Graph Theory in Quantum Information

Lecture 4

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Claim Let $X$ be a Latin square graph and let $Y$ be an induced subgraph of $X$. Then the char. poly of $X \setminus Y$ is determined by the char. poly of $Y$ and its complement.
Corollary: \(\phi(X \setminus \{i,j\})\) only depends on whether \(i\) and \(j\) are adjacent.
Symmetric powers

Input $X \times X \{k\}$ $wr$-subsets of size $k$ from $V(X)$

Edge $- \alpha \cap \beta \iff$ symmetric difference $\alpha \Theta \beta$ of $\alpha$ and $\beta$ is an edge of $X$. 
Symmetric powers

input \( X \) \( \{ e \}_{e \in \mathbb{Z}_2^k} \) \( \binom{X}{k} \)

\( \text{ws-subsets of size } k \text{ from } V(X) \)

edge - \( \alpha \cap \beta \leftrightarrow \text{symmetric difference} \)

\( \alpha \ominus \beta \) of \( \alpha \) and \( \beta \) is an edge of \( X \).
Does the spectrum of the k-th symmetric power determine the graph?
Does the spectrum of the k-th symmetric power determine the graph?
Does the spectrum of the $k$-th symmetric power determine the graph?

Discrete walker. An arc in a graph is an ordered pair of adjacent vertices $(1,2)$, $(2,1)$.
Line digraph of $X$

$V_X$ - arcs of $X$

"edges" $((i,j), (j,k), i,j,k \in X)$
Suppose $X$ is regular with degree $k$. Let $U = \frac{2}{k} A(x) - P$ orthogonal.
$S^+(M)$

$S^+(U)$ $S^+(U^2)$ $S^+(U^3)$

all strongly regular graphs on up to 40 vertices are determined by the spectrum of $S^+(U^3)$
Cai-Fuhrer-Immerman

Input \( X \) - min valency \( \geq 3 \)

\[ VA = \{ (v, x): v \in V(x) \} \]

\( \gamma \) is a subset of the neighbors of \( v \) with even size

\[ VB = \{ (v, w, i): v \sim w \in V(x), i = 0, 1 \} \]
\((n, \alpha) \sim (n, w, r)\)

\[u = \nu\]

\[
\begin{cases}
i = D, & w \in \mathcal{X} \\
i = 1, & w \notin \mathcal{X}
\end{cases}
\]

\text{k-regular on } \nu \text{ vars}

\[2^{k-1} \nu + 4e\]