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Department of Civil Engineering
THE UNIVERSITY OF BRITISH COLUMBIA
2324 MAIN MALL
VANCOUVER, BC V6T 1Z4
CANADA
Date: June 1998
ABSTRACT

The present study investigates the diffraction problem of waves interacting with a long rectangular breakwater with two side plates, for brevity it will be referred to as pi-type. The effect of the addition of two side-boards on the performance of a rectangular breakwater is the main aim of this study. The structure is assumed to be placed in water of uniform constant depth, and an eigenfunction expansion method was used in the solution. The fluid is assumed inviscid, and the flow irrotational, so that a velocity potential may be used to describe the flow. The exciting forces, added masses, damping coefficients, responses of the structure, and the transmission coefficient were all calculated for different cases and their results were analyzed. Different waves approaching the breakwater from any direction are considered. Comparisons between the present approach and different numerical solutions published previously were used to verify the developed numerical model. Other comparisons between the traditional rectangular breakwater and the proposed pi-type breakwater are also performed.

The present solution proved to be a fast accurate method as compared to other methods. The present study showed that maximum wave forces and heave response occur for the normal wave incidence, but the surge and pitch responses are not maximum for normal waves and different directions then need to be considered. The proposed breakwater experiences lower exciting forces when compared to a rectangular one having the same under-tip clearance, thus savings of materials and cost are achieved by using this type, provided its good performance with respect to wave transmission is assumed. The
numerical results showed good agreement with the measured transmission coefficient for both the fixed structure and freely floating structure cases.
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LIST OF SYMBOLS

For the sake of reducing the list of notations, only symbols representing physical quantities are listed here, while other symbols used in mathematical derivations are defined when mentioned or in the Appendix.

- $a$: half beam width
- $A_{ij}$: added mass coefficient
- $b$: plate height
- $B_{ij}$: damping coefficient
- $h$: breakwater's draft
- $F_{ij}$: exciting forces
- $F_{ij}$: reaction forces due to radiated waves
- $g$: acceleration due to gravity
- $GM$: metacenter to center of gravity distance
- $d$: water depth
- $H$: incident wave height
- $k$: incident wave number
- $ka$: diffraction parameter
- $K$: Keulegan-Carpenter number
- $L$: incident wave length
- $m$: mass of unit length of the breakwater
- $M_0$: polar mass moment of inertia of the breakwater's about y-axis per unit length
- $N, M$: truncation parameters
- $P$: hydrodynamic pressure
- $P_e$: wave energy flux
- $R, T$: reflection and transmission coefficients
- $s$: height of water gap under the breakwater
- $S_1$: horizontal stiffness of mooring cables
- $S_2$: vertical stiffness of mooring cables
- $X_j$: response amplitude operator
$Z_G$ : z-coordinate of the breakwater's centre of gravity

$Z_0$ : z-coordinate of the point which pitch motion is taken around

$\alpha$ : proportionality factor for oblique force calculation

$\beta$ : angle of wave incidence

$\phi$ : velocity potential

$\lambda$ : breakwater length

$\rho$ : fluid density

$\tau$ : exciting force reduction factor due to a finite length

$\omega$ : incident wave frequency

$\xi_j$ : responses of the breakwater

$\Xi_j$ : complex amplitudes of breakwater responses
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CHAPTER 1

INTRODUCTION

1.1 General

Floating structures have seen increasing use in coastal regions as breakwaters, berths, wave energy devices, and floating bridges. In some cases, floating structures have replaced fixed structures and offer several advantages over fixed structures, including reduced impact on marine life, improved water circulation, and reduced siltation in navigable channels; as well as being generally more economical especially in deep water or in the case of poor soil condition at the seabed. In addition, floating structures are transportable and can be relocated. Limitations of floating structures include their inability to provide adequate wave protection under severe wave conditions in the case of a breakwater, and the excessive motions of the structure in the case of a floating bridge. Therefore, such structures are generally used in relatively sheltered coastal areas.

Examples of floating structures in use include circular pipes, rectangular caissons, twin-pontoon (catamaran) breakwaters, and scrap tire breakwaters. Structures with rectangular cross-section are the most common because of their usable deck area. Various methods have been proposed to improve the performance of floating structures. As such, the present thesis considers the changes in the performance of a floating rectangular caisson due to the attachment of wave boards (vertical plates) to its up-wave and down-wave sides.
1.2 Literature Survey

Wave interactions with a large structure can be divided into two categories: the hydrodynamic problem associated with wave diffraction and radiation, and the structural response problem. The diffraction problem concerns the calculation of exciting forces induced by incident and scattered waves on the fixed body, while the radiation problem considers added mass and damping coefficients associated with forced motions of the structure. The structural response problem concerns a solution to the equations of motion of the structure with the hydrodynamic exciting force and reaction forces associated with forced problem as the driving force and the hydrostatic and mooring cables stiffnesses as the restraining forces, the literature survey for these two problems will be discussed separately below.

1.2.1 Diffraction/Radiation Problem

A number of authors have dealt with the problem of a long horizontal structure floating in water of finite or infinite depth with normal wave incidence. Regarding the cross section, rectangular floating caissons have been the subject of numerous studies, including Mei and Black (1969), and Black et al. (1971) who used a variational formulation and showed accurate results of selected far field properties; and Bai (1975) who studied the scattering of oblique waves. For floating circular cylinders, Leonard et al. (1983) considered the interference effects around multiple flexible floating cylinders, Garrison (1969) solved the wave-structure interaction problem for a shallow draft cylinder, and Garrison (1984) subsequently used a Green's function to solve for oblique
wave incidence on flexible cylinders. His results show significant variation in the added mass with wave direction. For an arbitrary cross section, the hydrodynamic added mass and damping coefficients were calculated by Andersen and Wuzhou (1985) for floating and submerged structures.

Some authors have tackled the wave-structure interaction problem experimentally. Measurements of the wave transmission and reflection for the tethered and freely floating cases were presented by Sutko et al. (1974), and the effect of uncoupling the problem into scattering and radiation subproblems on the accuracy of the results was investigated by Fugazza et al. (1988). Their tests showed that the uncoupling procedure may be generally valid except for predicting the heave damping coefficient.

The method of matching eigenfunction expansions with unknown coefficients has been used successfully in many studies. Using this approach, the interaction effects between two adjacent parallel floating bridges were studied by McIver (1985); a simplified solution for rectangular floating structures with deep draft and large beam in long waves was presented by Drimer et al. (1992); Mullarkey et al. (1992) used this method to study the performance of submerged rectangular breakwaters; and recently, Williams and McDougal (1996) utilized this approach to study energy dissipation by a submerged long tethered breakwater of rectangular cross section.

1.2.2 Structure Response Problem

From the viewpoint of a hydrodynamicist, the effect of the presence of a structure on the wave field is relatively important, whereas a structural designer will be more
interested to perform an accurate analysis of the responses of floating structures due to wave excitation. A number of studies have been conducted on the analysis of floating structures. Isaacson and Fraser (1979) studied the effect of slack mooring cables on the response of floating breakwaters and on the second-order drift force. The hydrodynamic problem was solved using a two-dimensional finite element analysis of the fluid domain, and the mooring system was modeled using a plane-frame structural analysis. Results showed that for the case of a moored body, the second-order drift force is different from that for an unrestrained body; and hence must be calculated iteratively.

For an actual 1200 m long floating curved concrete bridge, the Salhus bridge in Norway, Langen (1983) used the finite element method to represent the structure, while and the wave loading was derived using linear potential theory, with the incident waves considered to be short-crested. The spectral approach and Monte Carlo simulation methods were used to study the structural responses both in the frequency domain and time domain. Due to the deep water conditions at the bridge site, the bridge was not anchored, hence lateral forces from waves, currents, and winds were considered in the calculation of the bending, torque and axial forces induced in the main bridge sections, composed of rectangular pontoons.

Miller and Christensen (1984) studied the rigid body motions of a floating single rectangular breakwater in oblique waves, and compared the results of a frequency domain analysis to full scale field measurements. A frequency domain analysis was also used by Hutchinson (1984) to determine the covariance matrix of the six degrees of freedom of a floating structure.
Isaacson and Nwogu (1987) studied the effect of wave directionality on the loads and motions of long structures of arbitrary cross-section for variable water depth. The solution was obtained using a Green's function. Wave excitation and hydrodynamic coefficients were calculated to determine the body responses by solving the three coupled linear equations of motion. The solution considered both the flexible and rigid body cases.

1.2.3 Thin Plate Barriers

One method of protection from waves is possible by using a thin vertical plate to obstruct the wave field. This plate may be located at different levels within the water column. A selection of studies conducted with this type of breakwater are now summarized.

Wiegel (1960) proposed a simple prediction of the transmission coefficient for a vertical barrier extending from above the water surface to some distance above the seabed. Liu and Abbaspour (1982) used the boundary integral equation method to solve for an inclined thin breakwater at the water surface.

Three different types of vertical thin barriers that have been studied include a submerged barrier at the seabed, a full depth barrier with a horizontal slit, and another extending from above the water surface to some distance above the sea bed. For these three barrier types, Abul-Azm (1993) studied the case of for normal waves, while Losada et al. (1992) studied the case of oblique waves.
Hara and Suzuki (1991) appear to be the only authors who have studied a combined cylindrical structure with vertical plates. A small-scale model was used and assembled from a square cylinder with two vertical plates fixed at its ends parallel to wave direction. This horizontally oscillating model was used to model and measure the effects of end-plates on hydrodynamic forces.

1.3 Objectives and Scope of Work

The objective of the present thesis is to study numerically the wave interaction with a floating structure of rectangular cross section having vertical wave boards attached to the up-wave and down-wave sides of the structure.

A numerical method is utilized to solve the corresponding diffraction and radiation problems. This method is based on eigenfunction expansions with unknown coefficients in individual regions defined by the vertical sides of the breakwater, and matching conditions are used to evaluate the unknown coefficients. The resulting numerical model is then used to predict the transmission and reflection coefficients, the added mass and damping coefficients and the exciting forces. The equations of motion are solved to obtain the motion response of the structure.

The numerical model is verified by comparisons with previous published results based on other methods of solution. Since a literature survey indicates relatively few previous publications relating to this type of breakwater. Various consistency checks are also used to verify the model for the case of wave boards attachment In addition,
experimental testing of the transmission coefficient of such a structure in different wave conditions is also performed.

Finally the numerical model is used to investigate the effect of wave and structural parameters on the performance of the breakwater, including comparisons between the present configuration and a traditional rectangular breakwater.
CHAPTER 2

THEORETICAL FORMULATION

2.1 Wave Force Regimes

The solution of a wave-structure interaction problem first requires a determination of whether the flow is dominated by drag or by inertia force regimes. Both regimes depend mainly on the size of the structural member compared to the incident wave length and height. For slender structural members, the structure will not change any of the incident wave characteristics, the flow is then mainly dominated by drag forces, and in this case wave forces are calculated using the Morison equation. On the other hand, for large structural members, diffraction theory should be considered to take into account the changes in the wave field due to the presence of the member.

To distinguish between dominant flow regimes, Sarpkaya and Isaacson (1981) presented a relationship between Keulegan-Carpenter number \( K = \frac{U_m T}{D} \) and the diffraction parameter \( D/L \) or \( kD \), where \( U_m \) is the maximum particle velocity, \( T \) is the wave period, \( D \) is a characteristic dimension of the structure, \( L \) is the incident wave length, and \( k \) is the wave number. Fig. 2.1 shows the variation of \( K \) with \( D/L \); which shows that the present solution based on the potential flow assumption is only valid for \( D/L > 0.2 \) and \( K < 2 \): within these ranges the diffraction theory is applicable.
2.2 Problem Definition

An incident train of monochromatic, small amplitude waves of height $H$ and frequency $\omega$, propagate in water of constant water depth $d$ past a breakwater as shown in Fig. 2.2. A Cartesian coordinate system $(x,y,z)$ is defined with the $x$-$y$ plane at the undisturbed free surface, $y$ is directed along the breakwater axis and $z$ is measured vertically upwards. The direction of wave propagation $\beta$ is measured counterclockwise from $x$-axis. The breakwater is assumed impermeable and infinitely long, so that end effects can be neglected and the system is idealized as two-dimensional. The breakwater has a rectangular cross-section with vertical plates attached to the up-wave and down-wave faces as shown in Fig. 2.2. The characteristic dimensions of the breakwater are its beam $2a$, draft $h$, and plate height $b$ below the underside of the breakwater. The clearance between the seabed and the plate tip is denoted $s$ [such that $s = d - (h+b)$].

The water is assumed to be inviscid and incompressible, and the flow is irrotational, so that the flow field may be described in terms of a velocity potential. The fluid domain is divided into three regions: region 1 for $x \leq -a$, region 2 for $-a \leq x \leq a$, and region 3 for $x \geq a$, as shown in Fig. 2.2. The velocity potential $\Phi_p(x,y,z,t)$ in the $p$-th region may be expressed as

$$\Phi_p = \text{Re}[\phi_p(x,y,z)e^{-i\omega t}] \quad , \quad p = 1, 2, 3 \quad (2.1)$$

where $\text{Re}[\ ]$ is the real part of the complex expression, $i = \sqrt{-1}$, $\phi_p$ is the complex amplitude, and $p = 1, 2, 3$ denotes the applicable region. $\omega$ is related to the wave number $k$ through the linear dispersion relation:
\[ \omega^2 = g k \tanh(kd) \] (2.2)

where \( g \) is the acceleration due to gravity.

The wave height is assumed sufficiently small for linear wave theory to apply. The velocity potential is separated into contributions associated with the scattering of incident waves by a fixed structure, and radiation of waves due to motions of the structure. Thus

\[
\Phi_p = \text{Re} \left[ \left( -\frac{igH}{2\omega} \right) (\delta_{1,p} \phi_1(x,z) + [\phi_0(x,z)]_p) e^{iky \sin \beta} + \sum_{j=1}^{3} \Xi_j [\phi_j(x,z)]_p \right] e^{-i\omega t}, \quad p = 1, 2, \text{ and } 3 \tag{2.3}
\]

where \( \delta_{1,p} \) is the Kronecker Delta \((\delta_{1,p} = 1 \text{ for } p = 1, \text{ and } \delta_{1,p} = 0 \text{ otherwise})\), \( \phi_1 \) is the incident velocity potential, \( \phi_0 \) is the velocity potential associated with the scattering of waves around the fixed structure, \( \phi_j \) is the velocity potential of the radiated waves for each mode of motion, \( j = 1, 2 \text{ and } 3 \), and \( \Xi_j \) is the complex amplitude of motion. The incident potential is known and may be expressed as

\[
\phi_1(x,z) = \text{Re} \left[ \frac{\cosh[k(z+d)]}{\cosh(kd)} e^{i(k(x+a)\cos \beta)} \right] \tag{2.4}
\]

In addition to satisfying the Laplace equation within the whole fluid domain:

\[ \nabla^2 \Phi = 0 \] (2.5)

velocity potential components should satisfy the seabed boundary condition;

\[ \frac{\partial \phi_j}{\partial z} = 0 \quad z = -d, \quad -\infty \leq x \leq \infty \] (2.6)

and in regions 1 and 3, the potentials should also satisfy the free surface boundary condition:
The scattered potential and radiated potentials should satisfy a radiation condition at a large distance from the breakwater:

\[
\lim_{x \to \pm \infty} \left[ \frac{\partial}{\partial x} \pm \left( \frac{ik \cos \beta}{ik} \right) \right] [\phi_j]_p = 0 \quad \begin{cases} j = 0, & p = 1 \text{ and } 3 \end{cases}
\]

\[\frac{\partial [\phi_j]}{\partial z} = \frac{\omega^2}{g} [\phi_j]_p \quad z = 0, \quad -\infty \leq x \leq \infty, \quad p = 1 \text{ and } 3 \] (2.7)

and another boundary condition ensuring the continuity of normal velocity at the structure submerged surface

\[
\frac{\partial \phi_j}{\partial n} = V \cdot n \quad j = 0, 1, 2, \text{ and } 3 \quad (2.9)
\]

where \( V \) is the velocity vector of any point on the structure's submerged surface, and \( n \) is a unit normal vector on the structure's submerged surface pointing outwards of the fluid. The solution of the boundary value problem for each value of \( j \) must be continuous across each interface (\( x = -a \) and \( x = a, \quad 0 < z + d < s \)), and must satisfy therefore continuity of the potential and horizontal velocity. Thus at \( x = -a \)

\[
[\phi_j]_1 = [\phi_j]_2 \quad 0 < z + d < s, \quad j = 0, 1, 2, 3 \quad (2.10a)
\]

\[
\frac{\partial [\phi_j]_1}{\partial x} = \frac{\partial [\phi_j]_2}{\partial x} \quad 0 < z + d < s, \quad j = 0, 1, 2, 3 \quad (2.10b)
\]

Also, at \( x = a \), a structural boundary condition ensures the continuity of horizontal velocity at the vertical surfaces of the structure's submerged surface:

\[
\frac{\partial [\phi_j]_1}{\partial x} = \Omega_j \quad s < z + d < d, \quad j = 0, 1, 2, \text{ and } 3 \quad (2.10c)
\]

\[
\frac{\partial [\phi_j]_2}{\partial x} = \Omega_j \quad s < z + d < s + b, \quad j = 0, 1, 2, \text{ and } 3 \quad (2.10d)
\]
where $\Omega_j$ is the horizontal velocity of the breakwater surface, its value depending on the problem under consideration, and $j = 0, 1, 2, 3$ correspond to the scattering and the three radiation sub-problems, respectively. A similar set of potential and horizontal velocity conditions can be constructed at $x = a$, and should also be satisfied.

2.3 Scattering Problem

In this section the structure is assumed fixed and is subjected to a wave train propagating obliquely. Due to the periodic variation of the incident and scattered potentials in the $y$-direction, the governing equation may in this case be expressed by the modified-Helmholtz equation:

$$\frac{\partial^2 \phi_0}{\partial x^2} + \frac{\partial^2 \phi_0}{\partial z^2} - (k \sin \beta)^2 \phi_0 = 0$$

and the problem is then limited to finding a solution to equation (2.11) which satisfies equations (2.6) and (2.7) in the outer regions (1) and (3), and a solution in the inner region (2) for equation (2.11) with both (2.6) and a boundary condition that imposes the fixation of the structure. In the horizontal direction this condition is given by equations (2.10) c, d with $\Omega_0 = 0$ for this case, while in the vertical direction, the potential in region (2) should also satisfy

$$\frac{\partial [\phi_0]}{\partial z} = 0 \quad \text{on} \quad z = -h$$

The associated boundary value problem is solved by using eigenfunction expansions with unknown coefficients. These functions satisfy the governing equation and boundary
conditions. Since the structure has a vertical plane of symmetry, it is convenient to split the
potentials describing the flow into symmetric and antisymmetric parts

\[ \phi_0 = \phi_0^{(s)} + \phi_0^{(a)} \]

where the superscripts \((s)\) and \((a)\) denote symmetric (even function in \(x\)) and antisymmetric
(odd function in \(x\)) potentials. Thus the expression for \(\phi_0\) in region 2 contains two terms

\[ \psi_0(0) \sum_{n=0}^{\infty} \frac{(A_{0n}^{(s)} + A_{0n}^{(a)})}{\mu_n d} e^{\mu_n(x+a)} \psi_n(z) \] (2.13)

\[ [\phi_0(x,z)]_2 = \frac{1}{\psi_0(0)} \left\{ \sum_{m=0}^{\infty} \left[ B_{0m}^{(s)} \frac{\cosh(q_m x)}{\cosh(q_m a)} + B_{0m}^{(a)} \frac{\sinh(q_m x)}{\sinh(q_m a)} \right] \chi_m(z) \right\} \] (2.14)

\[ [\phi_0(x,z)]_3 = \frac{-1}{\psi_0(0)} \sum_{n=0}^{\infty} \frac{(A_{0n}^{(s)} - A_{0n}^{(a)})}{\mu_n d} e^{-\mu_n(x-a)} \psi_n(z) \] (2.15)

where \(A_{0n}^{(s)}, A_{0n}^{(a)}, B_{0n}^{(s)},\) and \(B_{0n}^{(a)}\) (\(n = 0,1,2,\ldots,\infty\)) represent four sets of unknown
coefficients which are yet to be determined, and

\[ \mu_0 = k_0 \cos \beta, \quad \text{and} \quad k_0 = -ik \] (2.16a)

\[ \mu_n = \sqrt{k_n^2 + k^2 \sin^2 \beta} \quad n = 1,2,3,\ldots \] (2.16b)

\[ q_n = \sqrt{p_n^2 + k^2 \sin^2 \beta} \quad n = 0,1,2,3,\ldots \] (2.17)

\[ p_n = \frac{n\pi}{(s+b)} \quad n = 0,1,2,3,\ldots \] (2.18)
and \( k_n, n = 1, 2, 3, \ldots, \infty \) are the positive real roots of the equation

\[
k_n d \tan(k_n d) = -\frac{\omega^2 d}{g}\]

(2.19)

Also, the depth dependent functions \( \psi_n(z) \) and \( \chi_m(z) \) are given by;

\[
\psi_n(z) = \left( \frac{4k_n \delta}{[2k_n d + \sin(2k_n d)]} \right) \cos[k_n(z + d)], \text{ for } n = 0, 1, 2, 3, \ldots \quad (2.20)
\]

\[
\chi_m(z) = \sqrt{\frac{\varepsilon_m d}{(s + b)}} \cos[p_m(z + d)], \text{ for } m = 0, 1, 2, 3, \ldots \quad (2.21)
\]

\( \varepsilon_0 = 1 \) and \( \varepsilon_m = 2 \) for \( m \geq 1 \)

where equations 2.13-2.15 have been obtained by the separation of variables method so as to satisfy the governing equation 2.11 within the fluid domain, as well as the seabed condition, the free surface boundary conditions and the radiation condition (in regions 1 and 3). The body surface boundary condition on the underside of the structure, the boundary conditions on the vertical surfaces of the structure (only \( x = -a \) and \( x = a \)) and the matching conditions for \( \phi \) and \( \frac{\partial \phi}{\partial x} \) are yet to be satisfied and are used to define the unknown coefficients. Also in equations (2.13) and (2.15) for regions 1 and 3 the first term in the expansions \( (n = 0) \) represent progressive reflected and transmitted waves, respectively; while the higher terms represent evanescent wave modes, which decay exponentially with the distance away from the breakwater. In different regions, each potential function should satisfy the boundary value problem described earlier (equations 2.4-2.10). To ensure the continuity of the solution for the potential over the domain \(-\infty \leq x \leq \infty\), matching of potential and horizontal velocities at the
two interfaces \( x = -a \) and \( x = a \) must be satisfied, from which the unknown four sets of coefficients \((A_{on}^{(s)}, B_{on}^{(s)}, A_{on}^{(a)}, \text{ and } B_{on}^{(a)})\) are determined.

Applying the matching and boundary conditions along \( x = -a \) and \( x = a \), equation (2.10) for both the symmetric and antisymmetric potentials, one may obtain the coefficients \( A_{on}^{(s)}, B_{on}^{(s)}, A_{on}^{(a)}, \text{ and } B_{on}^{(a)} \). (Other methods such as the least squares method or the collocation method have also been used to solve for a vertical plate barrier problems.) Since the breakwater is fixed (i.e., \( \Omega_0 = 0 \) in equations 2.10c and d), continuity of pressure at \( x = -a \) requires that equation (2.10a) be satisfied, hence;

\[
\frac{\psi_0(z)}{2} - \sum_{n=0}^{\infty} A_{on}^{(s)} \psi_n(z) = \sum_{m=0}^{\infty} B_{om}^{(s)} \chi_m(z) , \ -d \leq z \leq -(h + b) \quad (2.22)
\]

Continuity of mass flux, equations (2.10 b, c, and d) at \( x = -a \), after multiplying by \( d \) give

\[
\frac{ikd \cos \beta}{2} \psi_0(z) - \sum_{n=0}^{\infty} A_{on}^{(s)} \psi_n(z) = -\sum_{m=0}^{\infty} B_{om}^{(s)} q_m d \tanh(q_m a) \chi_m(z) , \ -d \leq z \leq -(h + b) \quad (2.23)
\]

\[
\frac{ikd \cos \beta}{2} \psi_0(z) - \sum_{n=0}^{\infty} A_{on}^{(s)} \psi_n(z) = 0 , \ -(h + b) \leq z < 0 \quad (2.24)
\]

\[
\sum_{m=0}^{\infty} B_{om}^{(s)} q_m d \tanh(q_m a) \chi_m(z) = 0 , \ -(h + b) \leq z < -h \quad (2.25)
\]

The two sets of the depth dependent functions \( \psi_n(z) \), and \( \chi_m(z) \) are orthonormal over the ranges \([-d,0]\) and \([-d,-h]\), respectively. That is:

\[
\int_{-d}^{0} \psi_n(z)\psi_L(z)dz = \begin{cases} 0 & n \neq L \\ 1 & n = L \end{cases} \quad (2.26a)
\]

\[
\int_{-d}^{-h} \chi_n(z)\chi_L(z)dz = \begin{cases} 0 & n \neq L \\ 1 & n = L \end{cases} \quad (2.26b)
\]
Multiplying equations (2.24) and (2.25) by $\psi_L(z)$, then integrating both sides with respect to $z$ over their valid ranges $[-d,-(h+b)]$ and $[-(h+b),0]$ respectively, and adding the resulting two equations, one obtains:

$$A_{L}^{(s)} = -\frac{\mu_0 d}{2} \delta_{0,L} + \sum_{m=0}^{\infty} B_{m}^{(s)} q_m d \tanh(q_m a) F_{L,m}, \quad -d < z < 0$$

(2.27)

where

$$F_{L,m} = \int_{-d}^{-(h+b)} \psi_L(z) \chi_m(z) dz$$

Substituting equation (2.27) into equation (2.22), yields;

$$\psi_0(z) \cos \left( \frac{\pi}{L} z \right) - \sum_{m=0}^{\infty} [q_m d \tanh(q_m a) \sum_{n=0}^{\infty} \frac{F_{n,m} F_{n,L,m}}{\mu_n d} \psi_n(z) + \chi_m(z)] B_{0m}^{(s)} = 0, \quad -d < z < -(h+b)$$

(2.28)

Multiplying equations (2.25) and (2.28) by $\chi_L(z)$, integrating both over their valid ranges $[-(h+b),-h]$ and $[-d,-(h+b)]$, respectively; and adding the resulting two equations, yields;

$$\sum_{m=0}^{\infty} \left\{ q_m d \tanh(q_m a) \left[ I_{L,m} + \sum_{n=0}^{\infty} \frac{F_{n,m} F_{n,L}}{\mu_n d} \right] + Q_{L,m} \right\} B_{0m}^{(s)} = F_0, \quad -d < z < -h$$

(2.29)

where:

$$I_{L,m} = \int_{-(h+b)}^{-h} \chi_L(z) \chi_m(z) dz$$

$$Q_{L,m} = \int_{-d}^{-(h+b)} \chi_L(z) \chi_m(z) dz$$

When the same matching procedure is performed on the asymmetrical potential at $x = -a$, a similar set of equations for $A_{0m}^{(a)}$ and $B_{0m}^{(a)}$ is obtained.
\[ A_{0L}^{(s)} = -\frac{\mu_0 d}{2} \delta_{0,L} - \sum_{m=0}^{\infty} B_{0m}^{(a)} q_m d \coth(q_m a) F_{L,m} \]  \quad \text{for} \quad -d < z < 0 \quad (2.30)

\[ \sum_{m=0}^{\infty} \left\{ q_m d \coth(q_m a) [I_{L,m} + \sum_{n=0}^{\infty} \frac{F_{n,m} F_{n,L}}{\mu_n d}] + Q_{L,m} \right\} B_{0m}^{(a)} = -F_{0,L}, \quad \text{for} \quad -d < z < -h \quad (2.31) \]

By truncating the infinite summations in equations (2.29) and (2.31) at \( n = m = M \), each summation over \( m \) leads to a set of \( M+1 \) linear simultaneous equations for the unknown complex coefficients \( B_{0m}^{(s)} \) and \( B_{0m}^{(a)} \), which can be solved by standard matrix inversion techniques. Then \( A_{0m}^{(s)} \) and \( A_{0m}^{(a)} \) can be deduced from equations (2.28) and (2.31) and the potential functions representing the scattering problem are then known.

2.3.1 Fixed Structure Reflection and Transmission Coefficients

In the case of a fixed breakwater, the potential function corresponding to the scattering problem consists of the sum of the symmetric and antisymmetric potentials that are now known. The expressions in equations 2.13-2.15 can be used to obtain the far field amplitudes of the reflected and transmitted waves. Hence the reflection coefficient \( R_0 \) and transmission coefficient \( T_0 \) for the fixed breakwater can be calculated from:

\[ R_0 = \frac{1}{2} (R^{(s)} + R^{(a)}) \]  \quad (2.32a)

\[ T_0 = \frac{1}{2} (R^{(s)} - R^{(a)}) \]  \quad (2.32b)

where \( R^{(s)} \) and \( R^{(a)} \) are the amplitudes of the symmetrical and antisymmetrical reflected waves per unit amplitude of incident waves. And may be calculated from

\[ A_{00}^{(s,a)} = -\frac{1}{2} R^{(s,a)} \mu_0 d \]  \quad (2.33)
2.3.2 Exciting Forces

The forces induced by the incident and scattered wave fields on the fixed structure are termed exciting forces, and may be calculated from an integration of the hydrodynamic pressure \( p \) around the structure's submerged surface. Using the linearized Bernoulli equation [see e.g., Sarpkaya and Isaacson, 1981], the hydrodynamic pressure may be written as

\[
p = -\rho \frac{\partial \Phi_p}{\partial t} = i\omega \rho \Phi_p , \quad p = 1, 2, \text{ and } 3
\]  

(2.34)

where \( \rho \) is the water density.

Appropriate integrations around the breakwater wetted surface \( S_o \) yield the exciting forces. These integrations may be expressed as:

\[
F_j = (i\omega \rho) \begin{cases} 
\int_{S_o} \Phi_p n_j ds 
& \text{for } j = 1, 2 \\
\int_{S_o} d^2 \Phi_p [ (z-Z_o) n_1 - x n_2 ] ds 
& \text{for } j = 3
\end{cases}
\]  

(2.35)

where \( n_j \) is the unit normal vector to the structure surface, and \( j = 1, 2, 3 \) denotes horizontal, vertical, and rotational directions, respectively. The variation of the exciting forces is sinusoidal in both time and the \( y \) direction, and may be expressed in complex notation as

\[
F_j = \text{Re} [ F_0 e^{i(k y \sin \beta - \omega t)} ] , \quad \text{for } j = 1, 2, \text{ and } 3
\]  

(2.36)

It is noted that the symmetric potential does not contribute to either the horizontal exciting force or the exciting moment about the \( y \)-axis. Also the antisymmetric potential does not contribute to the vertical exciting force. Substitution of the potentials from equations (2.13), (2.14) and (2.15) into equation (2.35) gives formulae for the amplitudes of each of the exciting force components per unit length of the structure. The horizontal, vertical exciting forces and the pitching moment around \( Z_o = 0 \) are thus given respectively by:
\[
\frac{F_{01}}{\frac{1}{2} \rho g H d} = \frac{2}{\psi'(0)} \left[ \frac{1+R(a)}{2} W_0 - \sum_{n=1}^{N} A_n^{(a)} \mu_n d W_n + \sum_{m=0}^{M} B_{0m}^{(a)} V_n \right] 
\]  
(2.37)

\[
\frac{F_{02}}{\frac{1}{2} \rho g H d} = \frac{2 \chi_0(z)}{\psi'(0)} d \left[ B_{00}^{(s)} \frac{\tanh(q_0a)}{q_0a} + \sqrt{2} \sum_{m=1}^{M} (-1)^m B_{0m}^{(s)} \tanh(q_ma) \right] 
\]  
(2.38)

\[
\frac{F_{03}}{\frac{1}{2} \rho g H d^2} = \frac{2}{\psi'(0)} \left[ \frac{1+R(a)}{2} W_0 - \sum_{n=1}^{N} A_n^{(a)} \mu_n d W_n + \sum_{m=0}^{M} B_{0m}^{(a)} \frac{-V_m + (-1)^m}{(q_md)^2} \left( q_ma \coth(q_ma) - 1 \right) \right] 
\]  
(2.39)

where:

\[
V_n = \int_{-h}^{0} \chi_n(z) \, dz 
\]  
(2.40a)

\[
W_n = \int_{-h}^{0} \psi_n(z) \, dz 
\]  
(2.40b)

\[
\overline{V}_n = \int_{-h}^{0} \chi_n(z) \, dz 
\]  
(2.40c)

\[
\overline{W}_n = \int_{-h}^{0} \psi_n(z) \, dz 
\]  
(2.40d)

2.3.3 Effect of Finite Structure Length

In the case of obliquely incident waves, the exciting forces are influenced by the spatial variation of the incident and scattered potentials in the y-direction. This effect must be included
in the integration of the pressure over the wetted surface of the structure and can be accounted for by a reduction factor $\tau$ given by Isaacson and Nwogu (1987) as

$$\tau = \text{Re} \left[ \frac{\lambda}{2} \int_{-\lambda/2}^{\lambda/2} \sin \beta \sin \left( \frac{k\lambda}{2} \sin \beta \right) \, dy \right] = \begin{cases} \frac{2}{k\lambda \sin \beta} \sin \left( \frac{k\lambda}{2} \sin \beta \right) & \beta \neq 0 \\ 1 & \beta = 0 \end{cases}$$

(2.41)

where $\lambda$ is the length of the structure in y-direction. Thus the exciting forces on a structure of finite length for obliquely incident waves are equal to forces amplitudes given by equations (2.37-2.39) multiplied by this reduction factor and the structure length $\lambda$.

2.4 Radiation Problem

Solutions for the velocity potentials of the radiated waves are now derived for forced two-dimensional motions of the structure. These are used in turn to derive the hydrodynamic added mass and damping coefficients to be used in the equations of motion. The two-dimensional motion of the rigid structure can be defined by a dimensionless displacement vector which is periodic in time and is given as

$$\xi_j = \text{Re} \left[ \Xi_j e^{-i\omega t} \right] , \quad j = 1, 2, \text{ and } 3$$

(2.42)

where $\Xi_j$ is the complex amplitude of motion. The response amplitude operator $X_j$ is equal to the complex amplitude of oscillation of the cylinder divided by the incident wave amplitude ($H/2$, $j = 1, 2$, or $H/2d$, $j = 3$), and $j = 1, 2, \text{ and } 3$ correspond to surge, heave and pitch motions respectively.
The wave radiation problems associated with forced surge, heave and pitch motions in otherwise still water sea are each solved separately. The velocity potential of the radiated waves may be given by:

\[ \Phi_R = \text{Re} \left[ \sum_{j=1}^{3} \phi_j(x, z) e^{-i \omega t} \right] \]  

(2.43)

where \( \phi_j \) satisfy the Laplace equation, together with the seabed boundary condition (2.6), the free surface condition (2.7), and the structural boundary condition (2.9). The structural boundary condition is applied at the equilibrium position of the cylinder rather than at the instantaneous position and may be written as

\[ \frac{\partial \phi_j}{\partial n_i} = v_i \quad \text{around the submerged structure surface,} \quad i, j = 1, 2, \text{ and } 3 \]  

(2.44)

where \( n_i \) is the unit vector normal to the surface of the structure, \( v_i \) is the velocity of the structure in the \( n_i \)'s direction and is given by:

\[ v_i = \frac{\partial \phi_j}{\partial t} = \text{Re} \left[ -i \omega \xi_j n_i e^{-i \omega t} \right] \]  

(2.45)

where:

\[ \begin{align*}
    n_1 &= \hat{i} \\
    n_2 &= \hat{k} \\
    n_3 &= (z - Z_o)\hat{i} - x\hat{k}
\end{align*} \]

and \( \hat{i} \) and \( \hat{k} \) are the direction cosines of the unit normal vector \( n \) on the wetted surface of the structure in the \( x \) and \( z \) directions, respectively. The point \((0, Z_o)\) is that point around which pitch motion is prescribed, and it is considered here as \( Z_o = 0 \). The solution to the radiation problem in region (1) is given in the form of an eigenfunction expansion with unknown coefficients as:
\[ [\phi_j(x, z)]_2 = -i\omega f_j \left\{ \sum_{n=0}^{\infty} \frac{A_j^{(s)}}{k_n d} e^{k_n(x+a)} \psi_n(z) - \sum_{n=0}^{\infty} \frac{A_j^{(a)}}{k_n d} e^{k_n(x+a)} \psi_n(z) \right\}, \quad j = 1, 2, 3 \] (2.46)

where \( f_j = a, d \) and \( a^2 \) for the surge, heave, and pitch problems, respectively. The amplitudes of the asymptotic radiated waves at a large distance from the structure per unit incident wave amplitude are given by

\[ a_j^{(s,a)} = -\frac{A_j^{(s,a)}}{k_0 d}, \quad \text{for } j = 1, 2, \text{ and } 3 \] (2.47)

The surge and pitch motions radiate waves that are asymmetric about \( x = 0 \), while the heave motion causes a symmetric wave as shown in Fig. 2.3. Hence, the potential in region (2) is given by an odd function in \( x \) for surge and pitch, and by an even function for heave. However, in order for the potential in region (2) to satisfy the non-homogeneous structural boundary condition at its keel, a particular solution denoted by \( \Gamma_j \) satisfying both the Laplace equation and the structural boundary condition is included in the solution. Thus the potential in regions (2) and (3) may be given by:

\[ [\phi_j(x, z)]_2 = -i\omega f_j \left\{ \frac{X_j^{(a)}}{a} \chi_0(z) + B_j^{(s)} \chi_0(z) + \sum_{n=1}^{\infty} B_j^{(a)} \frac{\sinh(p_n X)}{\sinh(p_n a)} \chi_n(z) \right. \\
+ \left. \sum_{n=1}^{\infty} B_j^{(s)} \frac{\cosh(p_n X)}{\cosh(p_n a)} \chi_n(z) + \Gamma_j \right\}, \quad j = 1, 2, \text{ and } 3 \] (2.48)

\[ [\phi_j(x, z)]_3 = -i\omega f_j \left\{ \sum_{n=0}^{\infty} \frac{D_j^{(s)}}{k_n d} e^{-k_n(x-a)} \psi_n(z) - \sum_{n=0}^{\infty} \frac{D_j^{(a)}}{k_n d} e^{-k_n(x-a)} \psi_n(z) \right\}, \quad j = 1, 2, \text{ and } 3 \] (2.49)
where $A^{(a)}_{jn} = B^{(a)}_{jn} = 0$ for the heave motion problem ($j = 2, n = 0, 1, 2, \ldots$), and $A^{(s)}_{jn} = B^{(s)}_{jn} = 0$ for both the surge and pitch motions ($j = 1$ and $3, n = 0, 1, 2, \ldots$). The symmetry relationships between the velocity potentials in regions 1 and 3 also apply to the radiation problem, and an explicit solution for region 3 is therefore not needed.

The particular solutions in region 2 are given by:

$$\Gamma_1 = 0$$  \hspace{1cm} (2.50)

$$\Gamma_2 = \frac{1}{2(s+b)d} \left[(z+d)^2 - x^2 \right]$$  \hspace{1cm} (2.51)

$$\Gamma_3 = \frac{x}{2(s+b)a^2} \left[rac{(z+d)^2 - x^2}{3} \right]$$  \hspace{1cm} (2.52)

The solution to each of the three boundary value problems associated with wave radiation proceeds by finding the unknown coefficients in the expansions in both regions 1 and 2, by matching velocity potentials, horizontal particle velocities, and structural boundary condition at $x = -a$ (equation 2.10 a-d). At the keel of the breakwater, the structural boundary condition is satisfied by the particular solution $\Gamma_j$, while at both sides of the plate it is satisfied in the horizontal direction by default during the matching process. Thus, for each mode of motion there is a set of linear simultaneous equations in the unknown coefficients in region 2, together with another equation for the unknown coefficients in region 1. These equations may be derived for each mode of motion. For surge these equations are given by:

$$\sum_{m=1}^{\infty} \left\{ p_m d \coth(p_m a) \left[ I_{L,m} + \sum_{n=0}^{\infty} \frac{F_{n,m} F_{n,L}}{k_n d} \right] + Q_{L,m} \right\} B^{(a)}_{jm} + \frac{B^{(a)}_{10}}{\sqrt{s+b}} \left[ I_L + \frac{d}{a} V_L + \sum_{n=0}^{\infty} \frac{U_n F_{n,L}}{k_n a} \right]$$

$$= \left[ \frac{d}{a} V_L - \sum_{n=0}^{\infty} \frac{W_n F_{n,L}}{k_n a} \right]$$  \hspace{1cm} (2.53)
For heave these equations are given by:

\[
A_{1L}^{(a)} = \frac{d}{a} \left[ \frac{U_L B_{10}^{(a)}}{\sqrt{S+b}} + \sum_{m=1}^{\infty} B_{1m}^{(a)} p_m a \coth(p_m a) F_{L,m} + W_L \right] 
\]

(2.54)

For pitch these equations are given by:

\[
A_{2L}^{(a)} = \sum_{m=1}^{\infty} B_{2m}^{(a)} p_m d \coth(p_m a) F_{L,m} - \frac{ad}{(S+b)h} U_L 
\]

(2.56)

\[
A_{3L}^{(a)} = \frac{d}{a} \left[ \frac{U_L B_{30}^{(a)}}{\sqrt{S+b}} + \sum_{m=1}^{\infty} B_{3m}^{(a)} p_m a \coth(p_m a) F_{L,m} + \frac{[U_L - a^2 U_n] F_{n,L}}{2(S+b) a} \right] - \frac{W_L}{a} 
\]

(2.58)

where

\[
-(h+b) \int_{-d}^{-(h+b)} \psi_n(z) dz
\]

(2.59a)

\[
U_n = \int_{-d}^{-(h+b)} (z+d) \psi_n(z) dz
\]

(2.59b)
The infinite series in the previous six equations are truncated at \( n = m = M \), and it is noted that \( L = 0, 1, \ldots, M \), and the elements of the right-hand-side vector of equations (2.53), (2.55), and (2.57) are then calculated. This leads to three sets of \( M+1 \) linear simultaneous equations for the unknown coefficients \( B_{jm}^{(s,a)} (j = 1, 2, 3) \), which are again solved by a complex matrix inversion procedure. The remaining unknown sets of coefficients \( A_{jm}^{(s,a)} \) can then be calculated from equations (2.54), (2.56), and (2.58), resulting finally in the velocity potential for each mode of motion.

### 2.4.1 Added Mass and Damping Coefficients

In addition to the exciting forces defined earlier in Section 2.3.2 and associated with the incident and scattered waves, there are reaction forces arising also from the radiated waves. These forces are proportional to the amplitude of the breakwater motions. The force in the \( j \)-th direction due to motion in the \( i \)-th mode is denoted \( F_{ij} \). These forces are calculated as the integration around the equilibrium position of the wetted surface rather than around the instantaneous position. This assumption is justified on the basis of small structural responses. These reaction forces are given by:
\[
F_{ij} = \begin{cases} 
  i \omega \rho d \int_{S_0} \Phi_i \rho n_j \, ds & \text{for } i = 1, 2 \\
  i \omega \rho d^2 \int_{S_0} \Phi_i \rho [(z - z_0) n_1 - x n_2] \, ds & \text{for } i = 3 
\end{cases}
\]

(2.60)

For example \( F_{13} \) is the pitching moment in the case of the surge mode of motion. Note that by a symmetry argument, some of these reaction forces are equal to zero. The surge and pitch modes excite antisymmetric waves (odd function in region 2), leading to a zero vertical force from these potentials, hence \( F_{12} = F_{32} = 0 \). Similarly the horizontal force and pitching moment arising from the heave potential are also zero, hence \( F_{21} = F_{23} = 0 \). Therefore, the reaction force matrix is reduced to five non-zero elements which may be calculated from

\[
\frac{F_{11}}{\rho h^2} = 2\omega^2 a \left\{ \sum_{n=0}^{N} \frac{A_{ln}^{(a)}}{k_n d} W_n - \sum_{m=0}^{M} B_{lm}^{(a)} V_n \right\} 
\]

(2.61)

\[
\frac{F_{13}}{\rho h^3} = 2\omega^2 a \left\{ -\sum_{m=0}^{M} B_{lm}^{(a)} V_n + \sum_{n=0}^{N} \frac{A_{ln}^{(a)}}{k_n d} W_n - \sqrt{2} \sum_{m=1}^{M} \frac{(-1)^m}{p_m d} B_{lm}^{(a)} \left[ \frac{a}{d} \coth(p_n d) - \frac{1}{p_n d} \right] 
  + \frac{a^2}{3d^2} B_{10}^a \right\} 
\]

(2.62)

\[
\frac{F_{22}}{\rho h^2} = 2\omega^2 h \left\{ -\frac{d}{(s+b)} \left[ \frac{a}{d} B_{20}^{(s)} + \sqrt{2} \sum_{m=1}^{M} B_{2m}^{(s)} (-1)^m \tanh(p_n a) \right] + \frac{a d^2}{2(s+b)h} \left[ \frac{s^2}{d^2} - \frac{a^2}{3d^2} \right] \right\} 
\]

(2.63)

\[
\frac{F_{31}}{\rho h^3} = 2\omega^2 a^2 \left\{ \sum_{n=0}^{N} \frac{A_{ln}^{(a)}}{k_n d} W_n - \sum_{m=0}^{M} B_{3m}^{(a)} V_n + \frac{d^2}{12a(s+b)} \left[ \frac{(s+b)}{d} - \frac{(s)}{d} - \frac{(a^2 b)}{d^3} \right] \right\} 
\]

(2.64)

\[
\frac{F_{33}}{\rho h^4} = 2\omega^2 a^2 \left\{ \sum_{n=0}^{N} \frac{A_{ln}^{(a)}}{k_n d} W_n - \sqrt{2} \sum_{m=1}^{M} \frac{(-1)^m B_{3m}^{(a)}}{p_m d} \left[ \frac{a}{d} \coth(p_n d) - \frac{1}{p_n d} \right] + \frac{a^2}{3d^2} B_{30}^{(a)} 
  - \sum_{m=0}^{M} B_{3m}^{(a)} V_n \right\} 
\]

(2.65)

An important property of the force matrix is its symmetry, which is a consequence of Green's reflection principle. The radiation potentials satisfy the following integral.
\[ \int_{S_c} \left[ \phi_i \frac{\partial \phi_j}{\partial n} - \phi_j \frac{\partial \phi_i}{\partial n} \right] ds = 0 \]  

where \( S_c \) is the contour bounding the fluid domain, and includes the structure wetted surface \( S_0 \), the seabed, the free surface, and two vertical imaginary surfaces at infinity. However, since the potentials of the radiated waves satisfy all boundary and radiation conditions, this integral is satisfied on the wetted surface only, hence

\[ \int_{S_0} \left[ \phi_i \frac{\partial \phi_j}{\partial n} - \phi_j \frac{\partial \phi_i}{\partial n} \right] ds = 0 \]  

which together with an alternative form of equation (2.61) given as

\[ F_{ij} = -\rho \int_{S_0} \phi_i \frac{\partial \phi_j}{\partial n} ds \]  

implies that \( F_{ij} = F_{ji} \). Thus there are only four elements that need be calculated in order to construct the reaction force matrix. This symmetry feature is used as a check for the numerical model \( (F_{13} = F_{31}) \). The reaction forces may now be decomposed into two components; one in phase with the acceleration and relating to as the added mass; and the other in phase with the velocity representing the radiation damping of the forced structure motion. Thus:

\[ F_{ij} = \omega^2 A_{ij} + i\omega B_{ij} \]  

where \( A_{ij} \) and \( B_{ij} \) are the added mass and damping coefficients per unit length, respectively.

2.5 Equations of Motion

The separate scattering and radiation problems may now be linked. The dynamic response of the structure is obtained by solving equations of motion of the structure. These have been given
for example by Sarpkaya and Isaacson (1981), and include both hydrostatic and hydrodynamic exciting and restoring forces as shown in Fig. 2.4. They may be written for a unit length of the structure for normal wave incidence and motion in surge, heave and pitch as:

\[
\begin{align*}
[-\omega^2 (m + A_{11}) - i\omega B_{11} + S_1]X_1 + \left[-\omega^2 (-mZ_G + A_{13}) - i\omega B_{13}\right]X_3 &= F_{01} \\
[-\omega^2 (m + A_{22}) - i\omega B_{22} + S_2]X_2 &= F_{02} \\
[-\omega^2 (-mZ_G + A_{31}) - i\omega B_{31}]X_1 + \left[-\omega^2 (M_0 + A_{33}) - i\omega B_{33} + mg\overline{GM}\right]X_3 &= F_{03}
\end{align*}
\]

(2.70)

(2.71)

(2.72)

where

\[F_{0j}\] = the vector of wave exciting forces amplitudes,

\[A_j\] and \[B_j\] = the added mass and damping coefficient matrices,

\[m\] = mass per unit length of the breakwater = \[m_1 + 2m_2\],

\[m_1\] = mass per unit length of the rectangular part of the breakwater,

\[m_2\] = mass per unit length of each protruding plate,

\[\overline{GM}\] = the arm of the hydrostatic restoring moment developed by small inclination

\[
= \frac{I_Y}{\forall} - \left[\frac{h}{2} - Z_G\right],
\]

\[I_Y\] = the moment of inertia of the structure’s area cut by the water surface,

\[\forall\] = displaced volume of water by a unit length of the structure,

\[Z_G\] = z-coordinate of the centre of gravity = \[h - \overline{KG}\],

\[\overline{KG}\] = the keel to centre of gravity distance,

\[M_0\] = mass moment of inertia about y-axis = \[mr_G^2\], and

\[r_G\] = roll radius of gyration.
The effect of any mooring lines per unit length of the structure are represented herein by their horizontal and vertical stiffness components $S_1$ and $S_2$, respectively. The plate thickness is considered very small, and its effect on the radius of gyration may be neglected. In the case of a freely floating structure the vertical stiffness will be due to buoyancy, and can be expressed in the following form

$$S_2 = 2\rho ga$$  \hspace{1cm} (2.73)

The heave equation of motion (2.71) may now be solved separately to obtain the heave response amplitude operator $X_2$, and equations (2.70) and (2.72) may be solved simultaneously to obtain the surge and pitch response amplitude operator $X_1$ and $X_3$, respectively. These responses may be calculated for any given wave frequency, and in case of oblique waves interacting with a structure of finite length, equations 2.70-2.72 may be used to get an estimate of the response, this is achieved by multiplying the exciting force amplitudes by the reduction factor $\tau$, in this case the structure not only exhibits surge, heave and pitch but also the other three modes (sway, roll, and yaw) have to be considered.

2.6 Total Reflection and Transmission Coefficients

Two of the major physical properties important in most ocean engineering applications are the reflection and transmission coefficients, which are defined respectively as the ratio of the reflected and transmitted wave heights to the incident wave height.

For a floating structure, the reflected and transmitted wave amplitudes are calculated as the linear superposition of five different wave components due to incident, scattered, and radiated waves corresponding to each mode of the three modes of motion. Therefore, referring to
equations 2.13-2.15 and equation 2.46, the total reflected and transmitted wave potentials in the far field for normal waves may now be written as:

$$\phi_R = Re \left\{ \frac{-igH \cosh[k(z + d)]}{2\omega \cosh(kd)} \left( \frac{A_{on}^{(s)} + A_{on}^{(a)}}{2} + \frac{\omega^2 \psi_0(0)}{g} \sum_{j=1}^{3} f_j a_j x_j \right) e^{-ik(x+a)} \right\}$$ (2.74)

$$\phi_T = Re \left\{ \frac{-igH \cosh[k(z + d)]}{2\omega \cosh(kd)} \left( \frac{A_{on}^{(s)} - A_{on}^{(a)}}{2} - \frac{\omega^2 \psi_0(0)}{g} \sum_{j=1}^{3} f_j \frac{D_{j0}}{k_0 d} x_j \right) e^{ik(x+a)} \right\}$$ (2.75)

Thus the complex reflection coefficient $R$ and transmission coefficient $T$ for a floating structure can now be calculated from:

$$R = \frac{(A_{on}^{(s)} + A_{on}^{(a)})}{2} + \frac{\omega^2}{g} \sum_{j=1}^{3} f_j a_j j_0$$ (2.76)

$$T = \frac{(A_{on}^{(s)} - A_{on}^{(a)})}{2} - \frac{\omega^2}{g} \sum_{j=1}^{3} f_j x_j \frac{D_{j0}}{k_0 d}$$ (2.77)

2.7 Energy Considerations

Since the mathematical formulation of the present solution is based on the assumption of an irrotational flow of an inviscid fluid, energy losses do not occur. It is important here to define wave energy flux as the average rate of energy transfer across a vertical plane of unit width normal to the wave propagation direction. An expression for the energy flux may be derived by considering the average rate of doing work by the dynamic pressure. Using Bernoulli equation, the energy flux $P_e$ may be given by

$$P_e = -\frac{\rho}{T} \int_{0}^{T} \int_{-d}^{d} \Phi_t \Phi_x dz dt$$ (2.78)
where \( T \) is the wave period, and the subscripts indicate differentiation with respect to time and wave direction propagation \((x')\), respectively. As linear wave theory is employed, the upper limit of the integration \( z = \eta \) may be replaced by \( z = 0 \). Substituting the incident potential from equation (2.4), equation (2.78) gives the incident wave energy flux for normal wave incidence \((\beta = 0)\) as

\[
[P_e]_I = \frac{\rho g c H^2}{16} \left[ 1 + \frac{2kd}{\sinh(2kd)} \right] \quad (2.79)
\]

which may be given as a function of the incident wave group celerity \((C_g)\) as:

\[
[P_e]_I = \frac{\rho g C_g H^2}{8} \quad (2.80)
\]

This equation shows that the energy flux is proportional to the square of the wave amplitude. Equation (2.78) may also be applied to the asymptotic complex potentials given by equations (2.74) and (2.75) to give the energy fluxes associated with the reflected and transmitted waves for normal wave incidence \((\beta = 0)\):

\[
[P_e]_R = \frac{\rho g C_g H^2}{8} \left| R_o + \frac{\omega^2 \psi_0(0)}{g} \sum_{j=1}^{3} f_j a_j X_j \right|^2 \quad (2.81)
\]

\[
[P_e]_T = \frac{\rho g C_g H^2}{8} \left| T_o - \frac{\omega^2 \psi_0(0)}{g} \sum_{j=1}^{3} f_j \frac{D_{10}}{k_0 d} X_j \right|^2 \quad (2.82)
\]

As the total energy is conserved, the sum of both the reflected and radiated wave energy fluxes should equal the incident wave energy flux:

\[
[P_e]_R + [P_e]_T = [P_e]_I \quad (2.83)
\]
hence
\[ |R|^2 + |T|^2 = 1 \]  \hspace{2cm} (2.84)

where \( R \) and \( T \) are the complex reflection and transmission coefficients, respectively. Under the same principle of energy conservation, the fixed structure reflection and transmission coefficients should also satisfy equation (2.84) for all wave directions. Therefore, for both cases this relationship may now be used to calculate either the reflection or the transmission coefficient if the other is known.

2.8 Haskind Relation

A very useful relation derived by Haskind (1957) enables the calculation of the exciting forces without the need to solve the wave scattering problem. This relation is used herein as a check on the accuracy of the present solution. Using Green’s theorem, the exciting forces may be written as

\[ F_{0j} = -\rho \int_{s_{\infty}} \left[ \phi_i \frac{\partial \phi_j}{\partial n} - \phi_j \frac{\partial \phi_i}{\partial n} \right] ds \]  \hspace{2cm} (2.85)

where \( \phi_j \), \( j = 1, 2, 3 \) represents the radiated velocity potentials corresponding to the forced motion in surge, heave, and pitch modes, respectively. Considering the vertical radiation boundary at a large distance from the structure, and substituting for the potentials, yields

\[ \frac{F_{0j}}{\rho g} = i \sqrt{2kd \tanh(kd)} \frac{f_j}{a_j} \left[ 1 + \frac{2kd}{\sinh(2kd)} \right]^{1/2} \]  \hspace{2cm} (2.86)
where $a_j$ is the amplitude of radiated waves far away from the structure per unit incident wave amplitude, and is defined by equation (2.47). Thus the horizontal exciting force, the vertical exciting force and the pitching moment may be calculated directly using this relation upon knowing the far field wave properties of the incident and radiated waves.
3.1 Dimensional Analysis

Experiments have been carried out in order to validate the numerical model for certain conditions. Prior to describing the test facilities and procedure, it is useful to perform a dimensional analysis of the problem at hand. This enables the physical parameters influencing the problem to be presented in a relatively compact form using a dimensionless representation.

The purpose of this experimental study is to measure the transmission coefficient of the breakwater for both the fixed and freely floating structure cases. However, since the numerical results also relate to the exciting forces, added mass and damping coefficients, and breakwater motions, the application of a dimensional analysis to these dependent variables are also considered. In the case of the transmission coefficient for a fixed structure, the relevant dependent variables include those shown as follows:

\[ H_t = f(\rho, g, a, H, h, d, b, L, v, \sigma), \quad j = 1, 2, \text{ and } 3 \]  \hspace{1cm} (3.1)

In fact, viscous forces may appear due to the presence of the plate or sharp corners where the flow can separate. This separation effect is neglected here. Surface tension forces will only be confined in the meniscus at both sides of the breakwater, their effect is of second
order and may be neglected for the present linearized problem. Employing the Buckingham Pi-theorem, equation (3.1) may then be reduced to

\[ T = f(ka, \frac{a}{h}, \frac{b}{a}), \quad j = 1, 2, \text{and } 3 \tag{3.2} \]

where \( ka = 2\pi a/L \). For the other variables similar relationships may be developed. The exciting forces may be assumed to vary linearly with the wave height, thus may be non-dimensionalised as follows: \( F_j / \frac{1}{2} \rho g H d \) (\( j = 1, 2 \)), \( F_j / \frac{1}{2} \rho g H d^2 \) (\( j = 3 \)), while the added mass and damping coefficients per unit length of the breakwater are represented in the following dimensionless parameters: \( A_{ij} / \rho d^2 \), \( B_{ij} / \rho d^2 \sqrt{gd} \) (\( i = j = 1, 2 \)), \( A_{ij} / \rho d^3 \), \( B_{ij} / \rho d^3 \sqrt{gd} \) (\( i = 1 \) and \( j = 3 \), or \( i = 3 \) and \( j = 1 \)), and \( A_{ij} / \rho d^4 \), \( B_{ij} / \rho d^4 \sqrt{gd} \) (\( i = j = 3 \)).

To study the transmission coefficient and amplitudes of motion, one needs to include the position of the center of gravity and the radius of gyration, so that an extended functional dependence of the amplitudes of motion may then given by:

\[ \Xi_j = f(\rho, d, h, b, L, H, z_G, m, r_G), \quad j = 1, 2, \text{and } 3 \tag{3.3} \]

A dimensional analysis now gives:
\[
\frac{\Xi_j}{H/2} = f\left( \frac{a}{d}, \frac{h}{d}, \frac{ka}{a}, \frac{z_G}{a}, \frac{m}{\rho ah}, \frac{r_G}{a} \right), \quad \begin{cases} j = 1 \text{ and } 2 \\ j = 3 \end{cases}
\] (3.4)

Considering the static equilibrium of the breakwater in still water, the mass per unit length \( m \) may be calculated as a function of the beam and draft neglecting the volume of two protruding portions of the plates compared to the total volume.

### 3.2 Wave Flume Setup

The model was set up and tested in the Plexiglas flume of the Hydraulics Laboratory in the Department of Civil Engineering at The University of British Columbia. The flume is 0.62 m wide and 20 m long measured from the wave maker to the holding tank at the downwave end of the flume. The wave maker generated the required wave conditions by the oscillatory movement of a paddle. This paddle was hinge-jointed to the bottom of the flume and controlled by a Digital Equipment Corporation (DEC) VAX station 3200 computer.

At the down-wave end of the flume, an artificial beach with a slope of about 1:15 covering a horizontal distance of 7.5 m was installed. The beach consisted of well graded sand on a plywood base and covered with artificial hair matting. The purpose of sand and hair matting is to absorb wave energy so as to damp waves approaching the beach and reduce wave reflection.
Wave heights in the down-wave region of the breakwater were measured by two capacitance type wave probes installed at distances of 1.0 m and 1.5 m from the model. In general, one probe is sufficient for the ordinary measurement of transmitted wave heights, but the two probes are so arranged away from the breakwater in order to exclude the effect of the evanescent wave modes resulting from wave scattering in vicinity of the breakwater on the measured transmitted wave height. The use of two probes also provides a backup for any malfunction in one of the probes during testing. A schematic diagram for the flume showing the wave probe positions and other components of the test system is shown in Fig. 3.1.

3.3 Model Assembly

A model of a pi-type floating breakwater was constructed from acrylic sheets of thickness 0.6 cm and specific gravity 1.17. Figure 3.2 shows a photograph of the model ready for tests for fixed breakwater case. The model has a beam of 32 cm, and a length of 53.5 cm. The cross section of the rectangular part of the model is 30.8 x 10.0 cm, with two screw-attached side plates of dimensions 53.5 x 22.0 x 0.6 cm. These plates protrude a distance of 12 cm from the bottom of the rectangular part of the breakwater. The whole box-shaped model was covered by a light aluminum plate of thickness 0.2 cm. They have dimensions of 53.5 x 9.75 cm. The specific gravity of the aluminum alloy used is 2.77. In order to reach the required draft of the model (5 cm), cylindrical weights are centered around two aluminum rods fixed to the bottom inside of the model. These rods were arranged in a symmetrical way, such that they do not move the position of the centre of
gravity from the z-axis. The z-coordinate of the centre of gravity and the radius of
gyration of the model around the y-axis of the model were \( z_G/a = 0.16 \) and
\( r_G/a = 1.22 \).

3.4 Instrumentation

After connecting each of the probes to one of the VAX-station channels, a
selection of prescribed wave conditions were specified, each in a separate computer file.
These files are used to generate waves with the required conditions using the wave paddle. Each probe senses the change in the water surface elevation and sends a signal to the computer which is placed in a separate output file. After calibrating the probes to read the change in water surface elevation, waves are generated by controlling the paddle through the GEDAP (Generalized Experiment Control, Data acquisition and Analysis Package) library of software and the associated RTC (Real Time Control) programs. These programs were developed at the National Research Council of Canada. For each test, the duration of data recording was set to 14 sec with a frequency of 50 Hz, corresponding to 700 points for each test, and the interval between two successive signals was 0.02 sec.

3.5 Model Testing Procedure

A total of 14 tests were conducted to study the effect of different wave conditions on the breakwater. The various wave conditions are as shown in Table 3.1. The table
shows the selected values of the diffraction parameters $ka$. For the fixed breakwater modeling, the model was clamped to the flume sides using a thin aluminum plate attached to its sides. On the other hand, for the floating breakwater modeling, the model was freely floating in the flume. In order to prevent the model from drifting, light rubber bands tied the model to the flume sides at two points on the model at the still water level on each side. Their inclination was almost horizontal, and the stiffness of these bands is negligible (about 4N/m), so that the model may be considered freely floating.
CHAPTER 4

RESULTS AND DISCUSSION

4.1 Verification of Numerical Model

The analytical solution presented in chapter two has been modeled numerically. It is appropriate initially to verify the model to the extent possible. Ideally, this would be achieved by comparing the present results with those of previous experimental and/or numerical studies. However, a literature survey indicates very few publications on such a type of floating breakwater. Consequently, the verification has been achieved simply by comparisons with some previously published results for rectangular floating breakwaters; and in addition various consistency checks has been applied to the model.

4.1.1 Rectangular Breakwater

Figure 4.1 compares predictions of the horizontal and vertical forces and the pitching moment based on the present numerical model for the case of normal wave incidence with the results of Black et al. (1971), who adopted a variational method of solution, for the case of a/d = 0.5 and h/a = 1.0. For a shallower body, Fig. 4.2 compares the horizontal and vertical forces predicted by the present method with the results of McIver (1983) based on an eigenfunction expansion method. The effect of the incident wave angle on the horizontal and vertical forces, and the pitching moment has also been studied. Figure 4.3 compares present results with those of Bai (1974) based on a finite
element formulation for the case $a/d = h/d = 0.5$ and for three different values of the diffraction parameter $ka$. All the comparisons given in Figs. 4.1-4.3 show excellent agreement. However, Fig. 4.3 shows that the present results overpredict the pitching moment for those of Bai by up to about 4%.

Figure 4.4 shows comparisons of the present results for the added mass and damping coefficients with those of Andersen and Wuzhou (1985) for a relatively shallow draft breakwater with $h/d = 0.1$. Present results show good agreement for all coefficients except for the heave added mass coefficient. The present heave added mass results are higher by 5%, and this difference is significant at high frequencies.

The response amplitude operators have been calculated and plotted as functions of the diffraction parameter. Figure 4.5 shows a comparison between the surge and heave RAO's predicted by the present method with the results of Isaacson & Nwogu (1987) for several wave directions and a rigid structure. Both results are in excellent agreement.

Figure 4.6 compares the fixed body reflection coefficient with Ursell (1947), and Black & Mei (1969), shown in Figs. 4.6 (a) and 4.6 (b), respectively. Normal wave incidence was only considered for different $h/a$ values. The reflection and transmission coefficients for a freely floating structure were verified with the results of McDougal & Sulisz (1987). Both coefficients are plotted versus relative depth $kd$ in Fig. 4.7. Two different cases were used with normally incident waves. Once more, this comparison shows an excellent agreement.
4.1.2 Pi-Type Breakwater

Due to the lack of published results for pi-type breakwaters, verification of the present solution is achieved by using two different checks. These are developed by using the far field properties of wave potential components. One of these checks is achieved by comparing the exciting forces calculated from the near-field pressure integration around the wetted surface with that calculated from the far-field properties using the Haskind relation. In this relation, the exciting forces may be calculated from a knowledge of the amplitudes of the incident wave and the radiated waves: this relation is as given in equation (2.86). This comparison is as given in Fig. 4.8, and indicates excellent agreement between both methods. Only the pitching moment calculated from the near field properties shows higher values than that based on the Haskind relation, this difference occurring near the peak of the curve.

The other check is a comparison of the cross added mass and damping coefficients calculated from the surge and pitch wave fields. Using the symmetry property of the added mass and damping coefficients matrices, the surge-pitch and pitch-surge coefficients should be equal. This comparison is shown in Fig. 4.9, which shows a slight difference in the added mass coefficient.

4.2 Effect of Plate Height

In order to assess the use of the pi-type breakwater, it is of considerable importance to study the variation of the various quantities of interest with plate height. These include the exciting forces, added masses, damping coefficients, response
amplitude operators, and the transmission coefficient. Figure 4.10 shows the
dimensionless exciting forces plotted as functions of $ka$ for different plate heights. The
dimensionless horizontal force reaches a peak at a certain value of the diffraction
parameter $ka$. This is consistent with the lower particle accelerations for long waves (low
$ka$), and less exposure for short waves (large $ka$). The horizontal force increases with the
increase of plate height as expected. The variation in the vertical force is somewhat
different. For very long waves ($ka \to 0$), the vertical force amplitude is associated with
the hydrodynamic pressure beneath a wave crest. More generally, the vertical force is a
maximum for the no-plate case; and an increase of the plate height reduces the amplitude
of dynamic pressure fluctuation at the breakwater bottom, giving a lower force. On the
other hand, the variation of the pitching moment is more or less similar to that of the
horizontal force. However the moment due to the pressure beneath the breakwater
generally opposes that due to the pressure on the breakwater sides; and these are in
balance giving rise to a zero pitching moment at a specific wave frequency. This depends
of course on the plate height, and for $b/a = 0.0$ occurs at $ka = 2.0$.

The corresponding added mass and damping coefficients are given in Fig. 4.11 for
different plate heights. The damping coefficients diminish for long waves, and also tend
to zero at high frequencies (large $ka$). The added mass coefficients are nearly constant for
$ka > 1.0$, and vary considerably otherwise. It is important to note that the surge, surge-
pitch, and pitch added mass and moment coefficients tend to maximum values at the long
wave extreme ($ka \to 0$). Also it is to be noted that, the presence of the plate results in
increased added mass and damping coefficients, except for the heave damping coefficient which shows an opposite trend.

The response amplitude operators (RAO's) of the breakwater have been calculated and are given in Fig. 4.12 as functions of $ka$ for $a/d = 0.5$, $h/a = 0.5$, $\beta = 0.0$ and various values of relative plate height $b/a$. For long waves ($ka \to 0$), the surge and heave RAO's correspond to motion amplitudes equal to the horizontal and vertical water particle motion amplitudes respectively, while the pitch RAO corresponds to a pitch amplitude equal to the maximum water shape. The heave RAO exhibits a typical damped single degree of freedom system response, where the maximum response occurs at an excitation frequency lower than the system natural frequency depending on the damping in the system (Fig. 4.12 (b)). The pitch RAO has a high peak at the value of $ka$ corresponding to the pitch natural frequency. Due to the surge and pitch response coupling, a significant increase in the surge motion occurs at the same value of $ka$.

It is important to note that the numerical results for the RAO's are based on the centre of gravity located at the still water level, the radius of gyration $r_o$ was calculated and for different plate heights assuming a very thin plate the ratio of the radius of gyration to water depth ranged around ($r_o/d = 0.45$), this was calculated based on a cross section having a uniform density. This is not the case for most practical applications. However for a real case, it is essential that the position of the centre of gravity and the mass moment of inertia or radius of gyration should all be calculated a priori.

Figure 4.13 shows the transmission coefficient as a function of $ka$ for both the fixed and freely floating cases. Once more results are shown for $a/d = 0.5$, $h/a = 0.5$, $\beta = 0.0$ and various values of relative plate height $b/a$. For long waves ($ka \to 0$), the surge and heave RAO's correspond to motion amplitudes equal to the horizontal and vertical water particle motion amplitudes respectively, while the pitch RAO corresponds to a pitch amplitude equal to the maximum water shape. The heave RAO exhibits a typical damped single degree of freedom system response, where the maximum response occurs at an excitation frequency lower than the system natural frequency depending on the damping in the system (Fig. 4.12 (b)). The pitch RAO has a high peak at the value of $ka$ corresponding to the pitch natural frequency. Due to the surge and pitch response coupling, a significant increase in the surge motion occurs at the same value of $ka$.
β = 0.0 and for various values of b/a. In both cases the structure is transparent for the long wave incidence as expected. For a fixed structure, Fig. 4.13 (a) indicates that T decreases monotonically with ka from T = 1.0 for long waves (ka → 0) to T = 0.0 for short waves (high ka). For a given ka the transmission coefficient decreases with an increase in relative plate height b/a, as expected. In the case of a freely floating structure (Fig. 4.13 (b)) the transmission coefficient decreases from unity in the long wave limit (ka → 0) to zero at the point of complete reflection, where the breakwater acts as a complete vertical barrier. As ka increases further, the transmission coefficient rises again, with a maximum value for the no-plate case. The sharp increases of the transmission coefficient beyond unity are spurious, and are associated with a failure of the method at the position of resonance, where the structure oscillates severely.

4.3 Effect of Wave Direction

In chapter 2, it was shown that the three-dimensional scattering problem for oblique waves can be reduced to a two-dimensional problem, by assuming a sinusoidal variation of the potential along the breakwater length. On the other hand, the radiation problem was assumed two-dimensional due to the assumption of rigid body motions.

Figure 4.14 shows the dimensionless exciting forces as functions of the diffraction parameter ka. For various values of incident wave direction β, the figure shows that maximum forces occur for normal wave incidence. The variation of the horizontal force and pitching moment with wave direction β becomes insignificant at higher values of ka (greater than about ka ≈ 1). However, the variation of the vertical force with β is more
significant for intermediate values of \( ka \). Recalling Fig. 4.3, where the exciting forces amplitudes are plotted as functions of \( \beta \), the horizontal force and pitching moment at a specific \( \beta \) may be related to the values for normal wave through \( \cos \beta \), so that only the normal wave exciting forces need to be calculated. For practical applications, these exciting forces may then be approximated as:

\[
F_{0j}(\beta) = \alpha F_{0j}(0) \cos \beta \quad (4.1)
\]

where:

\[
\alpha = \begin{cases} 
1.1 & \beta \geq 15^\circ \\
1.0 & \beta < 15^\circ 
\end{cases} \quad (4.2)
\]

On the other hand, the vertical force appears to exhibit a constant value for \( \beta < 45 \), provided \( ka < 0.4 \) also, so that the normal wave force may be used instead for this range of \( \beta \) and \( ka \). Outside these conditions the force has to be calculated directly in terms of \( \beta \) and \( ka \).

Due to the spatial variation of the velocity potential in case of oblique waves, integrations of the exciting forces along the length of an infinitely long breakwater give zero total exciting forces. This results in zero structural response, hence the transmission coefficient is the same for both the fixed breakwater and freely floating breakwater cases. The variation in the transmission coefficient is shown in Fig. 4.15.

4.4 Comparison of Two Types of Breakwaters

The performance of a pi-type breakwater with \( a/d = 1.0, b/a = 0.25, h/a = 0.25 \) is now compared with a rectangular breakwater which has the same dimensions except that
its draft corresponds to h/a = 0.5. These values were chosen in order to have the same gap under the structure for both types (s/d = 0.5).

The exciting forces for both breakwaters are compared in Fig. 4.16, which shows that the pi-type breakwater experience a lower horizontal force and the same vertical force over the whole range of ka, while the pitching moment is lower for the pi-type breakwater for the range ka < 2.4 and then is higher for higher values of ka.

Figure 4.17 shows the variations in the added mass and damping coefficients with ka for the two breakwaters. All the added mass coefficients are higher for the pi-type breakwater except for the heave coefficient which is the same for both types as expected. The increase in the added mass may be physically described by an extra mass of water confined between the two plates, were they close to each other.

RAO’s of the two breakwater types are compared in Fig. 4.18. The figure indicates slightly higher surge and heave responses for the pi-type breakwater over a large range of ka. Although the horizontal exciting force is lower for the pi-type breakwater, the surge response is higher than that of the rectangular breakwater, due to the larger mass of the rectangular breakwater. The comparison of pitch motions show that the two types have almost the same responses, but only their peaks occur at different values corresponding to different natural frequencies. As the heave added mass and damping coefficients (Fig. 4.17 (b)) are nearly equal for both types, it is expected they have also nearly equal responses. This is confirmed in Fig. 4.18, the slight difference being due to the higher inertia of the rectangular type, leading to a lower response.

For any type of breakwater, the transmission coefficient is a primary performance parameter, when comparing one type to another. Fig. 4.19 compares the transmission
coefficient for both the aforementioned types. The pi-type breakwater shows slightly higher values than that for the rectangular. This difference is almost constant for $ka > 3.0$, and is of order 0.1 for the freely floating case.

4.5 Experimental Results

The proposed pi-type breakwater was tested in the flume of the Hydraulics Laboratory of the Civil Engineering Department. The model was setup in the flume for both the fixed breakwater case and the freely floating case. The transmitted wave heights were measured for different incident wave characteristics as described in chapter 3. The wave steepness was held constant in all tests, and the transmission coefficient was calculated directly as the ratio between the incident and transmitted wave heights. Figure 4.20 shows a comparison between the measured transmission coefficients and the results predicted by the numerical model. This figure shows a reasonable agreement between the predicted transmission coefficients and those measured, however Fig. 4.20 (a) shows a better agreement between both sets of results, except at intermediate values of $ka$ where there are significant differences between both results. This difference may be attributed to wave reflection from the beach, end effects due to the gap between the model and the sides of the flume, to the presence of the evanescent waves at the probe locations, and to general experimental errors. Figure 4.20 (b) shows the freely floating case, where both the predicted and measured results are in excellent agreement, except that significant differences are noted at resonance ($ka = 0.54$), where the breakwater exhibits large motions and viscous effects are of primary importance at this stage.
4.6 Numerical Considerations

Consideration is not given to possible numerical errors associated with the truncation of the infinite series representation of the potentials in the numerical method that has been used. Each infinite series is truncated to $M$ terms leading to a set of $(M+1)$ linear simultaneous equations for each set of unknown coefficients of the potential in region 2.

To study the effect of $M$ on the accuracy of the results, the added mass and damping coefficients are shown as functions of $M$ in Fig. 4.22 for the case $ka = 1.0$, $a/d = 0.5$, and $h/a = 0.5$. These particular coefficients were chosen among the various output parameters since a greater number of evanescent modes are required to describe the flow field variation near the structure submerged surface in the radiation problem. Different values of relative plate height were used in the calculation. The slope of these curves may be considered as a measure of the stability of the results: i.e., higher accuracy is achieved as any curve becomes horizontal. For the case considered, most of these curves become horizontal for $M > 30$. It is also noted that for deeper plates ($b/a = 1.0$) and the same small value of $M$, the slope of the corresponding curve is steeper than other curves for shallower plates, so that the deeper the plate, the higher is the $M$ value that should be used. At the same time, for breakwaters with large beam and deep draft, the flow underneath the structure in long waves may be considered as quasi-uniform flow, and a smaller value of $M$ may then be used.
CHAPTER 5

SUMMARY AND CONCLUSIONS

5.1 Summary

The diffraction problem of waves interacting with a rectangular long breakwater with two side plates is solved. Different waves approaching the breakwater from any direction are considered. The structure is assumed to be located in water of uniform constant depth. Thus an eigenfunction expansion method was used in the solution. The exciting forces, added masses and damping coefficients, responses of the structure, and the transmission coefficient were all calculated for different cases and their results were analyzed. Comparisons between the present approach and different numerical solutions published previously have been made for all physical quantities of interest. A comparison between the traditional rectangular breakwater and the proposed pi-type breakwater is also performed.

5.2 Conclusions

From the analysis of the results of the present study, the following conclusions may be drawn

1- The present pi-type breakwater offers a good alternative for protection from moderate waves. This type of breakwater experiences lower wave forces than a rectangular
breakwater having the same under-tip clearance. This would enable the use of light
moorings, while its smaller volume would also lead to savings in material and
consequently total costs. Consequently, the addition of two plates to a new design or
to an existing rectangular breakwater would be recommended for wave transmission
reduction.

2- This type of breakwater may be recommended to be used for a range of diffraction
parameter of \( ka > 1.5 \).

3- The eigenfunction expansion solution offers an accurate and numerically efficient
method, but it only applies to rectangular fluid sub-domains, which limits its
applicability to certain configurations.

4- In the case of oblique waves approaching the structure, it is seen that wave forces are
a maximum for normal waves. Consequently, the heave response would also be a
maximum for the normal wave case. Therefore, this is not valid for the surge and/or
pitch responses, as these two responses are coupled. The maximum responses in this
case are strongly dependent on the ratio between the horizontal force and the pitching
moment, this ratio being different for different angles of wave incidence. Thus
different angles of incidence should be considered, if surge and/or pitch responses are
important.

In developing the present solution, linear wave theory was used. However, the
presence of the plate may be a source of nonlinearity and additional complexities. The
effect of flow separation at the tip of the plate long waves should be considered before
using this type in engineering applications.
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APPENDIX

EXPLICIT MATHEMATICALEXPRESSIONS

The following provides explicit expressions for various parameters developed in chapter 2:

\[ F_{m,n} = \frac{(-h+b)}{\int_{-d}^{d} \psi_m(z) \chi_n(z) dz} \]  (A.1)

\[ F_{0,0} = \frac{\sqrt{2} \sinh(k s)}{k d \left( s + b \right) \left[ 1 + \frac{\sinh(2 k d)}{2 k d} \right]^{1/2}} \]  (A.2)

\[ F_{0,n} = \frac{2 \left[ \cosh(k s) \sin(p_n s) + \frac{k}{p_n} \cos(p_n s) \sinh(k s) \right]}{p_n d \left( s + b \right) \left[ 1 + \frac{\sinh(2 k d)}{2 k d} \right]^{1/2} \left[ 1 + \left( \frac{k}{p_n} \right)^2 \right]} \]  (A.3)

\[ F_{m,0} = \frac{\sqrt{2} \sin(k_m s)}{k_m d \left( s + b \right) \left[ 1 + \frac{\sin(2 k_m d)}{2 k_m d} \right]^{1/2}} \]  (A.4)

\[ F_{m,n} = \frac{1}{p_n d \left( s + b \right) \left[ 1 + \frac{\sinh(2 k d)}{2 k d} \right]^{1/2}} \left[ \frac{\sin((p_n + k_m) s)}{p_n + k_m} + \frac{\sin((p_n - k_m) s)}{p_n - k_m} \right] \]  (A.5)
\[ Q_{m,n} = \int_{-d}^{d} \chi_m(z)\chi_n(z) \, dz \]  
\[ Q_{0,0} = \frac{s}{s + b} \]  
\[ Q_{0,n} = \frac{\sqrt{2}}{p_n (s + b)} [\sin(p_n s)] = Q_{n,0} \]  
\[ Q_{m,n} = \begin{cases} 
\frac{d}{s + b} \left[ \frac{\sin[(p_n + k_m)s]}{(p_n + k_m)(s + b)} + \frac{\sin[(p_n - k_m)s]}{(p_n - k_m)(s + b)} \right] & n \neq m \\
\frac{d}{s + b} \left[ \frac{s}{d} + \frac{1}{p_n d} \sin(2p_n d) \right] & n = m 
\end{cases} \]  
\[ I_{m,n} = \int_{-(h+b)}^{h} \chi_m(z)\chi_n(z) \, dz \]  
\[ I_{0,0} = \frac{b}{s + b} \]  
\[ I_{0,n} = \frac{\sqrt{2}}{p_n (s + b)} [-\sin(p_n s)] = I_{n,0} \]  
\[ I_{m,n} = \begin{cases} 
\frac{-d}{s + b} \left[ \frac{\sin[(p_n + k_m)s]}{(p_n + k_m)(s + b)} + \frac{\sin[(p_n - k_m)s]}{(p_n - k_m)(s + b)} \right] & n \neq m \\
\frac{d}{s + b} \left[ \frac{b}{d} - \frac{1}{p_n d} \sin(2p_n d) \right] & n = m 
\end{cases} \]  
\[ W_n = \int_{-(h+b)}^{0} \psi_n(z) \, dz = \frac{\sqrt{2}}{k_n d} \left[ \sin(k_n d - \sin(k_n s)) \right] \]  
\[ = k_n d \left[ 1 + \frac{\sin(2k_n d)}{2k_n d} \right] \]
\[ V_n = \begin{cases} \frac{1}{\sqrt{2}} \frac{b}{\frac{1}{d} \int_{[s+b]^2} (s + b)^{\frac{1}{2}}} & n = 0 \\ \frac{-h}{\frac{1}{d} \int_{[s+b]^2} (s + b)^{\frac{1}{2}}} \left[ -\sin(p_n s) \right] & n \neq 0 \end{cases} \] (A.15)

\[ \overline{W}_n = \begin{cases} \sqrt{2} \frac{\sin(k_n s)}{k_n \left[ 1 + \frac{\sin(2k_n d)}{2k_n d} \right]} & n = 0 \\ \frac{\sin(k_n s)}{k_n \left[ 1 + \frac{\sin(2k_n d)}{2k_n d} \right]} & n \neq 0 \end{cases} \] (A.16)

\[ \overline{V}_n = \begin{cases} \frac{-1}{\frac{1}{d} \int_{[s+b]^2} (s + b)^{\frac{1}{2}}} \left[ \frac{b}{\frac{1}{d} \int_{[s+b]^2} (s + b)^{\frac{1}{2}}} + 1 \right] & n = 0 \\ \frac{\sqrt{2} \left( \frac{h + b}{d} \sin(p_n s) + \frac{1}{p_n d} [(-1)^n - \cos(p_n s)] \right)}{p_n \left[ 1 + \frac{\sin(2k_n d)}{2k_n d} \right]} & n \neq 0 \end{cases} \] (A.17)

\[ U_n = \begin{cases} \sqrt{2} \frac{\sin(k_n s)}{k_n \left[ 1 + \frac{\sin(2k_n d)}{2k_n d} \right]} & n = 0 \\ \sqrt{2} \frac{\sin(k_n s)}{k_n \left[ 1 + \frac{\sin(2k_n d)}{2k_n d} \right]} & n \neq 0 \end{cases} \] (A.18)

\[ I_n = \begin{cases} \frac{1}{\sqrt{2}} \frac{s}{\frac{1}{d} \int_{[s+b]^2} (s + b)^{\frac{1}{2}}} \sin(p_n s) & n = 0 \\ \frac{\sqrt{2} \sin(p_n s)}{p_n \left[ 1 + \frac{\sin(2k_n d)}{2k_n d} \right]} & n \neq 0 \end{cases} \] (A.19)
\[
\tilde{U}_n = \frac{\sqrt{2}}{\sqrt{k_n d \left[1 + \frac{\sin(2k_n d)}{2k_n d}\right]^2}} \left[\left\{\frac{\sin(k_n s)}{k_n d} + \frac{2s}{k_n d} \cos(k_n s)\right\}\right]
\]

(A.20)

\[
\tilde{I}_n = \frac{\sqrt{2}}{\sqrt{p_n d \left[s + b\right]^2}} \left[\left\{\frac{\sin(p_n s)}{p_n d} + \frac{2s}{p_n d} \cos(p_n s)\right\}\right]
\]

(A.21)

\[
G_n = \frac{\sqrt{2}}{\sqrt{p_n d \left[s + b\right]^2}} \left[\left\{\frac{\sin(p_n s)}{p_n d} - \frac{2s}{p_n d} \cos(p_n s)\right\}\right]
\]

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Table 3.1. Test wave conditions.
Figure 2.1. Wave force regimes [Sarpkaya & Isaacson (1981)]
Figure 2.2. Schematic of the breakwater: (a) elevation, and (b) plan.
Figure 2.3 Radiation modes: (a) surge, (b) heave, and (c) pitch
Figure 2.4. Free body diagram for a breakwater section
Figure 3.1. Wave flume layout (vertical scale exaggerated)

(dimsions in m)

- Paddle
- Wave probes
- Artificial beach
- Holding tank
Figure 3.2. Photograph of model setup for the fixed breakwater case.
Figure 4.1. Comparison of the exciting forces with Black et al. (1971) for $a/d = 0.5$, $h/a = 1.0$, $\beta = 0.0$, and $b/a = 0.0$; (a) horizontal force, (b) vertical force, and (c) pitching moment.
Figure 4.2. Comparison of the horizontal and vertical exciting forces with McIver (1986) for \( b/a = 0.0 \), and \( \beta = 0.0 \); (a) \( a/d = 0.196 \), \( h/d = 0.05 \), and (b) \( a/d = 0.25 \), \( h/d = 0.06 \).
Figure 4.3. Comparisons between the present method and Bai (1974) for \( a/d = 0.5, h/a = 1.0, \) and \( b/a = 0.0; \) (a) horizontal force, (b) vertical force, and (c) pitching moment.
Figure 4.4. Comparisons of the added mass and damping coefficients with Andersen & Wuzhou (1985) for $a/d = h/d = 0.1$, and $b/a = 0.0$; (a) surge coefficients, (b) heave coefficients, (c) surge-pitch coefficients, and (d) pitch coefficients.
Figure 4. Continued.
Figure 4.5. Comparison of response amplitude operators with Isaacson & Nwogu (1987) for $a/d = 0.625$, $h/a = 0.4$, $b/a = 0.0$, $r_G/a = 2.6$, $z_G/a = 0.0$; (a) surge RAO, and (b) heave RAO.
Figure 4.6. Comparison of the reflection coefficient of a fixed structure with previous results for $\beta = 0.0$ and $b/a = 0.0$; (a) Ursell (1947), and (b) Black & Mei (1969).
Figure 4.7. Comparison of the reflection and transmission coefficients for a freely floating case with McDougal & Sulisz (1987); for a/d = 1.0, and b/a = \( \beta = 0.0 \); (a) h/a = 0.8, and (b) h/a = 0.6.
Figure 4.8. Comparison of the exciting forces between the present solution and Haskind relation; for $a/d = 0.5$, $b/a = 1.0$, $h/a = 0.5$, and $\beta = 0.0$.

Figure 4.9. Comparison of the surge-pitch and pitch-surge added mass and damping coefficients; for $a/d = 0.5$, $h/a = 1$, and $\beta = 0.0$.
Figure 4.10. Influence of the diffraction parameter on exciting forces for \( a/d = 0.5, h/a = 0.5, \) and \( \beta = 0.0; \) (a) horizontal force, (b) vertical force, and (c) pitching moment.
Figure 4.11. Influence of the diffraction parameter on added mass and damping coefficients for $a/d = 0.5$, $h/a = 0.5$, and $\beta = 0.0$; (a) surge coefficients, (b) heave coefficients, (c) surge-pitch coefficients, and (d) pitch coefficients.
Figure 4.11. Continued.
Figure 4.11. Continued.
Figure 4.11. Continued.
Figure 4.12. Influence of the diffraction parameter on response amplitude operators for $a/d = 0.5$, $h/a = 0.5$, $r_{G}/a = 0.9$, $z_{G}/a = 0.0$, and $\beta = 0.0$; (a) surge RAO, (b) heave RAO, and (c) pitch RAO.
Figure 4.13. Influence of the diffraction parameter on the transmission coefficient for $a/d = 0.5$, $h/a = 0.5$, $r_G/a = 0.9$, $z_G/a = 0.0$, and $\beta = 0.0$; (a) fixed structure, and (b) freely floating.
Figure 4.14. Influence of wave direction on exciting forces for $a/d = 0.5$, $h/a = 0.5$, and $b/a = 0.125$; (a) horizontal force, (b) vertical force, and (c) pitching moment.
Figure 4.15. Influence of wave direction on the transmission coefficient for $a/d = 0.5$, $h/a = 0.5$, and $b/a = 0.125$; fixed structure.
Figure 4.16. Comparison of exciting forces between rectangular and pi-type breakwaters for the same beam (a/d = 1.0), under-tip clearance (s/d = 0.5), and normal wave incidence; (a) horizontal force, (b) vertical force, and (c) pitching moment.
Figure 4.17. Comparison of added mass and damping coefficients between a rectangular and a pi-type breakwater for the same beam \((a/d = 1.0)\), and under-tip clearance \((s/d = 0.5)\), and normal wave incidence; (a) surge coefficients, (b) heave coefficients, (c) surge-pitch coefficients, and (d) pitch coefficients.
Figure 4.17. Continued.
Figure 4.18. Comparison of RAOs between a rectangular and a pi-type breakwater for the same beam (a/d = 1.0), under-tip clearance (s/d = 0.5), and normal wave incidence; (a) surge RAO, (b) heave RAO, and (c) pitch RAO.
Figure 4.19. Comparison of transmission coefficient between rectangular and pi-type breakwaters for the same beam (a/d = 1.0), under-tip clearance (s/d = 0.5), and normal wave incidence; (a) fixed structure, and (b) freely floating.
Figure 4.20. Comparison between measured and computed transmission coefficient for $a/d = 0.375$, $b/a = 0.752$, $h/a = 0.312$, $r_G/a = 1.22$, and $z_G/a = 0.152$; (a) fixed structure, and (b) freely floating.
Figure 4.21. Effect of the truncation parameter on the added mass and damping coefficients for $ka = 1.0$, $h/a = 0.5$; (a) surge coefficients, (b) heave coefficients, (c) surge-pitch coefficients, and (d) pitch coefficients.
Figure 4.21. Continued.