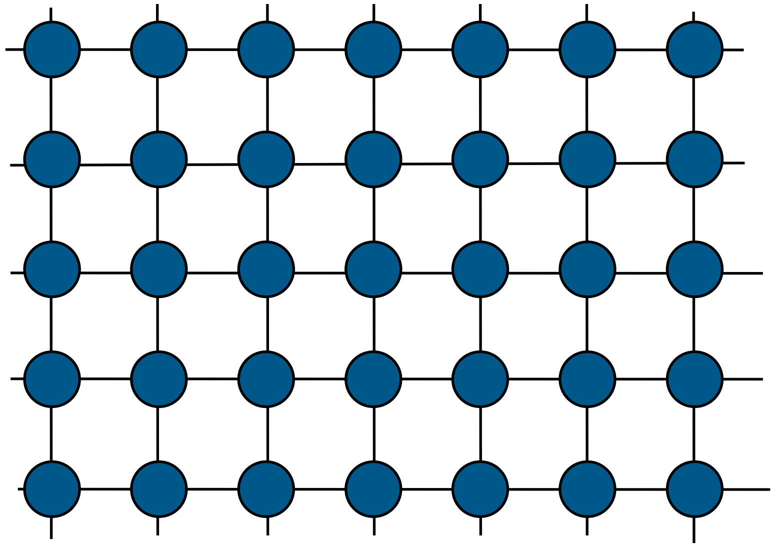
Measurement-based quantum computation

10th Canadian Summer School on QI



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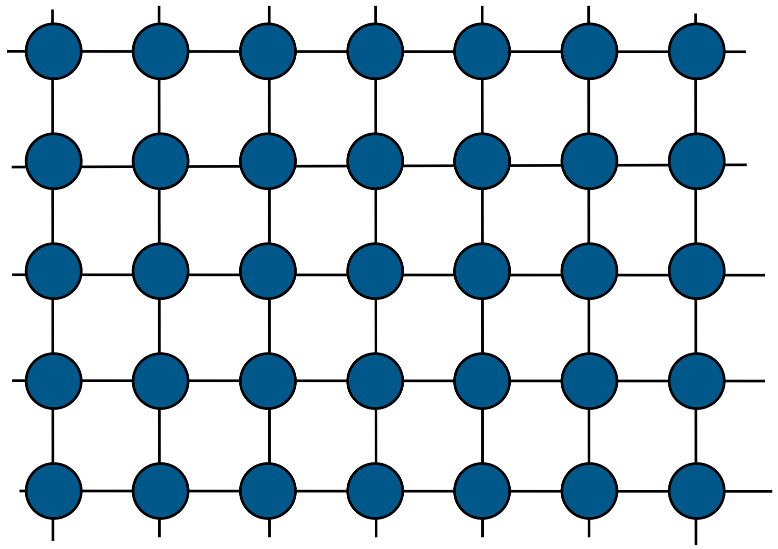
What is a quantum computer?

The one-way quantum computer

A multi-qubit entangled
"resource state"
E.g. cluster state



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Single-qubit
measurements

Choice of bases specify computation

Adaptive (some bases depend upon previous outcomes)

= A universal quantum computer

Overview of Lectures

- **1. Cluster states and graph states I**
  - What are they? Basic properties.
- **2. The one-way quantum computer**
  - What is the model? How does it work?
- **3. Cluster states and graph states II**
  - Stabilizer formalism and graphical representation
- **4. Cluster states and graph states III**
  - How do we build them?
- **5. Beyond cluster states** *(If time permits)*
  - (other models of MBQC)
- **6. (Robert Raussendorf) Fault tolerant MBQC**

Pauli group and Clifford Group

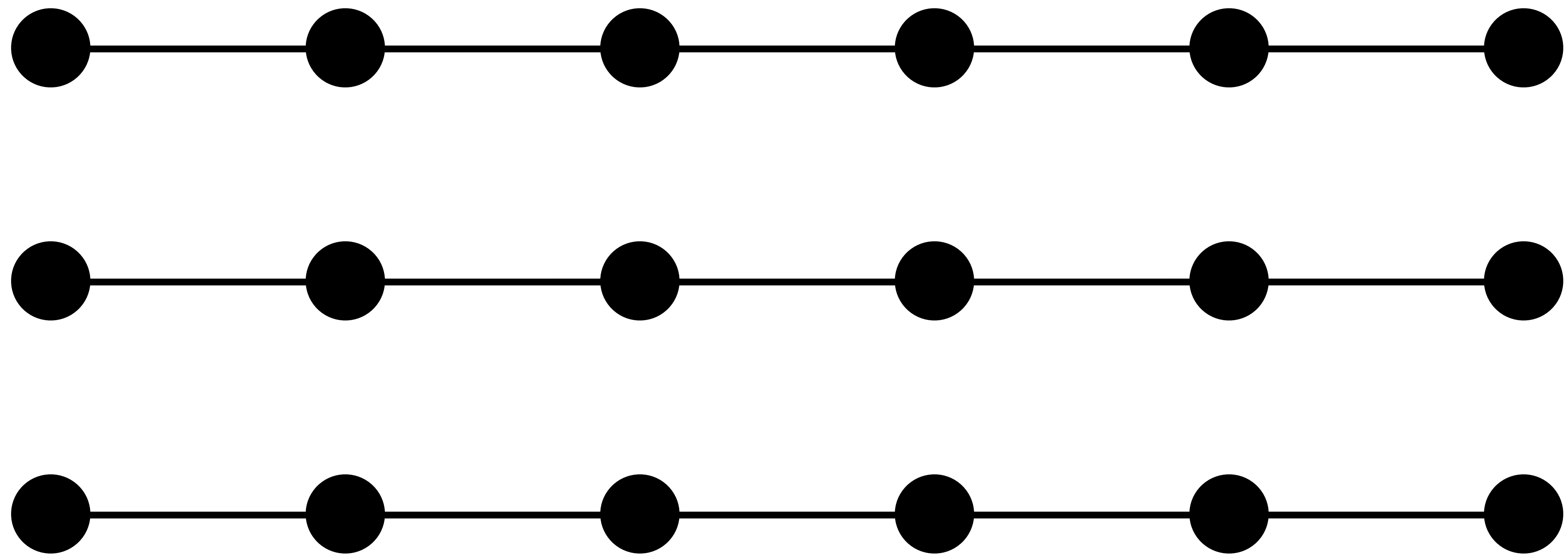
- Pauli group: $\mathbb{P}_n$
  - Set of all n-fold tensor products of **X**, **Y**, **Z** and **I**, with pre-factors **+1**, **-1**. **+i**, **-i** for group closure.
- Clifford group:
  - "Normalizer" of $\mathbb{P}_n$
    Set of unitaries C such that

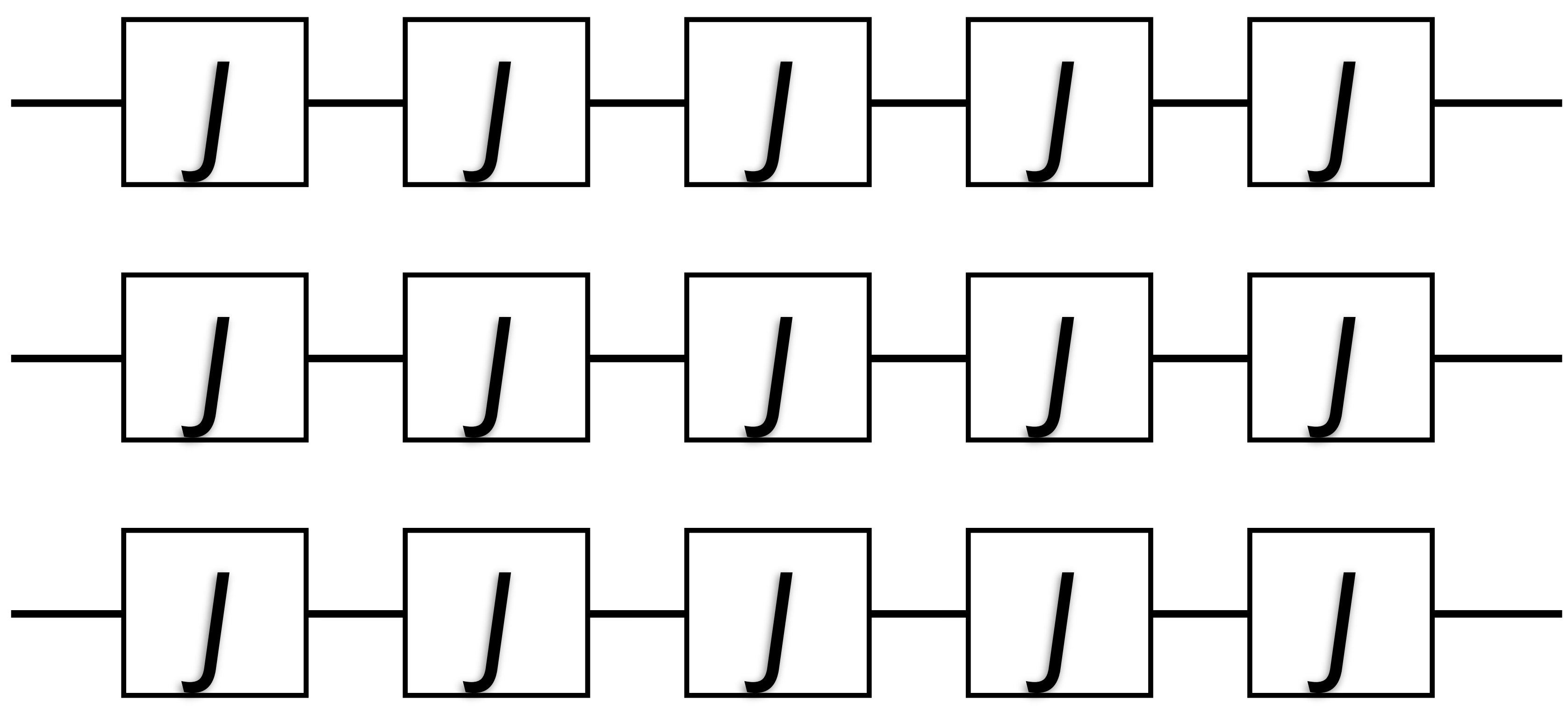
$$\forall \sigma_k \in \mathbb{P}_n \qquad C\sigma_k C^\dagger = \sigma_j \qquad \sigma_j \in \mathbb{P}_n$$

Equiv,: $C\sigma_k = \sigma_j C = (C\sigma_k C\dagger)C$

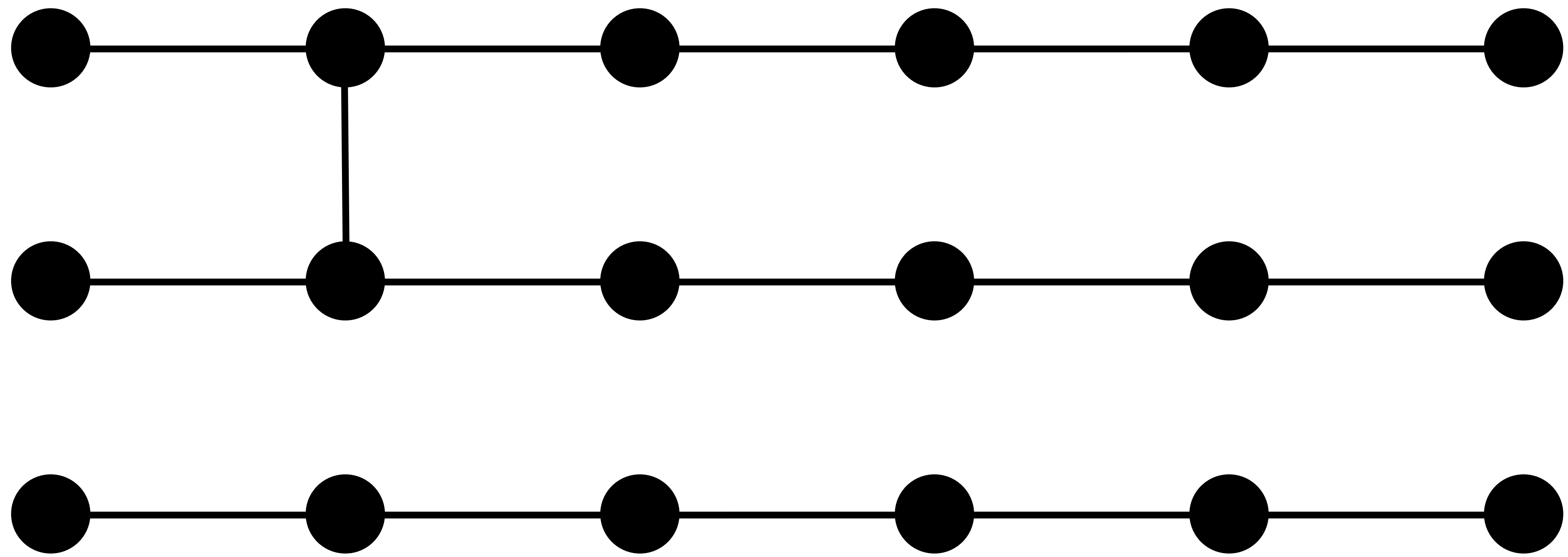
"Maps Pauli group onto Pauli group"

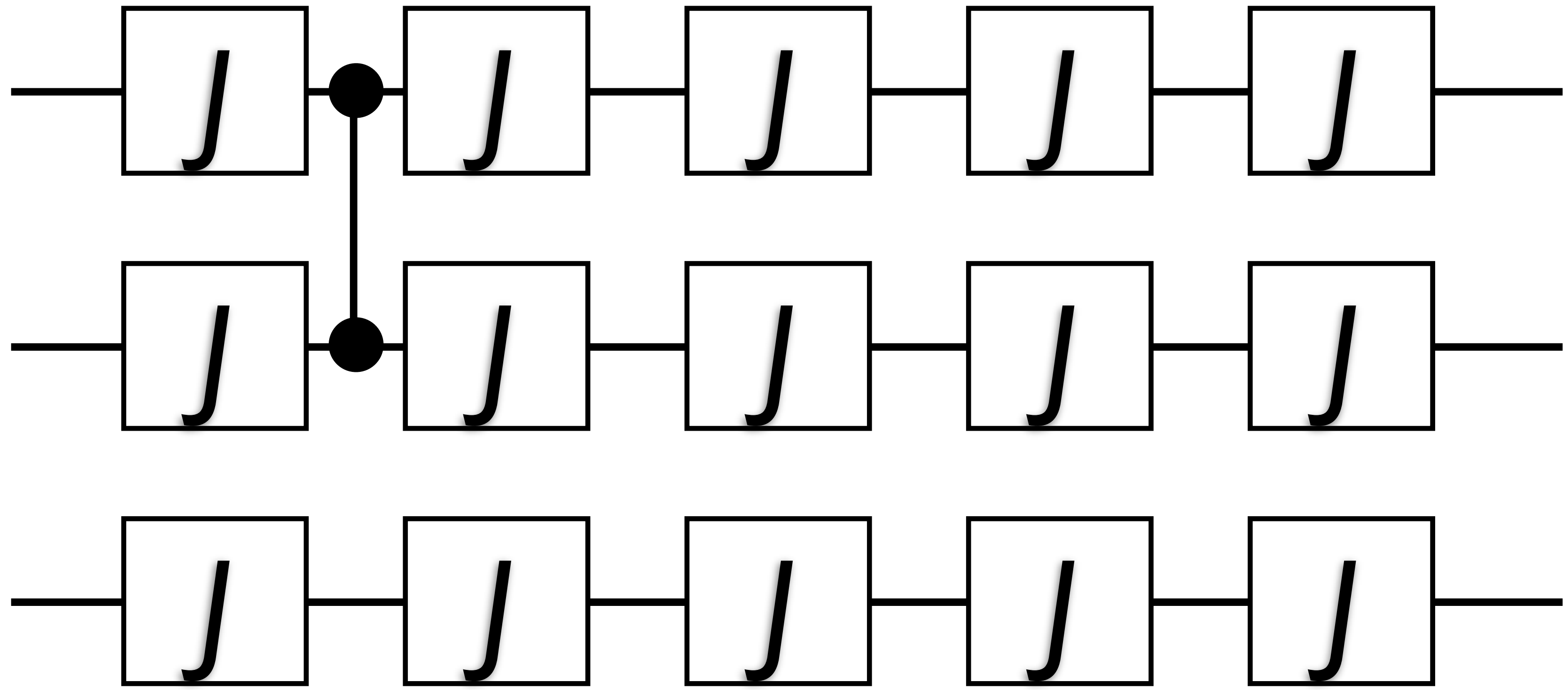
2-qubit gates and universal q.c.



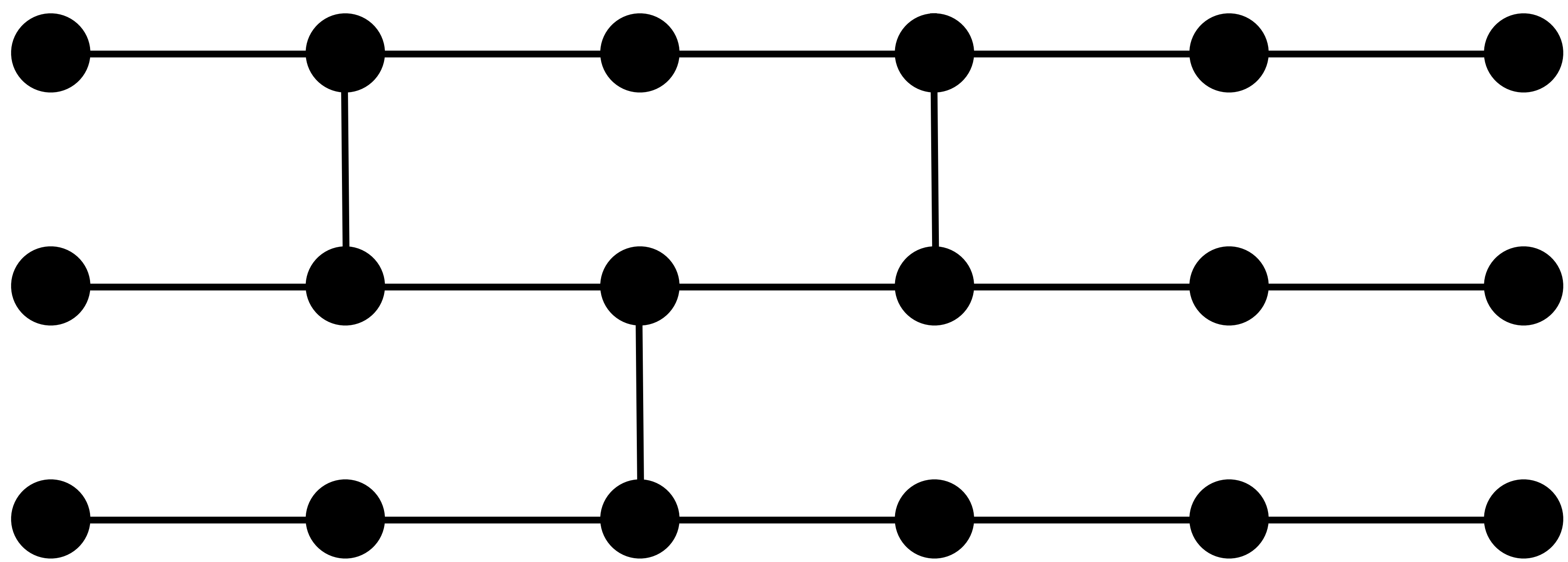


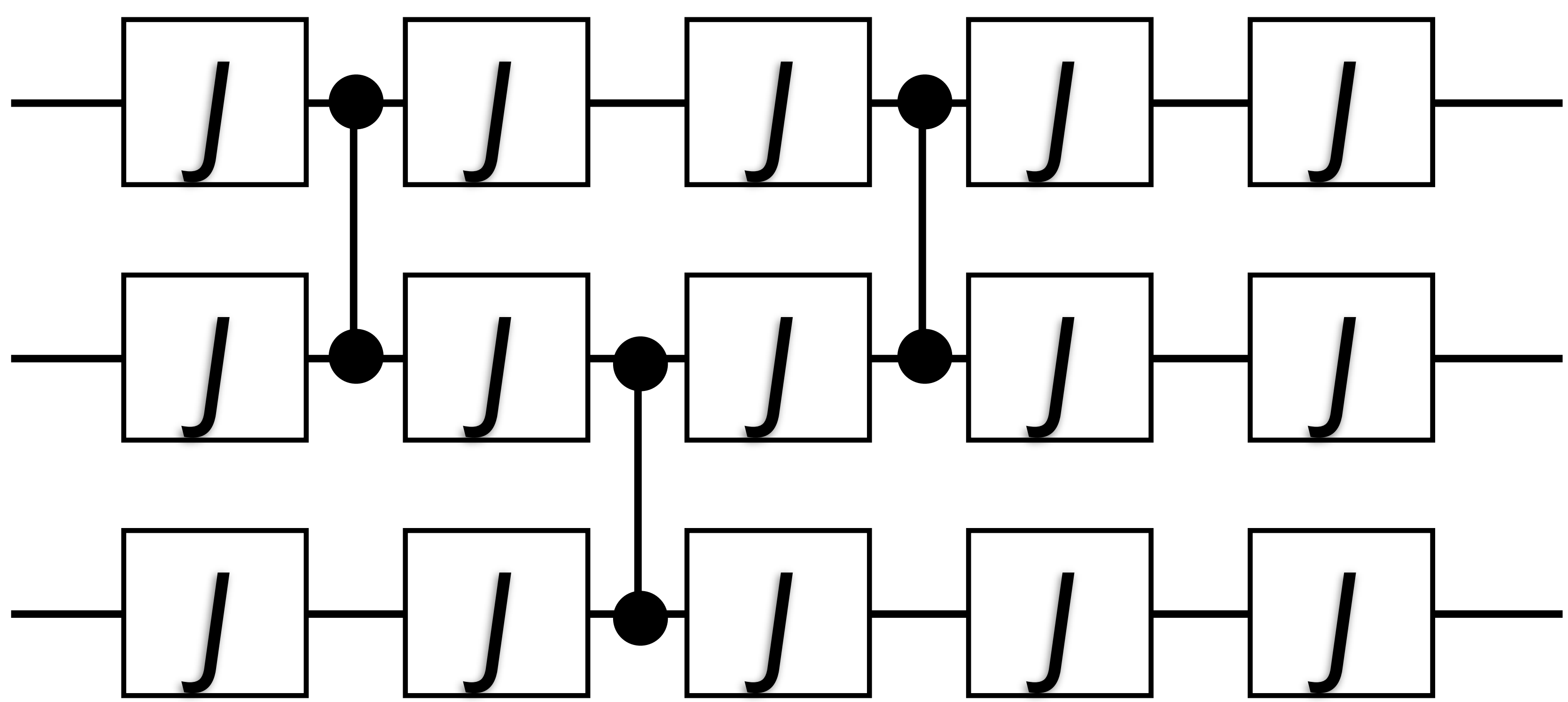
2-qubit gates and universal q.c.





2-qubit gates and universal q.c.





Etching with z measurements

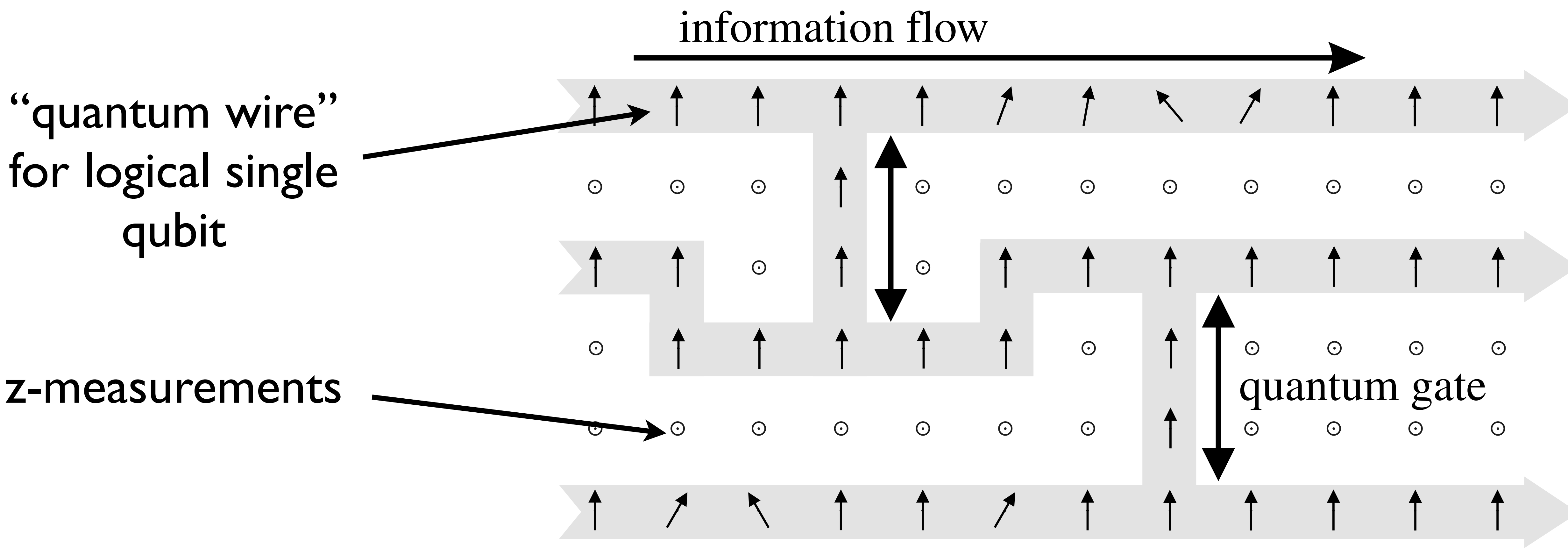
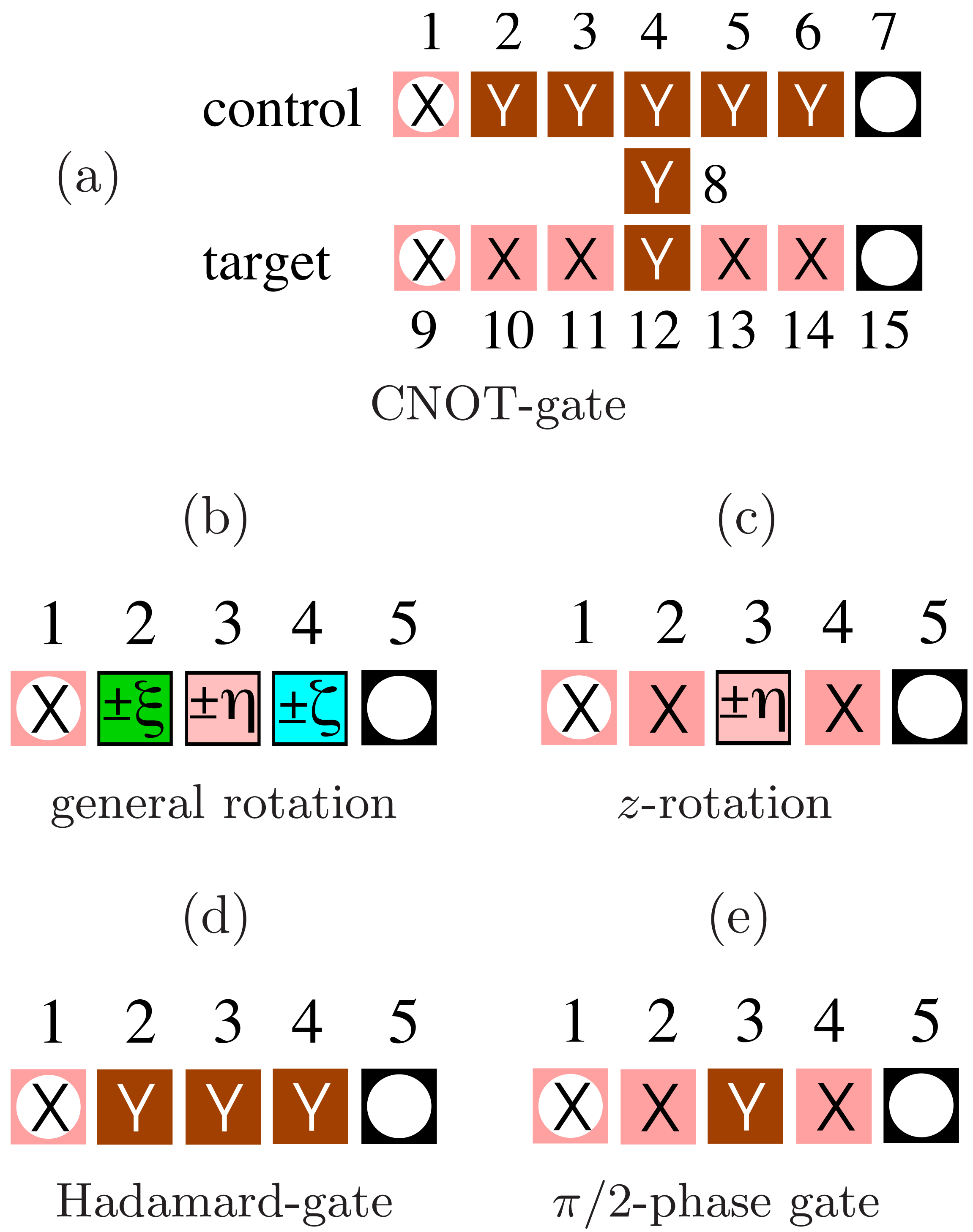


FIG. 1. Quantum computation by measuring two-state particles on a lattice. Before the measurements the qubits are in the cluster state $|\Phi\rangle_C$ of (1). Circles $\odot$ symbolize measurements of σ_z, vertical arrows are measurements of σ_x, while tilted arrows refer to measurements in the x-y plane.

- Figure 1in **H. J. Briegel** and **R. Raussendorf**, Phys. Rev. Lett. 86, 5188 (2001)

Measurement-patterns for gates in the one-way model



R. Raussendorf, D. E. Browne and H.J. Briegel, Phys. Rev. A 68: 022312 (2003)

LC-orbit of local Clifford equivalent states

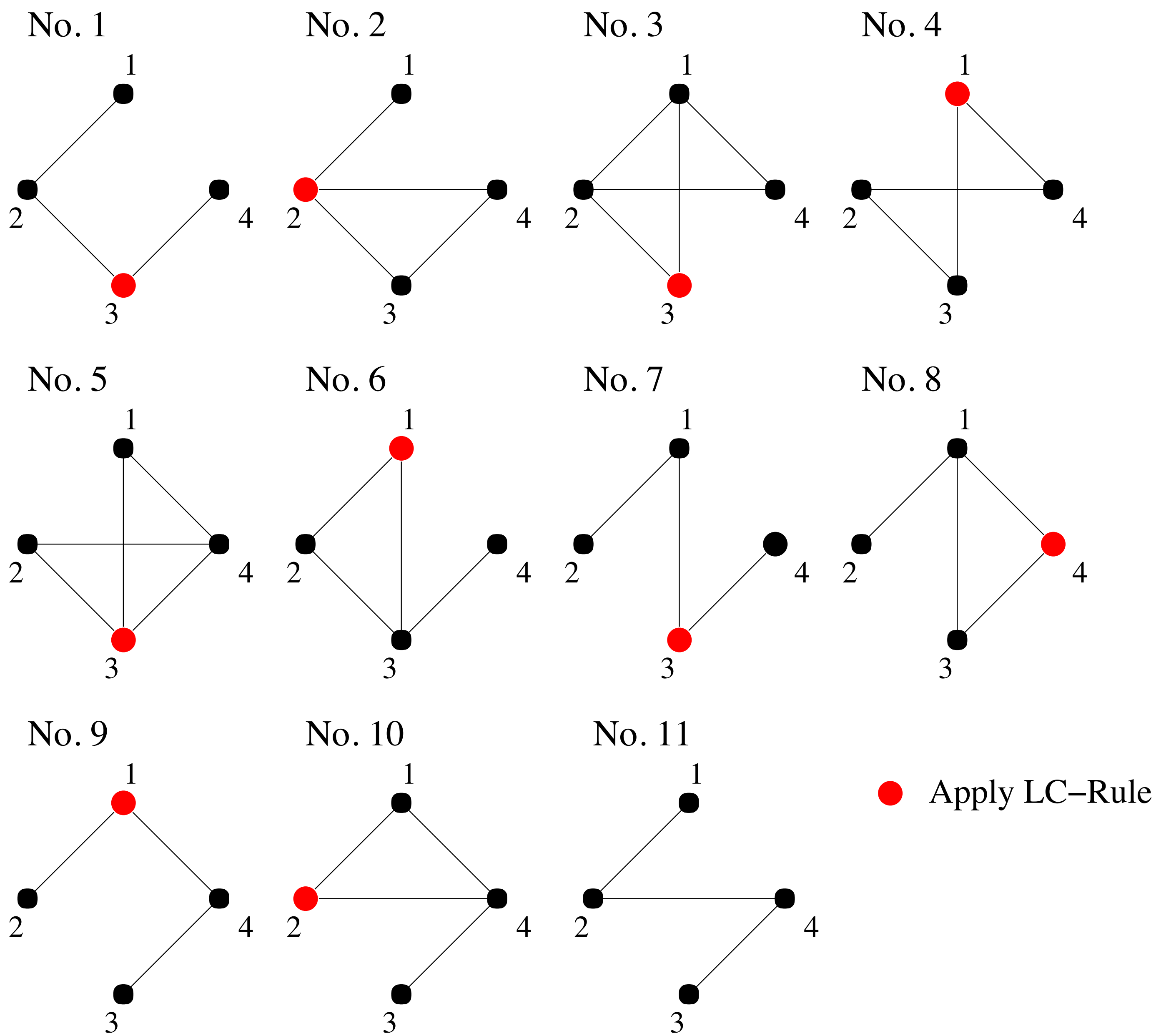
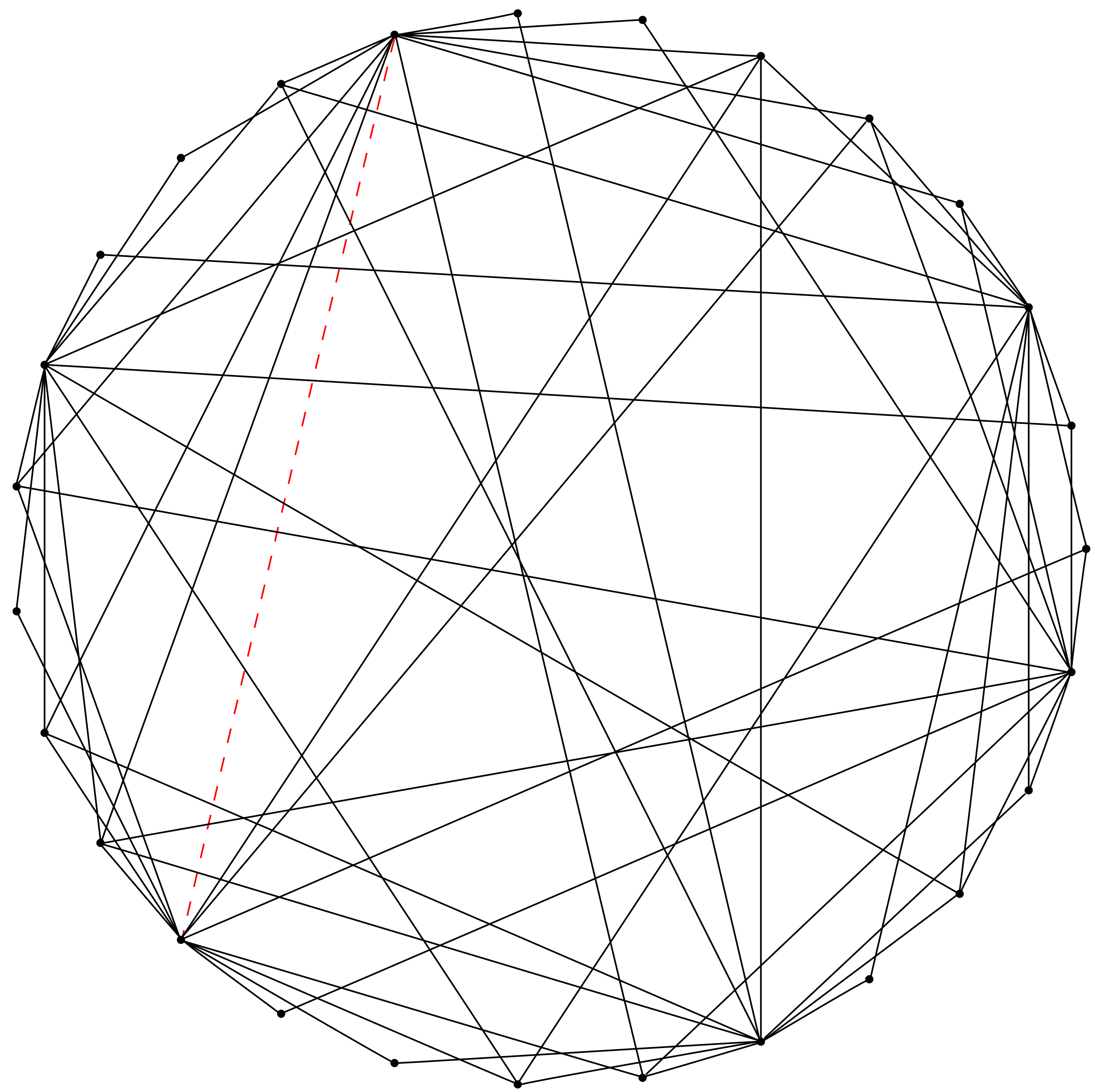


Fig. 4. – An example for a successive application of the LC-rule, which exhibits the whole equivalence class associated with graph No. 1. The rule is successively applied to the vertex, which is colored red in the figure.

- Figure 4 in **M. Hein, W. Dür, J. Eisert, R. Raussendorf, M. Van den Nest, H.-J. Briegel,** quant-ph/0602096 (Varena lectures)

Counter-example to LU-LC conjecture



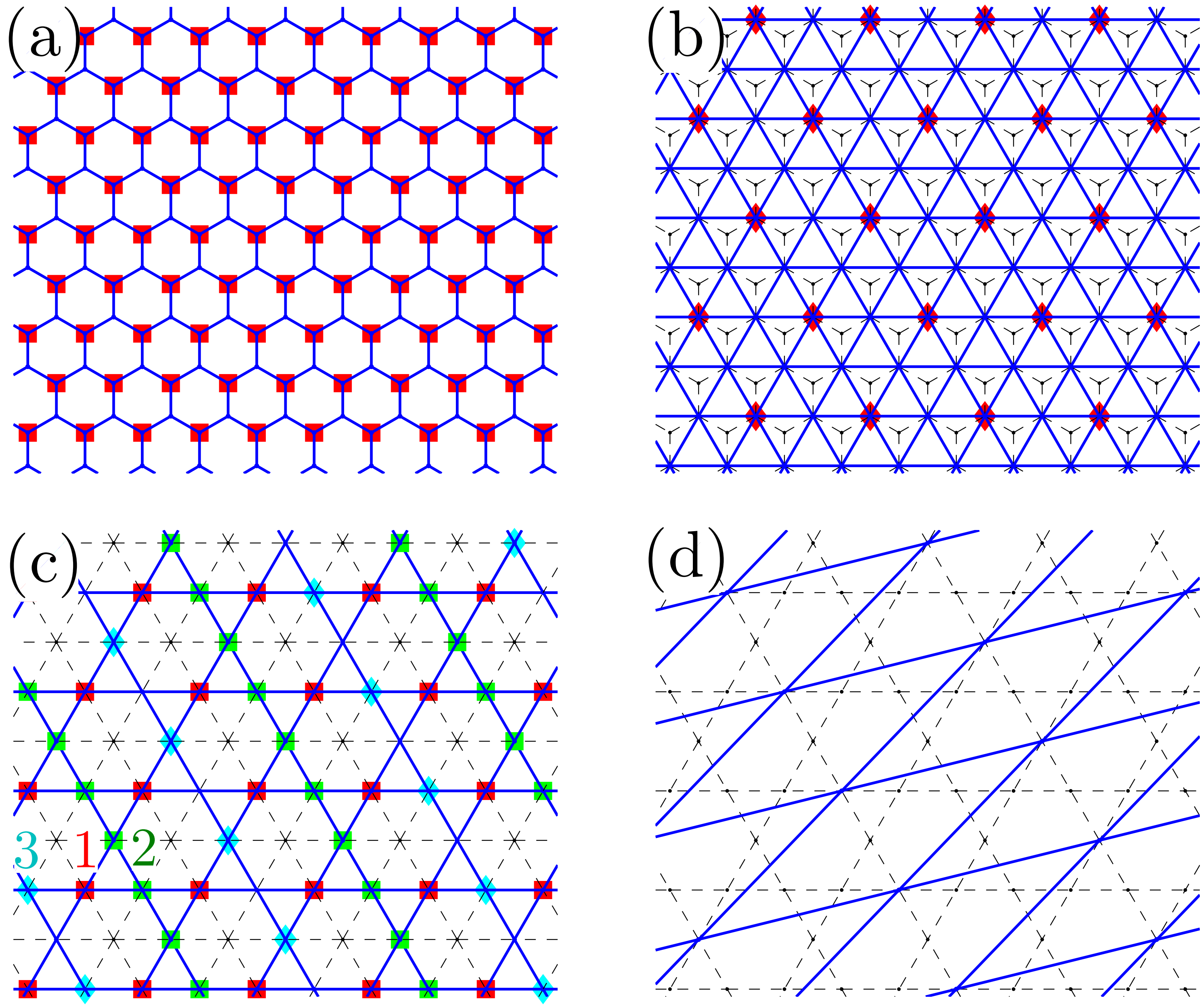
The two graphs (with / without the red edge) represent locally equivalent states, but are **not** related by the LC rule.

Zhengfeng Ji et al, Quantum Inf. Comput., Vol. 10, No. 1&2, 97-108, 2010

Universal resources derived via graphical rules

Hexagonal

Triangular



Kagome

Square

These regular graphs can all be mapped into square lattices via Pauli measurements - using the graphical rules. Hence **all** are resources for universal MBQC.

M. Van den Nest, et al, New J. Phys. 9 204 (2007).

Universal resources derived via graphical rules

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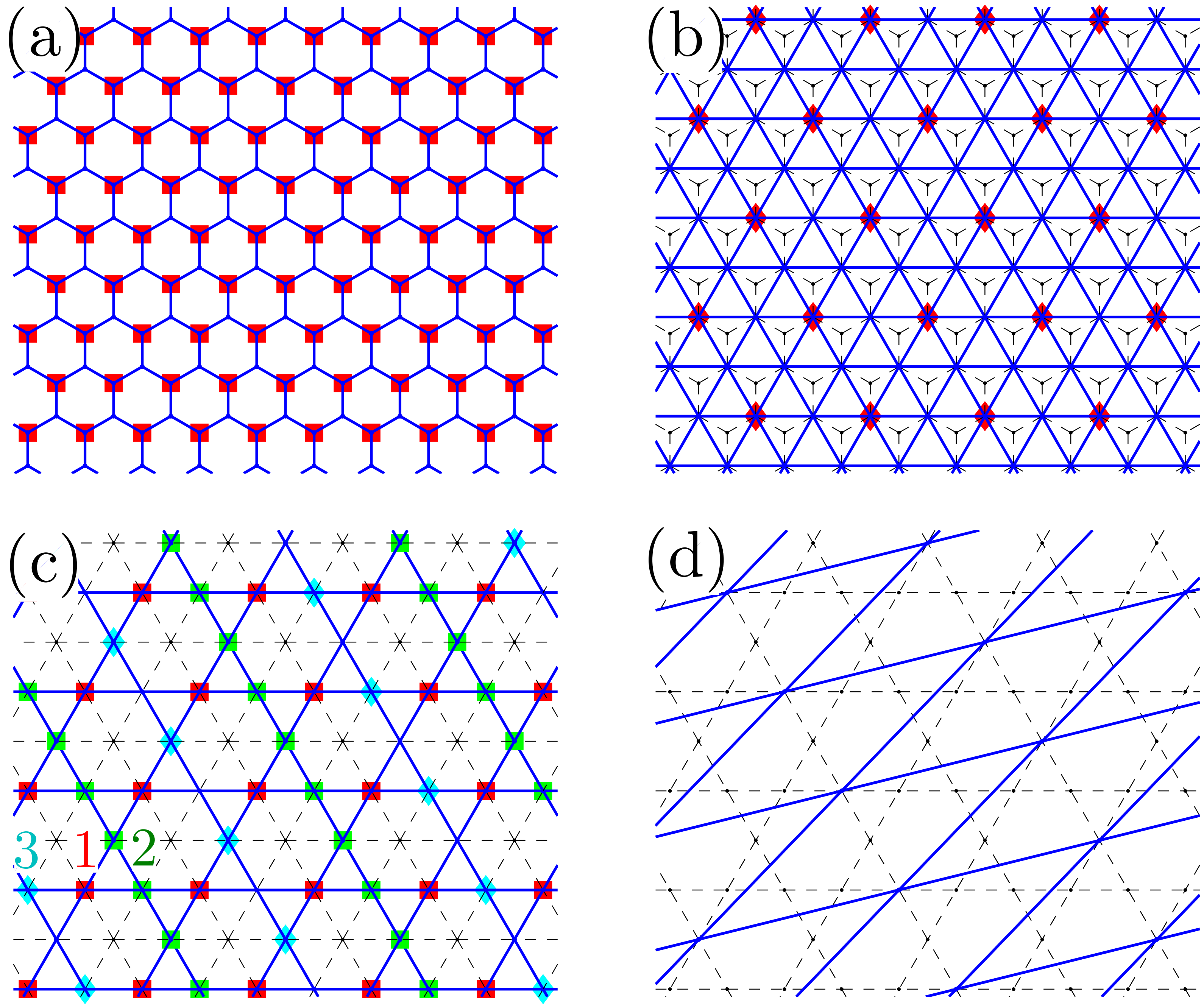

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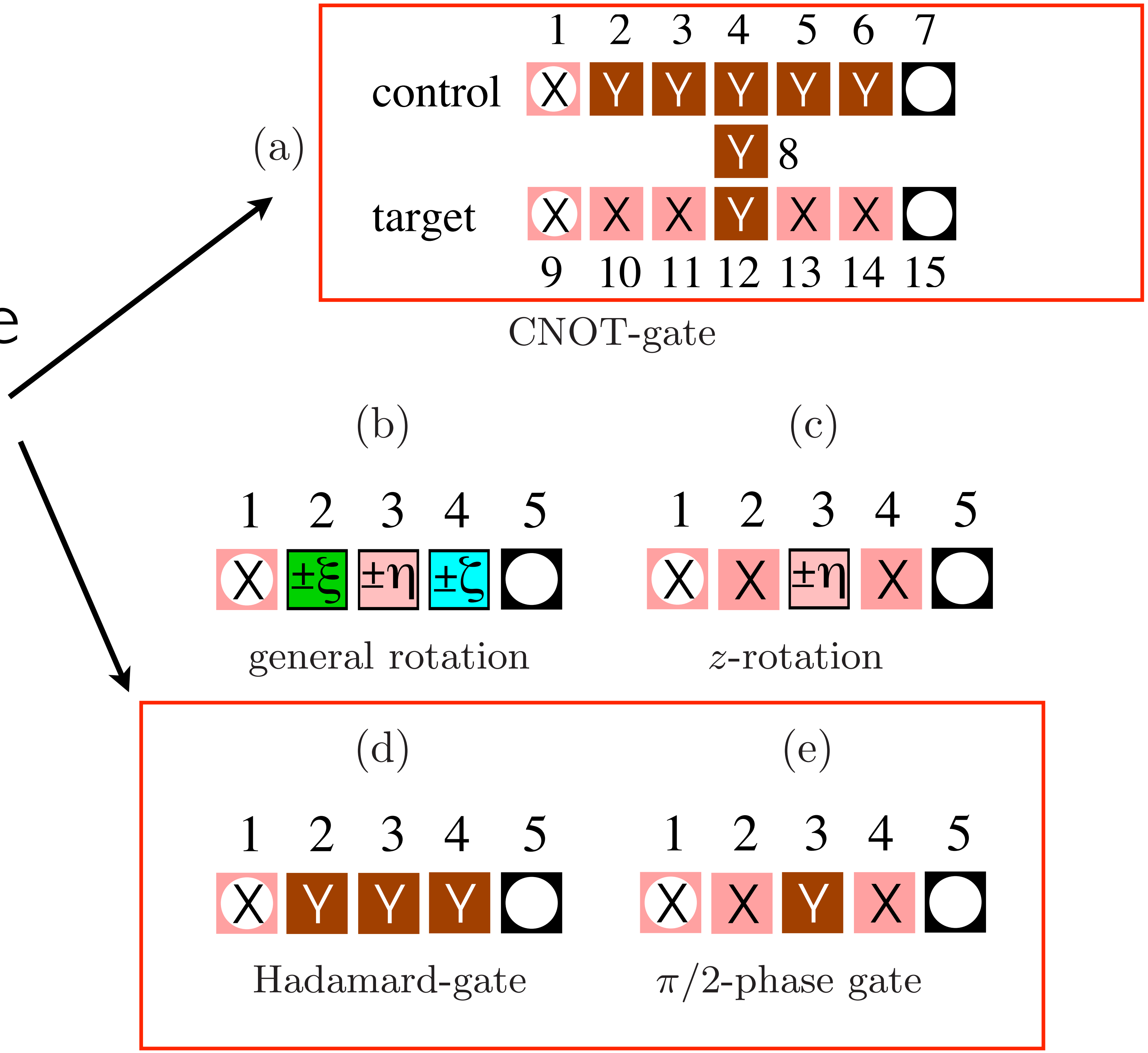
Measurement-patterns for gates in the one-way model

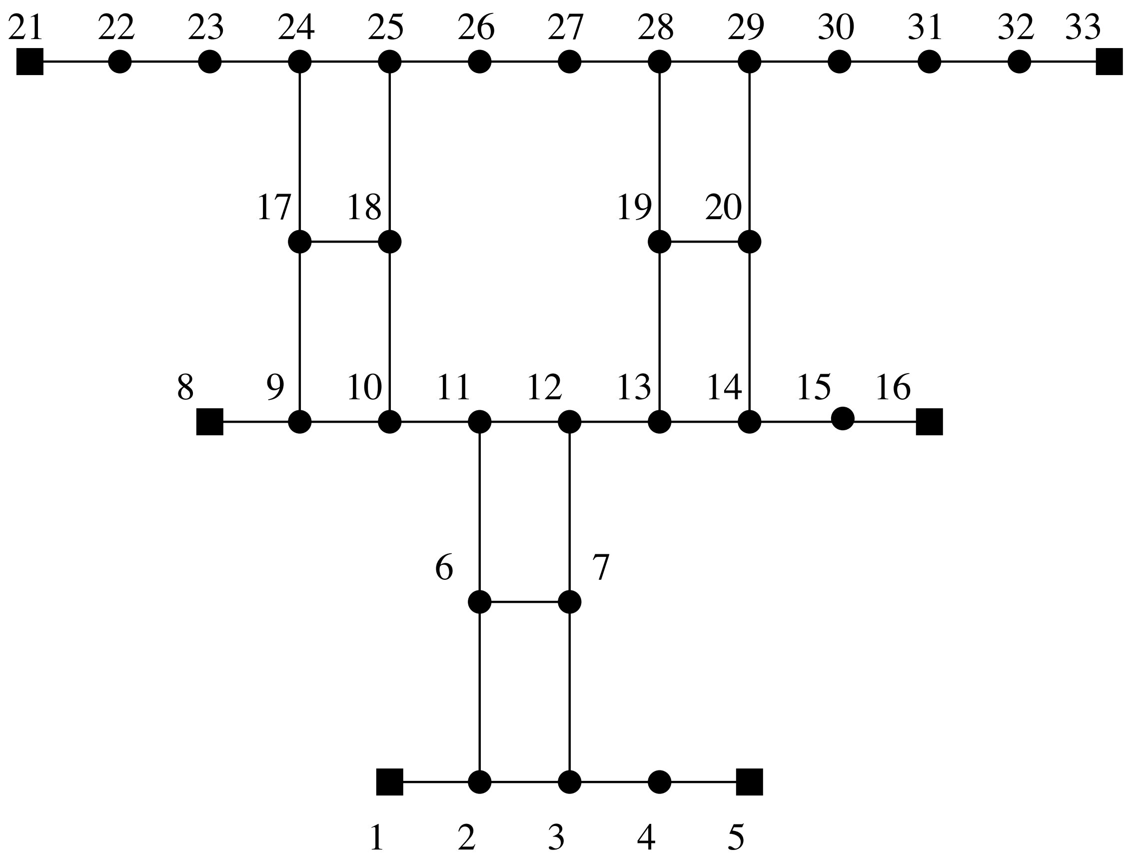
Generate the **full Clifford group**



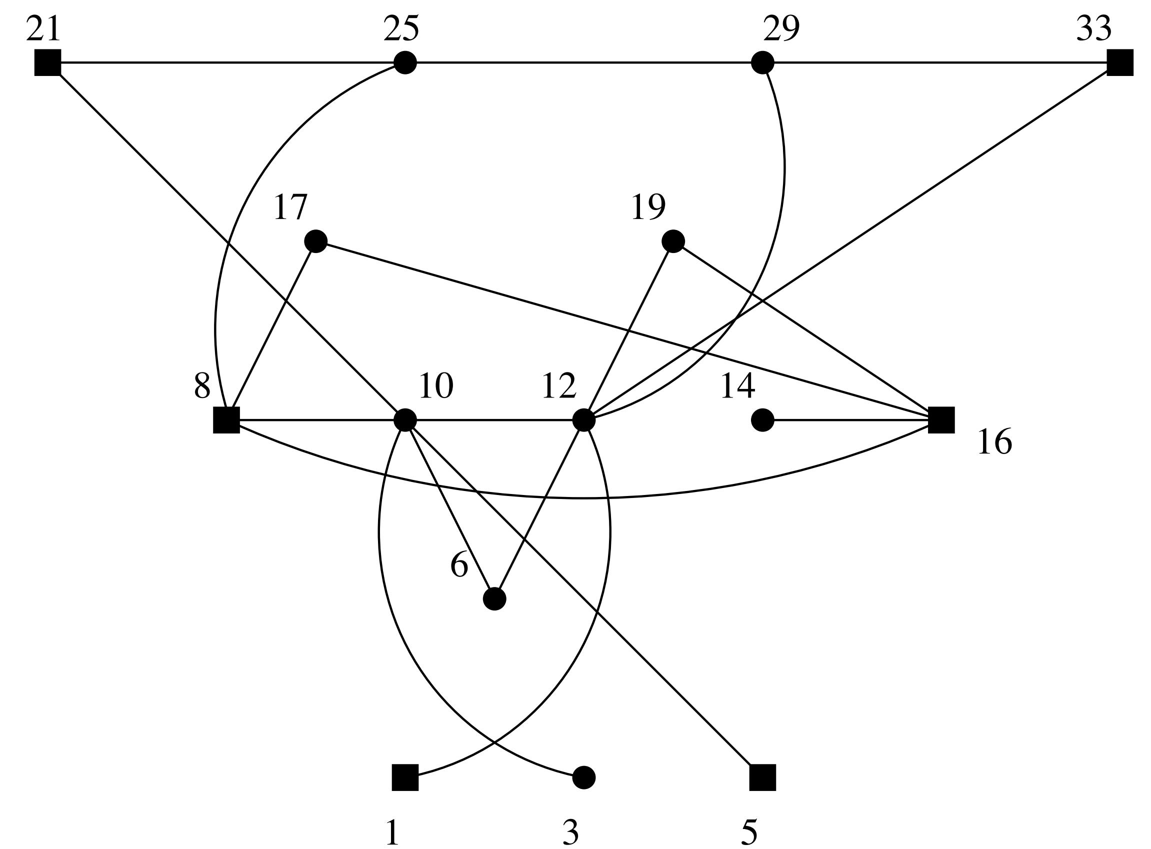
R. Raussendorf, D. E. Browne and H.J. Briegel, Phys. Rev. A 68: 022312 (2003)

Graphical "Gottesman-Knill Theorem"

Quantum Fourier Transform



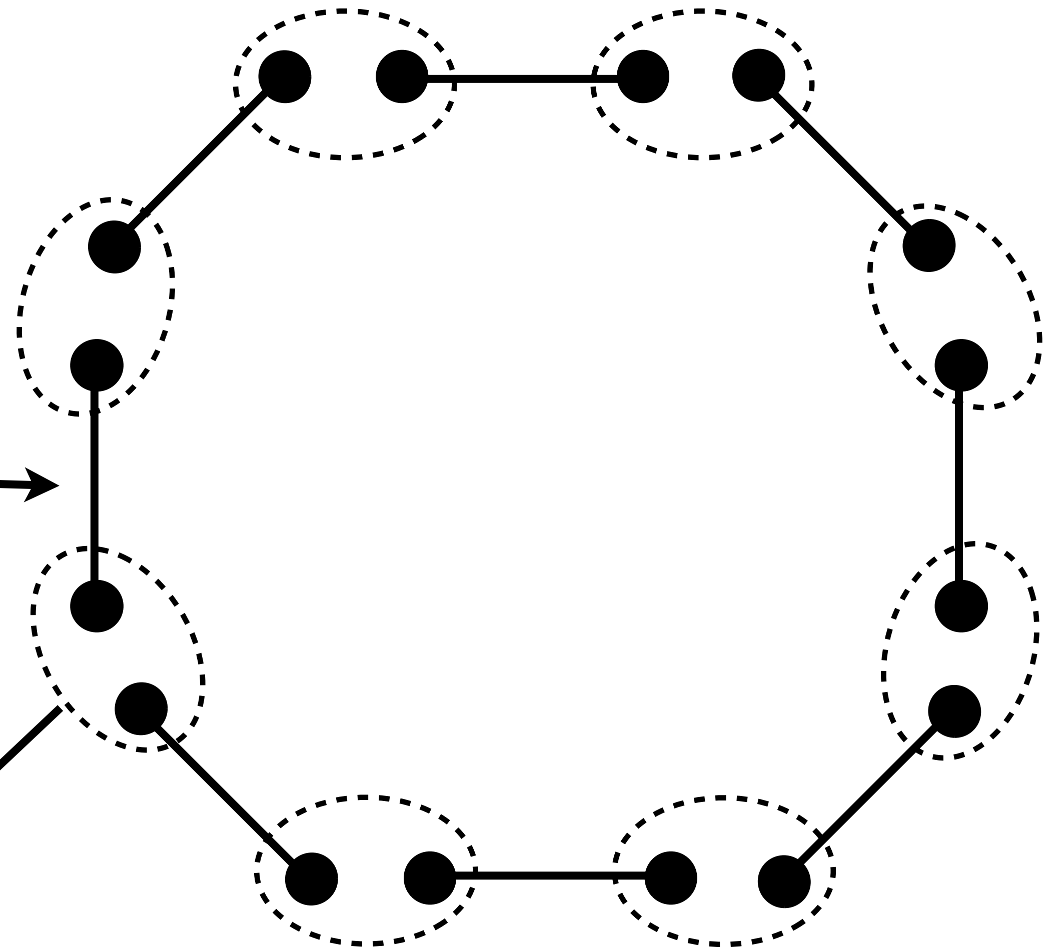
Standard-form
measurement pattern



Graph state **after all Pauli** **measurements performed**

M. Hein, J. Eisert and H.J. Briegel, Phys. Rev. A 69, 062311 (2004)

Valence bond PEPS to MPS



"Virtual Qudit pairs"

$$\sum_{n=1}^{d} |n\rangle|n\rangle$$

PEPS "projector"

$$\sum_{j=1}^{D} \sum_{p,q=1}^{d} A^j_{p,q} |j\rangle\langle p|\langle q|$$

$$M(j)_{m,n} = A^j_{p,q}$$

Constructs a **Matrix Product State MPS**

$$|\psi\rangle = \sum_{s_1,\ldots s_n} \mathrm{Tr}[M(s_1)M(s_2)\ldots M(s_n)]\ |s_1\rangle|s_2\rangle\cdots|s_n\rangle$$

References

- Progress Review
  - **H. J. Briegel, D. E. Browne, W. Dür, R. Raussendorf, M. Van den Nest,** Nature Physics 5 1, 19-26 (2009)
- Tutorials
  - **M. Hein, W. Dür, J. Eisert, R. Raussendorf, M. Van den Nest, H.-J. Briegel,** quant-ph/0602096 (Varena lectures on graph states)
  - **D. E. Browne** and **H. J. Briegel**, quant-ph/0603226
  - **M.A. Nielsen**, quant-ph/0504097
- And many more...
  - Search arxiv for **One-way, MBQC, Cluster States, Graph States, ...**