Tokimatsu and Seed, 1987; Ishihara and Yoshimine, 1992; Thomas, 1992).

Seed and Silver (1972) presented the results of a study on the factors affecting the settlement of sand due to earthquake shaking. The study was carried out using a large shaking table. Although they tested dry sands, the final effects of cyclic loading on the
volumetric change will be similar to those of saturated sands after dissipation of excess pore pressures. They showed that the settlement increases with increasing number of cycles, with increasing base acceleration, and with decreasing relative density.

Lee and Albaisa (1974) carried out a comprehensive study on the settlement characteristics of saturated sands following earthquakes. They found that the volumetric change due to dissipation of excess pore pressure increases with increasing cyclic shear strain, excess pore pressures and decreasing relative density. The results are in agreement with those of Seed and Silver since the increase in cyclic strain and pore pressure is basically proportional to the increase in number of cycles and base accelerations. Furthermore, they also found that the reconsolidation volumetric strain increases with increases in the grain size or grading of soils. This implies that soils with high fines contents may exhibit smaller post-liquefaction settlement in comparison with that of clean sand with the same penetration resistance.

Tatsuoka et al. (1984) studied the volumetric strain due to pore pressure dissipation after a condition of a transient zero effective stress is reached. They found that the amount of the settlement is relatively insensitive to the change in effective overburden stress but it is strongly affected by relative density and maximum shear strain developed during cyclic loading. Therefore, although there is not much change in the effective stress past liquefaction state, continued cyclic loading can increase the settlement after dissipation of excess pore pressure. Similar findings have been reported by Thomas (1992).

Tokimatsu and Seed (1987) plotted the data of Tatsuoka et al. together with data obtained by other investigators in terms of post-liquefaction volumetric strain as a function of relative density and cyclic shear strain (Fig. 2.16). The figure clearly shows the strong influence of both relative density and cyclic shear strain on the volumetric
strain after liquefaction. However, the figure also shows that for a given relative density, the increase in volumetric strain associated with the change in cyclic shear strain from 10 to 15 percent is much smaller than that due to the increase in strain from 5 to 10 percent or from 2 to 5 percent. The data suggest that for a given relative density, there may exist a maximum cyclic shear strain (upper threshold of cyclic shear strain) above which the change in cyclic shear strain does not affect the post-liquefaction volumetric strain which occurs after dissipation of excess pore pressures.

Fig. 2.16. Proposed relationship between volumetric strain due excess pore pressure dissipation and \((N_1)_{60}\) (Tokimatsu and Seed, 1986).
Recent results published by Ishihara and Yoshimine (1992) in fact support the concept of upper threshold of cyclic strain. Their combined data for different relative densities are shown in Fig. 2.17 in terms of post-liquefaction volumetric strain as a function of maximum shear strain. This figure clearly shows that the volumetric strain increases with increasing shear strain until maximum shear strain of about 8 percent is reached.

Fig. 2.17. Relationship between volumetric strain due to pore pressure dissipation and maximum amplitude cyclic shear strain for different relative densities (Ishihara and Yoshimine, 1992).
Above this value, however, the increase in maximum shear strain does not affect the amount of volumetric strain. It is also interesting to note that this upper threshold value of shear strain is the same for all samples regardless of relative density. Thus, while there exists a lower threshold cyclic strain below which no pore pressure can be generated during cyclic loading (Dobry et al., 1981), there also exists an upper threshold cyclic strain above which the volumetric strains after dissipation of excess pore pressures are independent of maximum cyclic strain.

Fig. 2.17 also suggests that the volumetric strains due to dissipation of excess pore pressures after the liquefaction depend on the axial strains developed during cyclic loading. If the criterion for liquefaction in the laboratory is the development of strains in the samples during cyclic loading tests, as generally used in practice, this post-liquefaction volumetric strain will therefore depend on the factor of safety against liquefaction. Ishihara and Yoshimine (1992) have prepared a chart to approximate the volumetric strains of the liquefied soils if both the relative density and the factor of safety against liquefaction are known, as shown in Fig. 2.18. The data presented in this chart were obtained from simple shear tests on clean sand. Consequently, correction for fines content, such as proposed by Seed and Harder (1990), should be applied before this chart can be used reliably for silty sand. Direct use of this chart for silty sand will tend to overestimate the volumetric strains of the soils.

2.5. Summary

The pre- and post-liquefaction behaviour of saturated sands have been reviewed. From the deformation analysis point of view, the deformation of soil structures due to earthquakes can be divided into two categories: 1) deformations resulted directly from the earthquake (undrained condition), and 2) deformations due to excess pore pressure
Chapter 2: Undrained Behaviour of Saturated Sands

Fig. 2.18. Relationship between post-liquefaction volumetric strains and factor of safety against liquefaction (Ishihara and Yoshimine, 1992)
dissipation after the earthquake (drained condition). Category 1 can further be divided into deformations that occur before liquefaction and those that occur after liquefaction. It has been shown in the foregoing discussion that the deformation before liquefaction is generally much smaller than that after liquefaction. Thus the deformation before earthquake can be neglected without introducing significant errors into the final results. Deformations in category 2 were found to be mainly a function of relative densities, fines contents and maximum shear strains due to earthquake shaking.

The complexity of modeling soil response during and after earthquakes can be avoided if the essential factors that affect the soil deformations can be captured. These factors are: the post-liquefaction stress strain response and the volumetric strain after dissipation of excess pore pressures. By capturing these governing factors, a simple yet reliable alternative method for predicting liquefaction-induced deformations of earth structures can be developed. With this concept in mind, a new simplified method will be developed and details of the method will be discussed in Chapter 4 and 5.
CHAPTER 3: CURRENT METHODS FOR PREDICTING LIQUEFACTION INDUCED DEFORMATIONS OF EARTH STRUCTURES

3.1. Introduction

Various methods for predicting seismic deformation of earth structures such as dams, embankments, river levees and natural slopes have been developed. These methods include the simplified one-dimensional (1-D) Newmark's method and its modified versions by Sarma (1975) and Makdisi and Seed (1978), the simplified two-dimensional (2-D) finite element approach proposed by Lee (1974), the more involved Seed's dynamic stress path analysis (Seed et al., 1969; 1973; Serff et al., 1976; Seed, 1979), the sophisticated rigorous effective stress dynamic analysis (e.g. Prevost, 1981; Zienkiewicz et al., 1981; Finn, 1985) and the empirical methods (Hamada et al., 1987; Bartlett and Youd, 1992).

These methods will be briefly reviewed and their applicability for estimating liquefaction induced ground deformation will be discussed. Newmark's method, as one of the simplest and most convenient methods to predict earthquake induced displacement, will be reviewed first, followed by the extended version of Newmark's method proposed by Makdisi and Seed. These are followed by the review of the two-dimensional analysis using finite element approach proposed by Lee(1974). Then, the more complicated dynamic effective stress path method proposed by Seed. A review of the dynamic
CHAPTER 3: Current Methods...

effective stress analysis using the rigorous stress-strain model follows. Finally, the available empirical methods will be discussed.

3.2. One-dimensional Simplified Method

The stability of earth-structures has traditionally been assessed based on the factor of safety of a potential sliding mass using a pseudo-static limit equilibrium analysis. The term factor of safety is usually defined as the ratio of shear strength of soil to the driving shear stress acting at the points on the potential sliding surface. The assumptions inherent in this approach (e.g. Bishop, 1955; Lowe and Karafiath, 1959; Morgenstern and Price, 1965) are that the factor of safety at every point on the sliding surface is the same. A computed factor of safety less than unity implies that the soil-structures are not stable since the sliding mass will accelerate and large displacement will occur. Although this may be true for static case where the loads act continuously on the soil mass, a factor of safety less than unity may be still acceptable for earthquake loading where the loads act for only a finite duration of time. Newmark was the first to advance a concept that the stability of an embankment during an earthquake should be assessed on the basis of the deformation produced instead of the traditional pseudo-static factor of safety (Newmark, 1965).

3.2.1. Newmark's Method

Newmark proposed a very simple method for evaluating the potential deformation of earth-structures due to earthquake shaking. Newmark modeled a potential sliding block of the dam (Fig. 3.1.a) as a rigid plastic single degree of freedom system which can be viewed as a rigid mass resting on an inclined plane as shown in Fig. 3.1.b.
Newmark (1965) assumed that the soil behaves in a rigid-perfectly-plastic manner in which the movement will only occur when the driving forces due to the earthquake base acceleration are sufficient to overcome the yield resistance of the block. The yield resistance of the block is the force at which the movement is initiated and is equal to NW where W is the weight of the block and N is the yield acceleration in fraction of gravity units. The yield resistance of soils for the uphill movement is usually much higher than that for downhill movement (Fig. 3.1.c).

Newmark (1965) further assumed that the direction of the earthquake force, m.g.A(t), the resisting force, m.g.N, and the relative motion of the mass, D, are to be co-linear (Fig. 3.1.b). For a constant acceleration pulse of magnitude A.g which acts for a duration \( t_0 \) (Fig. 3.1.d), the equation of motion is given by:

\[
\ddot{x} = m \cdot g \cdot A - m \cdot g \cdot N \quad (3.1)
\]

where \( \ddot{x} \) = relative acceleration of the mass,

\( m \) = mass of the sliding block,

\( g \) = gravity acceleration,

\( A \) = acceleration pulse in fraction of g units, and

\( N \) = yield acceleration of the block.

The velocity due to the acceleration \( A \cdot g \) at \( t = t_0 \) is equal to:

\[
\dot{x} = V = A \cdot g \cdot t_0 \quad \text{or,}
\]

\[
t_0 = \frac{V}{A \cdot g} \quad (3.2)
\]

At this time (\( t = t_0 \)), the mass has a net velocity which is equal to \( (A-N) \cdot g \cdot t_0 \). However, since the resisting force is constant, the velocity of the mass will be zero at \( t = t_1 \) (Fig. 3.1.e). Thus,

\[
V = A \cdot g \cdot t_0 - N \cdot g \cdot t_1 = 0 \quad (3.3)
\]

Substituting Eq. 3.2 and solving Eq. 3.3, one obtains:
The maximum displacement of the mass relative to the ground can be determined by computing the shaded area in Fig. 3.1.e, as follows:

\[ D = \text{Area } \overline{OCA} - \text{Area } \overline{OBA} \]
\[ D = \frac{1}{2} \cdot V \cdot t_1 - \frac{1}{2} \cdot V \cdot t_0 \]  
(3.5)

Substituting Eqs. 3.2. and 3.4. into Eq. 3.5, yields:

\[ D = \frac{V^2}{2 \cdot g \cdot N} \left\{1 - \frac{N}{A}\right\} \]  
(3.6)

In general form, this equation can be presented as:

\[ D = \frac{V^2}{2 \cdot g \cdot N} \cdot C \]  
(3.7)

where \( C \) is the number of effective pulse of a given earthquake which is dependent on earthquake acceleration time histories and yield resistance of soils. Eq. 3.7 indicates that the maximum displacement of a block due to earthquake loading is proportional to the square of the maximum ground velocity and the number of effective pulses.

For a given earthquake acceleration time history, the steps involved in performing Newmark's analysis can be summarized as follow (Fig. 3.2):

a. Determine the yield acceleration of a potential sliding block by means of pseudo-static limit equilibrium analysis. The yield acceleration is the one that gives factor of safety equal to unity for the potential sliding surface.

b. Obtain the relative velocity of the sliding block by integrating the prescribed base acceleration, \( a(t) \), minus yield acceleration at the time interval where \( a(t) \) exceeds the yield acceleration. In this case the yield acceleration used will be downstream yield acceleration (\( N_d \)) since this value is usually much lower than the uphill yield acceleration (\( N_u \)).
Fig. 3.1. Newmark sliding block model (after Newmark, 1965).
Fig. 3.2. Schematic illustration of computing displacements using Newmark's method.
c. Determine the displacement, $d(t)$, by integrating the relative velocity of the block at the time interval when $v(t)$ is greater than zero.

Newmark also proposed simple equations based on the computed displacement of a sliding block due to strong motion records of four West Coast US earthquakes (Newmark, 1965). The earthquake records were normalized to a maximum acceleration of 0.5 g and maximum ground velocity of 30 in/s (75 cm/s). Two cases were considered: symmetrical resistance where the soil resistance is similar in both directions and unsymmetrical resistance where the resistance in one direction is much higher than the other. The computed maximum displacements were plotted as function of the ratio of $N/A$ (Fig. 3.3). The results show that Eq. 3.6 yields a good estimate for symmetrical cases with $N/A$ ratio larger than 0.1. For unsymmetrical cases, the following relationships were found:

\[
N/A > 0.5 \quad D = \frac{V^2}{2 \cdot g \cdot N} \cdot (1 - \frac{N}{A}) \cdot \frac{A}{N} \quad (3.8)
\]

\[
0.13 < N/A < 0.5 \quad D = \frac{V^2}{2 \cdot g \cdot N} \cdot \frac{A}{N} \quad (3.9)
\]

\[
N/A < 0.13 \quad D = \frac{6 \cdot V^2}{2 \cdot g \cdot N} \quad (3.10)
\]

This approach has proven to be very useful in analyzing earthquake induced displacements of earth structures, provided the yield resistance of soils does not change significantly with resulting displacement or with pore pressure build up (Goodman and Seed, 1966; Elgamal et al., 1990). As shown by Goodman and Seed (1966), some allowance for variation of the yield acceleration with displacement must be provided when Newmark's approach is applied to compute the movement of an embankment of dry dense sand. Otherwise, the method will underestimate the resulting displacements. This is
Fig. 3.3. Relationship between displacements and acceleration ratio, N/A. a). Symmetrical resistance; b). Unsymmetrical resistance (Newmark, 1965).
due to the fact that dry dense sand exhibits soil softening behaviour during dry or drained monotonic loading.

Seed (1966) considered that the method of direct calculation of earthquake induced displacement would apply only for structures made of unsaturated granular soils where no pore pressure change will occur due to earthquake motions. Since the yield acceleration at any instant of time depends on the effective stress and hence on the pore pressure developed at that time, Seed suggested that it would be very difficult to determine the yield acceleration of saturated materials where the pore pressure is changing during dynamic loading. The fact that Newmark's approach does not consider the variation of yield acceleration due to pore pressure rise led Seed to the conclusion that Newmark's method can not be used to predict the earthquake induced displacement of saturated granular materials.

One of the great advantages of using Newmark's method is its simplicity. However, the method is only suitable for soils exhibiting rigid plastic behaviour and considerable judgement must be exercised when applying this method for estimating the potential liquefaction induced deformation of the ground. In addition, the method only gives one displacement value in the direction of the sliding mass and is not capable of giving an overall deformation pattern of earth structures. Therefore, while this method of analysis is still a very useful approach for predicting displacement of earth structures due to earthquake, it generally does not give satisfactory results for predicting liquefaction induced ground deformation.

3.2.2. Makdisi-Seed Simplified Method.

Newmark's method requires one to first determine the peak ground acceleration, the yield acceleration acting along the base of the potential sliding block, and the velocity
of the moving mass. Difficulties may arise in selecting the representative effective peak acceleration of a potential sliding mass. In the effort to solve this problem, Seed and Martin (1966) and Ambraseys and Sarma (1967) used the shear beam analysis to study the dynamic behaviour of dams under earthquake loading and presented a rational way to compute the seismic acceleration on a potential sliding mass in earth dams. They found that the effective peak acceleration acting on the potential sliding mass decreases with the increased depth of failure surface within the embankment. Based on this study, Makdisi and Seed (1978) modified Newmark's method by allowing for the variations in accelerations throughout the embankment. Moreover, they assume that the soil behaves in a non-linear elastic manner during the calculation of accelerations. However, when they compute the displacements, they use the basic assumptions of Newmark's method, i.e. the failure develops at a well defined surface and a rigid plastic behaviour of soil.

In general, the procedure developed by Makdisi and Seed involves the following steps:

a. Determine the yield acceleration of the earth-structures by means of pseudo-static limit equilibrium analysis.
b. Determine earthquake induced stresses for every element in the earth-structures using dynamic response analysis.
c. Based on the results of step b, determine the time history of average accelerations of various elements on several potential sliding masses.
d. Determine the earthquake induced displacement for a given potential sliding mass using a Newmark-type analysis previously described in Section 3.2.1.

By strictly following each step described above, one will find that this simplified method is actually quite time consuming. Makdisi and Seed (1978) have established charts to ease the process of analysis. By knowing the yield acceleration of a potential
sliding mass, herein defined by $k_y$, the first natural period of the embankment, $T_0$, and the earthquake magnitude, $M$, one can use the charts shown in Figs. 3.4.a and 3.4.b or 3.4.c to estimate the earthquake induced deformation of an embankment. In this method, $u$ is the displacement, $y$ is the depth of potential sliding surface, $h$ is the embankment height.

It should also be noted that the acceleration time histories of the elements along a potential sliding surface were computed using a total stress finite element analysis QUAD4 (Idriss et al., 1973) which uses an equivalent linear technique and small strain theory. For low values of yield acceleration, i.e. $k_y/k_{	ext{max}} < 0.1$, the calculated acceleration time histories do not represent the real field behaviour and the resulting displacements may not be realistic. Furthermore, due to the total stress assumptions, the charts may not be applicable for soils that develop significant pore pressure during earthquakes. Finn et al. (1978) found that when cyclic pore water pressures exceed about 30 percent of the initial vertical effective stress, total stress dynamic analysis would tend to overestimate dynamic forces and underestimate displacements.

Sarma (1975) extended the original Newmark's sliding block concept to take into account the effects of pore pressure rise on the soil strength, the yield acceleration and the subsequent displacement during earthquakes. These effects are introduced in terms of Skempton pore pressure coefficients. It was found that the yield accelerations of soil structures are quite sensitive to the variation of these coefficients. Thus, since the computed displacements are very sensitive to the yield accelerations, small changes in the pore pressure coefficients may cause large variations in the computed displacements. Moreover, the pore pressure coefficients are dependent on several factors including stress path, number of cycles and relative density of soil. Accurate determination of these coefficients becomes very difficult for random earthquake loading.
Fig. 3.4. Makdisi-Seed method. (a). Variation of effective peak acceleration, $k_{\text{max}}$, with depth of base of potential sliding mass. (b) and (c). Variation of yield acceleration with normalized permanent displacement (Makdisi and Seed, 1978).
The methods described above suffer the same limitation as Newmark's in the assumption used that a distinct failure surface occurs in earth structures. Thus, they are not capable of giving an overall deformation pattern for the earth-structures and not applicable for predicting liquefaction induced deformation of earth-structures.

3.3. Two-dimensional Simplified Finite Element Approach

The simplified methods proposed by Newmark, Ambraseys-Sarma and Makdisi-Seed and others, assume that the soil mass slides along a well-defined surface. Yet, in most cases in the field, soil deformations result from the accumulation of strain increments throughout the body of the soil structures. Consequently, seismic deformation in these structures will be more realistically estimated using a general deformation field rather than one restricted to a single surface. Simplified methods utilizing a finite element approach that are capable of giving the overall deformation pattern of earth structures due to earthquake loading have been proposed by several researchers including Seed et al., (1969, 1973), Lee (1974), Serff et al., (1976), Taniguchi et al., (1983), and Kuwano et al., (1991). Some of the available methods will be reviewed in the following section starting with the simplest one.

3.3.1. Modified Modulus Approach

Lee (1974) proposed an alternative method for assessing the seismic deformation of earth structures. He simulates the effect of an earthquake on the dam by a reduction in the modulus of the soil comprising the dam. The key assumptions in this method are that: the static shear stress in the soil element of the dam does not change due to earthquake loading, the post-earthquake stress-strain behaviour of soil can be represented by a hyperbola similar to that of the pre-earthquake stress-strain curve and, finally, the initial
moduli of the soil before and after the earthquake are directly proportional to the strain developed in the soil samples in the laboratory. Fig. 3.5 shows the principles of the analysis for linear and nonlinear stress-strain behaviour of soils.

The analysis basically involves the following steps:

a. Calculate the initial static shear stress and displacement of each soil element in the dam using one step gravity turn-on analysis. The deformations computed in this step are merely reference deformations and are not representative of any actual pre-earthquake deformations.

b. Compute the final displacement of each soil element in the dam using a reduced soil modulus.

c. Subtract the final displacement from the initial displacement for each element. The resulting displacements equal the displacements of the dam due to earthquakes.

Similar procedures have also been proposed by Yasuda et al. (1991) and Kuwano et al. (1992).

While the approach is quite simple and capable of estimating the overall deformation pattern of the dam, the approach tends to significantly underestimate the computed displacements as has been shown by Serff et al., (1976) and Purssell (1985). This is mostly due to the fact that this approach does not satisfactorily model the post-liquefaction stress-strain curve that shows a marked reduction in soil stiffness. As shown by Purssell (1985), stiffness reduction of about 1000 times was required to predict the deformations observed in the field. The model also does not take into account of the inertia forces of the earth structures during earthquake shaking. Furthermore, the
Strain

\[ E_i = \frac{\tau_i}{\varepsilon_i} \quad \text{Modulus before earthquake} \]

\[ E_f = \frac{\tau_f}{(E_i + \varepsilon_f)} \quad \text{Modulus after earthquake} \]

Fig. 3.5. Modified modulus approach. (a). Linear stress-strain curve; (b). Non-linear Stress-strain curve.
approach does not consider additional deformations due to post-liquefaction settlement which may significantly contribute to the total deformations of earth structures.

### 3.3.2. Dynamic Stress Path Method

An alternative method for estimating seismic deformation of earth structures that takes account of the inertia effects due to earthquake loading has been presented by Serff et al. (1976). The method is basically a result of continuous refinement of the method originally proposed by Seed et al. (1969) and Seed et al. (1973) and is usually called a dynamic stress path method.

This method has been used to study the performance of several dams whose performance during earthquakes is known (Seed, 1979). The predicted behaviour of these dams is in general agreement with the observations (Seed, 1979).

There are three key assumptions in this procedure (see Fig. 3.6.a). The first assumption is that the stress-strain behaviour of soils for pre- and post-earthquake conditions can be represented by hyperbolic curves. The second one is that the pre-earthquake stress-strain curve uses soil parameters obtained from consolidated drained tests whereas the post-earthquake stress-strain curve uses soil parameters from consolidated undrained tests. The last assumption is that the displacements due the differences between the undrained and drained curves can be neglected.

In general, the dynamic stress path method proposed by Seed and his co-workers involves the following steps:

a. Compute the initial static stress of the dam cross section using a finite element method such as FEADAM (Duncan et al., 1984).

b. Using the selected design earthquake and the characteristics of motions developed for the rock underlying the dam, compute the dynamic response
Fig. 3.6. Dynamic stress path method. (a). Determination of equivalent stress; (b). Determination of nodal point forces (Serff. et al., 1976).
of the dam and the dynamic stresses induced by the earthquake motion(s) in representative elements of the dam. The finite element programs to perform such analysis are available, e.g. QUAD4 (Idriss et al., 1973), FLUSH (Lysmer et al., 1975).

c. Subject representative samples of the dam materials to the combined effects of the initial static stresses and cyclic shear stresses to determine their effects in term of cyclic pore pressure and strain development. Perform a sufficient number of laboratory tests to allow evaluations for all elements comprising the dam.

d. Based on the element strain potentials obtained in step c, determine the overall deformation of the dam cross-section.

In this method, the effects of earthquake loading are represented by a set of equivalent nodal forces that are derived from the element strain potentials in conjunction with the non-linear stress-strain behaviour of soils. The strain potential of each element is obtained from laboratory tests by subjecting representative samples to the appropriate static and dynamic stresses. These forces are then applied to the nodes of the soil elements to compute the resulting displacements.

Serff et al. (1976) presented a method for estimating the equivalent nodal forces. They assumed that the distribution of shear stress over the area of a soil element is constant and uniform. They also assumed that the maximum induced dynamic shear stresses ($\Delta \tau_{\text{max}}$) act along a horizontal plane. Thus, since $\Delta \tau_{\text{max}} = 1/2 \sigma_d$, the shear stress can be determined once the deviator stress ($\sigma_d$) due to a strain potential in a soil element is known (Fig. 3.6.a). The nodal point forces can be estimated by multiplying this shear stress with the width and the height of a soil element (Fig. 3.6.b). These forces are
applied in the direction of initial static shear stress since the soil tends to move in this direction during the earthquake.

Despite the fact that the dynamic stress path method includes the effects of kinetic energy due to earthquakes, the method still considerably underestimated the displacements when it was applied to estimate the deformation of the Upper San Fernando Dam during the 1971 earthquake (Serff et al., 1976). The resulting pattern of deformation was in good agreement with the observation. However, without simulation of crack development at the crest, the predicted magnitude of displacements were only about 20 to 25 percent of the observed values. As mentioned previously, the method assumes that the post-earthquake stress-strain curve of the liquefied soils can be represented by pre-cyclic static undrained loading curve. Results of laboratory tests show that the initial shear moduli of granular soils during post-cyclic/post-liquefaction monotonic loading are significantly lower than those during pre-cyclic monotonic loading. This shear modulus reduction also depends on the excess pore pressure ratio ($\Delta u/\sigma_{vo}'$) at the end of cyclic loading (e.g. Thomas, 1992, Salgado and Pillai, 1993, Vaid and Thomas, 1994, Yasuda et al., 1994). Thus, modeling the post-liquefaction stress-strain curve with pre-cyclic static undrained curve is not realistic since the model neglects the significant stiffness reduction due to pore pressure rise. This causes gross underestimation of the predicted displacements. Moreover, similar to modified modulus approach, the approach does not take account for the deformation due to excess pore water dissipation of the liquefied soils.

Finally, the steps involved in this approach are quite tedious and considerable judgment is required in each step. As mentioned by Seed himself (1979), if the procedures were carried out without proper judgment, the results could be misleading. In
addition, the large quantity of work associated with the cost of the analysis, makes this approach applicable to large projects only.

**3.4. Non-linear Effective Stress Dynamic Analysis**

Predicting liquefaction induced deformations of soil structures is a very complex problem. To accurately predict the liquefaction induced deformation of earth-structures, the behaviour of soils under cyclic loading conditions must be correctly modeled and the appropriate earthquake time histories applied. The best approach for solving this problem is using non-linear effective stress dynamic analyses in which the actual soil behaviour during earthquake loading is simulated by means of constitutive relation of soils derived from experimental observations. Although it seems that no one model can be applied to simulate all types of soils, many constitutive soil relations work quite well in predicting the behaviour of soils during cyclic loading (e.g. Zienkiewicz et al., 1981; Towhata and Ishihara, 1985; Matsuoka and Sakakibara, 1987; Byrne and McIntyre, 1994).

Using the appropriate constitutive relations of soils in conjunction with the use of finite element techniques, a coupled effective stress dynamic approach has been proposed (e.g. Prevost, 1981, Zienkiewics et al., 1981, Iai et al., 1992). In this approach, the soil model used considers a full coupling between shear stress and volumetric strain. For saturated fine grained materials, the shear-induced volumetric strain causes an increase in pore pressure and a reduction in effective stress. Although the total pore pressure change during cyclic loading is due to the combination of the change in the mean normal stress ($\Delta\sigma'_m$) and the shear-induced pore pressure, only the latter causes a decrease in effective stress of the system. By considering the change in effective stress of the earth structures, the displacements are computed by solving the equation of motion by a direct step-by-step integration method (Clough and Penzien, 1975). Although this approach is clearly
more fundamental than the other previous ones, the complexity, lengthy computational
time and associated high costs usually hinder its use for practical engineering purposes.

More practical is the somewhat 'loose' shear-volume coupling used in the effective
stress dynamic analysis developed by Finn (1985), Finn et al. (1986). The approach is an
extension of a non-linear dynamic effective stress analysis developed for a level ground
condition by Finn et al. (1976). The stress-strain model used in this method treats the soil
as a non-linear hysteretic material exhibiting Masing behaviour during unloading and
reloading. The induced pore water pressure due to cyclic loading is computed based on
Martin-Finn-Seed (MSF) pore pressure model (Martin et al., 1975).

The MSF pore pressure model proposed by Martin et al. (1975) was based on the
results of strain-controlled cyclic simple shear tests on medium dense sand (relative
density of 45 percent). The model computes the shear-induced pore pressure at the end of
each cycle using the following relationships:

\[
(\Delta \varepsilon_v)_{cycle} = C_1 (\gamma - C_2 \varepsilon_v) + \frac{C_3 \varepsilon_v^2}{\gamma + C_4 \varepsilon_v}
\]  

(3.11)

\[
(\Delta u)_{cycle} = M \Delta \varepsilon_v
\]  

(3.12)

in which,

\[(\Delta \varepsilon_v)_{cycle}\] = the increment of volumetric strain per cycle in percent,

\[\varepsilon_v\] = the accumulated volumetric strain from previous cycles in

percent,

\[\gamma\] = the amplitude of shear strain in percent for the cycles in

question,

\[C_1, C_2, C_3, C_4\] = constants for the sand in question at given relative density.

\[(\Delta u)_{cycle}\] = the pore pressure increase per cycle in kPa or psf, and

\[M\] = the constraining modulus of the soil in kPa or psf.
Because of this, the approach only computes the pore pressure at the end of every cycle of the earthquake loading, instead of at every time increment. Using the same test data, more efficient pore pressure model has also been proposed by Byrne (1990). His model computes the pore pressure rise at every half cycle and only requires 2 constants.

The approach with MSF pore pressure model has been incorporated in the series of TARA computer programs (Finn et al., 1986) and has been validated quite extensively against centrifuge test data from Cambridge University (Yogendrakumar, 1988). The method gave reasonable agreement with the observations in terms of acceleration, residual pore pressure and displacement time histories. Although the approach proposed by Finn is clearly more efficient than the other approaches that consider full shear-volume coupling of soils, the method is still quite costly for practical design purposes.

3.5. Empirical Method

Hamada et al. (1987) presented the most comprehensive compilation of field data regarding the liquefaction induced displacements in Japan during past earthquakes. Their data were mainly obtained from sites that liquefied during the 1964 Niigata and the 1983 Nihonkai-Chubu earthquakes which involved earthquake magnitudes of about M7.5. The sites were mainly gently sloping ground which consists of layers of loose clean sands.

Based on their data, Hamada et al. (1987) found that liquefaction induced displacements are a function of several factors including the ground slope and the thickness of liquefied layer. Hamada et al. proposed an empirical relationship for predicting the displacements as follow:

\[ D = 0.75 H^{1/2} \theta^{1/3} \]  
(3.13)

in which, \( D \) = displacements, m,  
\( H \) = the thickness of liquefied layer, m and
\[ \theta = \text{the ground slope, or the slope of the base of the liquefied layer} \]

\[ \text{whichever is greater, percent.} \]

Hamada et al. (1987) have shown that Eq. 3.11 predicts the measured displacements in Niigata and Noshiro cities within minus 50 percent and plus 100 percent. The considerable scatter in their results is probably due to the fact that the factors influencing the actual displacements in the field are much more complex and could not be captured by using only two parameters, i.e. ground slope and liquefied layer thickness.

Youd and Bartlett (1988) studied the validity of the Hamada et al. formula by applying it to the United States (US) case histories of lateral ground deformations. They found very poor agreement between the observed and the predicted displacements. Youd and Bartlett argued that the discrepancies may be attributed to different particle size of the US sand and different earthquake magnitudes from those of Japanese data.

As pointed out by Byrne (1990), a serious limitation of Hamada et al.'s empirical formula is that it does not take into account the effects of relative density of liquefied soils. It is generally accepted that the soil relative density strongly influences the behaviour of soil during and after liquefaction, as discussed in Chapter 2. Moreover, Byrne pointed out that the displacements in their data are in fact proportional to the thickness of liquefied layer instead of to the square root of the thickness as presented in Eq. 3.11.

Bartlett and Youd (1992) derived a more comprehensive empirical relationship based on the extensive data base compiled from Japanese and US case histories of liquefaction and lateral spreads. Their model considers several additional parameters including earthquake magnitude, distance from the earthquake source, and the properties of liquefied layers. Bartlett and Youd found that models with different parameters are
required to predict lateral ground displacement for free face and ground slope conditions. Their formula for both cases are as follow:

a. For free face conditions,

\[
\log(D_h+0.01) = -16.366 + 1.178 M - 0.927 \log R - 0.013 R + 0.657 \log W + 0.348 \log T_{15} + 4.527 \log (100-F_{15}) - 0.922 D_{50,15}
\]

\[(3.14)\]

b. For ground slope conditions,

\[
\log(D_h+0.01) = -15.787 + 1.178 M - 0.927 \log R - 0.013 R + 0.429 \log S + 0.348 \log T_{15} + 4.527 \log (100-F_{15}) - 0.922 D_{50,15}
\]

\[(3.15)\]

in which:

- \(D_h\) = Horizontal displacement, m,
- \(M\) = Earthquake magnitude (moment magnitude),
- \(R\) = Horizontal distance from the seismic energy source, km,
- \(W = 100 \times (\text{height}(H) \text{ of the free face})/\text{distance (L) from the free face})\),
- \(S\) = Ground slope, percent,
- \(T_{15}\) = Cumulative thickness in meters of saturated layers with \((N_i)_{60} < 15\),
- \(F_{15}\) = Average fines content of saturated granular layers included in \(T_{15}\), percent, and
- \(D_{50,15}\) = Average mean grain size in layers included in \(T_{15}\), mm.

While Bartlett and Youd's model is superior to the previous empirical model proposed by Hamada et al. (1987), the model is sensitive to magnitude and earthquake distance, and to the average fines contents of saturated layers \(F_{15}\). Moreover, as with any other empirical model, their model is only valid for the cases similar to case histories on which the model is based. Barlett-Youd’s model is most applicable for regions with earthquake magnitudes between 6.0 and 8.0, affecting sites underlain by continuous layers of sandy materials with \(T_{15}\) greater than 0.3 m, depth of liquefiable layer less than
15 m, $F_{15}$ less than 50 percent and $(N_1)_{50}$ less than or equal to 15. Applications of the model for cases other than those mentioned above, e.g. discontinuous liquefied layer, their model may not yield reliable results.

3.6. Summary

Various available methods for predicting liquefaction induced deformation of earth structures have been reviewed. Most of the current procedures either employ so many simplifying assumptions that they can not realistically represent the general behaviour of earth structures under seismic loading or are so complex and expensive that their use may not be practical for design purposes. Therefore, there seems to be a need for a relatively simple method capable of capturing the essence of the problem and realistically predicting the seismic deformation of earth structures in terms of both magnitude and pattern of displacements. The proposed procedure discussed in the next chapter is an attempt to fulfill such a need.
CHAPTER 4:
ONE-DIMENSIONAL METHOD USING ENERGY CONCEPT

4.1. Introduction

As discussed in the previous chapters, the key parameters to a reliable prediction of liquefaction-induced deformation of earth structures using simplified methods are an identification of the zone of liquefaction, a realistic modeling of post liquefaction stress-strain response and a consideration of earthquake inertia forces. Most of the available simplified methods either fail to model the realistic post-liquefaction stress-strain response of soil or fail to include the effects of earthquake inertia forces. Based on energy principles, Byrne (1990) extended the Newmark method from a rigid plastic to a flexible soil system that combines the effects of softened liquefied soil and inertia forces from earthquakes. Byrne (1990) has verified his method to some extent against shaking table test data and against the available empirical relationship (Hamada et al., 1987). The results are very promising. However, his method has not been validated thoroughly against observed displacements during past earthquakes.

In this chapter, Byrne's method is carefully reviewed as it will be used as a basis for the proposed two-dimensional procedure that will be discussed in Chapter 5. Moreover, studies are carried out to validate his method against case histories of liquefaction-induced lateral spreads and the results are presented. Finally, based on these studies, modifications to improve the method are proposed.
4.2. Newmark's Model Based on Energy Concept

Energy principles require that the work done by the external forces ($W_{\text{ext}}$) minus the work done by the stress field ($W_{\text{int}}$) equals the change in kinetic inertia of the system. This principle can be expressed as:

$$W_{\text{ext}} - W_{\text{int}} = \frac{1}{2} M \cdot (V_f^2 - V^2) = -\frac{1}{2} M \cdot V^2$$  \hspace{1cm} (4.1)

where $V_f$ is the final resting velocity and is equal to zero, and $V$ is the specified initial velocity. This principle will now be applied to Newmark's sliding block model to provide a direct comparison with the extended Newmark that will be discussed in Section 4.3.

As discussed in Chapter 3, the Newmark method is based on modeling a block with mass $M$ resting on an inclined plane of slope $\alpha$ as a single-degree-of-freedom rigid plastic system. Newmark's model is shown again in Fig. 4.1.a and b. The velocity of the block ($V$) comes from earthquake forces when the forces exceed the yield resistance of the potential sliding surface of the block.

![Fig. 4.1 (a). Block on an Inclined Plane and (b). Rigid Plastic Behaviour in Newmark Model.](image-url)
The external force acting on the block is the gravity driving force, \( M g \cdot \sin \alpha \), and is constant with displacement as shown in Fig. 4.1.b. The work done by the external force is the area beneath the driving force line. Newmark assumed the soil behaviour to be rigid perfectly-plastic, and consequently the soil resistance is constant with displacements as is also shown Fig. 4.1.b. The work done by the internal forces depend on the stress-strain curve of the soils and is equal to the area beneath the soil resistance line. To achieve energy balance, the external work minus the internal work must equal the change in the kinetic energy of the system. The displacements required to get this energy balance can be obtained from Eq. 4.1.

From Fig. 4.1.b, the external work of the system can be computed from \( W_{\text{ext}} = (Mg \cdot \sin \alpha) \cdot D \). The internal work is obtained from \( W_{\text{int}} = S_r \cdot L \cdot b \cdot D \), where \( S_r \) is the shear strength on the sliding surface, \( D \) is the required displacement to obtain the energy balance, and \( L \) and \( b \) are respectively the length and the width of the sliding surface. If a single velocity pulse is considered, Eq. 4.1 becomes:

\[
(M \cdot g \cdot \sin \alpha - S_r \cdot L \cdot b) \cdot D = -\frac{1}{2} \cdot M \cdot V^2
\]

or

\[
D = \frac{\frac{1}{2}MV^2}{(S_r \cdot L \cdot b - Mg \cdot \sin \alpha)}
\]

Since the yield acceleration, \( N \), for the sliding block shown in Fig. 4.1.a. is given by:

\[
\frac{(S_r \cdot L \cdot b - Mg \cdot \sin \alpha)}{Mg}
\]

Eq. 4.3 reduces to:

\[
D = \frac{V^2}{2gN}
\]

When six velocity pulses are considered, Eq. 4.4 will be identical to Newmark's formula in Eq. 3.10. for asymmetrical case with \( \frac{N}{A} \leq 0.13 \).
4.3. Extended Newmark

In contrast to the basic assumption in the Newmark method, soil will not behave in a rigid plastic manner when triggered to liquefy. As discussed in Chapter 2, the liquefied soil will lose its stiffness when the pore pressure rise causes the effective stress to drop to zero. However, upon straining, the soil will dilate causing it to strain harden and regain both stiffness and strength. By incorporating the essentials of this stress-strain response in his model, Byrne (1990) extended the Newmark method to take account of the effects of stiffness reduction in liquefied soils. Idealized pre-cyclic and post-cyclic stress strain curves are shown in Fig. 4.2.

![Stress vs Strain Diagram](image)

**Fig. 4.2.** Idealized pre- and post-liquefaction characteristics of loose sand.

The presence of an initial static bias under laboratory loading condition generally curtails the rise in pore water pressure under cyclic loading and may prevent a zero effective stress state and a consequent temporary near zero stiffness condition. Such a
test element generally behaves in a stiffer manner than one without a static bias. However, in the field a soil element with an initial static bias may lose this bias under cyclic loading, since the static bias will be transferred to stiffer soil elements that suffer less pore pressure rise. Liquefaction case history in Nerlerk berm due to cyclic ice loadings (Jeffries et al., 1988) and the results of centrifuge tests presented by Steedman (1989) showed that 100 percent pore pressure rise did occur for a soil element which had an initial static bias. This indicates that the stress point moved to the zero effective stress state and the static bias was lost.

The strains required to bring a soil element to the zero effective stress state are generally less than 1 percent. Thus triggering of liquefaction is a small strain phenomenon (Byrne, 1990). However, if the soil element is subsequently loaded monotonically, for example due to self weight of non-liquefied soils above it, large deformations may occur due to the very low stiffness at zero effective stress. As the strain increases, the soil dilates causing a drop in pore pressure and an increase in effective stress and stiffness until it reaches the residual state point. If the static stress is larger than the residual strength, flow failure will occur. If the static stress is less than the residual strength, limited deformations will occur.

The stress-strain characteristic of post-liquefaction loose saturated sand shown in Fig. 4.2 will now be incorporated into the work-energy approach allowing an extension of Newmark's concept. Point P in Fig. 4.2 is the pre-earthquake stress state of a soil element in the earth structure. Upon liquefaction, the stress state of the soil drops from its static value P to Q as shown in Fig. 4.3. This stress change occurs at very low strain as previously discussed. Its resistance then increases with strain to a residual value $S_r$. The driving force from the ground slope generally remains constant so that the system accelerates as it deforms. Since the system accelerates, it has a velocity when the strain...
reaches point R where the resistance is equal to the driving stress. Thus, the strain keeps increasing until an energy balance (the external work done by the driving force ($\tau_{st}$) is balanced by the work done by the internal stresses) is reached at point S. If during this process, the system also has a velocity from the earthquake shaking, the soil would deform further until it reaches point T.

Fig. 4.3. Work-energy principles, extended Newmark.

Comparing the rigid plastic Newmark approach with the extension to a general stress-strain relation (Figs. 4.1.b and 4.3) it may be seen that the standard Newmark method neglects the displacement from P to S. This could be a very considerable displacement since strains of 20 to 50 percent are commonly required to mobilize the residual strength, $S_r$. It should be noted that Newmark derived his equation for rigid plastic soils, and it is therefore not applicable without correction to liquefied soils that are very flexible in shear.
Newmark simulated the effects of an earthquake by applying a series of velocity pulses (V), where the number of pulses depended on the N/A ratio but could go as high as six. The magnitude of V was taken to be equal to the peak ground velocity. In carrying out analyses where liquefaction is triggered, only one pulse is considered. This is felt to be justified because the displacements that occur due to pulses prior to liquefaction will, in general, be small compared to those that occur upon liquefaction. The displacements that occur due to pulses after liquefaction will be taken into account by the introduction of a factor of safety against liquefaction, $F_L$. The factor of safety against liquefaction will affect the determination of limiting strain or strain at residual strength that controls the soil stiffness. This will be explained in more detail in Section 4.6.2.

For a single-degree-of-freedom system, the displacement that satisfies the energy balance can be calculated directly by solving Eq. 4.1. The solutions for this have been given by Byrne (1990) and this will be briefly described in the section which follows. For a multi-degree-of-freedom system, the method of determining the displacements will be described in Chapter 5.

**4.4. Byrne's One-dimensional Method**

Byrne's method for predicting liquefaction induced deformation is based on modeling an infinite slope consisting of a uniform thickness of crust and liquefied layer as a block resting on an inclined plane, as depicted in Fig. 4.4. The model consists of a mass $M$ and a spring, $K_L$, representing the strength and stiffness of liquefied layer. The liquefied layer is assumed to follow either a linear elastic plastic or a non-linear plastic soil response as shown in Fig. 4.5.

By applying the energy principles into the model, Byrne (1990) derived formulas to estimate the liquefaction-induced deformation of soil system as follows:
Fig. 4.4. Byrne's one-dimensional model. (a). Idealized infinite slope; (b). Model.

For a linear stress-strain relation shown in Fig. 4.5:

For $D \leq D_L$:

$$D = D_{st} + (D_{st}^2 + \frac{MV^2}{K_L})^{1/2}$$  \hspace{1cm} (4.9)
Chapter 4: *One-dimensional Method Using Energy Concept*

For $D > D_L$

$$D = D_{st} + \frac{1}{2} (D_L - D_{st}) + \frac{(D_{st}^2 + \frac{MV^2}{K_L})}{(D_L - D_{st})}$$  \hspace{1cm} (4.10)

For a non-linear stress-strain relation:

For $D \leq D_L$

$$D = \frac{1}{\tau_{st}} \left( \frac{D_L^3K_L}{3D_L} - \frac{1}{2} MV^2 \right)$$  \hspace{1cm} (4.11)

For $D \geq D_L$

$$D = \frac{\left( \frac{1}{2} MV^2 - \frac{1}{3} K_L D_L^2 + S_r D_L \right)}{(S_r - \tau_{st})}$$  \hspace{1cm} (4.12)

in which,

$D = $ liquefaction-induced displacement,
Chapter 4: *One-dimensional Method Using Energy Concept*

\[ D_s = \text{static displacement which corresponds to point R in Fig. 4.3}, \]
\[ D_L = \text{displacement above which the soil resistance is equal to the residual strength and constant with increasing displacement, as depicted in Fig. 4.5}, \]
\[ K_L = \text{stiffness of the liquefied soil and is equal to } \tau_{st}/\gamma_L, \]
\[ M = \text{soil mass}, \]
\[ V = \text{velocity of the soil mass}, \]
\[ \tau_{st} = \text{initial static shear stress}, \] and
\[ \gamma_L = \text{limiting shear strain or strain at residual strength}. \]

Similar to Newmark's approach, Byrne (1990) extended this model for 2-D slope as a single-degree-of-freedom system. He introduced a factor of safety against stability to determine the static driving stress of the soil mass using:

\[ \tau_{st} = \frac{S_r}{F_{st}} \]  \hspace{1cm} (4.13)

where \( S_r \) is the appropriate average strength along the failure surface after liquefaction is triggered, and \( F_{st} \) is a factor of safety against stability of the initial geometry of the slopes determined from limit equilibrium analysis. However, the mode of deformation in a finite slope is generally different from that in an infinite slope and the static driving stress acting on the sliding mass may change significantly with increased deformation. Consequently, the predicted displacements using this method may not be as reliable as those computed for infinite slopes.

**4.5. Determination of Required Parameters**

The key parameters in the extended Newmark method are the zone of liquefaction, the residual strength of the soil, \( S_r \), the limiting strain or the strain at residual strength, \( \gamma_L \), the peak ground acceleration, \( A \), and the peak ground velocity of the mass, \( V \). The residual strength and the strain at residual strength are needed to define the stress-strain curve of liquefied soils. The peak ground acceleration is required to evaluate the
zone of liquefaction and the peak ground velocity is needed for estimating the kinetic energy of the earthquake. Methods to evaluate the zone of liquefaction and other parameters are described in the following section.

4.5.1. Zone of Liquefaction

The zone of liquefaction is determined by using the method proposed by Seed et al. (1985) which is commonly used in practice. This method involves the determination of the cyclic shear stress ratio (CSR) due to a given earthquake and the cyclic resistance ratio (CRR) of the soil. The factor of safety against triggering of liquefaction, $F_L$, is determined from:

$$F_L = \frac{\text{CRR}}{\text{CSR}} \quad (4.14)$$

The CSR is evaluated using a total stress equivalent dynamic analysis (e.g. Schnabel et al., 1972; Lysmer et al., 1975) and is equal to $0.65 \left( \frac{(\tau_{dy})_{\max}}{\sigma'_{vo}} \right)$, where $(\tau_{dy})_{\max}$ is the maximum shear stress computed from a dynamic analysis and 0.65 is a factor to convert from a random loading to an equivalent uniform cyclic loading.

Alternatively, the CSR is computed from the simplified liquefaction analysis proposed by Seed and Idriss (1971):

$$\text{CSR} = 0.65 \cdot \frac{a \cdot \sigma_{vo} \cdot r_d}{g \cdot \sigma'_{vo}} \quad (4.14.a)$$

in which, $\sigma_{vo}$ and $\sigma'_{vo}$ are respectively total and effective stress of a soil element at a depth $d$, $a$ is peak ground acceleration of the earthquake and $r_d$ is a reduction factor due to soil depth which decreases from a value of 1 at the ground surface to a value of 0.90 at a depth of about 10 m.

The CRR of soil is best determined from direct testing of undisturbed samples. However, because of the difficulty and the expense of obtaining high quality undisturbed samples, CRR is commonly evaluated from normalized SPT blow counts, $(N_t)_{60}$ together
with the data from the field experience. The procedures for determining CRR from \((N_1)_{60}\) has been presented by Seed et al. (1985) and Seed and Harder (1990).

### 4.5.2. Residual Strength

As discussed in Chapter 2, the residual strength of soils can be determined directly by carrying out laboratory tests on the samples from the site or indirectly by correlating the equivalent clean sand SPT blow counts, \((N_1)_{60-cs}\), with the residual strength using the relationship presented by Seed and Harder (1990), as shown in Fig. 2.15. Alternatively, the empirical formula (Eq. 2.1 and 2.2) proposed by Byrne (1990) can be used to determine the average residual strength for a given \((N_1)_{60-cs}\). These formula are rewritten for convenience below:

\[
S_r = 0.0284 \cdot Pa \cdot e^{0.173(N_1)_{60-cs}} \quad (2.1)
\]

with a lower bound value given by:

\[
S_r = 0.087 \sigma'_{vo} \quad (2.2)
\]

It should be noted that the above empirical equations were based on the compilation of residual strength of liquefied soils with \((N_1)_{60-cs}\) values less than about 15. The use of these empirical formulas for liquefied soils with \((N_1)_{60-cs}\) value greater than 15 should be carried out cautiously.

### 4.5.3. Limiting Strain or Strain at Residual Strength

The strain at residual strength can be determined directly from the results of laboratory tests or by correlation between \((N_1)_{60-cs}\) and the limiting strain proposed by Seed et al. (1984, 1985), as shown in Fig. 2.14. For small \((N_1)_{60-cs}\) (less than 10), however, the values of residual strength and limiting strains for each \((N_1)_{60-cs}\) vary quite significantly. Alternatively, the strain at residual strength, \(\gamma_{1s}\), for a given \((N_1)_{60-cs}\) can be
obtained from the empirical formula given in Eq. 2.4 proposed by Byrne(1990) and rewritten below:

\[ y_L = 10^{(2.2 - 0.05(N_1)_{60-cs})} \]  

(2.4)

4.5.4. Peak Ground Acceleration and Velocity

In current practice, there are basically two ways of obtaining the peak ground acceleration and peak ground velocity of the earthquake at a given site. The first method is by using the local building codes and the second is by conducting a site specific seismic hazard evaluation (Idriss, 1985). Either method can be used to determine the peak ground acceleration and velocity of the mass from the earthquake.

Local building codes usually contain a seismic zone map that include minimum required seismic design parameters. The map of seismic zone shows contours of expected acceleration and velocity at a given area. These maps are based on a statistical analysis of the earthquakes that have been experienced in that area. The statistical analysis is either based on the method of extreme value (quasi-deterministic) or probabilistic analyses such as proposed by Cornell (1968). The same statistical procedure is also used in a site specific seismic hazard evaluation.

4.6. Validations

4.6.1. Byrne's Method

To be meaningful, a method for predicting liquefaction induced ground deformation of earth structures must be validated against the case histories of ground deformations during past earthquakes. Bartlett (1992) compiled case histories of ground deformations due to earthquakes including those which occurred in North America and Japan. The measured displacements were obtained from various boundary conditions
including those from the sites where the ground displacements were restrained by surrounding non-liquefied zones or by structures such as bridges (Hamada et al., 1992a, 1992b; O'Rourke et al., 1992; Bartlett and Youd, 1992). The displacements used here are the average values of those reported in each case history. The method proposed by Byrne was used to predict lateral ground displacements at these sites using pertinent information available in the literature, and the results are presented in Appendix A. Comparisons with the measured values are shown in Figs. 4.6 - 4.10.

Fig. 4.6 presents the comparison between predicted and measured displacements for North American case histories using a linear stress-strain assumption. For clarity, the predicted displacements using a non-linear stress-strain assumption are presented separately. As can be seen in this figure, the method generally predicts the displacements in the range of 50 to 200 percent of the measured values. The method underestimates the displacements at Portage Creek due to the fact that the SPT N values at these sites were exceptionally high because of the effects of gravel content. These SPT values are questionable. On the other hand, the method overestimates the ground displacements at Mission and Ship Creeks. This is mainly due to the fact that the displacements at Mission Creek were restrained by the non-liquefied boundaries (O'Rourke et al., 1992) whereas the displacements at Ship Creek were restrained by the bridge truss structures (Youd and Bartlett, 1992). It is interesting to note that the empirical formula of Youd and Bartlett (1992) also underestimates the displacements at gravelly sites in Alaska due to the 1964 earthquake, and overestimates the ground movements at Mission Creek and South of Market Area during the 1906 San Francisco earthquake.

Fig. 4.7 presents the same comparison using a non-linear stress-strain assumption. The figure shows that the method generally overestimates the displacements by a factor
Fig. 4.6. Predicted and measured displacements using a linear stress-strain assumption.
Fig. 4.7. Predicted and measured displacements using a non-linear stress-strain assumption.
of about two. The results suggest that the linear stress-strain assumption seems to be more appropriate for North American case histories. This linear stress-strain assumption is in agreement with the laboratory data of Duncan dam foundation in British Columbia presented by Salgado and Pillai (1993) (see Fig. 2.13).

When all displacements affected by the boundaries are removed from the plot, as can be seen in Fig. 4.8, the predicted displacements using the linear stress-strain curve show a better agreement with the measured values than does the non-linear curve. Except for several points with measured displacements greater than 2 meters, all predicted displacements lie very close to the 100 percent line. This indicates that the method generally predicts displacements in good agreement with the measured values.

When the method is applied to predict the ground deformations in Noshiro city due to the 1983 Nihonkai Chubu earthquake in Japan, however, both linear and non-linear assumptions seem to underestimate the measured displacements, as can be seen in Fig. 4.9. The SPT values measured at these sites are generally higher than those measured in the United States case histories (see Appendix A). Two of the possible reasons for these discrepancies are: first, the case histories in Japan involved clean fine sand that may behave differently from most of the liquefied soils in the United States case histories (Youd and Bartlett, 1988), and second, the residual strength used in the above predictions was based on the average values of residual strength from Seed and Harder's chart that were obtained mostly from case histories in North America. It is possible that the residual strength of liquefied sand in the Japanese case histories lies below the values used in the analyses. If smaller values of residual strengths are used, Byrne's method will predict larger displacements for these sites than those computed using average values of residual strength.
Chapter 4: One-dimensional Method Using Energy Concept

Fig. 4.8. Predicted and measured displacements using a linear stress-strain assumption for cases without local boundary effects.

Note: All displacements are not restrained by the adjacent structures or non-liquefied soils.

- △ 1906 San Francisco
- ● 1964 Alaska
- ○ 1971 San Fernando
- ■ 1983 Borah Peak
- □ 1987 Imperial Valley

Fig. 4.8. Predicted and measured displacements using a linear stress-strain assumption for cases without local boundary effects.
Chapter 4: One-dimensional Method Using Energy Concept

Fig. 4.9. Predicted and measured displacements for Japanese case histories.
As discussed in Chapter 2, the cyclic strain in a loose sand keeps increasing with additional loading cycles. This fact suggests that the limiting strains for small \( (N_1)_{60-cs} \) values shown in Fig. 2.6 will keep increasing depending on the peak ground acceleration and earthquake duration. The effects of both peak ground acceleration and earthquake duration can conveniently be included in terms of factor of safety against liquefaction \( (F_L) \). The sites having low SPT values but high peak ground accelerations may have very large limiting strains due to very low \( F_L \). On the other hand, the sites having low SPT values and low peak ground acceleration that hardly triggers liquefaction at the site \( (F_L \) about 1.0), may have much lower strains at residual strength in comparison to the previous ones. Byrne's method, however, does not take into account these effects. Consequently, depending on \( F_L \) of soils at the sites, the method sometimes overestimates the displacements at the sites having relatively high SPT blow counts (e.g. at Jensen Plant site, see Appendix A), but underestimates the displacements at the sites having low SPT blow counts (e.g. at Heber Road site).

4.6.2. The Modified Method

The limiting strain chart proposed by Seed et al. (1984) in Fig. 2.15 suggests that the limiting shear strain or the strain at residual strength is a function of cyclic stress level and equivalent clean sand SPT blow counts, \( (N_1)_{60-cs} \). By introducing the effects of factor of safety against liquefaction, \( F_L \), in evaluating the strain at residual strength of liquefied soils, Byrne's method is expected to give better predictions. To include these effects on the limiting strain evaluation, the limiting strain chart (Fig. 2.15) was modified so that the strain at residual strength can be determined if both \( F_L \) and \( (N_1)_{60-cs} \) are known.

In the original limiting strain chart (Fig.2.15), the line with limiting shear strain of about 3 percent is the loci of the points where the onset of liquefaction (the condition of
100 percent pore pressure rise) was observed in the simple shear tests (De Alba et al., 1976; Seed et al., 1984). Thus, it is assumed that this line is the loci of the points having \( F_L \) of unity. In other words, the points located at this line have a Critical Stress Ratio (CSR) equal the Critical Resistance Ratio (CRR). The factor of safety against liquefaction, \( F_L \), for any point in this chart for a given \( N_1 \) is determined by using Eq. 4.14.

To provide sufficient data point for plotting the modified chart, three additional curves correspond to limiting strains of 15, 40 and 100 percent were interpolated from the existing curves. These interpolated curves are shown in Fig. 4.10 as dashed lines. Then, at a given \( N_1 \), the \( F_L \) of several points corresponding to several values of limiting shear strain were computed by using Eq. 4.14. For example, for \( N_1 \) of 20, the \( F_L \) at points \( a, b \) and \( c \) were calculated, i.e. \( F_{L,a} = \frac{CRR_a}{CSR_a} \), \( F_{L,b} = \frac{CRR_b}{CSR_b} \), etc. Point \( a, b \) and \( c \) correspond to limiting shear strains of 3, 10 and 15 percent. The computed \( F_L \) values for these points were then plotted against the limiting shear strain, as shown in Fig. 4.11 for \( N_1 \) of 20. The same calculations were carried out for \( N_1 \) values of 5, 6, 7, 8, 10 and 15. The results of these calculations are shown in Fig. 4.11.

Using the chart in Fig. 4.11, each case history is predicted again using the same residual strength obtained from Eqs. 4.17 and 4.18 and the results are included in Appendix A. The results of the predictions using the modified method on the case histories presented in Fig. 4.8 are plotted in Fig. 4.12. In comparison to those presented in Fig. 4.8, better agreement between the predictions and the measured values is observed. Thus, the modified procedure for determining residual strains or limiting strains based on the combination of \( F_L \) and \( N_1 \) seems promising in improving the predictions.
Fig. 4.10. Chart to determine limiting strains proposed by Seed et al. (1984). Dash-lines were interpolated from the existing curves.
Fig. 4.11. Chart for determining limiting strains based on the factor of safety against liquefaction ($F_L$) and $(N_1)_{60-cr}$. Modified from Fig. 4.10 (Seed et al. (1984)).
Chapter 4: One-dimensional Method Using Energy Concept

Note: All displacements are not restrained by the adjacent structures or non-liquefied soils

- △ 1906 San Fransisco
- □ 1987 Imperial Valley
- ● 1964 Alaska
- ○ 1971 San Fernando
- ■ 1983 Borah Peak

Fig. 4.12. Predicted displacements using the modified procedure.
4.6. Limitations

The main limitation of the method, as any other one-dimensional method, is that it only gives one displacement value for each problem. In reality, however, the displacement of the soil structures may vary depending on the geometry and local boundary effects. For slopes with free face (such as in the river edges), the displacements generally increase with decreasing distance to the free face, as has been reported in many case histories (e.g. Hamada, 1992; O'Rourke et al., 1992). Moreover, the zone of liquefaction is often much more complex than assumed in the model. Thus, there is a need to extend the method to a two-dimensional and a multi-degree-of-freedom system that allows the consideration of geometry and local boundary effects. Nevertheless, for a preliminary analysis, the method can give a reasonable approximation for the possible liquefaction induced displacement, provided that the method is applied properly.

4.7. Summary

Newmark's rigid plastic model has been reviewed based on the work-energy principles. Using the same principles, Byrne (1990) extended the model from a rigid plastic one to a more general elasto-plastic stress-strain model which allows more realistic predictions of liquefaction induced lateral ground deformations. In this study, his method was validated against case histories of lateral spreads during past earthquakes. The method underestimates the ground displacements measured in Noshiro city, Japan, during the 1983 Nihonkai Chubu earthquake. The discrepancies probably stem from the different behaviour of clean fine sand involved in the Japanese case histories (Youd and Bartlett, 1988) or smaller residual strength of liquefied sands than those used in the predictions. However, the method generally gives very good agreement with the measured displacements at sites in the United States.
Despite its ability to reasonably predict the lateral displacements at most sites in the US case histories, the method does not take account of the effects of factor of safety against liquefaction ($F_L$). Consequently, it sometimes underestimates the displacements at the sites with low SPT values, but overestimates the ground displacements at sites having relatively high SPT values. Thus, to improve its predictive capability, the effects of $F_L$ are introduced in the procedure for evaluating the strains at residual strength. The modified procedure seems to be promising and yields better agreement with the measured values.

Byrne's method was developed for one-dimensional problem and consequently, it does not give an overall displacement pattern. In this thesis, the method is extended to a two-dimensional system which allows for the predictions of overall displacement pattern of earth structures. A description of the two-dimensional method will be presented in Chapter 5.
CHAPTER 5:
PROPOSED TWO-DIMENSIONAL METHOD

5.1. Introduction

Byrne's one dimensional method for predicting liquefaction induced deformation of soil structures discussed in Chapter 4 has been extended to a two-dimensional system and incorporated in a finite element computer program SOILSTRESS-2. This program is a modified version of SOILSTRESS which was originally developed by Byrne and Janzen (1981). This chapter presents a description of the method of analysis and the inclusion of work-energy principles in the two-dimensional system.

5.2. Method of Analysis

The method of analysis is similar to that proposed by Lee (1974) in which the effects of an earthquake on soil structures are simulated by a reduction of soil modulus. The proposed analytical procedure uses two sets of stress-strain curves for the pre- and post-earthquake/liquefaction conditions, as depicted in Fig. 5.1. The pre-earthquake stress-strain curves are used to determine the initial deformations, and the post-earthquake stress-strain curves are used to determine the final stress and deformations of the soil structures. Then, the deformations due to earthquake are obtained by subtracting the final deformations from the initial deformations of the soil structure.

The stresses and deformations of the earth structures are analyzed using a one-step gravity turn on method. In this method, the actual constructions of earth structures such as a dam are simulated by a single step loading instead of several increments of
loading. Clough and Woodward (1967) and Lee and Idriss (1974) have shown that this method predicts stresses of a simple homogeneous dam within reasonable accuracy but fails to provide an accurate prediction of dam deformations. However, in the proposed method, the initial deformations are considered merely as reference deformations and are not representative of any actual static deformations of earth structures.

For liquefied soils, the initial deformations are usually very small in comparison to the final predicted deformations and can be neglected. For non-liquefied soils, however, the initial deformations may still be significant and may affect the computation of internal work of these materials. Neglecting these initial deformations will tend to overestimate the internal work done by the system, and consequently underestimate the deformations.

In this procedure, the final displacements are those that satisfy the energy principles as expressed by Eq. 4.1, in which the external work minus the internal work equals the change in kinetic energy of the system. For one-dimensional system, the displacement that satisfies this principle can be obtained directly from the solution of Eq. 4.1. For two-dimensional system, these displacements can be determined using a pseudo-dynamic finite element approach which was originally proposed by Anderson (1991), using the following equation:

\[ [K] \{\Delta\} = \{F+\Delta F\} \]  
(5.1)

where \([K]\) is the global stiffness matrix of the system, \(\{\Delta\}\) is the vector of nodal displacements, \(\{F\}\) is the static load vector of the system, and \(\{\Delta F\}\) is the additional load vector applied to produce displacements that satisfy the energy balance of Eq. 4.1.

If \(\{\Delta F\} = 0\), then for a single-degree-of-freedom system, a displacement corresponds to point R in Fig. 4.3. will be predicted. However, at this point, the energy balance is not achieved yet. An additional force is required to balance the energy and predicts points S or T. This additional force can be obtained by applying a horizontal or
Chapter 5: Proposed Two-dimensional Method

Shear strain, $\gamma$

Shear stress, $\tau$

(a) Pre-liquefaction

(b) Post-liquefaction

Fig. 5.1. Pre- and post-earthquake stress-strain model of liquefied soils.
a vertical seismic coefficient, \( k \), such that \( \{DF\} = \{kW\} \), where \( W \) is the weight of soil element. In this case, the seismic coefficient \( k \) is not related to the peak ground acceleration but is selected by an iterative procedure so as to balance the energy in accordance with Eq. 4.1.

For a multi-degree-of-freedom system, the internal work, \( W_{\text{int}} \), equals the work done by the element stresses and strains, and the external work, \( W_{\text{ext}} \), equals the work done by the static load vector (gravity and boundary loads), \( \{F\}\{\Delta\}^T \). The additional force is adjusted to give displacements, \( \{D\} \), so as to balance the energy and satisfy Eq. 4.1. This additional force is merely an artifice to obtain the appropriate displacements and is not included in the computation of the internal work. A more detailed description of the evaluation of internal and external work will be given in Section 5.4.

The stress-strain model and the finite element formulation employed in the procedure are described in Appendices B and C.

In general, the proposed method of analysis consists of the following steps:

1. Compute the initial deformations, \( \{\Delta_1\} \), by solving the following equation:

\[
[K_1]\{\Delta_1\} = \{F_1\} \tag{5.2.1}
\]

in which, \( \{F_1\} \) is the initial load vector, and \( [K_1] \) is the global stiffness matrix derived from pre-earthquake soil properties.

2. Compute the final deformations, \( \{\Delta_2\} \), that give the overall energy balance of the system from the solution of:

\[
[K_2]\{\Delta_2\} = \{F_1 + \Delta F\} \tag{5.2.2}
\]

in which, \( \{F_1 + \Delta F\} \) is the final load vector, \( [K_2] \) is the global stiffness matrix derived from post-earthquake soil properties and \( \Delta F \) is the additional force required to obtain the energy balance of the system as described previously.
3. Evaluate the total deformations that include the additional deformations due to post-liquefaction settlement, \( \Delta_3 \), from the solution of:

\[
[K_2] \{\Delta_3\} = \{F_1 + \Delta F\} + \{F_s\} \quad (5.2.3)
\]

in which, \( \{F_s\} \) is the load vector due to post-liquefaction volumetric strain, and other terms are the same as those in step 2. A detailed description of the method to compute post-liquefaction settlement is given in Appendix C.

4. For cases where settlement is very small or can be neglected, subtract the deformations computed in step 2, \( \{\Delta_2\} \), with those computed in step 1, \( \{\Delta_1\} \), to give the liquefaction-induced displacements, \( \{\Delta_e\} \):

\[
\{\Delta_e\} = \{\Delta_2 - \Delta_1\} \quad (5.3)
\]

5. For cases where settlement cannot be neglected, subtract the deformations computed in step 3, \( \{\Delta_3\} \), with those computed in step 1, \( \{\Delta_1\} \), to give the total liquefaction-induced displacements including the post-liquefaction settlement, \( \Delta_t \).

\[
\{\Delta_t\} = \{\Delta_3 - \Delta_1\} \quad (5.4)
\]

These steps have been incorporated in the SOILSTRESS-2 finite element program. Cases without change in geometry during iteration (small deformation theory) and cases with changes in geometry during iteration (large deformation theory) are both considered.

### 5.3. Evaluation of Required Parameters

Most of the important parameters required for the proposed two-dimensional method are the same as those for the one-dimensional method: the zone of liquefaction, the residual strength and limiting shear strain of the liquefied soil, and the maximum velocity of the earth structure. The procedure to determine these parameters was described in Section 4.4.
Chapter 5: Proposed Two-dimensional Method

The only additional parameter for the two-dimensional system is the reconsolidation volumetric strain of liquefied soils. It has been shown in Chapter 2, that the post-liquefaction volumetric strains mainly depend on relative density of soil (represented by \( N_d \)) and maximum shear strain developed during cyclic loading (represented by \( F_L \)). Based on these parameters, the post-liquefaction volumetric strain can be determined from the chart proposed by Tokimatsu and Seed (1987) or Ishihara and Yoshimine (1992). The latter chart is more convenient to use as it directly provides the post-liquefaction volumetric strain as a function of \( N_d \) and \( F_L \). A description of the method to calculate post-liquefaction settlement is given in Appendix C.

5.4. Incorporation of Work-Energy Principles

As mentioned in Chapter 4, the work-energy principles require that the internal work done by the system minus the external work done by the external forces should equal the change in kinetic energy of the system, as expressed in Eq. 4.1. For the one-dimensional system, the evaluations of the three components involved in the energy equation is straightforward. For the two-dimensional system, however, several additional factors such as the work due to boundary loads and plastic volumetric strains are included. The methods to incorporate the internal and external work for the two-dimensional system are described in the following section.

5.4.1. Evaluation of External Work

5.4.1.1. Cases without Changes in Geometry

The external work for the two-dimensional system is basically due to initial load conditions or static loads, namely, the gravity and boundary loads. The gravity loads are due to the weight of the soil masses. The boundary loads are due to pressure and nodal loads including water pressures, boundary pressures of liquefied soils, and loads from the
superstructure. For cases where the static driving stress does not change significantly with increased deformation such as infinite slope problems (Fig. 4.3), the total external work of the system, $W_{\text{ext},t}$, can be computed conveniently using:

$$W_{\text{ext},t} = \{F_{i}\} \cdot \{\Delta_{2} - \Delta_{1}\}^{T} \quad (5.5)$$

where,

- $\{F_{i}\}$ = nodal forces due to static loads including gravity and boundary loads,
- $\Delta_{1}$ = nodal displacements at the initial loading condition, and
- $\Delta_{2}$ = nodal displacements at the final loading condition.

Alternatively, the external work of the system is computed per element and then the results are summed to give the total external work of the system:

$$W_{\text{ext},e} = \{F_{e,1}\} \cdot \{\delta_{e,2} - \delta_{e,1}\}^{T} \quad (5.6)$$

$$W_{\text{ext},t} = \sum W_{\text{ext},e} \quad (5.7)$$

in which,

- $W_{\text{ext},e}$ = the external work of the soil element,
- $\{F_{e,1}\}$ = the element forces due to static loads,
- $\{\delta_{e,2}\}$ = the final displacement of the soil element, and
- $\{\delta_{e,1}\}$ = the initial displacement of the soil element.

Both approaches are incorporated in the SOILSTRESS-2 program and both give the same results as expected.

### 5.4.1.2. Cases with Changes in Geometry

For cases where large deformations occur, the static driving stress will decrease with increasing deformations. These cases usually occur on steep slopes when the factor of safety of the soil structure is less than unity. However, contrary to general belief, total
failure will not occur if the soil can deform such that the static driving stress reduces to a value less than or equal to the residual strength of the soil. Consequently, the factor of safety at the end of deformations is equal or larger than unity. For a one-dimensional problem, this is illustrated schematically in Fig. 5.2.

The soil element illustrated in Fig. 5.2. has an initial static shear stress larger than the residual strength of liquefied soils. If the soil is triggered to liquefy and the residual strength of soil is reached, then using a conventional assumption, flow failure is expected. However, this is not necessarily the case when the geometry changes are accounted for in the analysis.

The decrease in the static driving stress is dependent on the geometry of the soil structures. The external work of the soil elements in these cases can be approximated based on the average value between the forces acting on the system at the initial and the final conditions using the following equation:

\[
W_{\text{ext}} = \left\{ \frac{1}{2} (F_1 + F_2) \right\} \cdot (\Delta_2 - \Delta_1)^T
\]  \hspace{1cm} (5.8)

in which,

\( F_1 \) = nodal forces due to static loads at the initial loading condition,
\( F_2 \) = nodal forces due to static loads at the final loading condition,
\( \Delta_1 \) = nodal displacements at the initial loading condition, and
\( \Delta_2 \) = nodal displacements at the final loading condition.
Fig. 5.2. Decrease in the static driving stress due to changes in geometry during mass movements.
5.4.2. Evaluation of Internal Work

Fig. 5.3 shows a cubical soil element in the field having effective normal stresses, \( \sigma_x, \sigma_y, \sigma_z \), and shear stresses, \( \tau_{xy}, \tau_{xz}, \tau_{yz} \) (referred to rectangular axes x, y, z). If this soil element undergoes corresponding normal strain increments \( \delta\varepsilon_x, \delta\varepsilon_y, \delta\varepsilon_z \), and shear strain increments \( \delta\gamma_{xy}, \delta\gamma_{xz}, \delta\gamma_{yz} \), the internal work per unit volume of the element is computed using:

\[
\delta W = \sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \sigma_z \delta\varepsilon_z + \tau_{xy} \delta\gamma_{xy} + \tau_{xz} \delta\gamma_{xz} + \tau_{yz} \delta\gamma_{yz} \tag{5.9}
\]

In terms of the principal effective stresses, \( \sigma_1, \sigma_2, \sigma_3 \), and the principal strain increments, \( \delta\varepsilon_1, \delta\varepsilon_2, \delta\varepsilon_3 \), Eq. 5.9 can be written as:

\[
\delta W = \sigma_1 \delta\varepsilon_1 + \sigma_2 \delta\varepsilon_2 + \sigma_3 \delta\varepsilon_3 \tag{5.10}
\]

For plane strain conditions, \( \sigma_2 \neq 0 \), but \( \delta\varepsilon_2 = 0 \). Thus, Eq. 5.10 becomes,

\[
\delta W = \sigma_1 \delta\varepsilon_1 + \sigma_3 \delta\varepsilon_3 \tag{5.11}
\]

or,

\[
\delta W = \frac{(\sigma_1 - \sigma_3)}{2} (\delta\varepsilon_1 - \delta\varepsilon_3) + \frac{(\sigma_1 + \sigma_3)}{2} (\delta\varepsilon_1 + \delta\varepsilon_3) \tag{5.12}
\]

For plane strain conditions, the shear strain increment, \( \delta\gamma \), and volumetric strain increment, \( \delta\varepsilon_v \), are given by:

\[
\delta\gamma = \delta\varepsilon_1 - \delta\varepsilon_3 \tag{5.13}
\]

\[
\delta\varepsilon_v = \delta\varepsilon_1 + \delta\varepsilon_3 \tag{5.14}
\]
Fig. 5.3. Stress condition of a soil element in the field.
Hence, in terms of shear stress, \( \tau \), and mean effective normal stress, \( \sigma_m \), Eq. 5.12 becomes,
\[
\delta W = \tau \delta \gamma + \sigma_m \delta \varepsilon_v
\]  
(5.15)
in which,
\[
\tau = \frac{(\sigma_1 - \sigma_3)}{2} \quad \text{(5.16), and}
\]
\[
\sigma_m = \frac{(\sigma_1 + \sigma_3)}{2} \quad \text{(5.17)}
\]  

The total internal work for the system, \( W_{int} \), is obtained by integrating Eq. 5.15 over the volume of the soil elements:
\[
W_{int} = \{ \int_{\gamma_1}^{\gamma_2} \tau \cdot \delta \gamma + \int_{\varepsilon_{v1}}^{\varepsilon_{v2}} \sigma_m \cdot \delta \varepsilon_v \} \cdot \text{Volume} \quad \text{(5.18)}
\]  
The first term in the right hand side of the equation is the internal work due to changes in the plastic shear strain from \( \gamma_1 \) to \( \gamma_2 \). The second term is the internal work due to changes in the volumetric strains from \( \varepsilon_{v1} \) to \( \varepsilon_{v2} \). The methods employed to evaluate both components of internal work are described in the section which follows.

5.4.2.1. Internal Work Due to Shear Strain

For non-liquefiable soils that do not experience stiffness and strength reduction, the internal work due to changes in the plastic shear strain can be computed by direct integration of the hyperbolic stress-strain curve. The shear stress at any point in the curve is given by:
\[
\tau = \frac{\gamma \cdot G_m \cdot s}{s + R_f \cdot \gamma \cdot G_m} \quad \text{(5.19)}
\]  

With reference to Fig. 5.4.a, the internal work per unit volume can be computed using:
\[
W_{int} = \tau_{ult} \left( \frac{\gamma - \tau_{ult}}{G_m} \ln(R_f \cdot G_m \cdot \gamma + \tau_m) \right)_{\gamma_1}^{\gamma_2} \quad \text{(5.20)}
\]  

for \( \gamma_1 \) and \( \gamma_2 \leq \gamma_L \), and
Chapter 5: Proposed Two-dimensional Method

\[ W_{\text{int}} = \tau_{\text{ult}} \left( \gamma - \frac{\tau_{\text{ult}}}{G_m} \ln \left( R_f \cdot G_m \cdot \gamma + \tau_m \right) \right) \frac{\gamma_L}{\gamma_1} + s \cdot (\gamma_2 - \gamma_L) \quad (5.21) \]

for \( \gamma_1 \leq \gamma_L \) and \( \gamma_2 > \gamma_L \).

For non-liquefied soils that experience a stiffness reduction, it is assumed that cyclic loading causes the initial shear stress to drop to a lower value which lies on the post-earthquake stress-strain curve, as depicted in path PQ in Fig. 5.4.b. The drop of the shear stress along this path may not necessarily occur in the field. The shear stress of the soil element in the field could move following path PR, or any path between paths PQ and PR. The path followed is dependent on many factors including the material types and the cyclic stresses due to earthquakes. For the proposed simplified procedure, however, the assumed PQ path yields conservative results in terms of the computed deformations and is considered to be appropriate. Eqs. 5.20 and 5.21 for determining the internal work of the soil element are also valid here.

For liquefied soils, it is also assumed that upon liquefaction, the initial static shear stress of soils drops from P to Q, as depicted in Fig. 5.4.c. As discussed in Chapter 2, this assumption is valid for liquefied soils that experience stress-reversal during cyclic loading.

The internal work of liquefied soils can be computed by direct integration of the post-earthquake stress-strain curve in terms of hyperbolic soil parameters as given by Eqs. 5.20 and 5.21. Alternatively, the internal work can be determined by a simpler approach as follows:

For \( \gamma_1 \) and \( \gamma_2 \leq \gamma_L \),
\[ W_{\text{int}} = \frac{1}{2} (\tau_1 + \tau_2) \cdot (\gamma_2 - \gamma_1) \quad (5.22) \]

For \( \gamma_1 \leq \gamma_L \) and \( \gamma_2 > \gamma_L \),
\[ W_{\text{int}} = \frac{1}{2} (\tau_2 + S_r) \cdot (\gamma_L - \gamma_1) + S_r \cdot (\gamma_2 - \gamma_L) \quad (5.23) \]
Fig. 5.4. Internal work due to shear strains.
Both approaches are implemented in the SOILSTRESS-2 program. The internal work for non-liquefied soils are computed using direct integration approach, i.e. Eqs. 5.20 and 5.21. For liquefied soils, Eqs. 5.22 and 5.23 are used to compute the internal work.

5.4.2.2. Internal Work Due to Volumetric Strains

The relationship between mean normal effective stress, $\sigma_m$, and plastic volumetric strains, $\varepsilon_v$, is shown in Fig. 5.5. The internal work per unit volume due to changes in the plastic volumetric strains is the shaded area of Fig. 5.5 and is computed as follows:

$$W_{\text{int,vol}} = \int_{\varepsilon_{v1}}^{\varepsilon_{v2}} \sigma_m \delta \varepsilon_v$$

(5.24)

in which,

$$\varepsilon_{v1} = \frac{\sigma_{m1}}{B_1}$$

(5.25)

$$\varepsilon_{v2} = \frac{\sigma_{m2}}{B_2}$$

(5.26)

$$B = k_b \cdot P_a \cdot \left( \frac{\sigma_m}{P_a} \right)^m$$

(5.27)

$$\sigma_m = \frac{1}{2} (\sigma_1 + \sigma_3)$$

(5.28)

where,

$\varepsilon_{v1}$ and $\varepsilon_{v2}$ = the initial and final plastic volumetric strains,

$B_1$ and $B_2$ = the bulk modulus of soil at the initial and final conditions, as given by Eq. 5.27.

$\sigma_{m1}$ and $\sigma_{m2}$ = the mean normal effective stress at the initial and final conditions.

Since $B$, $\sigma_m$ and $\varepsilon_v$ are interrelated, the internal work is best computed using direct integration of Eq. 5.24 (Salgado, 1993) as follows:

$$W_{\text{int,vol}} = \sigma_{m2} \cdot \varepsilon_{m2} - \sigma_{m1} \cdot \varepsilon_{m1} - \frac{1}{(2-m)k_b P_a (1-m)} \left\{ \sigma_{m2} (2-m) - \sigma_{m1} (2-m) \right\}$$

(5.29)
Chapter 5: Proposed Two-dimensional Method

Volumetric strain, $\varepsilon_v$

$\sigma_{m1} = \text{Initial mean effective stress}$

$\sigma_{m2} = \text{Final mean effective stress}$

$\varepsilon_{v1} = \text{Initial volumetric strain}$

$\varepsilon_{v2} = \text{Final volumetric strain}$

Fig. 5.5. Internal work due to volumetric strains.
This equation is valid for both contraction (positive volume changes, $\varepsilon_{v2} > \varepsilon_{v1}$) and expansion (negative volume changes, $\varepsilon_{v2} < \varepsilon_{v1}$). Derivation of Eq. 5.29. is presented in Appendix D.

Alternatively, the internal work due to volumetric strains can be approximated by the following equation:

$$W_{\text{int,vol}} = \frac{1}{2} (\sigma_{m2} + \sigma_{m1})(\varepsilon_{v2} - \varepsilon_{v1})$$  \hfill (5.30)

Both methods are incorporated in the SOILSTRESS-2 program and give very good agreement.

### 5.4.3. Evaluation of Inertia Forces

The inertia forces of the system is computed using:

$$W_{\text{kin}} = \frac{1}{2} \sum M_e \cdot V_e^2$$  \hfill (5.31)

in which,

$M_e$ = mass of the soil element, and

$V_e$ = velocity of the soil element relative to the rigid base.

For some cases such as small earth dams and slopes with a thin layer of liquefied soils, it is often appropriate to assume that the velocity of soil elements is constant with depth. For a thick soil layer, however, the velocity of a soil element may not be constant and can be assumed to be proportional to displacement. It is further assumed that maximum velocity occurs in the soil elements at the surface. Thus, for elements in a column of soil, the velocity of the soil element decreases with depth as depicted in Fig. 5.6. The element velocity, $V_e$, can be computed using:

$$V_e = \left(\frac{\delta_e}{\delta_{\text{max}}}\right)V_{\text{max}}$$  \hfill (5.32)

where,
Chapter 5: Proposed Two-dimensional Method

GROUND SURFACE

NON- liquefied CRUST

LIQUEFIED LAYER

NON- liquefied LAYER

\[ V_{\text{max}} = \text{velocity at the surface at a given soil column} \]

\[ V_e = \text{velocity of soil element} \]

\[ \delta_{\text{max}} = \text{maximum displacement at a given soil column} \]

\[ \delta_e = \text{displacement of a soil element} \]

Fig. 5.6. Assumed velocity distribution in the liquefied soils.
\[
\delta_e = \text{the element displacement}, \\
\delta_{\text{max}} = \text{the maximum displacement of the elements in the soil column}, \\
V_{\text{max}} = \text{the maximum velocity at the surface}.
\]

5.5. Summary

A simplified two-dimensional method for predicting liquefaction induced displacements based on energy concept has been presented. The method of analysis, determination of the required soil parameters, and the incorporation of the work-energy principles have also been discussed. The proposed procedure is embodied in the SOILSTRESS-2 finite element program.

In general, the procedure to perform liquefaction induced deformation analysis discussed in this chapter can be summarized as follows:

1. Determine the geometry of the earth structures to be modeled by the finite elements. This includes the evaluation of soil types and their properties, and the determination of water table elevation at the site.
2. Determine the peak ground acceleration and peak ground velocity of the site.
3. Evaluate the zone of liquefaction based on the information in steps 1 and 2.
4. Determine the pre- and post-earthquake stress-strain parameters of each material type. For non-liquefied soils, the procedures to obtain these parameters are presented by Duncan et al. (1980) and Byrne et al. (1987). For liquefied soils, the procedures to evaluate the residual strength and the strain at residual strength are described in Section 4.4. The method to determine the post-liquefaction volumetric strain is presented in Section 5.4.
5. Using the SOILSTRESS-2 computer program and the input file obtained from steps 1 to 4, compute the deformations which give the energy balance of the
system. This is performed by applying a seismic coefficient to the system by trial and error until the energy convergence criterion is satisfied.
CHAPTER 6:
VERIFICATION

6.1. Introduction

As in any other analytical procedure, the proposed procedure described in Chapter 5, and embodied in the finite element program SOILSTRESS-2, must first be verified against the closed form solution to check the accuracy of the predictions. Moreover, to be useful in predicting the displacement behaviour of earth structures in the field, the procedure must also be validated against liquefaction-induced deformation case histories. It is understood that such validation against only one case history will not be sufficient, particularly for an analytical procedure for predicting displacement behaviour of critical structures such as earth dams. For this reason, the proposed procedure is validated against eight case histories: two liquefaction induced lateral spreads and six earth dam case histories.

This chapter presents the results of verification of SOILSTRESS-2 against Byrne's closed form solution on which this procedure is based. The validations against field case histories will be presented in Chapters 7 through 11.

6.2. Verifications Against Byrne's Closed Form Solution

6.2.1. Sloping Ground

In order to verify the proposed procedure, the slope similar to that used in deriving Byrne's formula in Chapter 4 is modeled using finite element. It is important that all the assumptions inherent in the closed form solution are sufficiently represented in the model. Otherwise, the results will not be useful for deriving any conclusions.
Chapter 6: Verification

The geometry of the slope used for the closed form solution and the finite element approach (SOILSTRESS-2) was: the crust thickness was 1.5 m, the liquefied layer thickness was 1.4 m, and water table was at the bottom of the crust. The velocity of the soil mass was taken to be 0.2 m/s. The finite element model of the slope is shown in Fig. 6.1.

To conform with the assumptions used in Byrne's closed form solution, the crust must be sufficiently rigid to prevent any significant strain development within the crust layer. This was achieved by assigning a very high shear modulus and soil strength. Moreover, the soil mass taken into account in determining the kinetic energy of the system in both the closed form and the finite element procedure must be similar. This can be accomplished by assigning velocity magnitudes proportional to displacements for each column of elements.

The stress-strain properties of the liquefied layer for both the closed form and the finite element analyses were obtained from the empirical formulas proposed by Byrne (1990, 1991), i.e. Eqs. 2.1 and 2.2 for determining residual strength and Eq. 2.4 for determining residual strains (Sections 2.4.3 and 2.4.4). The rigid plastic behavior of soil can be represented using a hyperbolic model by assigning $R_f$ value equal to zero. The undrained behaviour of soil was achieved by setting the bulk modulus to a very high number. The pre-earthquake soil properties used in this study are presented in Table 6.1 and the post-earthquake soil parameters are given in Table 6.2.
Fig. 6.1. Sloping ground finite element model. (a) Original geometry. (b) Deformed geometry.
Chapter 6: Verification

Table 6.1. Pre-earthquake soil properties for finite element analyses.

<table>
<thead>
<tr>
<th>Soil</th>
<th>kₙ</th>
<th>n</th>
<th>kₜ</th>
<th>m</th>
<th>ϕ</th>
<th>c</th>
<th>Rₕ</th>
<th>γ</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
</tr>
<tr>
<td>Crust</td>
<td>2000</td>
<td>0.5</td>
<td>20000</td>
<td>0.25</td>
<td>50</td>
<td>900</td>
<td>0.60</td>
<td>19.62</td>
</tr>
<tr>
<td>Liq.layer</td>
<td>200</td>
<td>0.5</td>
<td>20000</td>
<td>0.25</td>
<td>30</td>
<td>0.0</td>
<td>0.60</td>
<td>16.68</td>
</tr>
</tbody>
</table>

Table 6.2. Post-earthquake soil properties of liquefied layer for both closed form and finite element analyses for different (N₁)₆₀-cs.

<table>
<thead>
<tr>
<th>(N₁)₆₀-cs</th>
<th>Residual Strength, Sₘ(₁) kPa</th>
<th>Strain at Residual strength, ( γ_{L}(b) ) percent</th>
<th>kₙ</th>
<th>n</th>
<th>kₜ</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
</tr>
<tr>
<td>3</td>
<td>4.8</td>
<td>112.2</td>
<td>0.042</td>
<td>1.0</td>
<td>2000</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>5.7</td>
<td>100.0</td>
<td>0.057</td>
<td>1.0</td>
<td>2000</td>
<td>0.0</td>
</tr>
<tr>
<td>5</td>
<td>6.8</td>
<td>89.1</td>
<td>0.086</td>
<td>1.0</td>
<td>2000</td>
<td>0.0</td>
</tr>
<tr>
<td>6</td>
<td>8.1</td>
<td>79.4</td>
<td>0.10</td>
<td>1.0</td>
<td>2000</td>
<td>0.0</td>
</tr>
<tr>
<td>7</td>
<td>9.6</td>
<td>70.8</td>
<td>0.135</td>
<td>1.0</td>
<td>2000</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>11.4</td>
<td>63.1</td>
<td>0.181</td>
<td>1.0</td>
<td>2000</td>
<td>0.0</td>
</tr>
<tr>
<td>9</td>
<td>13.6</td>
<td>56.2</td>
<td>0.239</td>
<td>1.0</td>
<td>2000</td>
<td>0.0</td>
</tr>
<tr>
<td>10</td>
<td>16.2</td>
<td>50.1</td>
<td>0.324</td>
<td>1.0</td>
<td>2000</td>
<td>0.0</td>
</tr>
<tr>
<td>11</td>
<td>19.2</td>
<td>44.7</td>
<td>0.430</td>
<td>1.0</td>
<td>2000</td>
<td>0.0</td>
</tr>
</tbody>
</table>

a) \( Sₘ = 0.0284 \text{ Pa} e^{0.173 (N₁)₆₀-cs} \)

b) \( γ_{L} = 10^{(2.2-0.05 (N₁)₆₀-cs)} \)

As discussed in Chapter 5, the energy balance of the system in the finite element approach can be achieved by applying a horizontal or a vertical seismic coefficient to the structures (pseudo-dynamic analyses). In this problem, the energy balance was achieved
by two approaches: applying horizontal seismic coefficient \( k_h \) only and a combination of both vertical \( k_v \) and horizontal seismic coefficients. The typical deformed mesh of the slope obtained from the finite element analyses is shown in Fig. 6.1.b and the computed displacements for 10 percent sloping ground are shown in Fig. 6.2. For comparison, the displacements computed from the closed form solution are also plotted in this figure.

It can be seen from Fig. 6.2 that the use of \( k_h \) and the combination of \( k_v \) and \( k_h \) give similar results. More importantly, the predicted displacements using both approaches agree very well with the closed form solution. The deviations from the closed form results are less than 5 percent and these are within the convergence criterion for energy balance which is ± 5 percent.

A similar problem for a 5 percent slope was also analyzed using both closed form solution and SOILSTRESS-2. In this problem, energy balance was achieved by applying a horizontal seismic coefficient only. The results are presented in Fig. 6.3 and compared with those for 10 percent sloping ground. Again, excellent agreement is seen between closed form and SOILSTRESS-2 results. The figure also shows that the 5 percent slope results in smaller displacements than those obtained from a 10 percent slope as expected.

### 6.2.2. Level Ground

In the above examples, the external work was due to both the gravity loads and the velocity of the slope mass. It is also interesting to see the effects of velocity on the computed displacements. This requires an exclusion of the effects of external work due to gravity loads from the finite element model. This requirement can be satisfied by considering a level ground model. The finite element mesh used is shown in Fig. 6.4.a.

The soil properties used in the analyses were the same as those used in the preceding section for \( (N_1)_{60-cs} = 10 \). Both the thickness of the crust and the liquefied layer
Fig. 6.2. Computed displacements using finite element procedures and Byrne’s closed form solution for 10 percent slope. Energy balance achieved by applying $k_h$ and combination of $k_v$ and $k_h$. 

Closed form  SS-2 (kh only)  SS-2 (kv & kh)
Fig. 6.3. Computed displacements using finite element procedures and Byrne's closed form solution for 5 and 10 percent slope. Energy balance achieved by applying $k_n$ only.
Fig. 6.4. Flat ground finite element model. (a). Original geometry. (b). Deformed geometry.
Chapter 6: Verification

VELOCITY VERSUS MAXIMUM DISPLACEMENTS FROM SOILSTRESS-2 AND BYRNE'S CLOSED FORM SOLUTION

1. SOILSTRESS: Velocity distribution.
2. CLOSED FORM: Total mass assumption.
3. SOILSTRESS-2: Vo proportional to displacement.
4. CLOSED FORM: Mass = Crust + 0.5 Liq layer.

Maximum Velocity (m/s)

Displacement (m)

(N1)60-cs = 10; Sr = 16.20; kg = 0.32

- Crust thickness = 5.0 m; Liq. layer thickness = 5.0 m
- Water Table at the crust bottom

Fig. 6.5. Computed displacements from finite element procedures and Byrne’s closed form solution for different assumption of velocity distribution.
Fig. 6.6. Comparison between the displacements from finite element procedures and Byrne's closed solution. All data combined.
were respectively 5.0 m. Similar to the preceding problem, the water table was at the bottom of the crust. This problem with a special node numbering system (from top to bottom for each soil column) can be used as an option that considers distribution of velocity proportional to displacement of the soil elements. A typical deformed shape for this problem is shown in Fig. 6.4.b. The results of the finite element analyses are compared with those from the closed form solution in Fig. 6.5 for both assumptions of constant velocity and velocity proportional to the displacements.

Again, very good agreement between the displacements computed using SOILSTRESS-2 and the closed form solution for both assumptions can be seen in Fig. 6.5. As expected, the assumption of constant velocity for all elements yields larger displacements than the assumption of velocity proportional to the displacements.

All data obtained from level ground, and 5 and 10 percent sloping ground problems are combined and presented in Fig. 6.6, in terms of predicted displacement using SOILSTRESS-2 versus the displacements computed using the closed form solution. The figure clearly shows good agreement between the closed form and SOILSTRESS-2 results. Thus, it can be concluded that the one-dimensional model has been successfully incorporated in the two-dimensional finite element program SOILSTRESS-2.

6.3. Summary

The proposed two-dimensional simplified finite element procedure for predicting liquefaction induced displacement has been verified against the closed form solution for slope and ground conditions. The closed form solution is used for a single-degree-of-freedom system subjected to a velocity pulse, and the finite element solution is used for a multi-degree-of-freedom system that requires an iterative approach to obtain the energy balance of the system.
The results show very good agreement between the displacements obtained using the finite element procedure and those obtained from the closed form solution. It is concluded that the closed form one-dimensional method proposed by Byrne (1990) has been successfully incorporated into the two-dimensional finite element procedure.
CHAPTER 7:
LIQUEFACTION INDUCED DEFORMATIONS AT THE WILDLIFE SITE

7.1 Introduction

The Imperial Wildlife Management Area, here referred to as the Wildlife site, is located in the Imperial Valley approximately 36 km north of El-Centro (Fig. 7.1). The site lies on the west side of the incised Alamo river flood plain which liquefied during the 1930, 1950, 1957, and 1981 earthquakes (Holzer et al., 1989). Because of the high possibility of liquefaction reoccurring at this site, it was heavily instrumented in 1982 to monitor expected liquefaction in future earthquakes. The site, in fact, liquefied again during the November 24, 1987 Superstition Hill earthquake. The epicenter of the earthquake was about 32 km from the site and the recorded peak ground acceleration was about 0.21 g at the surface where g is the acceleration of gravity. The recorded accelerations and pore pressures time histories provide a unique set of data of a field liquefaction case history. The records have been extensively used to verify proposed analytical procedures for predicting pore pressure rise during earthquakes (e.g. Dobry et al., 1989; Keane and Prevost, 1989; Byrne and McIntyre, 1994) and liquefaction-induced lateral spreading (Dobry and Baziar, 1991; Gu et al., 1994). In this section, the recorded lateral ground deformation will also be used to verify the proposed analytical procedures discussed in Chapter 5.
Fig. 7.1. Location of the Wildlife site, Imperial Valley, California.
7.2. Effects of the 1987 Superstition Hill Earthquake

On November 23 and 24, 1987, the Wildlife site was subjected to two earthquakes; the M6.2 Elmore Ranch, and the M6.6 Superstition Hills. Locations of the epicenters are shown in Fig. 7.1. Only the Superstition Hills earthquake was strong enough to cause liquefaction at this site. The pore pressure records at the site indicate that a soil layer at a depth of about 3 m from the surface (layer B1) liquefied (Holzer et al., 1989).

Holzer et al. (1989) reported that extensive ground cracks and sand boils accompanied liquefaction were observed at the site. The ground displacements were measured based on the changes in the original bench-mark distances relative to each other. The top inclinometer casing was deflected about 18 cm relative to its base beneath the liquefied layer in a N15E direction. The estimated subsurface residual horizontal shear strain based on the curvature of the casing was approximately 4 percent in the top part of Unit B1. The ground cracks, sand boils and ground displacements due to the earthquake at this site are shown in Fig. 7.2. The magnitude of the displacements at the surface in the east-west direction are presented in Table 7.1.

<table>
<thead>
<tr>
<th>Distance from the river (m)</th>
<th>Measured displacement (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) 1.6</td>
<td>18.3</td>
</tr>
<tr>
<td>8.5</td>
<td>5.6</td>
</tr>
<tr>
<td>15.0</td>
<td>4.5</td>
</tr>
</tbody>
</table>
As can be seen in Fig. 7.2, sand boils were observed in many places at the instrumented site, including some in a zone about 50 m away from the river. However, no lateral ground displacement was observed at a distance of about 35 m from the river. Small lateral ground displacements were observed at a zone of about 15 m from the river and increased in magnitude closer to the river. A sudden increase in the displacement magnitude was observed at a distance of about 3 to 5 m from the river (Table 7.1), characterized by the development of intensive ground cracks in this zone. This pattern of ground deformation indicates some potential for localized failure at the river edge.

7.2. Soil Investigations

Bennet et al. (1984) conducted field and laboratory tests to investigate the subsurface soil condition at this site. The stratigraphy of the soil deposit and the corresponding soil properties at this site are shown in Fig. 7.3. and 7.4. In general, the soil deposit consists of 7 layers of different soil types. However, from the liquefaction analysis point of view, only the top 4 layers designated as Unit A, B1, B2 and C are of interest and will be described here. The more detailed description of the geotechnical properties of the soil deposit at this site can be found in Bennet et al. (1984).

Unit A lies between a depth of 0 and 2.5 m and consists of very loose and very soft interbedded micaceous sandy silt, silt and clayey silt. As shown in Fig. 7.4, most of the soil layer in this unit has sufficiently high clay content to be liquefaction resistant. Moreover, most of this unit is located above the water table which is at a depth of about 1.5 m from the surface.
Chapter 7: Liquefaction-induced Deformation at the Wildlife Site

Fig. 7.2. Sand boils, ground cracks, and lateral spreading due to the Superstition Hill earthquake (after Youd and Bartlett, 1988 and Holzer et al., 1989).
Fig. 7.3 Soil stratigraphy and map of the instrumentations (after Holzer et al., 1989)
Fig. 7.4. (a). Geotechnical properties; (b). Liquefaction resistance and induced stress ratio due to the 1987 M6.6 Superstition Hill earthquake (after Holzer et al., 1989).
Unit A is underlain by Unit B1 at a depth of 2.5 m to 3.5 m. Unit B1 consists of a loose moderately sorted sandy silt with Standard Penetration Test (SPT) blow counts of about 4 and silt contents of about 30-40 percent. This layer seems to be highly susceptible to liquefaction. In fact, there is a strong indication that this layer liquefied during the 1981 M5.9 Westmorland earthquake (Bennet et al., 1989). This layer also liquefied during the Superstition Hills earthquake (Holzer et al., 1989).

Unit B1 is underlain by Unit B2 which is about 3.5 m thick. This layer consists of a loose to medium dense silty sand to very fine sand. The SPT values for this layer range between 5 to 23 with the average value of about 9. The average Cone Penetration Test (CPT) tip resistance is about 57 kg/cm².

The underlying Unit C is about 5 m thick, between a depth of 6.8 m and 12 m. Unit C comprises of medium to stiff, clayey silt and silty clay. The average SPT values for this layer is about 20. Unit C is underlain by thick layers of silt with interbeds of clayey silt and poorly sorted silt. Based on their geotechnical soil properties, Unit C and the underlying layers are not prone to liquefaction.

7.4. Deformation Analysis

The proposed finite element pseudo-dynamic analysis discussed in Chapter 5 will be applied to estimate the liquefaction-induced displacement of the ground at this site. The computed deformations are then compared with the measurements in the field.

7.4.1. Slope Geometry and Soil Parameters Used in the Analysis

The geometry of the slope was taken from Holzer et al. (1989) and the finite element mesh for this geometry is shown in Fig. 7.5. In this case, Unit C and the
Chapter 7: Liquefaction-induced Deformation at the Wildlife Site

Fig. 7.5. Finite element mesh used in the analysis.
underlying soil layers were not modelled in the finite element mesh. The soil properties for pre-earthquake condition were obtained from Bennet et al. (1984) and Dobry et al. (1989). The maximum shear moduli of the soils were computed based on the shear wave velocity data and adjusted for static loading condition as recommended by Byrne et al. (1987). The resulting initial shear modulus in terms of hyperbolic parameters \( k_g \) and \( n \) were in good agreement with the published data for similar soils by Duncan et al. (1980). The constant bulk modulus, \( k_b \), were computed from \( k_g \) based on the assumed Poisson ratio of 0.35 for the soil above the water table and taken to be 2000 for submerged soils. The bulk modulus exponent \( m \) were taken to be 0.25 for all soils. The high bulk moduli for submerged soils were intended to simulate the undrained condition when little or no volumetric strain occurs during earthquake shaking.

As shown by Holzer et al. (1989), significant pore pressure rises were observed in Units B1 and B2. However, only Unit B1 liquefied (developed 100 percent excess pore pressure) during this earthquake. This observation was also confirmed by the results of simplified liquefaction analysis (Seed and Idriss, 1971) carried out by Holzer et al. (1989). The factor of safety against liquefaction (\( F_L \)) for Unit B1 was about 0.98 whereas \( F_L \) for Unit B2 is about 1.2. Thus, while there was soil stiffness reduction due to pore pressure rise in both Units B1 and B2, the post-earthquake strength of Unit B2 can be considered to be the same as the original. Only Unit B1 was assumed to lose most of its strength and therefore a residual strength corresponding to its equivalent clean sand SPT blow counts was used as the post-earthquake soil strength.

The stiffness reduction for Unit B2 was estimated from the relationship between pore pressure ratio versus stiffness reduction presented by Yasuda et al. (1991, 1994) and Thomas (1992) as given in Chapter 2. Based on this relationship, the reduced stiffness for this unit was taken to be about 60 percent of the original. Other hyperbolic soil
parameters for this unit were assumed to be the same as those for the pre-earthquake condition.

For the liquefied Unit B1, the post-liquefaction stress-strain curve was assumed to be elastic perfectly plastic (Byrne, 1990). This curve can be defined if the limiting strain and the residual strength of the soil are known. The limiting strain of the soil was determined from the relationship between $F_L$ and limiting strain for different $(N_1)_{60-cs}$ values, as shown in Fig. 4.11. The average SPT value for this layer was about 4 (Fig. 7.4). This value was assumed to have been corrected for the hammer energy ratio of 60 percent. Taking into account the effects of overburden stresses ($\sigma'_{vo} = 44$ kPa) and the fines contents (30 percent), the equivalent clean sand blow counts, $(N_1)_{60-cs}$, for Unit B1 using the procedures outlined by Seed and Harder (1990) was about 6. The corresponding average residual strength obtained in the relationship between residual strength versus $(N_1)_{60-cs}$ presented by Seed and Harder (1990) was about 6 kPa which corresponds to $0.14\sigma'_{vo}$. Byrne's empirical formula (Eq. 4.17) gave $S_r$ of 8.5 kPa (which was equal to $0.19\sigma'_{vo}$). The strain at residual strength for this layer with $F_L = 0.98$ and $(N_1)_{60-cs} = 6$ obtained from Fig. 4.11 was about 15 percent. The computed $k_g$ for post-liquefaction condition was therefore 0.4. The analysis was first carried out with $S_r$ of 6 kPa and $k_g$ of 0.4. A sensitivity study of the computed displacements on residual strength was also carried out by using several values of residual strength ratio.

The volumetric strains of the soil deposit due to dissipation of excess pore pressure following liquefaction were estimated from the chart presented by Ishihara and Yoshimine (1992). The corresponding volumetric strain based on $F_L$ and $(N1)_{60-cs}$ were 1.5 percent and 0.8 percent respectively for Units B1 and B2. The volumetric strains for other soil units together with the soil parameters for pre- and post-liquefaction conditions are presented in Table 7.2.
Table 7.2. Soil properties used in the analysis of the Wildlife site

<table>
<thead>
<tr>
<th>Soil unit (1)</th>
<th>kg (2)</th>
<th>n (3)</th>
<th>kb (4)</th>
<th>m (5)</th>
<th>φ deg (6)</th>
<th>Δφ deg (7)</th>
<th>c or Sr kPa (8)</th>
<th>Rf (9)</th>
<th>εv (%) (10)</th>
<th>γt kN/m3 (11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A11)</td>
<td>150</td>
<td>0.5</td>
<td>450</td>
<td>0.25</td>
<td>35.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.9</td>
<td>0.0</td>
<td>17.6</td>
</tr>
<tr>
<td>A2</td>
<td>93</td>
<td>0.5</td>
<td>2000</td>
<td>0.25</td>
<td>35.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.9</td>
<td>0.0</td>
<td>17.6</td>
</tr>
<tr>
<td>B12)</td>
<td>112</td>
<td>0.5</td>
<td>(0.0)</td>
<td>(0.25)</td>
<td>35.0</td>
<td>(0.0)</td>
<td>0.0</td>
<td>(0.14σvo')</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>B2</td>
<td>121</td>
<td>(0.5)</td>
<td>(2000)</td>
<td>(0.25)</td>
<td>35.0</td>
<td>(0.0)</td>
<td>0.0</td>
<td>(0.0)</td>
<td>0.9</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Note:
1) Unit A1 is above water table.
2) Brackets indicate soil properties after earthquake.

Table 7.3. Measured and computed displacements using Sr = 0.14σvo’ at the Wildlife site

<table>
<thead>
<tr>
<th>Distance from the river (m)</th>
<th>Measured displacement (cm)</th>
<th>Computed displacement (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>1.6</td>
<td>18.3</td>
<td>19.6</td>
</tr>
<tr>
<td>8.5</td>
<td>5.6</td>
<td>5.9</td>
</tr>
<tr>
<td>15.0</td>
<td>4.5</td>
<td>3.5</td>
</tr>
</tbody>
</table>
The maximum relative velocity to compute the kinetic inertia of the earthquake was taken from the ratio of $A/V$, where $A =$ maximum ground acceleration after liquefaction in gravity ($g$) units and $V =$ maximum velocity in m/s. The ratio of $A/V$ was assumed to be 1.0. The recorded peak ground acceleration after liquefaction was about 0.12 g and the maximum velocity was therefore 0.12 m/s.

7.4.2. Results of the Analysis

The results of the analyses are presented in Figs. 7.6.a and b in terms of deformed shape and the displacement vectors of the slope. The computed displacements at the surface are also compared with the measured values in Fig. 7.7 and the magnitude of displacements are presented in Table 7.3.

From these figures and also from Table 7.3, it can be seen that the predicted displacements agree well both in terms of the magnitude and displacement pattern. The predicted displacement at the distances of 1.6 m and 8.5 m from the river are respectively 19.6 cm and 5.9 cm which are in excellent agreement with measured values (18.3 cm and 5.6 cm). Figs. 7.6 and 7.7 also show that the analytical procedures used here have correctly predicted the possibility of localized failure at the river edge. This localized slope failure was observed in the field as extensive cracks and manifested by sudden increase in the surface displacements at this area. The deformation pattern of this type is typical of liquefaction-induced lateral spreading on gently sloping ground with free face condition as also has been observed at the Joseph Jansen Filtration Plant, Los Angeles, during the 1971 San Fernando earthquake (O'Rourke et al., 1989, 1992), and at numerous places in Niigata city during the 1964 Niigata earthquake (Hamada, 1992).
Chapter 7: Liquefaction-induced Deformation at the Wildlife Site

Fig. 7.6. Predicted deformation at the Wildlife site: (a) Deformed mesh, (b) Displacement vectors.
Fig. 7.7. Predicted and measured lateral displacements at the Wildlife site.
For the cases considered, the computed settlement after dissipation of excess pore pressures was too small to affect the total deformation due to the small thickness of the liquefied zone. For other cases involving a thicker liquefaction zone, however, the post-liquefaction settlement could be an important factor that should not be overlooked. This is particularly true for the case where lifeline facilities (water pipes, sewers, etc.) exist at the transition zone between the liquefied and the non-liquefied soils.

The sensitivity of the prediction to the variation of the undrained strength was also investigated. The residual strength considered was between 0.10σ'\_vo and 0.16σ'\_vo. The results are presented in Fig. 7.8. The decrease in residual strength from 0.14 σ\_vo' to a value of 0.10 σ\_vo' (about 30 percent decrease) resulted in an increase in maximum displacement from 19.6 cm to 25.3 cm; an increase of about 27 percent. Furthermore, the use of residual strength ratio obtained from Byrne's formula (S_r = 0.19 σ\_vo') resulted in the maximum displacements of about 17 cm which was still very close to the measured displacements. Thus, the magnitude and the pattern of displacements do not seem to be sensitive to the change in the residual strength.

7.5. Summary

The Wildlife site has been instrumented to study the mechanism of liquefaction and related ground deformation in the field. A set of data obtained from this site is helpful for understanding the liquefaction and ground deformation mechanism in the field. The data is also useful to verify proposed analytical procedures for predicting pore pressure rise and ground deformation in the soil deposit due to earthquakes.
The proposed two-dimensional simplified procedure was applied to predict the liquefaction-induced deformation at the Wildlife site. The pre-earthquake soil parameters for the analysis were determined from Bennet et al. (1984) and Dobry et al. (1991). The post-liquefaction soil properties for the liquefied layer were determined from SPT N values corrected for the effects of hammer energy, overburden stresses and fines contents.
The predicted displacements are in good agreement with the observed values both in terms of the magnitude and displacement pattern. Moreover, the computed displacements do not seem to be sensitive to the change in the residual strength at the range of interest.
Chapter 8: Liquefaction Induced Lateral Spreads at Heber Road Site

CHAPTER 8 :
LIQUEFACTION INDUCED LATERAL SPREADS AT HEBER ROAD SITE, IMPERIAL VALLEY, CALIFORNIA

8.1. Introduction

Heber Road site is located at about 20 km southwest of El-Centro, California, and about 1.6 km from the fault rupture of the Imperial Valley earthquake of October 15, 1979. Due to the earthquake, a lateral spread occurred along the road and shifted the road and a parallel unlined canal towards a depression in south edge of the road. The maximum displacements of the road and the unlined canal were respectively about 1.2 m and 4.2 m (Youd and Bennett, 1983; Dobry et al., 1992). The lateral spread affected an area of about 160 m wide along the road and about 100 m long from the edge of depression to the northernmost cracks in the field north of the road. Youd and Bennett (1983) conducted a simplified liquefaction analysis and concluded that this lateral spread was due to liquefaction of a very loose sand deposit, a remnant of the old stream channel that previously passed the area.

The observed displacements at this site were used to validate the newly developed procedures for estimating liquefaction-induced lateral movements of earth structures discussed in Chapter 5. The results were also compared with those obtained by other methods such as the Newmark sliding block method (Castro, 1987).
8.2. Ground Movements Due to the Earthquake

Youd and Bennett (1983) reported displacements of 2.2 m of the unlined canal towards the 2 m deep depression at the south side of the road. Recently, based on the survey carried out by Youd and Bartlett, Dobry et al., (1992) presented a map of elevation contours showing a more detailed deformation pattern at the site, as shown in Fig. 8.1. This deformation map was drawn based on the assumption that the road and the canal were originally straight before the earthquake.

Youd and Bennett (1983) presented an aerial photograph of the site taken two days after the earthquake which showed the locations of soundings and borings at the site. It could be seen from the photograph that the maximum movement took place between boring numbers 5 and 6 which were close to the eastern edge of the old channel. This maximum movement was likely due to the thickness variation of the liquefied Channel Fill (CF) sand which was the thickest in this region, i.e. about 4.2 m. Moreover, as can be seen in Fig. 8.1, the maximum movement of the northern edge of the canal was about 2.3 m and the southern edge of the canal moved up to 4.2 m. The two meter difference in the lateral movements between the two edges of the canal suggests that wide extensive cracks might have taken place inside the canal. In fact, these cracks were observed clearly in the aerial photograph of the site taken by Youd and Bennet (1983). Fig. 8.1 also gives some insight that bulging has occurred at the toe of the slope at the area where the maximum displacements occurred.

Unfortunately, there was no data available regarding the initial geometry of this road section to allow better measurements of the ground displacements. However, based on the elevation contours and lateral movements given in Fig. 8.1, an approximate pre-earthquake geometry was drawn and combined with the post-earthquake configuration in Fig. 8.2. Although this pre-earthquake geometry might not be accurate in representing
Fig. 8.1 Deformation pattern at Heber Road site due to the earthquake (surveyed by Youd and Bartlett, presented by Dobry et al., 1992)
Heber Road Site, Imperial Valley, California
Pre-and post-earthquake geometry
(based on elevation contours by Bartlett and Youd, 1992)
Elevations are relative to an arbitrary datum

Fig. 8.2 Cross section at Heber Road site where the maximum displacements occurred during the 1979 earthquake
the actual initial geometry of the road cross-section, it is useful to show that the lateral movement was accompanied by vertical movement at the crest and the toe of the slope as shown in Fig. 8.2.

8.3. Subsurface Soil Conditions

The soil condition at the site has been extensively investigated by field and laboratory tests (Kuo and Stokoe, 1982; Sykora and Stokoe, 1982; Youd and Bennet, 1983). Youd and Bennett (1983) have conducted a Standard Penetration Test (SPT) and a Cone Penetration Test (CPT) and some of the results are presented in Fig. 8.3.

The site consists of a top layer of a brownish loose and fine sandy fill with thickness varying between 0.9 m to 1.5 m and standard penetration resistance N-values (safety hammer) of about 3 to 6. Underlying this fill, a 3.4 to 4.0 m thickness of fine silty fluvial sands was encountered with three different sub-units characterized by different relative densities. The western part of the cross-section contains a medium to dense fine sand of point-bar origin with N-values of 29 to 36 (safety hammer) and 32 to 35 (donut hammer) with fines contents of about 10 percent. The central part of this section consists of a loose, very fine silty sand with fines content of about 20 percent and N-values of 1 to 7 (safety hammer) and 2 to 4 (donut hammer). This deposit is a natural channel fill deposited by the ancient stream (Youd and Bennet, 1983). A silty sand of medium density with fines content of about 15 percent lies in the eastern part of the deposit. This sub-unit is characterized by N-values of 9 to 13 (safety hammer) and 17 to 19 (donut hammer). This fluvial sand is underlain by continuous alternating layers of medium-stiff lacustrine clays and fine medium dense to dense fluvial sands.

Grain size distribution curves for the three fluvial deposits point bar sand (PB), channel fill sand (CF) and levee sand (L) are shown in Fig. 8.4. As shown in the figure, all
Fig. 8.3. Soil conditions at Heber Road site (Youd and Bennet, 1983).
three sand deposits have almost the same grain size distribution except that the fines contents are slightly different. Point bar sand has the smallest fines content and the fill channel sand has the highest. However, based on study carried out by Tsuchida (1970) on the effect of grain size distribution on the liquefaction potential, all three of these sands are liquefiable.

Laboratory investigations have been conducted to study the strength and index properties of the soil deposits (e.g. Kuo and Stokoe, 1982). Summary of these investigations are shown in Table 8.1. The results of resonant column tests at different confining pressures are also shown in Fig. 8.5. This figure, as well as the stiffness values shown in Table 8.1, clearly demonstrates the different stiffness of the soils associated with different densities.
Table 8.1. Summary of soil laboratory data at Heber Road site (After Vucetic and Dobry, 1986 reported by Dobry et al., 1992)

<table>
<thead>
<tr>
<th>Properties</th>
<th>Field Results</th>
<th>Laboratory Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sykora and Stokoe (1982)</td>
<td></td>
</tr>
<tr>
<td>Soil Description or classification symbols</td>
<td>PB</td>
<td>L</td>
</tr>
<tr>
<td>Dense fine sand</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Loose very fine sand and silty sand</td>
<td>0.112</td>
<td>0.081</td>
</tr>
<tr>
<td>Mean grain size, $D_{50}$ (mm)</td>
<td>167.3</td>
<td>164.8</td>
</tr>
<tr>
<td>Percentage of fines (%)</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Average SPT-N (blows/foot)</td>
<td>32.5</td>
<td>12.5</td>
</tr>
<tr>
<td>CPT resistance (kg/cm$^2$)</td>
<td>203</td>
<td>71</td>
</tr>
<tr>
<td>Dry Unit Weight</td>
<td>100.4</td>
<td>95.7</td>
</tr>
<tr>
<td>Void ratio, $e_0$</td>
<td>0.67</td>
<td>0.72</td>
</tr>
<tr>
<td>Maximum shear modulus, $G_{max}$ (ksf)</td>
<td>1283</td>
<td>786</td>
</tr>
<tr>
<td>Permeability (cm/sec)</td>
<td>4.2x10$^{-3}$</td>
<td>-</td>
</tr>
</tbody>
</table>

*$G_{max}$ values measured in the two laboratories are corrected to account for the in-situ effective stresses.

1 pcf = 16.02 kg/m$^3$ = 0.157 kN/m$^3$; 1 kg/cm$^2$ = 98.07 kPa; 1 ksf = 47.88 kPa.
8.4. Previous Ground Deformation Analysis

Castro (1987) analyzed the deformations at this site with a view to determining residual strength that would be compatible with the observed displacements. He used the Newmark rigid plastic analysis for this purpose (Fig. 8.6.a). Castro (1987) also used the SHAKE program (Schnabel et al., 1972) to estimate the maximum acceleration of the moving mass. The earthquake record used was obtained from Bonds Corner which had a peak ground acceleration of 0.8 g. Bonds Corner is located about 5.2 km from the southern end of the fault rupture whereas the Heber Road site lies only about 1.6 km from the fault. Consequently, the peak ground acceleration which occurred at the site during the earthquake might have been higher.
Fig. 8.6. (a). Rigid plastic assumption of liquefied sand (Castro, 1987). (b). Maximum displacements versus residual strength at Heber Road site (1 psf = 0.0479 kPa).
Castro applied this earthquake motion at the dense sand layer just beneath the liquefied soil. Based on the shear moduli obtained by Sykora and Stokoe (1982), Castro computed the average peak ground acceleration of the moving mass to be 0.47 g. His analysis predicts that significant attenuation has taken place due to the soft liquefied layer. Yield acceleration of the moving mass was computed using a pseudo-static stability analysis for different assumed residual strengths. Subsequently, Castro computed the maximum displacements for each assumed residual strength and the results are shown in Fig. 8.6.b. Based on these results and the average observed displacements between the road and the canal (about 1.7 m), Castro back-calculated the residual strength of the liquefied soil at this site to be 4.8 kPa.

8.5. Current Deformation Analyses

8.5.1. Cross Section and Cases Considered

The cross-section of the site was taken from the approximate pre-earthquake geometry. This cross-section represents a section that underwent maximum ground displacements during the earthquake. Unfortunately, no data were available to define the subsurface condition along this cross-section to the north of the canal. Therefore, the assumptions of both uniform (Case 1) and non-uniform thickness of liquefied soil (Case 2) along this cross-section (N-S direction) were taken in the analyses. These assumptions imply that soil parameters of liquefied layer were the same for each element in the liquefaction zone. The water table was assumed to be at a depth of 1.8 m from the fill surface as obtained by Youd and Bennett (1983) during site investigation. The finite element mesh for this cross-section for uniform thickness assumption is shown in Fig. 8.7. The analyses were carried out by assuming that a plane strain condition prevails in the field. As a consequence, the thickness of liquefied layer in the direction parallel to the
canal (E-W direction) was assumed to be uniform. In reality, however, the thickness of liquefied layer (Channel Fill sand or Unit A2 in Fig. 8.3) decreases with distance from this cross-section to the old channel boundaries forming a small bowl-shape valley. This assumption will lead to a somewhat softer soil response than the actual one which in turn might lead to an overestimation of the displacements, particularly in the area beneath the slope.

8.5.2. Soil Properties.

For the pre-earthquake condition, the data shown in Fig. 8.5 were used to compute the soil stiffness parameters $k_s$ and $n$. The soil stiffness parameters for post-earthquake condition were estimated from the curve proposed by Seed and Harder (1990) in conjunction with the limiting shear strain proposed by Seed et al. (1984). Undrained condition was assumed during the analyses and this was accomplished by taking bulk modulus parameters $k_b = 2000$ and $n = 0.25$.

The average Standard Penetration Test blow counts used to obtain the residual strength for the liquefied soil were obtained from Youd and Bennett (1983). The average SPT blow counts corrected for the effects of overburden stress, $(N_1)_{60-cs}$ was about 3. Applying fines correction factor 2 for 20 percent fines contents, the $(N_1)_{60-cs}$ value for the channel fill sand was taken to be 5.

For $(N_1)_{60-cs} = 5$, the corresponding lower bound residual strength in Seed and Harder's chart was almost zero and the upper bound value was 14.5 kPa which corresponds to $S_t/\sigma'_{vo} = 0.30$ ($\sigma'_{vo}$ was taken for a point at 3.5 m depth which was equal to 48 kPa). However, this value seemed to be very high in comparison to the average residual strength of 0.21 $\sigma'_{vo}$ obtained from simple shear test data on undisturbed samples with $(N_1)_{60-cs}$ value of about 13 (Salgado and Pillai, 1993; Byrne et al., 1991, 1993).
Chapter 8: Liquefaction-induced Lateral Spreads at Heber Road Site

Fig. 8.7. Part of the finite element mesh used in the analyses of Heber Road site. Case 1
Therefore, this upper bound value of Seed-Harder was not used in the analyses.

Byrne’s empirical formula (1990) gave a value of 6.8 kPa. This value was very close to the 33rd percentile of the residual strength from the chart (about 6.0 kPa). It was therefore decided to use a residual strength of 6.8 kPa or $S_r/\sigma_{vo} = 0.14$ for the analyses.

The limiting shear strain used to define the post-liquefaction shear modulus was obtained from the chart in Fig. 4.11. This chart is a modified version of the chart proposed by Seed et al., (1984) that takes account of the effect of factor of safety against liquefaction ($F_L$). Using $F_L = 0.17$ (Youd and Bennet, 1983) and $(N_1)_{60-c_s} = 5$, the strain at residual strength was about 135 percent for Channel Fill (CF) deposit. The post-earthquake soil properties for non-liquefied soils (PB and L) were assumed to be the same as the pre-earthquake properties. The pre- and post-earthquake soil parameters for the three soil types used in Cases 1 and 2 are presented in Table 8.2.

The velocity of the ground mass was based on the peak ground acceleration of 0.80 g and was taken to be 0.80 m/s. The analyses were also carried out using the mass velocity of 0.47 m/s based on the surface acceleration of 0.47 g obtained by Castro (1987).

8.8. Results of the Analyses

The analyses were carried out by applying horizontal seismic coefficients to the system to achieve the energy balance. The results of the analyses are presented in Fig. 8.8 for Case 1. The computed and measured displacements at several nodes of interest are plotted in Fig. 8.9 as a function of distance from an arbitrary point and the values are also presented in Table 8.3.
Table 8.2. Pre- and post earthquake soil properties used in the analyses

<table>
<thead>
<tr>
<th>Material type</th>
<th>$k_g$</th>
<th>$n$</th>
<th>$k_b$</th>
<th>$m$</th>
<th>$\phi$ deg.</th>
<th>$\Delta\phi$ deg.</th>
<th>$c$ kPa</th>
<th>$R_f$</th>
<th>$\gamma_s$ kN/m$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand Fill</td>
<td>209</td>
<td>0.43</td>
<td>1000</td>
<td>0.25</td>
<td>35</td>
<td>0.0</td>
<td>0.0</td>
<td>0.70</td>
<td>19.6</td>
</tr>
<tr>
<td>Channel Fill</td>
<td>153</td>
<td>0.43</td>
<td>2000</td>
<td>0.25</td>
<td>30</td>
<td>0.0</td>
<td>0.0</td>
<td>0.90</td>
<td>19.6</td>
</tr>
<tr>
<td>(0.05)</td>
<td></td>
<td></td>
<td>(0.0)</td>
<td></td>
<td>(0.0)</td>
<td></td>
<td>6.8</td>
<td></td>
<td>(0.0)</td>
</tr>
<tr>
<td>Sand</td>
<td>181</td>
<td>0.43</td>
<td>2000</td>
<td>0.25</td>
<td>35</td>
<td>0.0</td>
<td>0.0</td>
<td>0.80</td>
<td>19.2</td>
</tr>
</tbody>
</table>

Note: The number in brackets indicates the properties after liquefaction.

Table 8.3. Predicted and measured displacements at several locations
(for location of the nodes, see Fig. 8.7)

<table>
<thead>
<tr>
<th>Node</th>
<th>Predicted (m)</th>
<th>Measured (m)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uniform thickness (Case 1)</td>
<td>Non-uniform thickness (Case 2)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$v_0=0.47m/s$</td>
<td>$v_0=0.80m/s$</td>
<td>$v_0=0.80m/s$</td>
</tr>
<tr>
<td>71</td>
<td>1.92</td>
<td>2.24</td>
<td>3.36</td>
</tr>
<tr>
<td>92</td>
<td>1.59</td>
<td>1.90</td>
<td>2.04</td>
</tr>
<tr>
<td>110</td>
<td>1.57</td>
<td>1.88</td>
<td>1.07</td>
</tr>
</tbody>
</table>
Fig. 8.8 Typical deformed geometry obtained from the current analyses of Heber Road site for Case 1. Displacement magnification factor = 1.0.
As can be seen in Table 8.3, the proposed procedure using a maximum velocity of 0.80 m/s correctly predicted the large ground displacements at the canal. For uniform thickness assumption, the northern edge of the canal was predicted to move about 1.90 m which was very close to the measured 2.2 m, and the southern edge moved about 2.2 m in comparison to the measured 4.2 m. The use of a maximum velocity of 0.47 m/s resulted in slightly less lateral movement, i.e. 1.59 m at the northern edge and 1.92 m at the southern edge. This small difference might be due to the fact that the deformations which occurred at this site were mostly due to very low stiffness of the liquefied soils.

The predicted magnitude and pattern of displacements in Case 1 are almost uniform at the locations away from the slope as shown in Fig. 8.8. However, the magnitude of displacements start to increase at the northern edge of the canal (node 71). The sudden increase in the deformation at this zone might be due to the potential for localized failure of the slope into the depression, as can be seen clearly in this figure. This type of deformation pattern is typical of earthquake-induced lateral spreads for uniform thickness of liquefied soil where a plane strain condition prevails. This deformation pattern has been observed in numerous case histories in Japan and the United States including those in Niigata city due to the 1964 earthquake and at the Joseph Jansen Filtration Plant due to the 1971 San Fernando earthquake.

Fig. 8.8 also shows that bulging or uplift took place at the base of the slope. Moreover, a tension zone also developed inside the canal (between node 71 and 92) indicating the potential for crack development. In fact, bulging and cracks at these locations were observed in the field (Youd and Bennet, 1983).

Fig. 8.9 shows the comparison between the measured and predicted lateral displacements at this site. It can be seen that the measured displacements slowly increase closer to the depression and attain a maximum lateral displacement at the southern edge
of the unlined canal. This deformation pattern suggests that the liquefied soil deposit in this site might not be uniform, or the soil elements in the liquefied layer (CF) were stiffer further from the depression. The liquefied channel sand shown in Fig. 8.3 might be thinner or narrower with increasing distance from the depression.

Although the predicted magnitude of the displacement at the northern edge of the canal in Case 1 is close to the measured value (node 92 in Table 8.3), the displacement magnitudes at other locations do not agree well with the observation. This might be due to the assumption that the liquefied soil was uniform along the cross-section (N-S direction). This assumption tends to underestimate the displacements at the location close to the depression, but overestimate the displacements further away from the depression.

Significant vertical deformations were predicted in the area close to the depression. Although some vertical movements might have occurred at the site, the amount of settlement at the crest of the slope might be much less than predicted. Because of the three-dimensional effects of the bowl-shape valley of the liquefied soil (Fig. 8.3), the actual response of the soil in the field might have been stiffer than that assumed.

Case 2, assuming the non-uniform thickness of liquefied layer (in N-S direction), were carried out to simulate the stiffer response of the soil with increasing distance from the depression. The computed displacements at several nodes of interest are also presented in Fig. 8.9. In this case, the procedure predicts the displacement magnitudes in the three locations of interest in better agreement with the observations than those obtained in Case 1. Moreover, except for excessive settlement at the slope, the pattern of displacement was also in general agreement with the observation.
Fig. 8.9. Horizontal ground displacements at several locations of interest. Case 1 and Case 2.
8.8. Back-calculated Residual Strength

Poulos et al. (1985) suggested that the residual strength of liquefied soil is only dependent on initial void ratio of the soils. Thus, a unique value of residual strength is assumed for a given relative density. The residual strength back-calculated from liquefaction case histories is also assumed to be constant. However, laboratory data show that the residual strength of soil is a function of stress path (Vaid et al, 1989) and also initial vertical effective stress (Salgado and Pillai, 1993). In this section, an attempt is made to back-calculate the normalized residual strength of the liquefied soils using the proposed procedure. The liquefied soil was assumed to follow a linear elastic-perfectly-plastic stress strain curve as shown in Fig. 4.5.

To systematically back-calculate the residual strength of the liquefied layer at this site, several residual strengths normalized to their initial effective stresses were assumed and the displacements were computed for each residual strength. Moreover, several constant residual strengths were also assumed. The computed displacements were used to provide some comparison between the use of constant residual strength and normalized residual strength. The cross-section used in the back-calculation analyses was the non-uniform thickness of the liquefied layer (Case 2). The results are shown in Figs. 8.10, 8.11 and 8.12.

Figs. 8.10 and 8.11 show that the use of constant residual strength or normalized residual strength gives basically the same results. For example, for $S_r = 9.6$ kPa which corresponds to $S_r/\sigma'_{vo} = 0.2$, both residual strengths yield a computed displacement at the south edge of the canal of about 1.5 m. This was expected since the liquefied layer was not sufficiently thick. However, in a case involving a much thicker liquefied layer, constant residual strength must be used with caution since it tends to underestimate the strength at the bottom layer but overestimate the strength at the top soil layer.
Fig. 8.10. Horizontal ground displacements for several locations as a function of residual strengths.
Fig. 8.11. Horizontal ground displacements at several locations as function of normalized residual strengths.
Chapter 8: *Liquefaction-induced Lateral Spreads at Heber Road Site*

Fig. 8.12. Horizontal ground displacements for several residual strength.
Fig. 8.12 shows the computed displacements as a function of normalized residual strength. The observed displacements at the three locations of interest are also shown in this figure. Based on the observed values on each location, the residual strength was obtained from the figure. The possible residual strength at the site obtained using this procedure was between 0.13 and 0.14 $\sigma'_v$ and was equal to about 6 kPa for $\sigma'_v = 48$ kPa. This value is in the same order of magnitude as the residual strength at the Wildlife site which has similar soil deposit with $(N_1)_{60-CA}$ of about 5 (Chapter 7). However, it is slightly higher than that obtained by Castro (1987) using the Newmark method which is about 4.8 kPa or corresponds to $S_r/\sigma'_v = 0.10$. This discrepancy might be attributed to a different assumption in the potential failure surface in the Newmark method and in the proposed two-dimensional method. For liquefied soils, the assumption of distinct failure surface (rigid plastic assumption) tends to underestimate the back-calculated residual strength.

8.9. Summary

Deformation analyses of liquefaction-induced lateral spreads at the Heber Road site have been presented using the proposed simplified procedure. The soil parameters required for the analyses were obtained from the published routine field and laboratory data for this site. A plane strain condition was assumed to prevail. Uniform and non-uniform thickness of liquefied layers were also assumed. Despite its simplicity, the procedure correctly predicts the large movements of the soil deposit at the site due to the 1979 Imperial Valley earthquake. The good agreement between the predicted and the measured displacements at this site suggests that the proposed procedure has a potential for predicting this type of ground failure in other cases.
An attempt has also been made to back-calculate the residual strength of the liquefied layer of this site. It was found that the residual strength computed was about $0.14 \sigma_{vo}$. This value is slightly higher than the result obtained by Castro (1987) using Newmark’s method, but agrees well with the residual strength of a similar deposit at the Wildlife site.
CHAPTER 9:
DEFORMATION BEHAVIOUR OF THE SAN FERNANDO DAMS DURING THE 1971 CALIFORNIA EARTHQUAKE

9.1. Introduction

On February 9, 1971, an earthquake of magnitude 6.6 on the Richter scale hit the San Fernando Valley, California. One of the major effects of this earthquake was the damage inflicted on the Lower and the Upper San Fernando Dams due to liquefaction induced deformations. Liquefaction of the hydraulic fill materials within the body of the dam caused a flow slide to occur on the upstream part of the Lower Dam leaving only about 1.5 m of freeboard. In the Upper Dam, the slide movements resulting from liquefaction of the hydraulic fill within the dam were not as severe as those in the Lower Dam. However, the crest of the dam moved about 1.5 m downstream and settled about 0.8 m. In both cases, no water was released from the reservoir.

Extensive investigations have been carried out to study the causes and mechanism of the dam slides (e.g. Seed et al., 1973, 1975; Serff et al., 1976; Jong, 1988). These investigations provide an extensive data base of the geotechnical properties at the dam sites. These data bases together with excellent records of dam deformations provide an exceptional example of the field behaviour of an earth dam which can be used to validate any analytical procedures for estimating liquefaction-induced lateral movements.
9.2. The February 1971 San Fernando Earthquake

The earthquake occurred at 6:00 a.m. local time on February 9, 1971. It had a magnitude of 6.6 on the Richter scale and a focal depth of about 13 km. The epicenter of the earthquake was located approximately 14 km from the Lower San Fernando Dam.

Several stations within the area of high intensity successfully recorded the ground accelerations produced by the earthquake. One of the most interesting ground acceleration records was obtained at a station located about 8 km south of the epicenter, about 16 m above the left abutment of the 111 m high Pacoima Dam. The horizontal peak ground acceleration from this record was 1.25 g (g is the gravity acceleration) and the vertical peak ground acceleration was 0.72 g. Some investigators, however, suggest that this high peak ground acceleration may be due to the peculiarity of the topographic condition of the recording station, that substantially amplifies the ground motion of the dam base (Seed et al., 1973).

Seed et al. (1973, 1975) also reported another set of ground accelerations that are of importance to the deformation analysis of the dams. The records were obtained from two seismoscopes located, respectively, on the east abutment and on the crest of the Lower Dam. Scott (1972) carefully examined the recorded motions from the east abutment and observed that some of the small regular waves on the trace were a peculiarity of the seismoscope. This observation enabled him to ingeniously convert the recorded motions into a time history of acceleration. The horizontal peak ground acceleration obtained from this record was 0.80 g. However, due to uncertainties in the measurements involving the condition when the pen went off the scale, Scott suggested that the range of peak ground acceleration of the earthquake in the vicinity of the dam was about 0.55 to 0.60 g. These peak ground accelerations are in accordance with the recorded peak ground accelerations in the other stations located various distances from
the epicenter (Seed et al., 1973). In the present analysis, therefore, the peak ground acceleration at the dam sites will be assumed to be 0.60 g.

9.3. Geometry of the Dams

The Lower and the Upper San Fernando Dams are located in Los Angeles, California. These two dams form the major part of the Van Norman Reservoir Complex, involving one additional water storage dam, some smaller dikes and several kilometers of channels. Both dams are founded on a recent alluvial deposit consisting of stiff clay with lenses of sand and gravel. The maximum thickness of the alluvium is approximately 11 meters.

The layer of alluvium is underlain by shales and silt stones of the Upper Miocene age. The western part of the alluvium is underlain by a massive friable sandstone of the Pico Formation of the Middle Pliocene age. The geological condition of the dam sites has been elaborated by Seed et al. (1973).

The cross-sections of the Lower and Upper Dams before the sliding are shown in Fig. 9.1.a and b. The Lower and Upper San Fernando Dams were built directly on the alluvium without stripping off the original soil surface. Although dam construction involved the use of rolled fill materials, the major portions of the two dams were built by using a hydraulic fill method of construction. Seed et al (1973) have described in more detail the method of construction of these two dams. In the following sections, only the information pertinent to this study will be highlighted.

9.3.1. The Lower San Fernando Dam

The dam mainly consists of hydraulic fill materials which form the bottom part of the dam to the elevation of about 329 m (1080 ft) at the axis and 332 m (1090 ft) at the
Fig. 9.1. Cross-section through the Lower and Upper San Fernando Dams.
upstream and downstream edges (Fig. 9.1.a). This hydraulic fill material is capped by a rolled earth fill composed of shales taken from the east abutment of the dam. Before the sliding, the dam had a height of about 43 m (142 ft), a crest width of 6 m and a length of 634 m. The upstream slope of the dam was 2.5:1.0, and the downstream slopes were 2.5:1.0 and 4.5:1.0.

9.3.2. The Upper San Fernando Dam

Similar to the Lower Dam, the main body of the dam, which comprises of hydraulic fill materials, was constructed directly above the foundation soil to an elevation of 366 m (1200 ft). The materials used for this construction were obtained from the valley floor. On the upstream side of the dam, the hydraulic fill is underlain by compacted dry fill obtained from the borrow pit to an elevation of about 371 m (1218 ft). Fig. 9.1.b shows the geometry of the dam before the earthquake. It had a height of about 15 m (50 ft), a crest width of 6 m (20 ft) with downstream and upstream slopes of 2.5:1.0 and a 30 m wide berm at an elevation of 366 m (1200 ft).

9.4. Effects of the 1971 Earthquake

9.4.1. The Lower San Fernando Dam

The earthquake caused major slides in the upstream slope and the upper part of the downstream slope of the Lower Dam. As a result, only about 1.5 m height of freeboard remained immediately after the slide which prevented the water from overtopping the dam. Concerns arose that further sliding might occur especially due to possible major after-shocks. Therefore, some 80,000 people living downstream from the dam were evacuated until the water was drawn down to a safe elevation.

After the water was sufficiently drawn down, the extent of damage to the dam became more easily identified. At this time, much of the slide debris was visible. Seed et
al. (1973) reported that the upstream slope with concrete face had moved horizontally outward and vertically downward into the reservoir. They also observed that the surface of the slide debris consisted of scarps suggesting the occurrence of multiple shear failure zones in the dam. Sand boils were also observed in the some of the depressions in the lower portions of the slides.

Extensive site investigations including boring and large trench excavations were carried out (Seed et al., 1973). The large trenches were very useful in identifying the soil layers involved in the slide movements. From the observations of these trenches, they found strong evidence that the slides were caused by the liquefaction of the hydraulic fill materials near the base of the upstream shell. Moreover, the results of observations on the type of soils involved in the slides enabled them to reconstruct the initial configuration of the dam. The cross-sections of the dam after the slides and the reconstructed cross-section are shown in Fig. 9.2.

Based on careful observations of the field evidence, Seed et al (1973) postulated the sliding mechanism in the Lower Dam. The slide movements were initiated by the liquefaction of the hydraulic fill near the base of the embankment, as shown by the black area in Fig. 9.2. As the sliding progressed, the soil mass above the liquefied layer broke into blocks (blocks 5 to 11) and displaced horizontally through a distance varying from 9 to 45 m into the reservoir. This movement removed the support from the upstream side of the clay core. Since the clay core could no longer support the overlying soil mass, secondary slide movements were initiated. These slides removed the crest of the dam and the upper part of the downstream slope (blocks 1 to 4) and left the dam in a precarious condition.
Fig. 9.2. Cross-sections Through the Lower San Fernando Dam: (a) Conditions After the 1971 Earthquake, and (b) A Schematic Reconstruction of the Failed Section (After Seed and Harder, 1990). Note: 1 ft = 0.3048 m.
9.4.2. The Upper San Fernando Dam

As previously mentioned, the effects of the earthquake on the Upper Dam were not as severe as those on the Lower Dam. However, large horizontal and vertical displacements were also observed. The crest of the dam settled about 0.8 m and was displaced about 1.5 m horizontally downstream. Several longitudinal cracks were observed along almost the full length of the dam. These cracks were similar in nature to those observed in the Lower Dam suggesting multiple shear zones of failure due to the downstream movement of the main body of the dam. A vertical longitudinal crack was also observed in the downstream slope of the rolled fill section and sand boils were noticed below the downstream toe.

Shortly after the earthquake, measurements of dam deformations were taken at several points on the maximum section of the dam. These points were the upstream parapet wall (point A), the midpoint of the downstream slope of the rolled fill section (B), the upstream and downstream ends of the berm (C and D), the midpoint of the downstream slope (E) and the downstream toe of the dam (F). The measured displacements are presented in Table 9.1 and the directions are shown in Fig. 9.3.

Table 9.1. Measured Displacements of the Upper San Fernando Dam (after Serff et al., 1976).

<table>
<thead>
<tr>
<th>Points</th>
<th>Measured Displacements (m)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal</td>
<td>Vertical</td>
</tr>
<tr>
<td>A</td>
<td>1.49</td>
<td>-0.76</td>
</tr>
<tr>
<td>B</td>
<td>1.49</td>
<td>-0.51</td>
</tr>
<tr>
<td>C</td>
<td>1.95</td>
<td>-0.06</td>
</tr>
<tr>
<td>D</td>
<td>2.19</td>
<td>-0.43</td>
</tr>
<tr>
<td>E</td>
<td>1.76</td>
<td>-0.54</td>
</tr>
<tr>
<td>F</td>
<td>1.09</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

Note: - Negative vertical displacements indicate downward movements.
- See Fig. 9.3 for locations of the points.
Chapter 9: Deformation Behaviour of the San Fernando Dams

Fig. 9.3: Displacements of the Upper San Fernando Dam due to the 1971 Earthquake (After Serff et al., 1976).
The horizontal displacements of the dam were increased toward the downstream berm whereas the vertical displacements were greatest at the crest. The horizontal and vertical displacements at the crest were respectively 1.50 m downstream ward and 0.76 m downward. The berm of the dam displaced 1.95 m at the upstream end and 2.19 m at the downstream edge. The settlements at these points were 0.06 and 0.43 m. The horizontal displacements at the downstream slope were 1.75 m at the middle of the slope and 1.10 m at the slope toe. The vertical movements at these points were 0.54 m downward and 0.06 m upward respectively.

9.5. Soil Investigations

9.5.1. The Lower San Fernando Dam

The results of the Standard Penetration Test on the Lower Dam reported by Seed et al. (1973) show that the SPT values of the hydraulic fill materials in the upstream shell of the dam is between 10 and 20 blows per foot. The results for the downstream shell, however, show slightly higher SPT values ranging between 13 to 25 blows per foot. The fines content (grain size diameter < 0.076 mm) of the hydraulic materials for both the upstream and downstream shell obtained from the grain size analyses varies from about 5 percent to about 25 percent with a mean value of 15 percent.

A number of static triaxial tests were carried out on the samples obtained from both the hydraulic fill and the alluvium from the Lower Dam.

In terms of effective soil stress parameters, the results of Consolidated Undrained (CU) and Consolidated Drained (CD) triaxial tests indicated that an internal friction angle of the hydraulic fill obtained from these tests was 37 degrees with zero cohesion intercept. The angle of internal friction of the alluvium was 38 degrees also with zero cohesion intercept.
Seed et al. (1988) analyzed the SPT data of the 1971 and the 1985 field investigation and computed the corresponding $(N_1)_{60-CS}$ values for certain sublayers of the dam. The representative $(N_1)_{60-CS}$ values for downstream and upstream shell of the Lower Dam were found to be 12.5 and 11.5. These values correspond to $(N_1)_{60-CS}$ of 14.5 and 13.5 with a fines contents correction of 2.

Based on extensive laboratory tests and the steady state procedures proposed by Poulos et al. (1985), Seed et al. (1988) found that the average residual strength of the upstream hydraulic fills materials was about 40 kPa. This value was approximately the same as the average initial driving stress in the upstream shell of the dam. Only relatively small deformation of the dam can be expected if this value is used for computing dam deformation. Since in fact the dam developed very large deformation, the residual strength of hydraulic fills must have been much less than 40 kPa. Seed et al. (1988) determined the residual strength to be between 14 to 24 kPa based on the final configuration of the upstream slope of the dam after movement had stopped.

Using the same steady state approach and considering the dynamics of the slides and various sources of uncertainty, Davis et al. (1988) estimated an average residual strength for the upstream hydraulic fill to be about 26 kPa. Baziar and Dobry (1991) and Dobry et al. (1992) carried out laboratory tests on the samples from the Lower San Fernando Dam. They used a discontinuous sample preparation method to simulate the stratification of soil layers in the field. They found that the ratio of residual strength to its overburden stress, $S_r/\sigma'_{vo}$, was in the range of 0.12 to 0.145.

### 9.5.2. The Upper San Fernando Dam

Soil investigations including boring, trenching, and seismic surveys as well as laboratory tests, were carried out in the Upper San Fernando Dam shortly after the earthquake. Seventeen bore holes and three trenches were made along three cross-
sections of the dam. One more trench was excavated several months later. Seed et al. (1973) and Lee et al. (1975) have reported these investigations in more detail.

The rolled fill section at the top of the dam generally consisted of clayey fine sand to sandy clay with penetration resistance of 15 to 25 blows/foot. The hydraulic fill materials at the upstream part of the dam mainly consisted of alternating layers of clean sand and silty sand with occasional lenses of silty clay. The penetration resistance of this material varied from 10 to 25 blows/foot. The core of the dam predominantly consisted of clay with interbedded silt and silty fine sand. The blow counts of the core materials were lower than those measured in any other part of the dam shell. The SPT blow counts were about 5 to 15 blows/foot. The boundary between the hydraulic fill materials and the foundation alluvium was clearly identified by a significant increase in blow count. The alluvium comprised of layers or pockets of sandy gravel, clayey gravel and gravely silty sand. The erratic distributions of blow counts in this layer reflects the heterogeneity of this layer.

A number of grain size analyses were carried out on the samples taken from several points between the outer shell and the center of the dam. It was found that the soil samples became progressively finer from the downstream shoulder towards the center of the dam. The sample in the downstream edge of the berm was predominantly coarse gravelly sand whereas near the central part of the dam was mainly silt and clay. The average fines content of the samples near the center of the dam was about 10 percent. Based on the SPT blow counts corrected for the effects of effective overburden stress and fines content, Seed and Harder (1990) took a value of $(N_{1})_{60-cs}$ of 15 for the hydraulic fill materials.

Seed et al. (1973) presented the results of CU and CD triaxial tests on coarse and fine sands from the hydraulic fill. The data show no appreciable difference between the
strength parameters of the coarse and fine sands determined from both tests. The results indicated zero cohesion intercept, $c'$, and an angle of internal friction, $\phi'$, of 37 degrees.

A similar trend was also observed in the results of static triaxial tests on undisturbed samples of the alluvium. There was virtually no difference between the parameters of the coarse and fine sands. The results indicated an angle of internal friction, $\phi'$, of 38 degrees and zero cohesion intercept, $c'$.

9.6. Deformation Analysis of the Lower San Fernando Dam

9.6.1. Review of Previous Studies

Previous investigations on the sliding of the Lower San Fernando Dam (Seed et al., 1973; 1975a; 1975b; Seed, 1979) showed that liquefaction had occurred in the lower section of the upstream shell of the dam. Seed et al. (1973) have successfully reproduced the occurrence of liquefaction in these zones by using the dynamic effective stress approach. Using the base input motion obtained from the abutment seismograph, Seed et al. (1973) evaluated the average cyclic stress developed in the soil elements in the dam after 10.5 seconds of shaking. This is an approximate time when the peak ground acceleration occurred. The stresses induced in the soil elements after the first 10.5 seconds of shaking were represented by 2 cycles of an equivalent uniform cyclic stress. The computed stresses were subsequently compared with the laboratory cyclic stress required to cause liquefaction in 2 cycles of equivalent uniform cyclic stress. In this case, the triggering of liquefaction was defined as the attainment of 5 percent double amplitude cyclic strains in triaxial tests.

Results of the analyses showed that after 10.5 seconds of shaking, liquefaction developed in an extensive zone in the interior portion of the upstream shell and in a smaller portion of downstream shell, as shown by a black area in Fig. 9.4. An additional
Fig. 9.4. Geometry of the dam, water table and estimated zone of liquefaction of the Lower San Fernando Dam (after Seed et al., 1973).
zone of liquefaction was also predicted at the end of the earthquake (after 15 seconds of the earthquake shaking) in the upstream shell but only a small new zone developed in the downstream shell of the dam.

As discussed in Chapter 2, one of the most serious consequences of soil liquefaction is the severe reduction of soil stiffness and strength at small strains. Liquefied soil may temporarily lose its ability to support the soil elements above it. As a result, the static driving stress initially acting on this soil will be transferred to adjacent non-liquefied soil elements. This in turn may result in large deformation of soil mass. The liquefied soil may regain its stiffness and possibly its strength at large strains depending on the relative density of the granular soils. It was this liquefied soil that was primarily responsible for the major deformations of the dam.

Of special interest in the Lower San Fernando Dam failure is that the fact that the slide movements took place after the earthquake had stopped. Evidence collected by Seed (1979) revealed that the sliding occurred approximately 30 seconds after the cessation of the earthquake. Consequently, the sliding was driven only by its own gravity loads and no inertia forces from the earthquake were acting during the sliding. This implies that at the end of the earthquake shaking, the dam was at the condition of incipient failure (Seed, 1979). Any small change in soil strength at the potential failure zone would lead to a progressive failure of the dam.

The fact that the dam did not fail under strong ground accelerations also implies that despite an extensive liquefaction zone in the upstream shell of the dam, the dam was sufficiently strong to withstand any significant deformation that might lead to a catastrophic failure during severe ground shaking. This case represents one of the very rare examples of the deformation behaviour of liquefied soils in the field. Knowledge obtained from the laboratory regarding the behaviour of soil under cyclic loading is very
important to understand the actual behaviour of liquefied soils in this dam during earthquake.

The interesting behaviour of the Lower San Fernando Dam during this earthquake have challenged many researchers (e.g., Seed et al., 1975; Seed, 1979; Gu et al., 1993). Seed (1979), for example, advanced a hypothesis that the high stability of the dam during the earthquake was due to dilative response of the zone of non-liquefied soil near the toe of the dam. During the earthquake, the compacted and dilative material at this zone developed high undrained strength that prevented the slide from occurring during the shaking although some of the hydraulic fills have liquefied. As water migrated from the reservoir to this zone, the soil strength gradually decreased from its undrained strength to its drained value. This water migration ultimately resulted in the initiation of the sliding when the soil strength sufficiently decreased to a value lower than the driving stress of the dam.

Seed (1979) presented the results of limit equilibrium analyses for two conditions: (1) the condition immediately after the earthquake; and (2) a short period after the cessation of the earthquake. Thus, both analyses were carried out under static condition and no inertia forces from the earthquake were included. These results of analyses are shown in Figs. 9.5.a and b.

Seed argued that for the zone under the conditions of very high back-pressure such as in the zone near to the upstream toe of the dam, the tendency of negative pore pressure development during cyclic loading would give high undrained strength value which would also be independent of the initial effective confining stress. For Case 1, Seed used an undrained strength ($S_u$) value of 172 kPa (3600 psf) obtained from the results of laboratory tests on the samples from the toe area. Seed also took a value of 158 kPa (3300 psf) for the rolled fill material and assumed zero strength in the liquefied zone. The
Chapter 9: Deformation Behaviour of the San Fernando Dams...

Fig. 9.5. Limit Equilibrium Analyses of the Lower San Fernando Dam (After Seed, 1979).

(a) Stability of Lower San Fernando Dam immediately after earthquake. Note: undrained loading of all zones; shear resistance drops to zero in zone where \( r_u = 1 \) (condition of cyclic mols).

(b) Stability of Lower San Fernando Dam a short time after earthquake motion stop. Note: undrained loading of all zones; shear resistance drops to zero in zone where \( r_u = 1 \) (condition of cyclic mols).
computed factor of safety against static stability for the upstream slope was about 1.4 indicating a stable condition at this stage (Fig. 9.5.a).

Seed (1979) argued that when the earthquake stopped, water would tend to migrate from the zone of high pore pressure to the dilatant zone where the pore pressure was negative or less than hydrostatic. As a result, the soil strength would decrease from its undrained value to a smaller drained value. This condition was assumed for Case 2. Using the strength parameters shown in Fig. 9.5.b, the computed factor of safety against sliding was about 0.80 indicating that flow failure of the dam would be expected to occur under its own weight. However, a factor of safety slightly less than unity may still be acceptable since the driving stresses would decrease with increasing deformations. Although large deformations may occur, the movements would eventually stop when the driving stresses were equal to or less than the soil resistance. There is no simple way of knowing these deformations from the results of limit equilibrium analyses. Thus, for cases involving a factor of safety less than unity in which large deformations may occur, limit equilibrium analysis alone may not be sufficient to assess the stability of dams. Deformation criteria as proposed by Newmark (1965) will provide better information on dam stability. This is particularly true for cases involving liquefied soils of loose to medium dense sands whose stress-strain characteristics are totally different than those of the pre-earthquake condition.

The deformations of the Lower San Fernando Dam due to the 1971 earthquake were analyzed using the extended Newmark procedure discussed in Chapter 5. This study was intended to check the capability of the proposed procedure in predicting the liquefaction-induced deformation of this dam.
9.6.2. Soil Parameters Used in the Analyses

The pre-earthquake soil parameters for the analyses were taken from Seed et al. (1973). The post-earthquake soil parameters were based on the empirical formula proposed by Byrne (1990) in conjunction with the assumption of post-liquefaction stress-strain curve discussed in Chapter 4. The zone of liquefaction was assumed to be that computed by Seed et al. (1973).

The average factor of safety against triggering of liquefaction ($F_L$) in the liquefaction zone was approximately 0.5. This was determined from the ratio of the Critical Resistance Ratio (CRR) of the hydraulic fill materials and the Critical Stress Ratio (CSR) due to earthquake computed by Seed et al. (1973). From Fig. 4.11, the limiting strain of the liquefied soils with average ($N_d')_{60-cs}$ value of 13.5 and $F_L$ of 0.5 was about 35 percent.

The residual strength of the liquefied soil computed using Byrne's formula (Eqs. 2.1 and 2.2) for ($N_d')_{60-cs}$ of 13.5 was about 29 kPa. For comparison, the average residual strengths computed by Seed et al. (1988) and Davis et al. (1988) by using the steady state approach were respectively about 40 kPa and 26 kPa.

Fig. 9.2 shows that the average height of the soil column above the liquefied zone was about 20 m. By assuming the unit weight of the overlying soil to be 19.8 kN/m3, the effective overburden stress for this soil element was about 200 kPa. Therefore, the ratio of undrained strength to the initial effective stress is 0.145. This value was in reasonable accord with the results of laboratory investigations carried out by Baziar and Dobry (1991) and Dobry et al. (1992) which was in the range of 0.12 to 0.145.

Recent laboratory data show that the residual strength of liquefied soils depends on the effective confining stress (Ishihara, 1993; Salgado and Pillai, 1993). Variation of residual strength can be expected for a thick liquefaction zone involving different initial effective stresses at different elevations of soil layers. Accordingly, it seems reasonable to
use a residual strength ratio \( (S_0/\sigma_{vo}) \) instead of a constant residual strength for cases involving a thick liquefied zone. For the present deformation analyses, it was decided to use both constant residual strength and residual strength as a function of initial effective stress. A constant residual strength of 29 kPa and a residual strength ratio of 0.14 were considered.

The hyperbolic stress-strain soil parameters for the liquefied soil were obtained by assuming the post-liquefaction stress-strain curve to be linear elastic plastic (Fig. 4.5). This assumption implied an \( R_f \) value of zero. The shear modulus exponent, \( n \), was taken to be zero. Based on the residual strength of 29 kPa and the limiting strain of 35 percent, the shear modulus number, \( k_s \), was computed to be 0.8.

The bulk modulus number, \( k_b \), for pre- and post earthquake condition was set to be large to simulate undrained condition during the sliding. In this case, the bulk modulus number, \( k_b \), was taken to be 2000 and bulk modulus exponent, \( m \), of 0.25.

Since the increase in pore water pressure also occurred in the other part of the shell and in the core of the dam, although not as high as those in the liquefied zone, the soil elements here must also have experienced a stiffness reduction at the end of the earthquake shaking. This stiffness reduction was taken into account in the analyses by reducing the \( k_s \) value as much of 50 percent for the shell and 70 percent for the core from their pre-earthquake values. This reduction was estimated from the laboratory data presented by Yasuda et al. (1991) and Thomas (1992).

The inertia forces from the earthquake was considered to be zero because the movements took place at a period of no shaking.

The finite element mesh used in this analyses is shown in Fig. 9.6. Material types, zone of liquefaction and approximate water table during the earthquake are also shown in
this figure. The pre- and post-earthquake soil properties used in the analyses are presented in Table 9.2.

9.6.3. Cases Considered

Four cases were considered in the analyses. Case 1 was the case immediately after the cessation of earthquake shaking. In this case, the soil elements at the toe of the dam were assigned a high undrained strength value as suggested by Seed (1979), i.e. 172 kPa. Case 2 was the condition a short time after the earthquake ceased. For this case, the soil elements at the toe were assigned their drained strength and the soil properties were assumed to be the same as those of the shell materials. In both cases, the undrained strength of liquefied soil was taken to be 29 kPa. Case 3 was the same as Case 2 except that the residual strength of liquefied soil was taken to be 0.14 $\sigma'_{vo}$. In the three cases above, the changes in geometry were considered in the analysis. Case 4 was the same as Case 2 except that changes in geometry were not considered in the analysis.

9.6.4. Results of the Analysis

Typical results are shown in Fig. 9.7 in terms of deformed mesh and vector displacements at each node. The magnitude of displacements computed at the crest (node 270) and at the toe (node 168) for all cases considered are shown in Table 9.3.

As can be seen in Table 9.3, the proposed procedure predicted significant displacements at the crest and the toe in all cases including those in Case 1. In Case 1, the crest moved about 0.7 m upstream and settled about 1.2 m whereas the toe moved about 1.4 m upstream and 0.3 m upward. These deformations are larger than the those reported by Seed (1979).
Chapter 9: Deformation Behaviour of the San Fernando Dams

Fig. 9.6. Finite element mesh used in the analyses of the Lower San Fernando Dam
Chapter 9: *Deformation Behaviour of the San Fernando Dams*

### Table 9.2. Soil Properties used in the analyses of the Lower San Fernando Dam

<table>
<thead>
<tr>
<th>Material Type</th>
<th>$k_f$</th>
<th>$n$</th>
<th>$k_b$</th>
<th>$m$</th>
<th>$\phi'$ degree</th>
<th>$c'$ kPa</th>
<th>$R_f$</th>
<th>$\gamma_s$ kN/m$^3$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Foundation</td>
<td>150</td>
<td>0.40</td>
<td>2000</td>
<td>0.25</td>
<td>38</td>
<td>0</td>
<td>0.76</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>2. Dam Shell</td>
<td>230</td>
<td>0.50</td>
<td>2000</td>
<td>0.25</td>
<td>37</td>
<td>0</td>
<td>0.72</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>3. Dam Core</td>
<td>230</td>
<td>0.50</td>
<td>2000</td>
<td>0.25</td>
<td>37</td>
<td>0</td>
<td>0.72</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>4. Rolled Fill</td>
<td>150</td>
<td>0.76</td>
<td>1000</td>
<td>0.25</td>
<td>25</td>
<td>125</td>
<td>0.90</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>5. Liq. Soils</td>
<td>230</td>
<td>0.50</td>
<td>2000</td>
<td>0.25</td>
<td>37</td>
<td>0</td>
<td>0.72</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

*Note: The numbers in brackets indicate the properties after earthquake.*

### Table 9.3. Magnitude of displacements computed at the crest and at the toe of the Lower San Fernando Dam

<table>
<thead>
<tr>
<th>Case</th>
<th>$S_r$ (kPa)</th>
<th>Toe (node 168)</th>
<th>Crest (Node 270)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Hor. Displ. (m)</td>
<td>Vert. Displ. (m)</td>
<td>Hor. Displ. (m)</td>
</tr>
<tr>
<td>1</td>
<td>29</td>
<td>-1.28</td>
<td>0.28</td>
<td>-0.70</td>
</tr>
<tr>
<td>2</td>
<td>29</td>
<td>-2.76</td>
<td>0.14</td>
<td>-1.21</td>
</tr>
<tr>
<td>3</td>
<td>0.14 $\sigma_{vn}'$</td>
<td>-2.69</td>
<td>0.13</td>
<td>-1.10</td>
</tr>
<tr>
<td>4</td>
<td>29</td>
<td>-4.98</td>
<td>0.09</td>
<td>-1.14</td>
</tr>
</tbody>
</table>
Fig. 9.7 Typical results: Deformed mesh and displacement vectors of the Lower San Fernando Dam.
Seed et al. (1973, 1975) reported that the first slide movements in the Lower Dam occurred due to liquefaction of the hydraulic fill material. The second one was initiated due to the loss of support of the soil above the clay core. As shown in Fig. 9.7, the proposed procedure correctly predicted the first slide movement in the upstream part of the dam due to soil liquefaction. The predicted horizontal and vertical displacements at the crest of the dam in Case 2 and 3 were in the order of 1 m. The horizontal displacements at the toe were about 3 m for both cases. The predicted horizontal and vertical displacements at the crest in Case 4 was about 1 m. At the toe, however, the horizontal displacements was very large, i.e. about 5 m. The displacements in the order of 3 to 5 m can be interpreted as a failure, as will be discussed later. The pattern of displacement shown in Fig. 9.5 is in general agreement with the observed cross-section of the dam after failure (Fig. 9.2).

It is also interesting to note that in this case, the use of constant residual strength and residual strength as a function of initial vertical effective stress did not result in significant difference in the computed displacements. For both Cases 2 and 3, the computed displacements at the dam crest were about 1.2 m horizontal, and about 1.0 m vertical. The predicted horizontal and vertical displacements at the toe were about 3.2 m and 0.14 m. This was probably because the soil stiffness in the liquefied zone in Cases 2 and 3 were not significantly different.

For cases involving large displacements of soil masses under water, such as in the case of Lower Dam, the computed displacements in the order 3 m should be interpreted cautiously. In terms of shear strains, the 3 m displacement at the toe was approximately equal to 20 percent. These large strain might have caused some cracks in the upstream face of the dam such as evidenced by the cracks observed between block 5 to block 11 in Fig. 9.2. At the toe area, these cracks would allow the water from the reservoir to get
into the sliding mass and weaken the soil. Thus, while the driving stresses of the dam were reduced due to changes in geometry during sliding, the soil resistance also decreased with increased deformations. This case was similar to the case with constant driving stress and constant soil resistance as depicted in Fig. 4.3.

As expected, the displacements computed using constant driving stress in Case 4 (no change in geometry during iteration) were larger than those computed when changes in geometry was considered (Case 3). The computed displacements using the assumption of constant driving stress were, respectively, 4.98 m horizontal to the upstream direction and 0.09 m vertically upward at the toe area. At the crest, the computed displacements were 1.14 m horizontally upstream and 0.71 vertically downward.

9.7. Deformation Analyses of the Upper San Fernando Dam

Similar to the Lower Dam, the deformation which developed in the Upper San Fernando Dam due to the 1971 earthquake was predicted using the proposed simplified method. Previous studies on the deformation of this dam using the same method have been reported by Byrne et al. (1992).

9.7.1. Review of Previous Studies

Seed et al. (1973) evaluated the dynamic response of the Upper San Fernando Dam using the total stress dynamic-finite element analysis. Using the modified Pacoima record as input base rock motion, they computed the dynamic stress which developed in the soil elements in the dam. They showed that an extensive zone of liquefaction developed after the first 6 seconds of the shaking, as shown in Fig. 9.8. An additional shaking (from 6 seconds to 15 seconds of earthquake shaking) caused an increase in the extent of liquefaction within the dam.
Fig. 9.8. Geometry of the dam, water table and estimated zone of liquefaction of the Upper San Fernando Dam (after Seed et al., 1973).
Seed et al (1973) carried out limit equilibrium stability analyses to assess the post-earthquake stability of the dam. For the analyses, they used strength parameters from CU tests and neglected the seismic forces. The results show that the computed minimum factor of safety against sliding was about 1.75. This indicates that the dam would be stable even under the effects of small inertia forces that might develop during aftershocks.

9.7.2. Soil Properties used in the Analyses

The pre-earthquake soil parameters were based on the data reported by Seed et al. (1973) whereas the post-earthquake soil parameters were based on the corrected SPT value reported by Seed and Harder (1990), as will be described later. The zone of liquefaction, as shown in Fig. 9.8, were based the results of dynamic analyses performed by Seed et al. (1973).

Although the zone of liquefaction shown in Fig. 9.8 consisted of several separated zones, most of the liquefied zones shown in this figure liquefied at about 6 seconds of earthquake shaking. Thus, the post-earthquake parameters for the liquefied zones can be considered the same. Moreover, since the earthquake lasted about 15 seconds, it would seem reasonable to assume an average factor of safety against triggering of liquefaction ($F_L$) of 0.4 for all the liquefied zones. Using the relationship between residual strength and $(N_1)_{60-CS}$ value in Fig. 4.11, the limiting strain of the liquefied soils for $(N_1)_{60-CS}$ of 15 and $F_L$ of 0.4 was taken to be 30 percent.

The computed residual strength for $(N_1)_{60-CS}$ of 15 using Byrne's formula (Eqs. 2.1 and 2.2) was about 38 kPa. The average height of a soil column above the soil elements in the middle of liquefied zones was about 8 m. Considering the elevation of water table in the dam, the approximate overburden stress here was 110 kPa. Therefore, the normalized residual strength ratio ($S_r/\sigma_{vo}$) was about 0.34. This value was felt to be too high for residual strength of soils. In comparison, the normalized residual strength of
undisturbed samples from Duncan Dam foundation was about 0.21 $\sigma'_{vo}$ with an equivalent clean sand SPT blow counts of about 13 (Salgado and Pillai, 1993, Byrne et al., 1993). The residual strength value at the 33rd percentile obtained from Seed-Harder's chart is about 24 kPa (500 psf) for $(N_1)_{60-cs}$ of 15. This leads to a normalized residual strength of 0.22 $\sigma'_{vo}$. In the present analysis, both constant residual strength of 24 kPa and normalized residual strength of 0.22 $\sigma'_{vo}$ were used.

Seed et al. (1973) reported high increase in pore pressure in the core and the shell of the dam due to the earthquake. The stiffness reduction of the soils due to pore pressure increase was accounted for by reducing the shear modulus number, $k_g$, of these materials. The post-earthquake $k_g$ of the core and the shell were respectively taken to be 30 percent and 50 percent of their original values. The other post-earthquake parameters for these materials were assumed to be the same.

The post-earthquake soil properties for rolled fill and alluvium were considered to be the same as the original values.

The settlement due to pore pressure dissipation of the soils in the liquefied zones were taken into account by specifying the post-earthquake potential volumetric strains. For $(N_1)_{60-cs}$ of 15 and $F_L$ of 0.4, the chart prepared by Ishihara and Yoshimine (1992) yielded a volumetric strain of 2.5 percent. This value was comparable with the results of 2 percent volumetric strain used by Byrne et al. (1992) based on Tokimatsu-Seed chart.

The peak ground acceleration of 0.6 g obtained from the modified Pacoima records was used to determine the velocity of the soil mass. The mass velocity was required to estimate the kinetic energy of the earthquake. The initial velocity was taken to be 0.6 m/s. A constant velocity of 0.60 m/s was assumed for each soil element within the dam. This assumption was reasonable since no significant ground amplification in the dam was reported (Seed et al., 1973).
The finite element mesh, approximate water table during the earthquake and the material types used in the analyses are shown in Fig. 9.9. The pre- and post-earthquake soil properties are presented in Table 9.4.

9.7.3. Cases Considered

Several cases were considered in the analyses and these are listed in Table 9.5.

9.7.4. Results of the Analyses

Typical results in terms of deformed mesh and vector displacements are shown in Fig. 9.10. The magnitude of displacements for several locations in the dam are presented and compared with the measured values in Table 9.6.

Comparing Fig. 9.10 with Fig 9.3, it can be seen that the procedures predict the pattern of displacements of the dam reasonably well. The magnitude of displacements obtained from application of horizontal seismic coefficients (Cases 2 to 5) are also in the same order of magnitudes with the measured values. However, the prediction using a combination of vertical and horizontal seismic coefficients to achieve energy balance (Case 1) resulted in much smaller displacements than those measured in the field. For this case, the use of a horizontal seismic coefficient to balance the external energy seems to be more appropriate. This is probably due to the geometry of the Upper Dam that tends to be more flexible under horizontal seismic loading than under vertical seismic loading. Consequently, the external energy tends to be dissipated in the simple shear mode of deformations.
Fig. 9.9. Finite element mesh, water table and material types used in the analyses of the Upper San Fernando Dam.
Table 9.4: Soil Properties used in the analyses of the Upper San Fernando Dam.

<table>
<thead>
<tr>
<th>Material Type</th>
<th>ks</th>
<th>n</th>
<th>kb</th>
<th>m</th>
<th>φ' degree</th>
<th>c' kPa</th>
<th>Rf</th>
<th>γs kN/m³</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Foundation or Alluvium</td>
<td>117</td>
<td>0.80</td>
<td>2000</td>
<td>0.40</td>
<td>37</td>
<td>0</td>
<td>0.66</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>2. Dam Shell</td>
<td>175</td>
<td>0.52</td>
<td>2000</td>
<td>0.26</td>
<td>37</td>
<td>0</td>
<td>0.78</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(87.5)</td>
<td>(0.52)</td>
<td>(2000)</td>
<td>(0.26)</td>
<td>(37)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Dam Core</td>
<td>175</td>
<td>0.52</td>
<td>2000</td>
<td>0.26</td>
<td>37</td>
<td>0</td>
<td>0.78</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(52)</td>
<td>(0.52)</td>
<td>(2000)</td>
<td>(0.26)</td>
<td>(37)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Rolled Fill</td>
<td>125</td>
<td>0.76</td>
<td>1000</td>
<td>0.38</td>
<td>25</td>
<td>125</td>
<td>0.90</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>5. Liq. Soils</td>
<td>175</td>
<td>0.50</td>
<td>2000</td>
<td>0.26</td>
<td>37</td>
<td>0</td>
<td>0.78</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.7)</td>
<td>(0.0)</td>
<td>(2000)</td>
<td>(0.26)</td>
<td>(0)</td>
<td></td>
<td>(24)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>and</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.22σv₀</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Note: Numbers in brackets indicate the properties after the earthquake.
Chapter 9: Deformation Behaviour of the San Fernando Dams

Fig. 9.10. Typical results: Deformed mesh and displacement vectors of the Upper San Fernando Dam.
Chapter 9: Deformation Behaviour of the San Fernando Dams...

Table 9.5. Cases considered in the analyses of the Upper San Fernando Dam

<table>
<thead>
<tr>
<th>Case</th>
<th>$S_\sigma$, kPa</th>
<th>Type of seismic coefficients</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24</td>
<td>kv+kh</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>kh</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$0.22\sigma_{vo}'$</td>
<td>kh</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>kh</td>
<td>reported by Byrne et al. (1992)</td>
</tr>
</tbody>
</table>

Table 9.6. Predicted and measured displacements at several locations in the Upper San Fernando Dam

<table>
<thead>
<tr>
<th>Points</th>
<th>Horizontal Displacements (m)</th>
<th>Vertical displacements (m)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured 1</td>
<td>Case 2</td>
<td>Case 3</td>
</tr>
<tr>
<td>A</td>
<td>1.49</td>
<td>0.13</td>
<td>1.44</td>
</tr>
<tr>
<td>B</td>
<td>1.49</td>
<td>0.22</td>
<td>1.58</td>
</tr>
<tr>
<td>C</td>
<td>1.95</td>
<td>0.26</td>
<td>1.84</td>
</tr>
<tr>
<td>D</td>
<td>2.19</td>
<td>1.86</td>
<td>2.22</td>
</tr>
<tr>
<td>E</td>
<td>1.76</td>
<td>0.55</td>
<td>2.16</td>
</tr>
<tr>
<td>F</td>
<td>1.09</td>
<td>0.41</td>
<td>1.11</td>
</tr>
</tbody>
</table>

Note: See Fig. 9.10. for locations of the points.
As shown in Table 9.6, the computed horizontal displacements in Cases 2 to 4 at the locations of interest (points A to F) are in good agreement with the measured values. The predicted vertical displacements, however, are generally less than the measured values. Only at the crest do the vertical displacements agree quite well with the measurements.

The lower vertical displacements predicted at the downstream toe of the dam suggest that liquefaction might also have developed at this area. This is confirmed by the observation of Seed et al. (1973) who reported that sand boils, longitudinal cracks and bulging developed at the toe area which indicated that the liquefaction zone likely extended to the toe of the dam. In the present study, however, the possibility that the liquefaction zone extended to the toe area was not investigated further.

Case 4 in Table 9.6 presents the results of previous studies reported by Byrne et al. (1992). Different assumptions were taken in the post-earthquake properties of the dam shell and the core. However, the post-earthquake properties of the liquefied soils were similar. Only slight differences in the post-liquefaction volumetric strain exists. In spite of these differences, the results computed by Byrne et al. (1992) are essentially the same as those obtained in the current studies. The results confirm that the post-earthquake stress-strain parameters, particularly shear modulus and residual strength, are the most important factors in predicting liquefaction-induced deformations.

As in the case of Lower Dam, the use of constant residual strength resulted in a similar displacement pattern and magnitudes to those obtained using residual strength as a function of effective overburden stresses. The thickness of the liquefied soils in this case probably does not cause a significant difference between the soil strength (and also stiffness) at the top and the bottom layer of the liquefaction zones.
9.8. Summary

The proposed procedure for predicting liquefaction-induced deformations of earth structures have been applied to predict the deformation of the Lower and Upper San Fernando Dams due to the 1971 San Fernando earthquake. The key parameters for the analyses are the post-earthquake stress-strain parameters of liquefied soils. These parameters can be determined based on data from routine site investigations and laboratory tests, in conjunction with the available correlations between $(N_1)_{60-\text{cs}}$ values with residual strength, limiting shear strains and volumetric strains.

The proposed simplified procedure predicts the upstream slide movements of the Lower San Fernando Dam which are in agreement with the observations. The procedure also predicts the downstream movements of the Upper San Fernando Dam that are in good agreement with the observations in terms of both the pattern and magnitude of displacements. The success of these predictions again suggests that the proposed two-dimensional simplified method provides a good alternative for predicting liquefaction induced ground deformations of earth dams.
10.1. Introduction

During the Izu-Ohshima-Kinkai earthquakes of January 1978, two tailings dams associated with Mochikoshi gold mining company failed due to liquefaction of the tailings materials behind the dams. Dam No. 1 failed almost simultaneously with the occurrence of the main shaking. Dam No. 2, on the other hand, failed about 24 hours after the main shocks (Ishihara, 1984; Okusa et al., 1984). The delayed failure of the No. 2 dam was postulated to be due to upward movement of the phreatic surface resulting from the liquefaction of the tailings deposits behind the dam.

Shortly after the earthquake, Japanese engineers conducted extensive site investigations and laboratory testing programs to study the causes and mechanism of the dam slides (Okusa and Anma, 1980; Ishihara, 1984; Okusa et al., 1984). The information about the geotechnical condition and soil properties at the dam sites, in addition to the reported failure mechanism of both dams, made this case one of the several unique case histories of dam failure caused by earthquake-induced liquefaction. This case can be used to check the validity of the proposed procedures for predicting the deformation behaviour of earth-structures under earthquake loading.
10.2. The January 1978 Izu-Ohshima-Kinkai Earthquake

Mochikoshi tailings dams are located in the southeastern area of Izu peninsula, about 120 km southwest of Tokyo, Japan. The tailings dams were used to store gold mine waste and constructed using an upstream construction method.

On January 14, 1978, the southeastern area of the Izu peninsula was shaken by an earthquake of magnitude 7.0 on the Richter scale. The epicenter of this earthquake was located at Sagami Bay, about 35 km east of the Mochikoshi tailings dams. The earthquake was followed by a number of after shocks, of which had a magnitude larger than 4.0 on the Richter scale. The largest aftershocks occurred on January 15, 1978 with their epicenters located approximately in the middle of Izu peninsula and only about 10 km from the locations of the dams. The focal depths of the earthquakes were estimated to be about 10 km.

The ground accelerations produced by the earthquakes were recorded at several stations. Unfortunately, none of the recording stations were located in areas of high shaking. Ohashi et al. (1980), as reported by Ishihara and Nagao (1983), studied the movement of the tombstones at a number of cemeteries to obtain the distribution of peak ground accelerations at this area. Based on these results, Ishihara (1984) estimated the peak ground acceleration at the dam sites was about 0.15 to 0.25 g.

Locations of the dams, epicenters of the earthquakes and estimated contours of peak ground accelerations due to the main shock at the area adjacent to the dams are shown in Fig. 10.1.

10.3. Effects of the Earthquakes on the Tailings Dams

The Mochikoshi tailings dams were constructed on the weathered deposit of cobble-containing tuffs. The original ground surface was stripped off in a saw tooth shape to provide a rough surface for the starter dam. The starter dam itself was constructed by
Fig. 10.1. (a). Epicenters of the main shock and aftershocks. (b). Locations of the tailings dams and estimated peak ground accelerations. (After Ishihara and Nagao, 1983).
compacting local volcanic soils. Due to the high permeability of the original soil deposit, no drainage system was installed at the bottom of the tailings pond. The permeability of the foundation soils was considered sufficient to provide drainage for the water resulting from the consolidation process of the tailings deposits.

The dams were raised by placing the local volcanic soils using an upstream construction method at a rate of approximately 2 m/year. The plan of the Mochikoshi tailings dams is shown in Fig. 10.2. The cross-sections of Dams No. 1 and No. 2 are presented in Fig. 10.3 and show the geometry of the tailings dams before and after the failure. An approximate water table during earthquake is also shown in Fig. 10.3.

10.3.1. Mochikoshi Tailings Dam No. 1

Dam No. 1 has a maximum height of 28 m and length of 73 m with a crest width of 5 m. The water table during the earthquake was approximately 3 m below the slope surface. The tailings dam failed catastrophically during the main shaking of the earthquake. The cause of the failure was the liquefaction of the tailings behind the dam.

The guardian of the dam witnessed this catastrophic event. According to him, as reported by Ishihara (1984), within about 10 seconds of the shaking, the frontal wall of the dam swelled causing excessive vertical movements at the crest. At this time, the tailings presumably had liquefied, temporarily lost their strength and stiffness, and behaved like liquid. The tailings materials beneath the upper part of the dam could no longer support the dam weight resulting in excessive settlements at the crest. Consequently, a breach occurred in the upper part of the dam close to the left abutment. A huge mass of tailings materials then flowed down to the river and took away the whole upper part of the dam.
Fig. 10.2. Plan of the Mochikoshi tailings dams after earthquake. (After Ishihara, 1984).
Fig. 10.3. Cross sections of the dams before and after the failures. (a). Dam No. 1. (b). Dam No. 2 (Ishihara, 1984).
10.3.2. Mochikoshi Tailings Dam No. 2

This tailings dam had a crest width of 5 m, a height of 22 m and a length of approximately 120 m. The original ground surface was slightly sloping to the upstream direction of the dam. The elevation of water table at the occurrence of the earthquake was similar to that of Dam No. 1, i.e. about 3 m below the slope surface. Although the geometry, water table condition, and relative density of the tailings were similar to those of Dam No. 1, this dam did not fail in the same way as Dam No. 1 did during the earthquake. Instead, the dam failed about one day after the cessation of the main shocks, or about 5 hours after the M5.8 and M5.4 aftershocks. The delayed failure of this dam was postulated to be due to the upward movement of the phreatic surface resulting from the redistribution of excess pore pressure of the liquefied tailings behind the dam (Okusa et al., 1984; Ishihara, 1984).

About one hour after the M5.4 aftershock, the guardian of the dam observed longitudinal cracks about 1 to 3 m long and about 5 mm wide at the downslope face of the dam. About one hour later, another longitudinal crack about 5 m long and 5 cm wide was observed in the middle of the downstream slope. These cracks were a strong indication that the dam has started to deform very slowly. These movements were probably triggered by the two aftershocks which might have increased the degree of disturbance in the liquefaction zone beneath the dam. At about 5 hours after the M5.4 aftershock, the guardian noticed a gradual sinking of the central part of the dam into the tailings causing serious loss of free-board. Then, the tailings behind the dam overtopped the dam crest, with about 20 m width, leading to the release of the large volume of tailings mass. The erosion that occurred during this process widened the dam breach to about 65 m. As result of the failure, the whole top portion of the dam were taken away leaving only about a half of the starter dam as shown in Fig. 10.3.
10.4. Site Investigations and Laboratory Tests

Almost immediately after the failure of the tailings dams, site investigations including boring and sampling were carried out. A total of 12 borings were made in the tailings pond and in the dam. The results of Standard Penetration Tests (SPT) at several locations are presented in Fig. 10.4. Static and cyclic laboratory tests were also carried out on undisturbed samples from the dam. Some of the results have been reported by Ishihara et al. (1980) and Ishihara (1984).

10.4.1. Tailings Materials

The tailings materials consisted of 3 to 7 cm thick layers of silt with a plasticity index of 10 and a non-plastic sandy silt with an average fine contents of 80 percent. The average standard penetration resistance of the tailings materials was zero until a depth of 15 m and about 3 at greater depth, as shown in boring No. 10 in Fig. 10.4. Ishihara (1984) noted that this low penetration resistance might have been due to soil remolding of the liquefied soils during the shaking. As reported by Byrne (1990), mixing of sandy silt soils may significantly reduce its strength. Furthermore, this was the case that occurred in the seismic failure of El-Cobre Dam in Chile (Dobry and Alvarez, 1967). The tailings significantly lost stiffness and strength due to soil mixing after liquefaction took place and flowed like liquid. For this reason, it was likely that the original blow counts of the tailings in this case were higher than 1, probably about 2 or 3.

The penetration resistance of the tailings deposit behind Dam No. 1 were slightly higher than those in the middle of the pond as shown in Boring No. 4 in Fig. 10.4. Near the surface, the blow counts were also zero which might also have been due to soil mixing caused by liquefaction and shaking. At greater depth, the SPT blow counts increased with depth and had an average value of about 3, suggesting the effects of
Fig. 10.4. The soil profiles and penetration resistance at several bore holes (Ishihara, 1984).
soil disturbance due to the shaking might have not occurred here, or at least were not significant. The higher blow counts in the lower half of the tailings are attributed to the consolidation process towards the permeable bed soils that somewhat stiffened the tailings. Therefore, it seems reasonable to assume that the average blow counts of the tailings deposit behind Dam No. 1 were about 3. By applying correction factors due to the effects of different SPT energy, overburden stresses, and fines contents, the equivalent clean sand value, \((N_{1})_{60-cs}\), of the deposit was found to be about 9.

Boring No. 7 in Fig. 10.4. shows the penetration resistance of the tailings deposit located at the original crest of Dam No. 2, at the dam abutment that did not fail. It is very unlikely that the soil at this location was disturbed as much as that in the middle of the pond. The penetration resistance from this bore hole can then be considered to represent the original SPT blow counts of the materials behind Dam No. 2. The average blow counts of the tailings deposit here (below water table) was also about 3, and thus the corresponding \((N_{1})_{60-cs}\) value was about 9.

The results of consolidated undrained static triaxial tests indicated a zero cohesion intercept, \(c\), and an angle of internal friction, \(\phi'\), varying between 30 and 39 degrees (Ishihara, 1984).

10.4.2. Dam Materials

The dam materials consisted of a mixture of the weathered tuffs and volcanic ashes obtained from the borrow pit adjacent to the dam. The average penetration resistance of this material was approximately 5. This material was above the water table and therefore was considered non-liquefiable.

The results of laboratory static triaxial tests on block samples of this material indicated an angle of internal friction, \(\phi'\), of about 35 degrees, and a cohesion intercept, \(c\), of 25 kPa (Ishihara, 1984).
10.5. Deformation Analyses

The proposed two-dimensional simplified method was applied to predict the deformation behaviour of the Mochikoshi tailings dams during the 1978 Izu-Ohshima-Kinkai earthquakes. Dam No. 1 failed during the earthquake and consequently inertia forces of the earthquake were included in the analyses. On the other hand, no earthquake inertia forces was considered in the analyses of Dam No. 2 since it failed during the period of no shaking. The analyses were carried out using the SOILTRESS-2 computer program.

10.5.1. Soil Properties Used in the Analyses

Since the data were not sufficient for determining the pre-earthquake hyperbolic stress-strain parameters, the properties of similar soils published by Duncan et al. (1980), were used in the present analyses. The estimation of hyperbolic stress-strain parameters in this way was considered satisfactory since the pre-earthquake soil properties do not significantly affect the end results.

The post-earthquake stress-strain parameters of the non-liquefied materials were kept the same as those of the pre-earthquake values. The post-earthquake soil parameters of the liquefied tailings were determined based on the corrected SPT values, \((N_{1})_{60-cs}\) using the procedures described previously. As a control, the results were compared with the post-earthquake properties of liquefied soils in other sites such as Wildlife and Heber Road sites (Chapters 7 and 8).

Ishihara (1984) reported the results of liquefaction analyses of the tailings materials. His results indicate that the tailings could liquefy to a depth of 7 m from the surface of the pond. The SPT resistance data obtained from the site investigation after the earthquake, however, suggest that the extent of liquefaction could have been much deeper than 7 m. For example, the effects of soil remolding due to the shaking was
observed in the log of Boring No. 10 (Fig. 10.4) to a depth of 15 m from the surface of the pond. This is a strong indication that liquefaction took place to a depth of 15 m and it could have possibly extended to a level deeper than this. In fact, the results of liquefaction analyses using Seed's simplified procedures carried out by the writer indicated that liquefaction could occur at the full depth of the tailings with an average factor of safety against liquefaction ($F_L$) of about 0.80. In the present analyses, it was therefore assumed that the tailings liquefied to their full depth.

Using the value of $(N_1)_{60-cs}$ of 9 in conjunction with Byrne's empirical formula, the residual strength of the tailings materials behind Dam No. 1 was found to be 13 kPa. The average effective overburden stress for the liquefied tailings in this zone was about 100 kPa. The corresponding normalized undrained strength, $S_o/\sigma'_{vo}$, was therefore 0.13. For comparison, the residual strength of a soil deposit at the Wildlife and Heber Road sites having $(N_1)_{60-cs}$ about 5 and 6 was about 0.14 $\sigma'_{vo}$. In fact, the final geometry of the tailings after the failure as shown in Fig. 10.3.a, indicated a slope of 1:7 which yielded a back-calculated residual strength of about 0.14 $\sigma'_{vo}$. Therefore, a residual strength ratio of 0.14 was assumed in the present analyses.

The strain at residual strength (limiting shear strain) determined from Fig. 4.11 using $F_L$ of 0.80 and $(N_1)_{60-cs}$ of 9 was found to be about 22 percent. Thus, the computed shear modulus number, $k_g$, of the liquefied materials was 0.6.

For Dam No. 2, the above stress-strain properties were also used. The residual strength, however, was taken to be 0.16$\sigma'_{vo}$ corresponding to the final slope of the tailings of 1:6 (see Fig. 10.3.b).

The inertia forces of the earthquake were taken from the average peak ground acceleration reported by Ishihara (1984) of about 0.17 g. The corresponding maximum velocity of the mass was taken to be 0.17 m/s.
In these analyses, the deformations due to post-liquefaction volumetric strains were not considered since both dams failed totally and the post-earthquake settlements were small in comparison to the maximum undrained deformations.

The pre- and post earthquake soil properties for both Dam No. 1 and No. 2 are given in Table 10.1.

10.5.2. Cases Considered

Three cases were considered in these analyses. Case 1 is the condition of Dam No. 1 during the main shaking after liquefaction was triggered. Case 2 is the condition of Dam No. 2 at the same moment as that in Case 1. Case 3 is the condition of Dam No. 2 about 5 hour after the M5.8 and M5.4 aftershocks or about 24 hours after the main shock. At this time, according to Okusa et al. (1984) and Ishihara (1984), the dam phreatic surface had moved sufficiently upward bringing the dam to a critical stability condition with factor of safety close to unity. It was at this time, the guardian of the dam observed a gradual sinking of the dam crest which was followed by the breach of the dam releasing the impounded tailings sludge.

The finite element mesh used in for each case showing material types and water table are presented in Fig. 10.5. for Dam No. 1 (Case 1) and Fig. 10.6. for Dam No. 2 (Cases 2 and 3).
Table 10.1. Soil properties used in the analyses of Mochikoshi tailings dams.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>(k_g)</th>
<th>(n)</th>
<th>(k_b)</th>
<th>(m)</th>
<th>(\phi)</th>
<th>(\Delta\phi)</th>
<th>(c) or (S_e) kPa</th>
<th>(R_f)</th>
<th>(\gamma_t) kN/m³</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Fill 1</td>
<td>173</td>
<td>0.5</td>
<td>1000</td>
<td>0.25</td>
<td>35.0</td>
<td>0.0</td>
<td>25.0</td>
<td>0.70</td>
<td>15.70</td>
<td></td>
</tr>
<tr>
<td>2. Tailings</td>
<td>135 (0.6)</td>
<td>0.5 (0.0)</td>
<td>2000 (2000)</td>
<td>0.25 (0.25)</td>
<td>34.0 (0.0)</td>
<td>0.0</td>
<td>0.0 (0.14(\sigma_{we}'))</td>
<td>0.80 (0.0)</td>
<td>18.40</td>
<td>Dam No. 1</td>
</tr>
<tr>
<td>3. Fill 2</td>
<td>173</td>
<td>0.5</td>
<td>2000</td>
<td>0.25</td>
<td>35.0</td>
<td>0.0</td>
<td>25.0</td>
<td>0.70</td>
<td>15.70</td>
<td>Dam No. 2</td>
</tr>
<tr>
<td>4. Starter dam</td>
<td>135</td>
<td>0.6</td>
<td>2000</td>
<td>0.25</td>
<td>40.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.80</td>
<td>19.00</td>
<td></td>
</tr>
<tr>
<td>5. Foundation</td>
<td>200</td>
<td>0.6</td>
<td>2000</td>
<td>0.25</td>
<td>35.0</td>
<td>0.0</td>
<td>25.0</td>
<td>0.80</td>
<td>15.70</td>
<td></td>
</tr>
</tbody>
</table>

Note: The number in brackets indicate the post-earthquake soil properties.
Fig. 10.5. Finite element mesh used in the analyses of Mochikoshi Dam No. 1 (Case 1).
Fig. 10.6. Finite element mesh used in the analyses of Dam No. 2. Phreatic surface \( a \) and \( b \) indicates Case 2 and Case 3.
10.5.3. Results of the Analyses

The results of the deformation analyses of all cases are presented in Fig. 10.7 to 10.9 in terms of deformed geometry and displacement vectors of the nodes. The magnitude of the displacements at the crest are also presented in Table 10.2.

Table 10.2. Predicted Displacements at the Dam Crest

<table>
<thead>
<tr>
<th>Case</th>
<th>Upstream Crest</th>
<th>Downstream Crest</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hor. Displ. (m)</td>
<td>Vert. Displ. (m)</td>
<td>Hor. Displ. (m)</td>
</tr>
<tr>
<td>1</td>
<td>6.8</td>
<td>-5.4</td>
<td>6.8</td>
</tr>
<tr>
<td>2</td>
<td>0.16</td>
<td>-0.57</td>
<td>0.16</td>
</tr>
<tr>
<td>3</td>
<td>0.30</td>
<td>-0.77</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Note: Negative sign indicates downward vertical movement (settlement) or leftward horizontal movement.

10.5.3.1. Mochikoshi Dam No. 1

In the analyses of Dam No. 1, zero seismic coefficient was applied. However, the resulting displacements were excessive although the changes in geometry were considered in the analyses. The computed vertical and horizontal displacements at the crest were in the order of 6 m (Table 10.2). The deformations in this magnitude are clearly an indication of flow failure. In this case, the average driving stress was much larger than the residual strength of soils and the changes in geometry did not sufficiently reduce the driving stress to a value equal to the residual strength.

As can be seen in Fig. 10.7.a, the dam experienced excessive vertical and horizontal movements at the crest, accompanied by the swelling of the frontal wall of the tailings dam. The liquefied tailings tended to push the dam up, as shown in Fig. 10.7.b, which presumably caused extensive cracks in the frontal wall of the dam. These
Fig. 10.7. Results of the analyses of Mochikoshi Dam No. 1 (Case 1). (a). Deformed geometry (displacement magnification factor = 1); and (b). Displacement vectors.
excessive cracks led to the breach of the dam which was followed by the movements of large masses of tailings materials down to the valley. The mechanism of deformation predicted in the analyses is in good agreement with the observation of the dam guardian, as reported by Ishihara (1984).

10.5.3.2. Mochikoshi Dam No. 2

As shown in Table 10.2, despite the liquefaction of the tailings behind the dam, the crest movements at Case 2 were only in the order of 0.5 m. These small deformations were hardly noticeable in the field by the guardian of the dam. There was no report of damage in this dam due to these deformations after the main shock had stopped.

Figs. 10.8.a and b show the deformed mesh and the vector of displacements of this dam. The displacement vectors of the tailings beneath the dam suggest that the liquefied tailings tended to push the dam downstream and upward. However, the thickness of the tailings behind this dam was only about 15 to 20 m which resulted in less pushing energy, in comparison to that created by the 28 m tailings behind the Dam No. 1. Furthermore, the self weight of the Dam No. 2, which happened to be thicker than Dam No. 1, exerted balancing forces against the push from the liquefied tailings. Because of this interaction, the crest of the dam only underwent small settlements and upstream horizontal movements. The frontal wall of the dam also experienced slight upward movement in the order of 0.1 m, as can be seen in Fig. 10.8.b. These small deformation, however, did not seem to be sufficient to bring about cracks in this zone. At the end of the earthquake, the dam was still stable, as observed in the field.

Case 3 represents the condition when the dam was at a critical stability condition due to the raising of the phreatic surface within the dam, as postulated by Okusa et al. (1984) and Ishihara (1984). In this case, all soil parameters used were the same as those used in Case 2. No kinetic energy of the earthquake was considered.
Fig. 10.8 Results of the analyses of Dam No. 2 during earthquake (Case 2). (a) Deformed geometry (magnification factor = 2). (b) Displacement vectors.
Fig. 10.9. Results of the analyses of Dam No. 2 after the cessation of earthquake (Case 3). (a). Deformed geometry (magnification factor = 2). (b). Displacement vectors.
As expected, the predicted displacements in Case 3 were larger than those computed in Case 2. The crest moved about 0.60 m upstream-ward and settled about 1.2 m. There was a strong tendency for crack development in the frontal wall of the dam, as can be seen in the vector displacements in Fig. 10.9.b. However, the displacement magnitudes in this zone were only about 0.5 m. The deformations of this magnitude would not cause the total failure of the dam. Thus, from deformation analysis point of view, the dam was predicted to be stable although the phreatic surface had increased from position \(a\) to \(b\) (Fig. 10.6).

The computed deformations in Cases 2 and 3 are consistent with the results of limit equilibrium analyses performed by Ishihara (1984) for different positions of the phreatic surface. For the phreatic surface as used in Cases 2 and 3, the computed factor of safety against stability \((F_s)\) was about two and unity, respectively. The \(F_s\) of about two would correspond to very small deformations (Case 2) whereas the \(F_s\) of unity would correspond to relatively larger deformations than those in Case 2, as predicted in the present analyses. However, as mentioned previously, the dam deformations for \(F_s\) of unity might still be tolerable by the dam since the movement would stop with increasing deformations due to changes in geometry during the movement. The \(F_s\) would need to be much less than unity to cause significant deformations that would lead to total failure of the dam.

From the preceding discussion, it seemed that the increase in phreatic surface alone was not sufficient to cause the flow failure of the dam. Due to limited information available regarding the actual mechanism that occur in the field, several different reasons may be possible to explain the mechanism of the dam failure. One of these possible explanations is proposed in the following paragraphs.
As reported by Ishihara (1984), there were two large aftershocks with Richter magnitudes of 5.8 and 5.4 about 5 hours before the failure. These aftershocks might have caused further disturbance of the liquefied tailings. As a result, the tailings stiffness might have further decreased causing the earth fill above these tailings (Material type 1 in Fig. 10.6) to sink slowly under its own weight. The first sign of dam deformation was observed in the field in form of longitudinal cracks in the frontal wall of the dam, about 1 hour after the aftershocks. Therefore, in addition to the upward movement of the phreatic surface, it was postulated that there was also a further decrease in the stiffness of liquefied soils as results of these aftershocks.

In terms of factor of safety against liquefaction, the two large aftershocks have caused the $F_L$ to reduce from 0.80 to a much lower value, say 0.1. As a consequence, the limiting strain determined from Fig. 4.11 and $(N_{1})_{60-cs}$ of 9 was about 50 percent. The computed modulus number of the liquefied tailings was therefore 0.3. Re-analyses of Case 3 using this lower $k_g$ value, resulted in large deformations, that might lead to the failure of the dam. The crest of the dam sank into the tailings causing a serious loss of freeboard. The settlement of the crest was in the order of 3 m. The results are shown in Fig. 10.10.

Although the deformations predicted, based on this postulate, correctly indicated that a flow failure may occur in this dam, this postulate is only one of the many possibilities that might occur in the field. A more rigorous dynamic effective stress analysis is required to investigate the actual reasons for the delayed failure of the Mochikoshi Dam No. 2.
Fig. 10.10. Results of re-analyses of Case 3 using smaller modulus values for liquefied soils. (a). Deformed geometry (magnification factor = 2). (b). Displacement vectors.
10.6. Summary

The proposed simplified procedure discussed in Chapter 5 has been applied to predict the deformations of Mochikoshi tailings dams due to the Izu-Ohshima-Kinkai earthquake. Based on the soil parameters and other required parameters obtained from the literature, the procedure correctly predicted the failure of the Mochikoshi Dam No. 1 and also stability of Dam No. 2 despite the liquefaction of the tailings materials behind the dam.

It was also found from this study that the upward movement of the phreatic surface alone did not seem to be sufficient to bring Dam No. 2 to total failure. It was postulated that further stiffness reduction might have taken place in the liquefied soils due to two large aftershocks, 5 hours before the failure. This further stiffness reduction might have contributed to the large deformations of the dam that eventually led to the failure of the dam.
11.1. Introduction

In 1985, Central Chile was shaken by an earthquake of magnitude 7.8 on the Richter scale. Many small earth dams within 90 km of the epicenter suffered some damage varying from minor cracks to major deformations. Fortunately, only two dams suffered serious damage. La Marquesa Dam underwent upstream and downstream slope failures as well as excessive crest settlement leading to 1.8 meter freeboard loss. The other one, La Palma Dam, suffered extensive cracking in the upstream slope causing the upstream crest to settle about 0.8 m. These large deformations were postulated to be due to liquefaction of soils within the dam.

Extensive site investigations were carried out to study the failure mechanisms of La Marquesa and La Palma Dams (De Alba et al., 1987, 1988). The deformations of these dams were measured and the initial geometry of the dams were reconstructed. Furthermore, based on the final geometry of the dams, the residual strength of the liquefied sands was back-calculated. The deformation behaviour of La Marquesa and La Palma Dams will be analyzed using the proposed method to check whether or not the method is capable of predicting the dam deformations due to earthquake induced liquefaction.
11.2. The 1985 Chilean Earthquake

On March 3, 1985, at 22:47 GMT, an earthquake of Richter magnitude 7.8 occurred in Chile. The earthquake was produced by the slippage between the Nazca plate and the South American plate that forms a subduction zone at a shallow angle. The peak ground accelerations at La Marquesa and La Palma Dams were estimated to be 0.60 g and 0.46 g respectively (De Alba et al., 1987, 1988). The location of the epicenter and the recorded peak ground accelerations at several stations in the vicinity of the dams are shown in Fig. 11.1.

11.3. Effects of the Earthquake on the Dams
11.3.1. La Marquesa Dam

La Marquesa Dam is located about 60 km from Santiago and about 40 km from the epicenter of the earthquake, as shown in Fig. 11.1. The dam had an original height of 10 m, a crest length of 220 m and a crest width of 5 m. The dam was built on a sandy clay and clayey sand foundation. The shell of the dam consisted of silty clayey sands obtained from the borrow pits in the reservoir area. The shell was underlain by a layer of silty sand in both upstream and downstream slopes. The core of the dam comprised of more plastic material and extended down to the base of the dam.

La Marquesa Dam experienced large deformations during the 1985 earthquake. Both the upstream and downstream shells of the dam underwent major sliding in the middle third of the dam. Excessive settlement occurred in the crest, resulting in 1.8 m freeboard loss. Extensive longitudinal cracks were also observed, particularly in the upstream slope, with crack width up to 0.8 m and crack depths of up to 2 m. The upstream and downstream shells experienced maximum settlements of about 3.5 m and 2.5 m respectively. The maximum horizontal displacements at the upstream and
Fig. 11.1. Location of the epicenter and the recorded peak ground accelerations.
Fig. 11.2. Post-earthquake geometry and reconstructed initial geometry of La Marquesa Dam (De Alba et al., 1988)
downstream shells were about 11 m and 6.5 m respectively. The geometry after sliding and the reconstructed pre-earthquake geometry are shown in Fig. 11.2.

De Alba et al. (1987, 1988) conducted extensive study on the causes of the dam failure. They postulated that the major cause of the sliding was the liquefaction of the saturated silty sand layer which was located immediately above the base sandy clay. Under an earthquake motion with peak ground acceleration of 0.6 g, the loose silty sand layer liquefied both in the upstream and downstream sides at an early stage of the earthquake shaking. The liquefaction of this layer were followed by temporary loss of its stiffness and strength. As a result, major sliding occurred in both sides of the slopes. These large slope movements were mainly driven by the weight of the shell of the dam. At this time, the dam shells probably did not liquefy. Instead, they broke into several blocks and slid downwards and outwards on the liquefied layer, both in the upstream and downstream slopes.

11.3.2. La Palma Dam

La Palma Dam is located about 75 km from the epicenter. The dam was apparently built using the same construction procedures as those of La Marquesa Dam. Thus, the geometry of the dam is similar to that of La Marquesa. The original maximum height of the dam was about 10 m, a crest length of about 140 m and a crest width of 5 m. The dam has a sandy clay core supported by clayey and silty sand shells. Similar to La Marquesa Dam, the upstream shell and part of the downstream shell are underlain by a layer of loose silty sand.

La Palma Dam suffered somewhat similar damage to that of La Marquesa Dam. However, only the upstream part of the dam experienced serious damage. A major longitudinal crack developed along the crest with a maximum width of 1.2 m and length of 80 m. The maximum settlement at the crest was about 0.8 m relative to the
downstream side. Large displacements developed in the middle third of the upstream slope with the upstream toe moved about 5 m. The cause of the failure was also postulated to be due to liquefaction of the silty sand layer underlying the upstream shell (De Alba et al., 1987, 1988). The pre- and post earthquake geometry of the dam is shown in Fig. 11.3.

11.4. Soil Investigations

11.4.1. La Marquesa Dam

Retamal (1985) as reported by De Alba et al. (1987) presented the results of two SPT borings and a test pit carried out at La Marquesa Dam. De Alba et al (1987) carried out additional four SPT borings and a test pit to confirm these results and to further investigate the soil conditions beneath the dam.

The boring logs obtained from these investigations show that the foundation of the dam is clayey sand/sandy clay with corrected SPT values, $(N_1)_{60}$, ranging from 10 to 50. This layer is overlain by a layer of loose silty sand both in the upstream and downstream slope with thickness varying between 1.4 to 1.7 m. The average corrected SPT blow counts, $(N_1)_{60}$, of this layer beneath the downstream slope was about 9 with fines content of about 20 percent. Allowing for the effects of fines contents as proposed by Seed (1987), the equivalent clean sand value, $(N_1)_{60-cs}$, was therefore 11. The silty sand layer beneath the upstream slope was found to be looser than that in the downstream slope. The average corrected penetration resistance, $(N_1)_{60}$, at this location was about 5 (Fig. 11.2) with fines contents of 30 percent. The corresponding equivalent clean sand value, $(N_1)_{60-cs}$, of this layer is 7.

The loose silty sand layer is underlain by silty and clayey sand shells. Although the average corrected SPT values of this material is about 5, the dam shells are considered to
Fig. 11.3. Pre- and post-earthquake geometry of La Palma Dam (De Alba et al., 1988)
be non-liquefiable due to their high clay contents, which are similar to the foundation soil. More detailed descriptions of the soil comprising the dam has been presented by De Alba et al. (1988).

11.4.2. La Palma Dam

Site investigations were carried out using the same procedures as those used in La Marquesa Dam (De Alba et al., 1987). The elevation of the water table at the time of investigation was about 0.5 m below the elevation when the earthquake occurred.

The foundation of the dam is clayey sand with \((N_1)_{60}\) values ranging from 5 to 10. Despite low SPT blow counts, this layer is not liquefiable due to the clay content in the soil. A layer of silt and silty sand with some gravel was found immediately above the foundation soil in the upstream slope of the dam. This layer had corrected SPT values varying from 8 to 14 and apparently did not liquefy during the earthquake (De Alba et al., 1988). This layer is overlain by a thin layer of loose silty sand with thickness varying between 0.5 to 1.0 m. The average corrected SPT blow counts, \((N_1)_{60}\), of this layer was about 4 (Fig. 11.3) with fines contents of 15 percent. The corresponding equivalent clean sand blow counts, \((N_1)_{60-cs}\), was therefore 5. This is the layer that was postulated to liquefy during the earthquake. The same layer of loose silty sand was also found in the downstream slope of the dam. However, this layer was above the water table and apparently did not liquefy during the earthquake.

11.5. Deformation Analyses of La Marquesa Dam

The proposed method was applied to predict the deformation of La Marquesa Dam due to the 1985 earthquake. Similar predictions using this method have been carried out and the results has been presented by Jitno and Byrne (1994). However, somewhat
different assumptions in the post-earthquake stress-strain of liquefied soils were taken in the present study.

11.5.1. Soil Parameters Used in the Analyses

The pre- and post-earthquake soil properties are listed in Table 11.1. The pre-earthquake soil properties were determined based on the \((N_1)_{60}\) values following the approach outline by Seed et al. (1983) and Byrne et al. (1987). The shear modulus parameters for these soils agree well with those of similar materials published by Duncan et al. (1980). The 10 kPa cohesion of the dam core was back-calculated from the final geometry of the dam after the earthquake. Due to lack of data, some assumption were made on the values of angle of internal friction, \(\phi\), and unit weight of the soils, \(\gamma_s\). The estimation of these parameters is usually sufficient since these parameters do not significantly affect the end results.

The residual strength of liquefied soil was determined using Byrne's empirical formula (Eq. 2.1). For liquefied soil beneath the upstream shell, the equivalent clean sand SPT blow counts, \((N_1)_{60-cs}\), were about 7 and the computed residual strength was therefore 10 kPa. For liquefied soil beneath the downstream shell, the residual strength for \((N_1)_{60-cs}\) of 11 was about 19 kPa.

Using simplified procedures proposed by Seed and Idriss (1971), the factor of safety against liquefaction of the loose silty sand layer beneath the upstream and downstream shell was computed to be 0.21 and 0.28 respectively. Using these values and \((N_1)_{60-cs}\) in conjunction with the chart in Fig. 4.11, the limiting strains of the upstream and downstream silty sands were found to be 97 and 83 percent. The corresponding shear modulus numbers, \(k_g\), were therefore 0.14 and 0.23, respectively, for upstream and downstream silty sand layers. These post-earthquake properties are slightly different from
### Table 11.1. Soil Properties Used in the Analyses of La Marquesa Dam

<table>
<thead>
<tr>
<th>Material Type</th>
<th>$k_s$</th>
<th>$n$</th>
<th>$k_b$</th>
<th>$m$</th>
<th>$\phi$ degree</th>
<th>$c$ kPa</th>
<th>$R_f$</th>
<th>$\gamma_s$ kN/m$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Shell, clayey sand</td>
<td>173 (86)</td>
<td>0.5</td>
<td>1000</td>
<td>0.25</td>
<td>35</td>
<td>0.0</td>
<td>0.70</td>
<td>19.0</td>
</tr>
<tr>
<td>2. D/S silty sand (liq.)</td>
<td>135 (0.23)</td>
<td>0.5</td>
<td>2000</td>
<td>0.25</td>
<td>35</td>
<td>0.0</td>
<td>0.80</td>
<td>16.5</td>
</tr>
<tr>
<td>3. U/S silty sand (liq.)</td>
<td>135 (0.14)</td>
<td>0.5</td>
<td>2000</td>
<td>0.25</td>
<td>35</td>
<td>0.0</td>
<td>0.80</td>
<td>16.5</td>
</tr>
<tr>
<td>4. Core, sandy clay</td>
<td>173</td>
<td>0.5</td>
<td>2000</td>
<td>0.25</td>
<td>35</td>
<td>10.0</td>
<td>0.70</td>
<td>19.0</td>
</tr>
<tr>
<td>5. Found., sandy clay</td>
<td>200</td>
<td>0.5</td>
<td>2000</td>
<td>0.25</td>
<td>35</td>
<td>0.0</td>
<td>0.70</td>
<td>19.0</td>
</tr>
</tbody>
</table>

Note: Number in the brackets indicates the post-earthquake soil properties.

### Table 11.2. Cases Considered

<table>
<thead>
<tr>
<th>Case</th>
<th>Upstream silty sand layer</th>
<th>Downstream silty sand layer</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_r$ (kPa)</td>
<td>$k_s$</td>
<td>$S_r$ (kPa)</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>0.14</td>
<td>19</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
<td>0.17</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
<td>0.2</td>
<td>29</td>
</tr>
</tbody>
</table>
the values used in the previous studies reported by Jitno and Byrne (1994).

Previous analyses (Jitno and Byrne, 1994) used the upper bound values of residual strength obtained from De Alba et al. (1987) and the assumption of 100 percent residual strains for the liquefied soils at the upstream and downstream dam shells. The use of upper bound residual strength for this case (i.e. 29 kPa and 17 kPa respectively for downstream and upstream liquefied layers) might be more reasonable since the slope movements occurred under the influence of earthquake inertia forces. Therefore, these residual strengths were also considered in the present analyses and the corresponding modulus numbers, $k_g$, were 0.35 and 0.17 respectively.

The upstream and downstream shells of the dam were not considered to liquefy but were assumed to experience a 50 percent reduction in stiffness. Their post-earthquake shear strengths, however, were considered to remain the same as the pre-earthquake values. This assumption is supported by the post-cyclic laboratory data shown by Thiers and Seed (1969), Andersen (1984), and recently by Jitno and Vaid (1991).

The maximum velocity of the dam was determined from the peak ground acceleration of 0.6 g. Assuming the ratio of $A/V = 1$, where $A$ is the acceleration in gravity units and $V$ is the maximum velocity in m/s, the maximum velocity of the mass was taken to be 0.60 m/s. It was also considered that the velocity is uniform for all soil elements within the dam. This assumption seems reasonable for small dams such as La Marquesa Dam where not much amplification of the ground acceleration is expected.

In this analysis, the post-liquefaction settlement was not considered since the liquefied layer was thin and consequently the magnitude of the settlement would be negligible in comparison to the undrained deformations.

The finite element mesh used in the analyses are shown in Fig. 11.4. The material types and the approximate water table during the earthquake are also shown in this figure.
11.5.2. Results of the Analyses

The analyses were carried out by applying vertical seismic coefficient to achieve energy balance of the system. The application of vertical seismic coefficient is reasonable since the slope movements in this dam were driven mainly by the weight of the dam itself, as also noted by De Alba et al. (1988). Typical predicted deformations of La Marquesa dam are presented in Fig. 11.5 in terms of deformed geometry and displacement vectors. The magnitude of displacements at several locations are also presented in Table 11.3.

<table>
<thead>
<tr>
<th>Locations</th>
<th>Measured (m)</th>
<th>Predicted (m) Case 1</th>
<th>Predicted (m) Case 2</th>
<th>Predicted (m) Case 3</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 231</td>
<td>-2.5</td>
<td>-4.2</td>
<td>-1.4</td>
<td>-2.3</td>
<td>D/S shell</td>
</tr>
<tr>
<td>Node 254</td>
<td>-1.8</td>
<td>-2.6</td>
<td>-0.9</td>
<td>-1.4</td>
<td>Crest(core)</td>
</tr>
<tr>
<td>Node 346</td>
<td>-1.8</td>
<td>-4.7</td>
<td>-1.1</td>
<td>-1.5</td>
<td>Crest(core)</td>
</tr>
<tr>
<td>Node 415</td>
<td>-3.5</td>
<td>large</td>
<td>-2.3</td>
<td>-3.3</td>
<td>U/S shell</td>
</tr>
</tbody>
</table>

Note: Negative signs indicate downward movements. See Fig. 11.4 for the node locations.

As can be seen in Fig. 11.5, both upstream and downstream shell undergo large deformations due to the soft liquefied layer beneath the dam shells. Excessive settlement is also observed in the dam crest. Furthermore, tensions are observed in the soil elements in the contact plane between the core and the dam shells. These tensions suggest that cracks would occur in this zone. These cracks were actually observed in the field (De Alba et al., 1988).
CHAPTER 11: Two Dam Failures During the 1985 Chilean Earthquake

Fig. 11.4. Finite element mesh used in the analyses, material types and approximate water table during the earthquake. La Marquesa Dam.
Fig. 11.5. Results of the Analyses. (a) Deformed geometry. (b) Displacement vectors.
The use of average residual strength from Seed and Harder’s chart tends to overestimate the deformations of the dam, as can be seen for Case 1 in Table 11.3. It must be recalled that the residual strength data in their chart were obtained from back-calculation of case histories with different orientation of failure planes that failed under static or dynamic loading. The upper and lower bound values were, respectively, for cases with and without considering the earthquake inertia forces. The results of the present analyses suggest that the upper bound values of residual strength might be more appropriate to use for cases that developed large deformations during the earthquake, such as this dam.

Case 2 underestimated the magnitude of displacements at all locations of interest. The computed deformations were only about 50 percent of the measured values. In this case, the effects of soil mixing in the liquefied layer due to earthquake shaking were not considered. It is reasonable to expect that soil mixing in the liquefied silty sand layer would occur under this severe earthquake shaking (peak ground acceleration = 0.6 g). As in the case of the Mochikoshi tailings dams (Chapter 10), soil mixing can have profound effects on the stiffness of the liquefied soils. Unfortunately, these effects are difficult to quantify and may depend on several factors including the relative density and fines contents of sands. To the writer’s knowledge, no systematic study has been carried out to investigate the effects of soil mixing on the post-liquefaction stress-strain behaviour of soils.

Although the predicted magnitude of deformations are less than those observed in the field, the damage due to these deformations may have similar impacts on the dam. The dam would need extensive repair before it could be used again. Furthermore, the pattern of deformations is in good agreement with that observed in the field. Thus, from a
practical point of view, the proposed method has successfully predicted the damage of the dam due to the 1985 Chilean earthquake.

The best prediction both in terms of magnitude and pattern of displacements was achieved in Case 3, as reported by Jitno and Byrne (1994). As mentioned before, this case used upper bound residual strengths from De Alba et al. (1987) and 100 percent residual strains for upstream and downstream liquefied layers. This 100 percent strain was intended to take into account the effects of soil mixing due to earthquake shaking and was based on engineering judgement. The predicted vertical deformations at the downstream and upstream shells in contact with the dam core (Node 231 and 415) are respectively about -2.2 m and -3.3 in comparison to the measured displacements of about -2.5 m and -3.5 m. Good agreement was also achieved at the crest and downstream shell. The crest is predicted to settle about 1.5 m in comparison to the measured settlement of 1.8 m.

11.6. Deformation Analyses of La Palma Dam

Similar to La Marquesa Dam, the deformation that developed in La Palma Dam during the earthquake of 1985 was predicted using the proposed method. The zone of liquefaction was that postulated by De Alba et al. (1988). The analyses were carried out using the computer program SOILSTRESS-2.

11.6.1. Soil Properties Used in the Analyses

Two cases were considered. The pre- and post-earthquake soil properties used are listed in Table 11.3. Case 1 used residual strength from Byrne’s empirical formula and Case 2 used the upper bound value of De Alba et al. (1987). The procedures for obtaining the soil parameters are similar to those used in the study of La Marquesa Dam.
Using Byrne’s empirical formula for \((N_1)_{60-\alpha}\) of 5, the residual strength of the liquefied silty sand layer was found to be about 7 kPa. The factor of safety against liquefaction based on the simplified procedure of Seed and Idriss for this layer was about 0.25. The corresponding residual strain was therefore about 100 percent. Accordingly, the computed shear modulus number, \(k_g\), was about 0.1.

An alternative residual strength from De Alba et al. (1988) was also considered (Case 2). The range of their residual strength values were between 6 and 14 kPa. The upper bound value of 14 kPa represents the residual strength with consideration of kinetic energy of the earthquake. Since the movements took place during the earthquake, it would be reasonable to take the upper bound value of 14 kPa for the residual strength used in the analyses. The corresponding \(k_g\) value for this residual strength was 0.14.

The shell of the dam was considered to experience 50 percent stiffness reduction due to severe earthquake loading. Similar to La Marquesa Dam, however, the shell was assumed to retain its original strength after the earthquake. The post-earthquake parameters for other soils were assumed to be the same as those of pre-earthquake values.

The post-earthquake/liquefaction volumetric strain was very small for very thin liquefied layer such as the loose silty sand layer in this dam. Thus, the post-earthquake settlement was not considered in the analyses.

Using the same procedure as that used for La Marquesa Dam, the maximum velocity of the dam was taken based on the value of the peak ground acceleration of 0.47 g. This value led to a maximum velocity, \(V\), of 0.47 m/s.

The finite element mesh used in the analyses showing the material types, the approximate water table and the liquefaction layer are presented in Fig. 11.6.
Table 11.4. Soil Properties Used in the Analyses of La Palma Dam

<table>
<thead>
<tr>
<th>Material Type</th>
<th>$k_s$</th>
<th>$n$</th>
<th>$k_s$</th>
<th>$m$</th>
<th>$\phi$ degree</th>
<th>$c$ kPa</th>
<th>$R_f$</th>
<th>$\gamma_s$ kN/m$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Shell, clayey sand</td>
<td>142 (71)</td>
<td>0.5</td>
<td>1000</td>
<td>0.25</td>
<td>35</td>
<td>0.0</td>
<td>0.70</td>
<td>20.5</td>
</tr>
<tr>
<td>2. D/S silty sand (liq.)</td>
<td>142 (71)</td>
<td>0.5 (0.5)</td>
<td>2000 (2000)</td>
<td>0.25 (0.25)</td>
<td>35 (35.0)</td>
<td>0.0 (0.0)</td>
<td>0.80 (0.80)</td>
<td>19.0</td>
</tr>
<tr>
<td>3. U/S silty sand (liq.)</td>
<td>142 (0.14)</td>
<td>0.5 (0.0)</td>
<td>2000 (2000)</td>
<td>0.25 (0.25)</td>
<td>35 (0.0)</td>
<td>0.0 (14.0)</td>
<td>0.70</td>
<td>19.0</td>
</tr>
<tr>
<td>4. Core, sandy clay</td>
<td>235</td>
<td>0.5</td>
<td>2000</td>
<td>0.25</td>
<td>35</td>
<td>10.0</td>
<td>0.80</td>
<td>20.5</td>
</tr>
<tr>
<td>5. Found., sandy clay</td>
<td>235</td>
<td>0.5</td>
<td>2000</td>
<td>0.25</td>
<td>35</td>
<td>0.0</td>
<td>0.70</td>
<td>18.0</td>
</tr>
</tbody>
</table>

Note: Number in the brackets indicates the post-earthquake soil properties.

Table 11.5. Cases Considered

<table>
<thead>
<tr>
<th>Case</th>
<th>Upstream silty sand layer</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k_s$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>0.10 Average $Sr$ from Byrne's formula</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>0.14 Upper bound $Sr$ from De Alba et al (1987)</td>
</tr>
</tbody>
</table>
Fig. 11.6. Finite element mesh, material types and approximate water table during the earthquake. La Palma Dam.
11.6.2. Results of the Analyses

The predicted dam deformations are presented in Fig. 11.7 in terms of deformed mesh and displacement vectors. Only the results for Case 2 are shown. The magnitude of the displacements for each case are presented in Table 11.6.

<table>
<thead>
<tr>
<th>Locations</th>
<th>Measured (m)</th>
<th>Predicted (m)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node 392</td>
<td>-1.0</td>
<td>large(failed)</td>
<td></td>
</tr>
<tr>
<td>Node 417</td>
<td>-1.1</td>
<td>-1.4</td>
<td></td>
</tr>
</tbody>
</table>

As shown in Fig. 11.7, the procedure correctly predicted the upstream movements of the dam due to liquefaction of the silty sand layer beneath the shell. Most of the deformations developed in the loose liquefied layer. However, large deformations were also observed in the contact elements between the upstream shell and the core resulting in tensile stresses within the soil elements. As in the case of La Marquesa Dam, the tensions in these elements indicate a potential for crack development in this zone which in fact was observed in the field (Fig. 11.3). Furthermore, the downstream shell only suffered minor deformations which also agrees well with the field observation.

Similar to the results of La Marquesa Dam analyses, the use of average residual strength (Case 1) tends to overestimate the computed deformations, as can be seen in Table 11.6. The computed deformations at the crest (node 392) and the shell (node 417) are very large (>4 m). This is an indication of flow failure. On the other hand, the computed deformations in Case 2 (upper bound $S_r$) are very close to the measured displacements. The predicted settlement at the crest and the upstream shell are
Fig. 11.7. Results of the analyses of La Palma Dam. (a). Deformed mesh (b). Displacement vectors.
-1.4 m and -1.1 m respectively in comparison to the measured -1.0 and -1.1 m. These results are consistent with those obtained in La Marquesa Dam analyses that the upper bound residual strength is more appropriate for cases of slope movements that occur under the influence of dynamic loading.

11.7. Summary

The proposed procedure has been applied to predict the deformation of La Marquesa and La Palma Dams during the 1985 Chilean earthquake. The average residual strength from Seed and Harder's chart (incorporated in Byrne's formula) and the upper bound values of residual strength from De Alba et al. (1987) were used. The results show that the procedure overestimates the dam displacements when the average residual strengths were used, but closely predicts the field deformations if the upper bound values of residual strength were considered. Moreover, the procedure is capable of correctly predicting major movements of the upstream slope of the dam due to liquefaction of silty sand layer beneath the shell.
One of the most damaging effects of earthquakes on earth structures is ground deformation due to liquefaction of loose saturated sands. This type of ground displacement has caused failures of numbers of earth dams including the classic failure of the Lower San Fernando Dam in California during the 1971 San Fernando earthquake. Large liquefaction induced ground deformations also occur in cases where static shear stresses do not exceed the residual strength of soils. Although the latter type of ground deformation does not involve a flow failure, it is potentially damaging and it has caused over one hundred million US dollar worth of damage in past earthquakes in California.

In practice, the earthquake induced deformation of earth structures is usually determined using the one-dimensional Newmark method. Despite its popularity among geotechnical engineers, the method suffer limitations in that it is only applicable for non-liquefiable soils whose strength and stiffness do not change markedly due to earthquake loading. Other similar methods such as Ambraseys-Sarma and Makdisi-Seed suffer the same limitations.

More rigorous methods employing a finite element approach are also used. These methods either use a simplified pseudo-dynamic approach incorporating pre- and post-earthquake stress-strain curves or use a full dynamic effective stress analyses incorporating complex stress-strain relationships of soils. However, complexities in
CHAPTER 12: Summary and Conclusions

determining the required soil parameters and the associated high cost generally limit these methods to research tools or to very large projects.

This thesis presents a simplified two-dimensional method for predicting liquefaction-induced ground displacements of earth structures. The method is based on mechanic principles and uses soil parameters that can be estimated from conventional soil tests. It has been verified by comparison with field experience.

The proposed procedure is basically an extension of Newmark method from a rigid plastic single-degree-of-freedom to a flexible multi-degree-of-freedom system. It is based on the assumption that the deformation prior to liquefaction is small in comparison to the deformation after liquefaction occurs. It takes account of the key factors for determination of liquefaction induced ground deformations: the zone of liquefaction, the realistic modelling of the post-earthquake (post-liquefaction) stress-strain behaviour of soils, the inclusion of the inertia forces of the earthquake, and the estimation of the post-earthquake settlement of soils. The proposed method has been incorporated into the SOILSTRESS-2 finite element program.

The proposed procedure requires determination of the pre- and post-earthquake stress-strain curves and the zone of liquefaction within earth structures. The pre-earthquake stress-strain curves are represented by hyperbolic curves, whereas the post-earthquake stress-strain curves are modelled by elastic perfectly-plastic curves. The zone of liquefaction may be determined using any accepted liquefaction analysis procedures. Where specific soil data is not available, the pre-earthquake stress-strain properties can be determined from the published hyperbolic stress-strain properties for similar soils. This estimate is acceptable since it does not significantly affect the end results. The stress-strain properties of liquefied soils are determined from the Standard Penetration Test data in conjunction with the available relationships of the corrected
CHAPTER 12: Summary and Conclusions

SPT blow counts versus residual strength and strain at residual strength. Alternatively, these parameters can be obtained from the results of post-cyclic monotonic loading of the soil samples in the triaxial, simple shear or torsional shear tests.

The analyses are carried out by computing the displacements that satisfy the requirement of energy balance of the system. For one-dimensional (1-D) problems, the displacements are computed from the closed solution proposed by Byrne (1990). For two-dimensional (2-D) problems, the displacements are computed using pseudo-dynamic finite element approach in which horizontal or vertical seismic coefficients are applied to the system by trial and error.

In this thesis, the 1-D solution proposed by Byrne (1990) has been validated against case histories of liquefaction ground displacements in the US and Japan. The computed displacements from this method are generally in good agreement with the measured displacements in the US case histories. The method, however, underestimates the measured displacements in Japanese sites. The discrepancy may be attributed to different behaviour of clean fine sand or smaller residual strength of the soil involved in Japanese case histories.

The proposed 2-D method has been validated against eight case histories of liquefaction-induced ground deformations. The predicted deformations in all cases are in good agreement with the observations in terms of both the magnitude and the pattern of displacements. The success of these predictions leads to a conclusion that the proposed method is a promising alternative for predicting liquefaction induced ground deformations of earth structures.

12.1. Suggestions for Future Research

Based on the results of the current studies, several topics for future research are suggested:
1. More validation against other case histories are needed to further check the general applicability of the proposed method. Numerous examples of lateral spreads in Japan during the 1964 earthquake, and in the US during the 1971 San Fernando earthquake may represent good case histories that deserve analytical studies using the proposed method.

2. Although the proposed method was developed for predicting ground deformations related to liquefaction of soils, the method may also be applicable for cases where no liquefaction takes place. More studies are suggested to investigate the applicability of the proposed method for non-liquefiable soils. Examples of the case histories of earthquake induced ground deformations involving non-liquefiable soils include the deformation of the La Villita and the El-Infiernillo dams during past earthquakes.

3. Little information is available on the post-earthquake stress-strain properties of gravelly soils. Recent findings show that these materials are also prone to liquefaction. Predictions of the deformations in these materials require more extensive data bases regarding their residual strengths and strains at residual strength. More research in this aspect, both in the laboratory and in the field, are needed.

4. The proposed procedures developed in this study was based on the small strain theory. Since liquefaction induced ground deformations usually involve large strains, the large strain theory is more appropriate. The incorporation of the large strain theory into the finite element method is suggested for further study.
BIBLIOGRAPHY


Byrne, P.M. (1991)."A Model For Predicting Liquefaction Induced Displacements," *Proc. 2nd Int'l. Conf. on Recent Advances in Geotech. Earthquake Eng. and Soil Dyn.*, St. Louis, 1027-1035. Also published as *Soil Mechanics Series No. 147*, Department of Civil Engineering, University of British Columbia, Vancouver, September 1990.


BIBLIOGRAPHY


BIBLIOGRAPHY


BIBLIOGRAPHY


BIBLIOGRAPHY

Simplified Procedure For The Analysis of Permanent Ground Displacement due to

Verification," PhD Thesis, Department of Civil Engineering, University of British
Columbia, Vancouver, Canada, 289 pages.


Ground Displacements," *Proc. 1st Japan-US Workshop on Liquefaction, Large
Ground Deformation and Their Effects on Lifeline Facilities*, National Center For
Earthquake Engineering Research, State University of New York at Buffalo, 22-31.


Analyses For Earthquakes: Numerical Solution and Constitutive Relations for Non-
linear (damage) Analysis," *Dams and Earthquakes*, Thomas Telford Limited, London,
179-194.
### Appendix A: Summary of Case Histories

#### Summary of Case Histories of Liquefaction Induced Displacements

<table>
<thead>
<tr>
<th>Cases</th>
<th>Tc (m)</th>
<th>TL (m)</th>
<th>Vo (m/s)</th>
<th>Slope (%)</th>
<th>FOS_L (N1)60_cs</th>
<th>Dmeas (m)</th>
<th>Liqdis1 (m)</th>
<th>Liqdis2 (m)</th>
<th>Restr. by non-liq. soils?</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>DNL2 structures or.</td>
<td></td>
</tr>
<tr>
<td>A. 1906 SAN FRANSISCO EQ.:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Valencia St</td>
<td>3</td>
<td>4.7</td>
<td>0.6</td>
<td>1.8</td>
<td>0.11</td>
<td>5.2</td>
<td>2.1</td>
<td>2.79</td>
<td>3.97</td>
<td>4.1</td>
</tr>
<tr>
<td>2. South of Market</td>
<td>3.65</td>
<td>4.9</td>
<td>0.6</td>
<td>0.8</td>
<td>0.24</td>
<td>8.8</td>
<td>1.3</td>
<td>1.15</td>
<td>1.89</td>
<td>0.94</td>
</tr>
<tr>
<td>Area</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Salinas River</td>
<td>4</td>
<td>7.2</td>
<td>0.28</td>
<td>2</td>
<td>0.6</td>
<td>6.9</td>
<td>1.8</td>
<td>3.19</td>
<td>4.91</td>
<td>2.07</td>
</tr>
<tr>
<td>4. Mission Creek</td>
<td>3.1</td>
<td>4.7</td>
<td>0.6</td>
<td>0.8</td>
<td>0.11</td>
<td>4.6</td>
<td>0.3</td>
<td>2.16</td>
<td>3.4</td>
<td>3.65</td>
</tr>
<tr>
<td>5. Coyote Creek</td>
<td>2</td>
<td>1.5</td>
<td>0.32</td>
<td>10</td>
<td>0.48</td>
<td>7.3</td>
<td>0.9</td>
<td>1.08</td>
<td>1.43</td>
<td>0.8</td>
</tr>
<tr>
<td>B. 1964 ALASKA EQ.:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Knik River #K-5a</td>
<td>7.6</td>
<td>10.2</td>
<td>0.21</td>
<td>0.5</td>
<td>0.6</td>
<td>11.9</td>
<td>0.6</td>
<td>0.57</td>
<td>1.51</td>
<td>0.5</td>
</tr>
<tr>
<td>7. Knik River #K-8</td>
<td>0.9</td>
<td>16.4</td>
<td>0.21</td>
<td>0.5</td>
<td>0.5</td>
<td>11.8</td>
<td>0.23</td>
<td>0.6</td>
<td>1.97</td>
<td>0.53</td>
</tr>
<tr>
<td>8. Matanuska MP147.1#M3</td>
<td>0</td>
<td>12.9</td>
<td>0.21</td>
<td>0.6</td>
<td>0.55</td>
<td>13.6</td>
<td>0.3</td>
<td>0.37</td>
<td>1.23</td>
<td>0.34</td>
</tr>
<tr>
<td>9. Matanuska MP147.1#M4</td>
<td>0</td>
<td>18</td>
<td>0.21</td>
<td>0.1</td>
<td>0.59</td>
<td>14</td>
<td>0.3</td>
<td>0.37</td>
<td>1.15</td>
<td>0.33</td>
</tr>
<tr>
<td>10. Matanuska MP147.4#M10</td>
<td>0</td>
<td>23.4</td>
<td>0.21</td>
<td>0.1</td>
<td>0.44</td>
<td>14</td>
<td>0.25</td>
<td>0.49</td>
<td>1.55</td>
<td>0.48</td>
</tr>
<tr>
<td>11. Matanuska MP147.5#M12</td>
<td>0</td>
<td>9.9</td>
<td>0.21</td>
<td>0.2</td>
<td>0.45</td>
<td>10.4</td>
<td>0.17</td>
<td>0.33</td>
<td>1.02</td>
<td>0.29</td>
</tr>
<tr>
<td>12. Matanuska MP148.3#M20</td>
<td>0</td>
<td>18.3</td>
<td>0.21</td>
<td>0.7</td>
<td>0.54</td>
<td>10.9</td>
<td>1.3</td>
<td>1.23</td>
<td>3.56</td>
<td>0.96</td>
</tr>
<tr>
<td>13. Matanuska MP148.3#M20</td>
<td>0.1</td>
<td>12.8</td>
<td>0.21</td>
<td>0.8</td>
<td>0.64</td>
<td>14.1</td>
<td>0.23</td>
<td>0.37</td>
<td>1.24</td>
<td>0.32</td>
</tr>
<tr>
<td>14. Resurrection R. MP3.1#3</td>
<td>0</td>
<td>4.5</td>
<td>0.41</td>
<td>0.1</td>
<td>0.61</td>
<td>26.4</td>
<td>0.3</td>
<td>0.03</td>
<td>0.07</td>
<td>0.03</td>
</tr>
<tr>
<td>15. Resurrection R. MP3.2#6</td>
<td>0</td>
<td>8.7</td>
<td>0.41</td>
<td>0.3</td>
<td>0.23</td>
<td>12.7</td>
<td>0.25</td>
<td>0.42</td>
<td>1</td>
<td>0.42</td>
</tr>
<tr>
<td>16. Resurrection R. MP3.3#8</td>
<td>0</td>
<td>3.8</td>
<td>0.41</td>
<td>0.3</td>
<td>0.28</td>
<td>15.4</td>
<td>0.75</td>
<td>0.12</td>
<td>0.3</td>
<td>0.12</td>
</tr>
<tr>
<td>17. Portage Crk. MP63.0#2</td>
<td>4.5</td>
<td>8.3</td>
<td>0.31</td>
<td>0.2</td>
<td>0.84</td>
<td>21.7</td>
<td>1.9</td>
<td>0.12</td>
<td>0.29</td>
<td>0.12</td>
</tr>
<tr>
<td>18. Portage Crk. MP63.0#4</td>
<td>3.2</td>
<td>4.3</td>
<td>0.31</td>
<td>0.2</td>
<td>0.51</td>
<td>10</td>
<td>1.85</td>
<td>0.38</td>
<td>0.82</td>
<td>0.35</td>
</tr>
<tr>
<td>19. Ship Creek MP114.3#16</td>
<td>4.7</td>
<td>1.6</td>
<td>0.18</td>
<td>3</td>
<td>0.12</td>
<td>6</td>
<td>0.24</td>
<td>0.99</td>
<td>1.38</td>
<td>1.24</td>
</tr>
<tr>
<td>20. Snow River Br.605A#1A</td>
<td>0</td>
<td>28.2</td>
<td>0.39</td>
<td>0.1</td>
<td>0.31</td>
<td>9.7</td>
<td>2.4</td>
<td>2.1</td>
<td>5</td>
<td>1.94</td>
</tr>
<tr>
<td>C. 1971 SAN FERNANDO EQ.:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21. Joseph Jensen Filtration Plant</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Section A-A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22. JJFP B-B</td>
<td>7.5</td>
<td>2.5</td>
<td>0.55</td>
<td>3.7</td>
<td>0.58</td>
<td>15.5</td>
<td>0.57</td>
<td>0.4</td>
<td>0.58</td>
<td>0.28</td>
</tr>
<tr>
<td>23. JJFP C-C</td>
<td>10.3</td>
<td>5.1</td>
<td>0.55</td>
<td>3.5</td>
<td>0.49</td>
<td>10.6</td>
<td>1.7</td>
<td>2.52</td>
<td>3.24</td>
<td>1.97</td>
</tr>
<tr>
<td>24. JJFP D-D</td>
<td>15.9</td>
<td>1.5</td>
<td>0.55</td>
<td>1</td>
<td>0.61</td>
<td>10.8</td>
<td>0.53</td>
<td>0.59</td>
<td>0.72</td>
<td>0.46</td>
</tr>
<tr>
<td>25. Juvenile Hall Middle Sect.</td>
<td>3.35</td>
<td>4.95</td>
<td>0.55</td>
<td>5.2</td>
<td>0.51</td>
<td>10</td>
<td>1.85</td>
<td>1.95</td>
<td>2.67</td>
<td>1.43</td>
</tr>
<tr>
<td>26. Juvenile Hall South Section</td>
<td>3.2</td>
<td>4.22</td>
<td>0.55</td>
<td>5.2</td>
<td>0.58</td>
<td>12.7</td>
<td>0.7</td>
<td>0.82</td>
<td>1.31</td>
<td>0.7</td>
</tr>
<tr>
<td>27. Juvenile Hall North Section</td>
<td>5.8</td>
<td>1.43</td>
<td>0.55</td>
<td>5.2</td>
<td>0.5</td>
<td>10.4</td>
<td>0.62</td>
<td>0.72</td>
<td>0.9</td>
<td>0.58</td>
</tr>
</tbody>
</table>
### Summary of Case Histories of Liquefaction Induced Displacements (CONTINUED)

<table>
<thead>
<tr>
<th>Cases</th>
<th>Tc</th>
<th>TL</th>
<th>Vo</th>
<th>Slope</th>
<th>$FOS_L (N1)_{60 _cs}$</th>
<th>Dmeas</th>
<th>Liqdis1</th>
<th>Liqdis2</th>
<th>Restr. by Remarks</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(m)</td>
<td>(m)</td>
<td>(m/s)</td>
<td>(%)</td>
<td>(m)</td>
<td>(m)</td>
<td>(m)</td>
<td>(m)</td>
<td>(m)</td>
<td>(m)</td>
</tr>
<tr>
<td>D. 1983 BORAH PEAK EQ.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28. Pence Ranch-HY2</td>
<td>1.5</td>
<td>3.8</td>
<td>0.47</td>
<td>1</td>
<td>0.47</td>
<td>11.7</td>
<td>0.3</td>
<td>0.28</td>
<td>0.64</td>
<td>0.26</td>
</tr>
<tr>
<td>29. Whiskey Spring Bor.WS-1</td>
<td>1</td>
<td>2.6</td>
<td>0.6</td>
<td>12</td>
<td>0.26</td>
<td>8.9</td>
<td>1</td>
<td>1.21</td>
<td>1.64</td>
<td>1.04</td>
</tr>
<tr>
<td>30. Whiskey Spring Bor. WS-3</td>
<td>7</td>
<td>4.3</td>
<td>0.6</td>
<td>6.7</td>
<td>0.44</td>
<td>11.4</td>
<td>1.2</td>
<td>2.42</td>
<td>3.11</td>
<td>2</td>
</tr>
<tr>
<td>E. IMPERIAL VALLEY EQ.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31. Wildlife</td>
<td>2.5</td>
<td>1</td>
<td>0.21</td>
<td>1</td>
<td>0.95</td>
<td>6.3</td>
<td>0.18</td>
<td>0.21</td>
<td>0.42</td>
<td>0.08</td>
</tr>
<tr>
<td>32. Heber Road</td>
<td>1.3</td>
<td>3.7</td>
<td>0.8</td>
<td>1</td>
<td>0.16</td>
<td>5</td>
<td>3.2</td>
<td>1.69</td>
<td>2.56</td>
<td>2.2</td>
</tr>
<tr>
<td>F. 1983 NIHONKAI CHUBU EQ. (JAPAN)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33. Section N1-1L</td>
<td>1.8</td>
<td>4.1</td>
<td>0.25</td>
<td>0.5</td>
<td>0.74</td>
<td>15.1</td>
<td>1.2</td>
<td>0.13</td>
<td>0.36</td>
<td>0.11</td>
</tr>
<tr>
<td>34. Section N2-2L</td>
<td>2.9</td>
<td>7.9</td>
<td>0.25</td>
<td>2.5</td>
<td>0.69</td>
<td>13.3</td>
<td>1.2</td>
<td>0.68</td>
<td>1.59</td>
<td>0.43</td>
</tr>
<tr>
<td>35. Section N2-3L</td>
<td>1.6</td>
<td>8</td>
<td>0.25</td>
<td>2.5</td>
<td>0.62</td>
<td>12.9</td>
<td>2</td>
<td>0.63</td>
<td>1.56</td>
<td>0.51</td>
</tr>
<tr>
<td>36. Section N3-1L</td>
<td>1.8</td>
<td>4.8</td>
<td>0.25</td>
<td>8.8</td>
<td>0.81</td>
<td>16.6</td>
<td>2</td>
<td>0.32</td>
<td>0.72</td>
<td>0.58</td>
</tr>
<tr>
<td>37. Section N3-2L</td>
<td>2.9</td>
<td>4.4</td>
<td>0.25</td>
<td>1</td>
<td>0.83</td>
<td>15.7</td>
<td>0.9</td>
<td>0.15</td>
<td>0.39</td>
<td>0.11</td>
</tr>
<tr>
<td>38. Section N4-2L</td>
<td>3</td>
<td>6.4</td>
<td>0.25</td>
<td>2</td>
<td>0.63</td>
<td>13.3</td>
<td>1.8</td>
<td>0.46</td>
<td>1.13</td>
<td>0.33</td>
</tr>
<tr>
<td>39. Section N4-3L</td>
<td>1.4</td>
<td>3.5</td>
<td>0.25</td>
<td>5.8</td>
<td>0.63</td>
<td>14.3</td>
<td>3</td>
<td>0.25</td>
<td>0.6</td>
<td>0.17</td>
</tr>
<tr>
<td>40. Section S2-1L</td>
<td>5.7</td>
<td>1.5</td>
<td>0.25</td>
<td>15</td>
<td>0.67</td>
<td>12.4</td>
<td>2.7</td>
<td>1.16</td>
<td>1.53</td>
<td>0.78</td>
</tr>
<tr>
<td>41. Section S3-1L</td>
<td>2.7</td>
<td>2.7</td>
<td>0.25</td>
<td>3</td>
<td>0.83</td>
<td>17.3</td>
<td>3.5</td>
<td>0.39</td>
<td>0.61</td>
<td>0.09</td>
</tr>
<tr>
<td>42. Section S4-1L</td>
<td>4.7</td>
<td>4.1</td>
<td>0.25</td>
<td>10</td>
<td>0.84</td>
<td>14.7</td>
<td>2.5</td>
<td>0.85</td>
<td>1.23</td>
<td>0.37</td>
</tr>
<tr>
<td>43. Section S5-1L</td>
<td>4.8</td>
<td>4.8</td>
<td>0.25</td>
<td>9</td>
<td>0.84</td>
<td>14.9</td>
<td>1.3</td>
<td>0.87</td>
<td>1.33</td>
<td>0.39</td>
</tr>
<tr>
<td>44. Section S6-1L</td>
<td>4.7</td>
<td>3.7</td>
<td>0.25</td>
<td>9</td>
<td>0.88</td>
<td>15</td>
<td>1.2</td>
<td>0.63</td>
<td>0.98</td>
<td>0.35</td>
</tr>
<tr>
<td>45. Section S7-1L</td>
<td>4.5</td>
<td>5.3</td>
<td>0.25</td>
<td>9.3</td>
<td>0.81</td>
<td>14.3</td>
<td>1.8</td>
<td>1.19</td>
<td>1.7</td>
<td>0.79</td>
</tr>
<tr>
<td>46. Section S7-2L</td>
<td>2.1</td>
<td>6.1</td>
<td>0.25</td>
<td>1.4</td>
<td>0.26</td>
<td>5.7</td>
<td>2.5</td>
<td>1.88</td>
<td>3.66</td>
<td>2.35</td>
</tr>
<tr>
<td>47. Section S7-3L</td>
<td>0.8</td>
<td>3.4</td>
<td>0.25</td>
<td>0.8</td>
<td>0.32</td>
<td>5.2</td>
<td>1</td>
<td>0.52</td>
<td>1.31</td>
<td>0.69</td>
</tr>
<tr>
<td>48. Section S7-4L</td>
<td>0.7</td>
<td>3.9</td>
<td>0.25</td>
<td>0.8</td>
<td>0.35</td>
<td>7.9</td>
<td>0.5</td>
<td>0.35</td>
<td>0.93</td>
<td>0.28</td>
</tr>
<tr>
<td>49. Section S12-1L</td>
<td>2.1</td>
<td>2.1</td>
<td>0.25</td>
<td>5.5</td>
<td>0.56</td>
<td>10.5</td>
<td>1.5</td>
<td>0.41</td>
<td>0.76</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Note: $T_c$=Crust thickness; $T_L$=Thickness of liq. layer; $V_o$=maximum velocity; $D_{L1}$=Linear Liqdisp;$D_{L2}$=Nonlinear Liqdisp; $D_{NL1}$=Linear Mod. Liqdisp; $D_{NL2}$=Non-linear Mod. Liqdisp.
The stress-strain model of soil used in both the original SOILSTRESS (Byrne and Janzen, 1981) and the SOILSTRESS-2 computer programs is a hyperbolic model (Kondner, 1963; Kondner and Zelasko, 1963). This model has also been used by many researchers to develop their analytical procedures (e.g. Kulhawy et al., 1969; Duncan and Chang, 1970; Serff et al., 1976; Finn et al., 1986). Part of its popularity is that it is simple and the required parameters can readily be obtained from the conventional laboratory triaxial tests. Moreover, hyperbolic soil parameters for many different types of soils are available (Duncan et al., 1980) and they can be used to approximate the soil parameters where specific stress-strain data is lacking.

Hyperbolic soil parameters are usually obtained from laboratory triaxial compression tests. The soil moduli obtained directly from these tests are Young's modulus, $E$, and bulk modulus, $B$. However, it is often convenient to work in terms of shear modulus, $G$, rather than $E$, as it is used in this procedure. $E$, $B$ and $G$ are interrelated with Poisson's ratio, $\mu$, according to the following equation:

$$G = \frac{E}{2(1 + \mu)} \quad \text{ (B.1)}$$

$$B = \frac{E}{3(1 - 2\mu)} \quad \text{ (B.2)}$$

and,

$$\mu = \frac{1}{2}(1 - \frac{E}{3B}) \quad \text{ (B.3)}$$

The relationship between shear stress, $\tau$, and shear strain, $\gamma$, for a hyperbolic model shown in Figs. 5.1.a and b is given by:
Appendix B: Stress-Strain Model of Soil

\[ \tau = \frac{\gamma}{\left( \frac{1}{G_m} + \frac{\gamma \cdot R_f}{s} \right)} \]  \hspace{1cm} (B.4)

where,

\[ R_f = \frac{s}{\tau_{ult}} \]  \hspace{1cm} (B.4.1)

The soil parameters appearing in the above equations are:

- \( G_m \) = the maximum shear modulus as shear strain, \( \gamma \), approaches 0,
- \( R_f \) = the failure ratio,
- \( \tau_{ult} \) = the ultimate shear stress predicted from the hyperbola when the shear strain goes to infinity,
- \( s \) = the shear strength of soil. This will be discussed in further details in Section B.2.

The failure ratio, \( R_f \), is a measure of how well the stress-strain curve approaches a hyperbola. \( R_f \) value for non-liquefied soils are usually ranging from 0.6 to 0.9 and the typical stress-strain curve is shown in Fig. 5.1.a. A value of \( R_f \) equal to unity represents a true hyperbolic curve. A value of \( R_f \) equal to zero represents an elasto-plastic stress-strain curve, as shown in Fig. 5.1.b.

B.2. Failure Criterion

Soils, as many other geologic materials, exhibit a stress dependent behaviour. Both the soil stiffness and strength generally increase with increasing effective overburden stress. The stress dependency of soil strength can closely be represented by Mohr-Coulomb failure criterion and this criterion is used in the proposed procedures. The Mohr-Coulomb failure criterion is given by:

\[ s = c + \sigma \tan \phi \]  \hspace{1cm} (B.5)

where,
Appendix B: Stress-Strain Model of Soil

\[ s = \text{the shear strength of soil or the shear stress at failure,} \]
\[ c = \text{the soil cohesion,} \]
\[ \phi = \text{the angle of internal friction, and} \]
\[ \sigma = \text{the effective normal stress at failure surface.} \]

In conventional drained compression triaxial tests, failure is reached by keeping the minor effective principal stress, \( \sigma_3 \), constant while increasing the major effective principal stress, \( \sigma_1 \). Based on this, it is sometimes assumed that a soil element in the field fails following this stress path. However, a soil element in the field does not always follow the stress path similar to that in conventional compression triaxial tests. It is often

Fig.B.1. Mohr failure criterion using constant mean effective normal stress.
Appendix B: Stress-Strain Model of Soil

The failure criterion for this condition can be expressed in terms of principal effective stresses. With reference to Fig. B.1., the following relationship is obtained:

\[ \tau_{\text{max}} = AB + BF \]  \hspace{1cm} (B.6)

or,

\[ \tau_{\text{max}} = OA \cdot \sin \phi + c \cdot \cos \phi \]  \hspace{1cm} (B.6.1)

Hence, \[ \tau_{\text{max}} = \left( \frac{\sigma_1 + \sigma_3}{2} \right) \cdot \sin \phi + c \cdot \cos \phi \]  \hspace{1cm} (B.6.2)

where,

\[ \tau_{\text{max}} = \text{the maximum shear stress at failure}, \]

\[ \sigma_1 = \text{the major principal stress}, \]

\[ \sigma_3 = \text{the minor principal stress}. \]

If it is assumed that \( \tau_f \) is the shear stress at failure on the failure plane, then:

\[ s = \tau_f = \tau_{\text{max}} \cdot \cos \phi = c + \sigma_f \cdot \tan \phi \]

as given in Eq. B.5.

For any combination of stresses, \( \sigma_x, \sigma_y, \tau_{xy} \), the factor of safety (FS) of a soil element in the field is given by:

\[ FS = \frac{\tau_{\text{max}}}{\tau_m} \]  \hspace{1cm} (B.7)

where,

\[ \tau_m = \sqrt{\left( \frac{\sigma_y - \sigma_x}{2} \right)^2 + \tau_{xy}^2} \]  \hspace{1cm} (B.7.1)

\[ \tau_m = \text{the maximum shear stress of a given soil element.} \]

The angle of internal friction in the above equations is assumed to be dependent on the effective confining stress and is given by:

\[ \phi = \phi_1 - \Delta \phi \cdot \log \left( \frac{\sigma_e}{\text{Pa}} \right) \]  \hspace{1cm} (B.8)

in which,

\[ \phi_1 = \text{the effective internal friction angle at a confining stress of 1 atmosphere}, \]

\[ \Delta \phi = \text{the decrease in friction angle for a ten fold increase in confining stress, and} \]
Appendix B: Stress-Strain Model of Soil

\( \sigma_c \) = the effective confining stress.

The strength parameters, \( c \) and \( \phi \), for pre-earthquake stress-strain curve are usually obtained from the results of laboratory triaxial tests. However, in case of lacking of laboratory data, the shear strength can also be obtained from cone penetration or standard penetration data. For example, the correlations between cone penetration resistance, \( q_c \), with strength parameters such as proposed by Robertson and Campanella (1986) can be used for this purpose.

B.3. Shear Modulus

As mentioned previously, the soil stiffness increases with increasing confining stress. Laboratory data suggest that the relationship between the initial shear stiffness or maximum shear modulus, \( G_m \), and the effective mean normal stress, \( \sigma_m \), can be expressed as follow:

\[
G_m = k_g \cdot Pa \cdot \left( \frac{\sigma_m}{Pa} \right)^n
\]  

(B.9)

in which,

- \( G_m \) = the maximum shear modulus,
- \( k_g \) = the shear modulus constant,
- \( Pa \) = the atmospheric pressure,
- \( \sigma_m \) = the effective mean normal stress, and
- \( n \) = the shear modulus exponent.

The secant shear modulus of a soil element at any strain level depends on the maximum shear modulus and the shear strength of soil, as given by:

\[
G = G_m \cdot (1 - \frac{\tau \cdot Rf}{s})
\]  

(B.10)

Introducing Eqs. B.5. and B.9 into Eq. B.10, Eq. B.10. becomes:

\[
G = k_g \cdot Pa \cdot \left( \frac{\sigma_m}{Pa} \right)^n \cdot (1 - \frac{\tau \cdot Rf}{c + \sigma_m \tan \phi})
\]  

(B.11)
Appendix B : Stress-Strain Model of Soil

The pre-earthquake hyperbolic stiffness parameters, $G_m$, $k_g$, and $n$, are obtained from the results of laboratory triaxial tests. The method to determine these parameters are described by Duncan et al. (1980) and Byrne et al. (1987).

B. 4. Bulk Modulus

Similar to shear modulus, bulk modulus is also stress dependent. Laboratory data demonstrate that the bulk modulus increases with increasing confining stress. The relationship between the bulk modulus, $B$, and the mean normal stress, $\sigma_m$, is given by:

$$B = k_b \cdot \text{Pa} \left(\frac{\sigma_m}{\text{Pa}}\right)^m$$

(B.12)

in which,

$k_b$ = the bulk modulus constant, and

$m$ = the bulk modulus exponent.

In the proposed simplified two-dimensional procedure, the bulk modulus of the soils is assumed to be very high to simulate undrained condition. This is implemented by assigning high $k_b$ values for both pre and post earthquake conditions. The pre-earthquake stress conditions are commonly computed based on the assumption that drained condition prevails during the construction of soil structures. In this assumption, lower soil bulk moduli are used. The use of high bulk moduli for pre-earthquake condition causes higher Poisson's ratio and slightly different stress conditions in the element. However, the shear stress is only slightly change and the difference in the computed displacements are insignificant.

If smaller bulk moduli such as used in the common assumption are employed in the pre-earthquake condition, extra volumetric strains will be predicted due to the difference between the bulk moduli of the pre and the post-earthquake conditions. This will cause additional external work into the system which in turn will give erroneous predicted displacements.
APPENDIX C:
FINITE ELEMENT PROCEDURE

The finite element method is one of the numerical techniques for solving problems of continuum mechanics with an accuracy acceptable to the engineers (Desai and Christian, 1977). In this procedure, soil continuum is discretized into an equivalent system of smaller continua called finite elements. These elements are interconnected by nodal points. In this way, the spatial variability of soil structure systems can be represented by a number of finite elements with corresponding soil properties. This finite element technique was employed to extend Byrne's one dimensional method (Byrne, 1990) to two-dimensional method that is capable of giving deformation patterns of soil structures.

Different shapes of finite elements are available. For one dimensional analyses, line elements either linear or curved is employed. For two dimensional engineering problems, triangular and quadrilateral elements are generally used to discretize the soil continuum. For three dimensional problems, the common elements used are tetrahedra and hexahedra.

The problems in solid mechanics can be formulated by one of the following: displacement method, stress or equilibrium method and mixed method that combines displacement and stress methods. In the displacement method, displacement is primary unknown and stress and strain are secondary unknowns. In the stress method, stress is the primary unknown whereas in the mixed method, both stress and displacement are assumed to be unknowns.

Among the three methods described above, the displacement method is the most popular method for solving geotechnical engineering problems. In the displacement method, it is relatively simple to establish an approximate function that satisfies compatibility requirements. Moreover, the displacement method offers smaller number and bandwidth of the final stiffness equations than those produced by the other methods (Desai and Christian, 1977). This method is used in SOILSTRESS-2 computer program.
To be acceptable, any numerical formulation must give a solution that converges to the exact solution. In the displacement method, it has been shown in the past that under certain circumstances this approach provides an upper bound to the true stiffness of the structure (Desai and Abel, 1972). This means that the computed displacement is smaller than the exact solution. However, as the finite element mesh is made finer, the solution will converge to the exact answer from below. To achieve this, the elements must satisfy the requirements of compatibility and completeness.

The finite elements are called compatible or conforming if the displacements are continuous within the element and compatible between adjacent element. The elements are called complete if the displacement model includes the rigid body displacement and constant strain states of the element.

In the following sections, the element shape and displacement model used in SOILSTRESS-2 will be discussed. In addition, the nonlinearities arising from both non-linear behaviour of soil as well as geometric non-linearity will be discussed.

C.1. Formulation of the Finite Element Model

The SOILSTRESS-2 computer program uses both triangular and quadrilateral elements to discretize soil structures. The element type used is a six degree-of-freedom constant strain triangle (CST), as shown in Fig. C.1. The quadrilateral element is basically an assemblage of four constant strain triangles. The CST element uses linear displacement field and is compatible and complete. Thus, the computed displacement represents a lower bound value and will converge to the exact solution when finer finite elements are used.

Using the principle of virtual work, the relationship between the element nodal displacements, \{\delta\}, element stiffness, \([k]\), and element nodal forces, \{f\}, are given by:

\[
[k] \{\delta\} = \{f\} \quad \text{(C.1)}
\]
Since soil response is governed by effective instead of total stress, the element stiffness matrix $[k]$ in Eq. C.1 is formulated based on effective stress-strain relation and leads to the following equation:

$$[k] \{\delta\} = \{f\} - \{k^*\} u \quad (C.2)$$

in which:

$[k] =$ the element stiffness matrix,

$\{\delta\} =$ the element nodal displacement,

$\{f\} =$ the element nodal forces, and

$[k^*] =$ a vector associated with the element pore pressure.

The element stiffness and nodal forces are then combined to form the global stiffness and the load vector of the system. This is given by:
Appendix C: Finite Element Procedure

\[ [K] \{\Delta\} = \{F\} - [K^*] \{u\} = \{F'\} \]  
(C.3)

in which,

- \([K]\) = the global stiffness matrix of the system,
- \(\{\Delta\}\) = the nodal displacements of the system,
- \(\{F\}\) = the total load vector of the system,
- \([K^*]\) = a vector associated with the element pore pressures,
- \(\{u\}\) = the elements pore pressures, and
- \(\{F'\}\) = the effective load vector.

As mentioned before, the analysis is based on small strain and plane strain assumptions. For this condition, the relationship between strains and displacements is represented by:

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = 
\begin{bmatrix}
\frac{\partial u}{\partial x} \\
\frac{\partial v}{\partial y} \\
\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}
\end{bmatrix}
\]  
(C.4),

in which,

- \(\varepsilon_x, \varepsilon_y, \gamma_{xy}\) = the element strains, and
- \(u, v\) = the nodal displacements in \(x\) and \(y\) direction.

The nodal displacements computed from Eq. C.3 are then used to determine the stresses and strains of each element in the system. The element strains are first computed using:

\[ [\varepsilon] = [C] \{\delta\} \]  
(C.5)

Subsequently, the element stresses are computed from:

\[ [\sigma] = [D] \{\varepsilon\} \]  
(C.6)

in which,

- \(\{\varepsilon\}\) = the matrix of element strains,
[C] = the strain displacement matrix that depend on the geometry of the element and Eq. C.4,

{\sigma} = the matrix of element stresses, and

[D] = the elasticity matrix of soils.

For plain strain condition, the elasticity matrix [D] is given by:

\[
[D] = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1-\mu & \mu & 0 \\ \mu & 1-\mu & 0 \\ 0 & 0 & \frac{1-2\mu}{2} \end{bmatrix}
\] (C.7)

C.2. Non-linearity

The formulation described in the previous section is based on the assumptions that the stress-strain curve of soil is linear elastic and the displacements are small. In most cases, however, the stress-strain behaviour of soil is inelastic and highly non-linear. Moreover, the analyses of liquefaction induced deformation usually involve large soil movements. Because of these, two problems of non-linearity arise. The first is material non-linearity which arises from the non-linear constitutive relations of soils. The second is geometric non-linearity which results from the finite changes in geometry of the deforming mass. The analyses must therefore be carried out by considering these non-linearities in order to yield realistic results.

There are three basic techniques for solving non-linear problems in the finite element method: incremental method, iterative method and combination of the two. Many variants of these methods are also available (Desai and Abel, 1972; Cook, 1981). Each method has its advantages and limitations over the other methods. Thus, the selection of the method basically depends on several factors including the stress-strain law of the materials considered.
Appendix C: Finite Element Procedure

Fig. C.2. a) Incremental method. b) Iterative method.
The incremental method is performed by dividing the applied total load into smaller load increments, as shown in Fig. C.2.a. These load increments are usually of the same magnitude although they are not necessarily equal. The load increments are applied one in a time and the equations are assumed to be linear during this process. Displacement due to this increment is computed based on tangent stiffness. This incremental process is repeated until all the load increments are applied. The total displacement is obtained from the accumulation of the displacement at each increment. Difficulty arises when this method is used for strain softening or elastic-perfectly plastic material (Desai and Abel, 1972). Thus, this method is not used in this analysis procedure since the proposed procedure uses an elastic-perfectly plastic stress-strain model for post-earthquake response of liquefied soils.

The iterative method is carried out by applying the full load at each step but varying the moduli until strain compatible moduli are achieved in each element. This method is used in SOILSTRESS-2 to tackle both material and geometric non-linearities, as will be described later. The method used here is schematically shown in Fig. C.2.b.

The iterative method employed here uses a secant modulus approach and requires that the stiffness matrix is updated at each iteration. However, the load vector is kept constant during the iteration process. At the first iteration, an initial stiffness matrix is formed and the corresponding nodal displacements are solved. Based on these results, a new stiffness matrix is built and new displacements are computed. This process is repeated until the prescribed convergence criteria for displacement and shear modulus are satisfied. The advantage of using this method over the incremental method is that it is more stable for strain softening material or elastic plastic materials when all the soil strength is fully mobilized.

The geometric non-linearity is also solved using this iterative approach. At each iteration, the stiffness matrix is updated based on new values of secant moduli, and load vector is also updated based on the new geometry of the deforming mass.
C.3. Volume Correction

During the analyses, the bulk modulus of the soils are computed from the value of shear moduli based on Eq. B.12. For a very high bulk modulus value (relative to the shear modulus), the computed Poisson’s ratio, $\mu$, will be close to 0.5 and this will lead to a numerical instability in the elasticity matrix $[D]$. To avoid this, $\mu$ is defaulted to a value close to 0.5 such as 0.495. This defaulted Poisson’s ratio leads to a smaller bulk modulus (moduli is computed using Eq. B.2) which in turn results in an additional volumetric strain (Fig. C.3). This additional volumetric strain, $\Delta e_v^*$, can be computed from:

$$\Delta e_v^* = \left( \frac{\sigma_m}{B'} - \frac{\sigma_m}{B} \right)$$

in which,

$\sigma_m$ = the change in mean effective stress,

$B$ = the bulk modulus of soil computed from Eq. B.12, and

$B'$ = the bulk modulus of soil computed from Eq. C.2 using defaulted $\mu$ of 0.495.

Although this additional volumetric strain is not significant for soils with high shear moduli, it will seriously overestimate the total volumetric strain for cases involving liquefied soils whose soil moduli are very small. This volumetric strain is corrected by introducing an additional pore pressure load vector, $\{f\}_b$, in Eq. C.3 so the equation becomes:

$$[K]_{\text{end}} \{\Delta\} = \{F'\} + \{f\}_b$$

$$\{f\}_b = [K] \{\Delta u\}$$

$$\Delta u = B \Delta e_v^*$$

in which,

$[K]$ and $[F']$ = the global matrix and load vector of the system at the end of iteration, and

$[\Delta]$ = the corrected nodal displacements.
Fig. C.3. Additional volumetric strain due to defaulted bulk modulus.
C.4. Evaluation of Post-earthquake Settlement

The excess pore pressure due to the shaking would drain sometimes after the earthquake causing post-earthquake volumetric strains, $\varepsilon_v$, and settlements. These additional settlement are evaluated using similar procedure as that used to perform volume correction due to the use of default Poisson’s ratio as explained in the preceding section. An additional pore pressure load vector due to post-earthquake volumetric strains, $\{f\}_s$, is added to Eq. C.9. The equation now can be written as:

$$[K] \{\Delta\} = \{F'\} + \{f\}_b + \{f\}_s \quad (C.10),$$

$$\{f\}_s = [K] \{\Delta u\}_s \quad (C.10.1),$$

$$\Delta u_s = -B \varepsilon_v \quad (C.10.2)$$

Negative sign in Eq. C.10.2 indicates that the nodal forces due to these volumetric strains are acting in compression to yield downward vertical displacements.
APPENDIX D:

By referring to Fig. 5.5, the internal work due to volumetric strains, $W_{\text{int,vol}}$, is computed as follows:

$$W_{\text{int,vol}} = \int_{\varepsilon_v}^{\varepsilon_v f} \sigma_m d\varepsilon_v$$  \hspace{1cm} (D.1)

in which,

$$\varepsilon_v = \frac{\sigma_m}{B}$$  \hspace{1cm} (D.2)

and,

$$B = k_b P_a \left( \frac{\sigma_m}{P_a} \right)^m$$  \hspace{1cm} (D.3)

Eq. D.1 can also be expressed as:

$$W_{\text{int,vol}} = \sigma_m f \cdot \varepsilon_v f - \sigma_m i \cdot \varepsilon_v i - \int \varepsilon_v d\sigma_m$$  \hspace{1cm} (D.4)

Hence,

$$W_{\text{int,vol}} = \sigma_m f \cdot \varepsilon_v f - \sigma_m i \cdot \varepsilon_v i - \int \frac{\sigma_m}{B} d\sigma_m$$  \hspace{1cm} (D.5)

$$W_{\text{int,vol}} = \sigma_m f \cdot \varepsilon_v f - \sigma_m i \cdot \varepsilon_v i - \int \frac{\sigma_m f}{\sigma_m} d\sigma_m$$  \hspace{1cm} (D.6)

Finally,

$$W_{\text{int,vol}} = \sigma_m f \cdot \varepsilon_v f - \sigma_m i \cdot \varepsilon_v i - \frac{1}{k_b (P_a)^{(1-m)} (2-m)} (\sigma_m f^{(2-m)} - \sigma_m i^{(2-m)})$$  \hspace{1cm} (D.7)