Classical Simulation of Quantum Systems

Lectures 3 & 4

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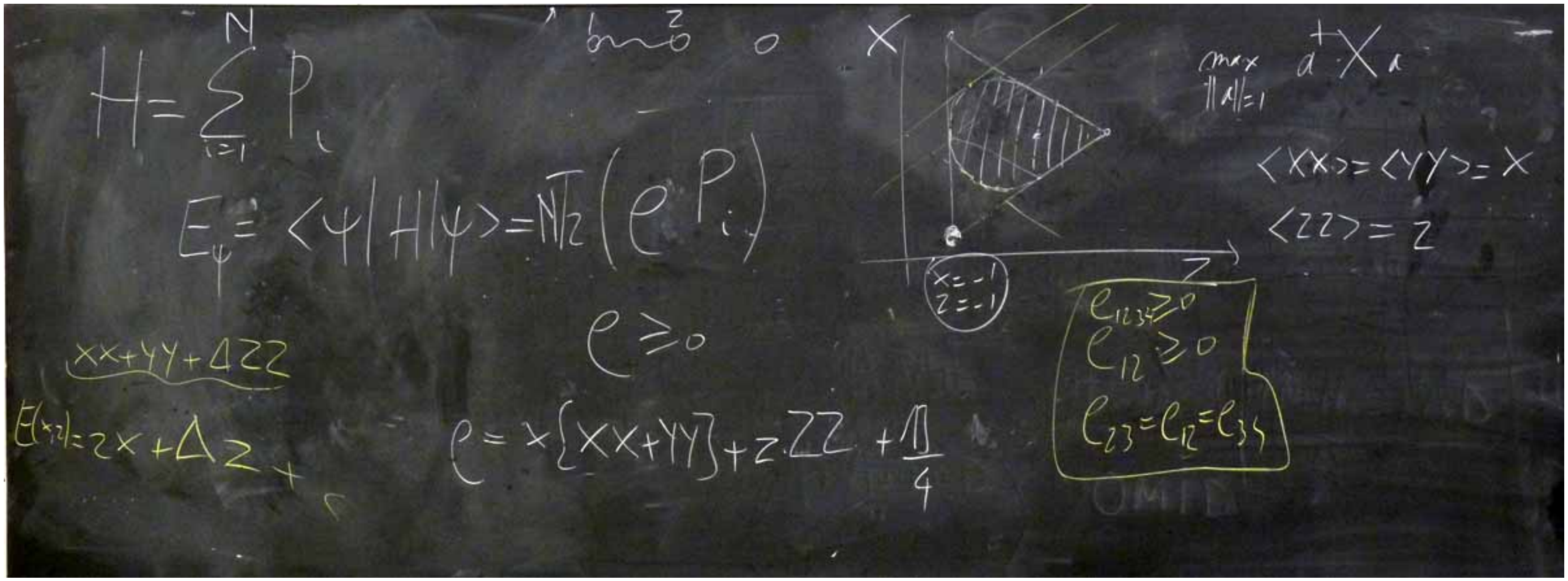
$$\mathcal{H}_{\text{HUBBARD}} = t\sum_{i} a_i^\dagger a_j + \underline{U\sum_i n_i(n_i - 1)}$$

$$H_{\text{Heis}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

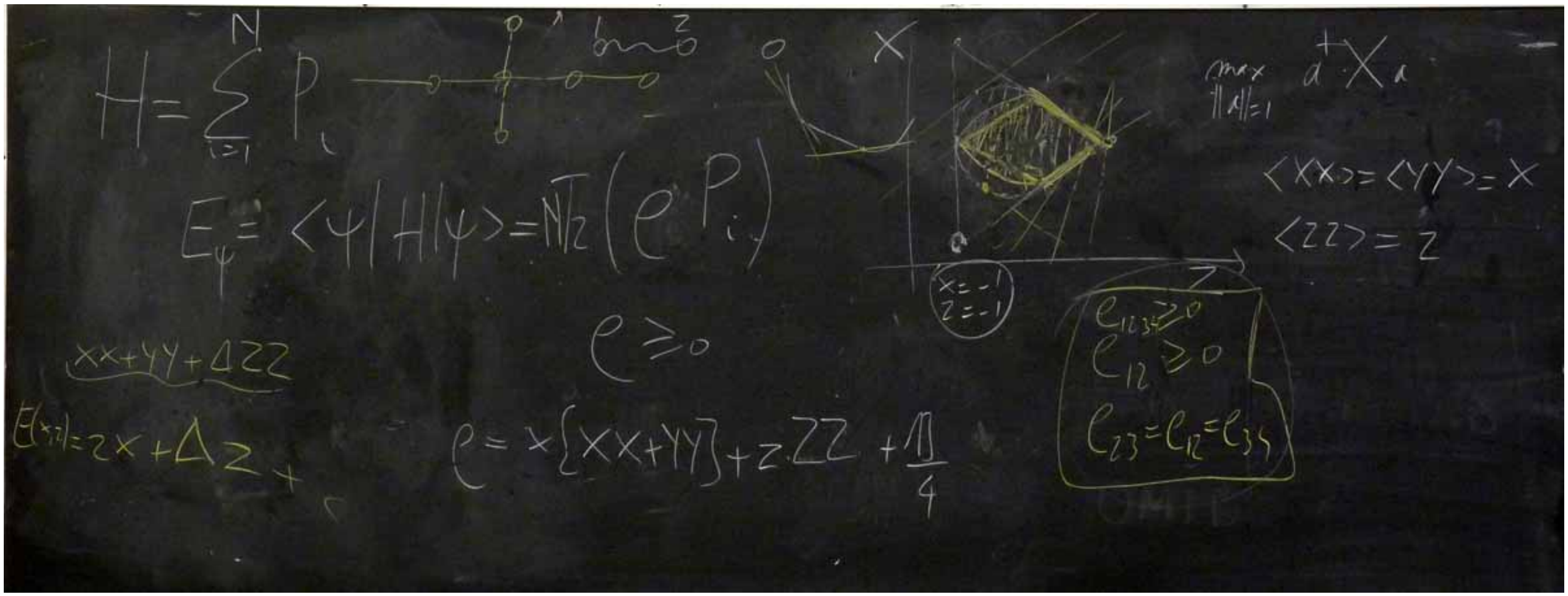
$$\underline{H = \sum P_i}$$

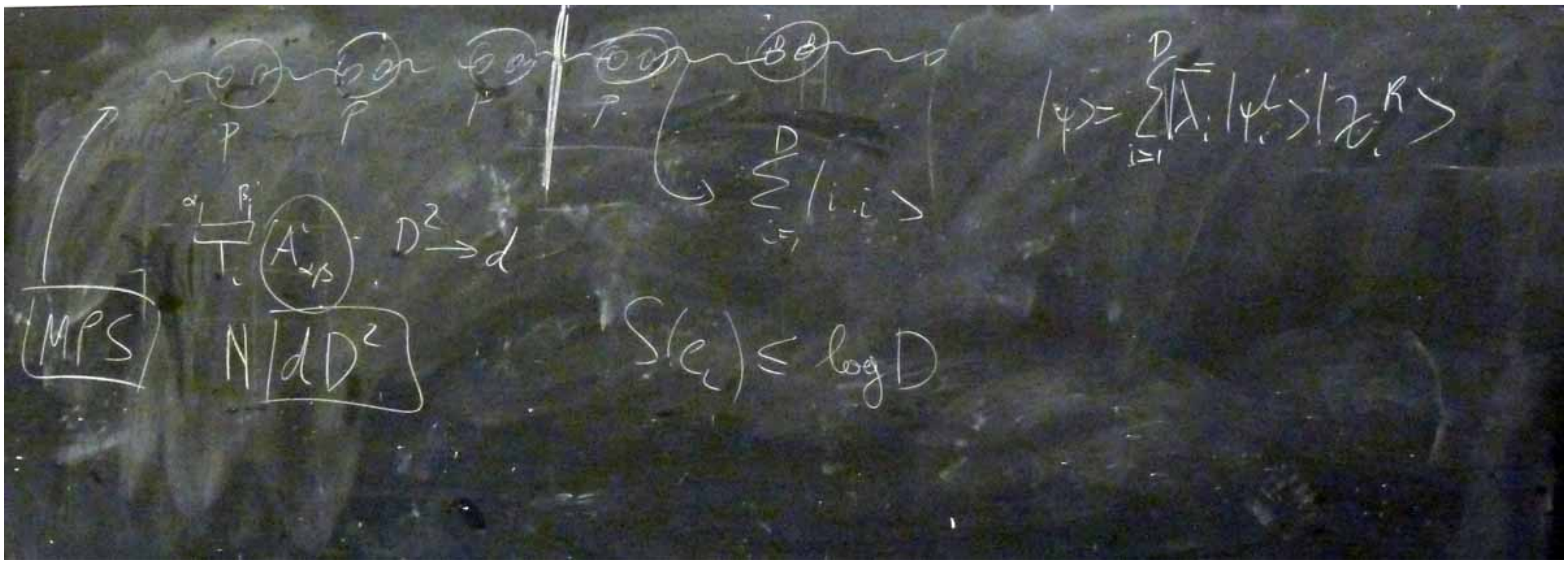
$$\to \sum_i \vec{\sigma}_i \cdot \vec{\sigma}_{i+1}$$

$$X_iX_{i+1} + Y_iY_{i+1} + Z_iZ_{i+1} \begin{cases} 1 & (3\times) \\ -3 & (1\times) \to \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right) \end{cases}$$



$H = \sum_{i=1}^{N} P_i$
$E_\psi = \langle\psi|H|\psi\rangle = N/2 \, (\rho P_i)$
$\rho \geq 0$
$XX + YY + \Delta ZZ$
$E(x,z) = 2x + \Delta z + c$
$\rho = x\{XX+YY\} + z\, ZZ + \frac{1}{4}$
X
Z
$x = -1$
$z = -1$
$\max_{\|a\|=1} a^\dagger X a$
$\langle XX\rangle = \langle YY\rangle = x$
$\langle ZZ\rangle = z$
$\rho_{1234} \geq 0$
$\rho_{12} \geq 0$
$\rho_{23} = \rho_{12} = \rho_{34}$


H = Σ_{i=1}^{N} P_i
E_ψ = ⟨ψ|H|ψ⟩ = N/2 (ρ P_i)
ρ ≥ 0
XX + YY + ΔZZ
E(x,z) = 2x + Δz +
ρ = x{XX+YY} + z ZZ + 𝟙/4
X
Z
x = -1
z = -1
ρ_1234 ≥ 0
ρ_12 ≥ 0
ρ_23 = ρ_12 = ρ_34
max_{‖α‖=1} α† X α
⟨XX⟩ = ⟨YY⟩ = x
⟨ZZ⟩ = z


MPS
$N\, dD^2$
$A^i_{\alpha\beta}$
$D^2 \to d$
$\sum_{i=1}^{D} |i,i\rangle$
$S(\rho_L) \leq \log D$
$|\psi\rangle = \sum_{i=1}^{D} \sqrt{\lambda_i}\, |\psi_i^L\rangle |\chi_i^R\rangle$

- gapped Hamiltonians $\longrightarrow$ Hastings: $S_\alpha(L) \sim$ cst

- gapless systems $\longrightarrow$ critical $\boxed{\frac{c}{3}\ln L}$

$\boxed{\text{Renyi}}$

$S_\alpha(\rho) = \frac{1}{1-\alpha}\log \mathrm{Tr}(\rho^\alpha) \xrightarrow{\alpha=1} -\mathrm{Tr}(\rho \log \rho)$

$S_\alpha(L) \leq c_1 \ln L \Rightarrow \exists\, |\psi_D\rangle$:

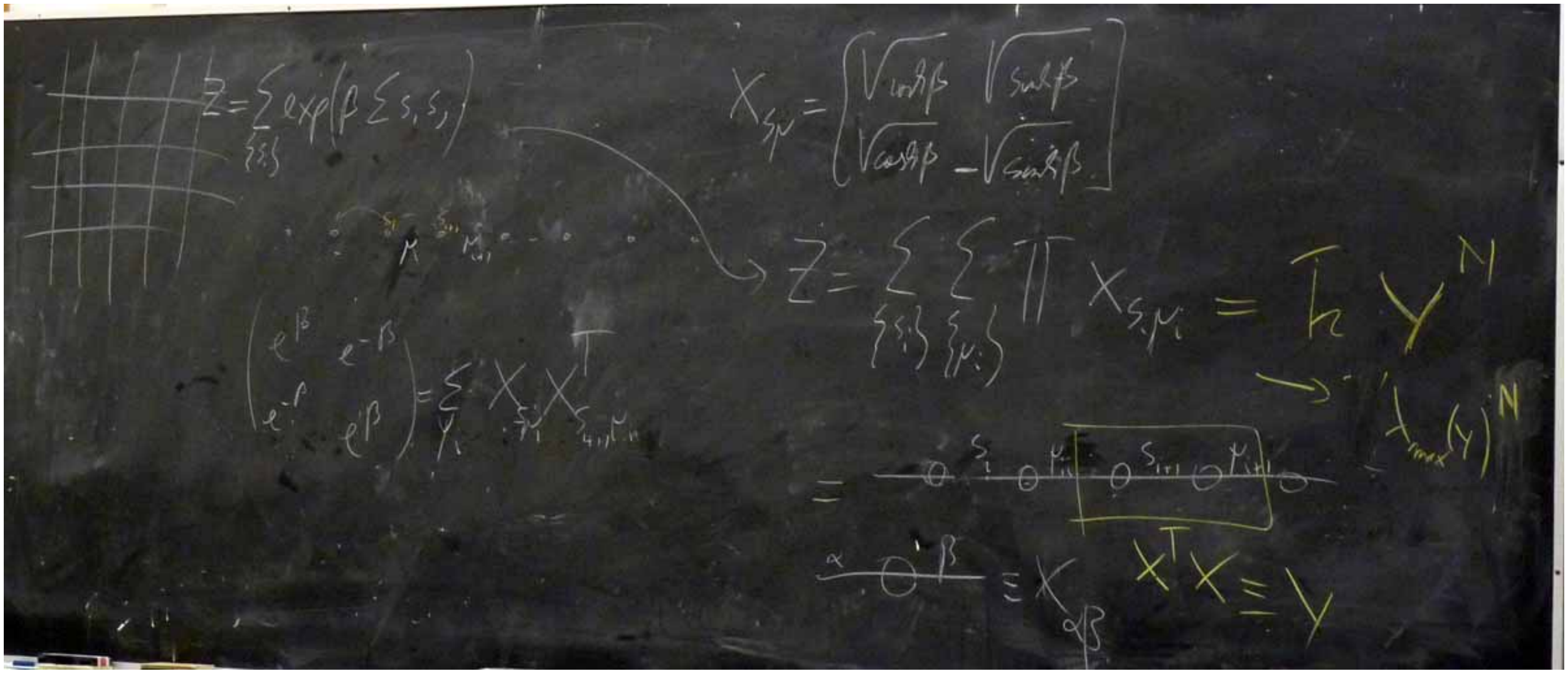
$\| |\psi_D\rangle - |\psi_{ex}\rangle \| \leq \varepsilon$; $D \leq \frac{c_2}{\varepsilon} N^{\frac{\alpha}{1-\alpha}}$

$$|\langle O_1 O_2\rangle - \langle O_1\rangle\langle O_2\rangle| \sim \exp(-L/\xi)$$

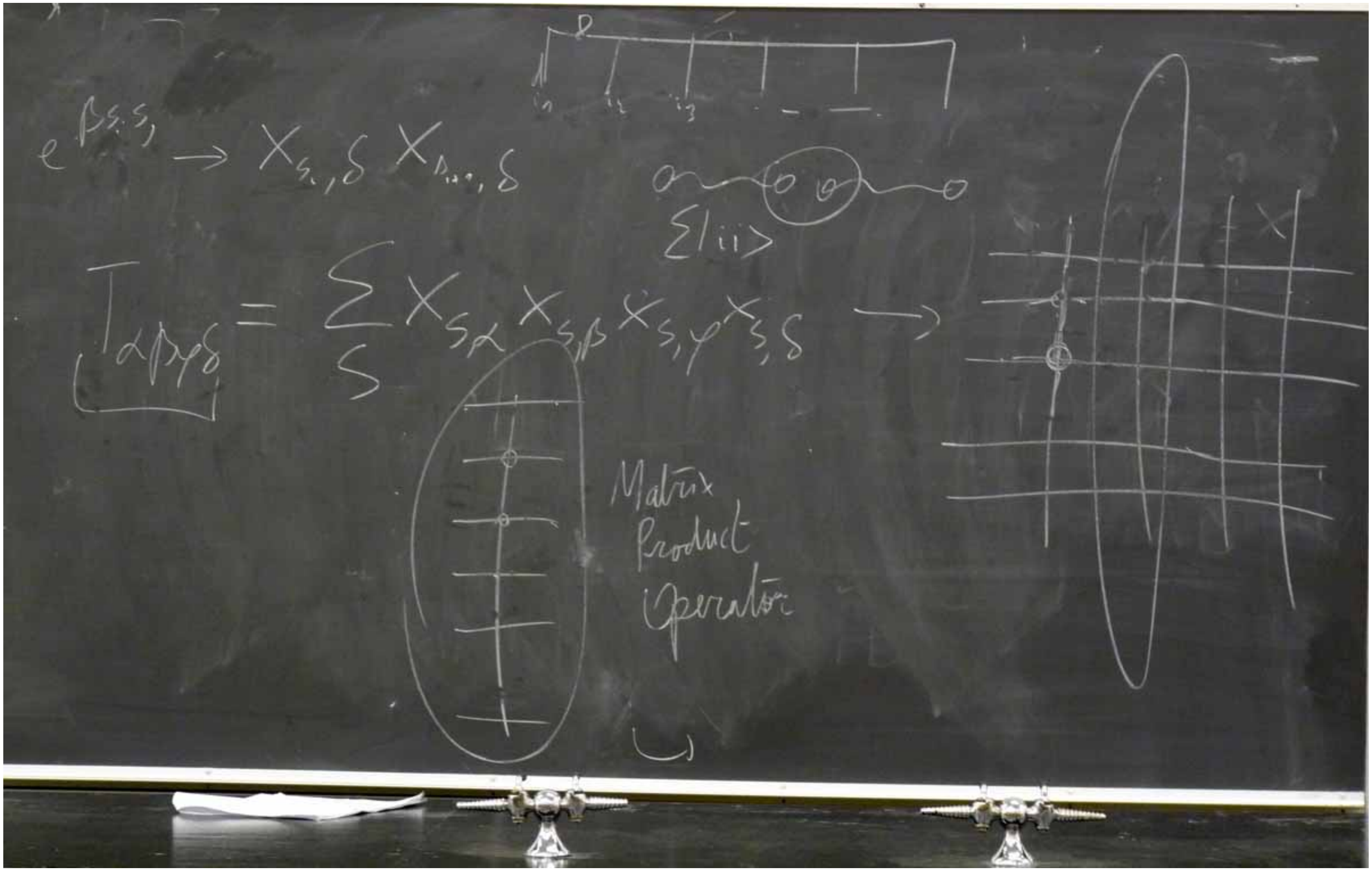
$$\rho_A = X \omega X^\dagger$$
$$= Y\, Y^\dagger$$
$$X_{\alpha\beta} = |\psi_{\alpha\beta}\rangle$$

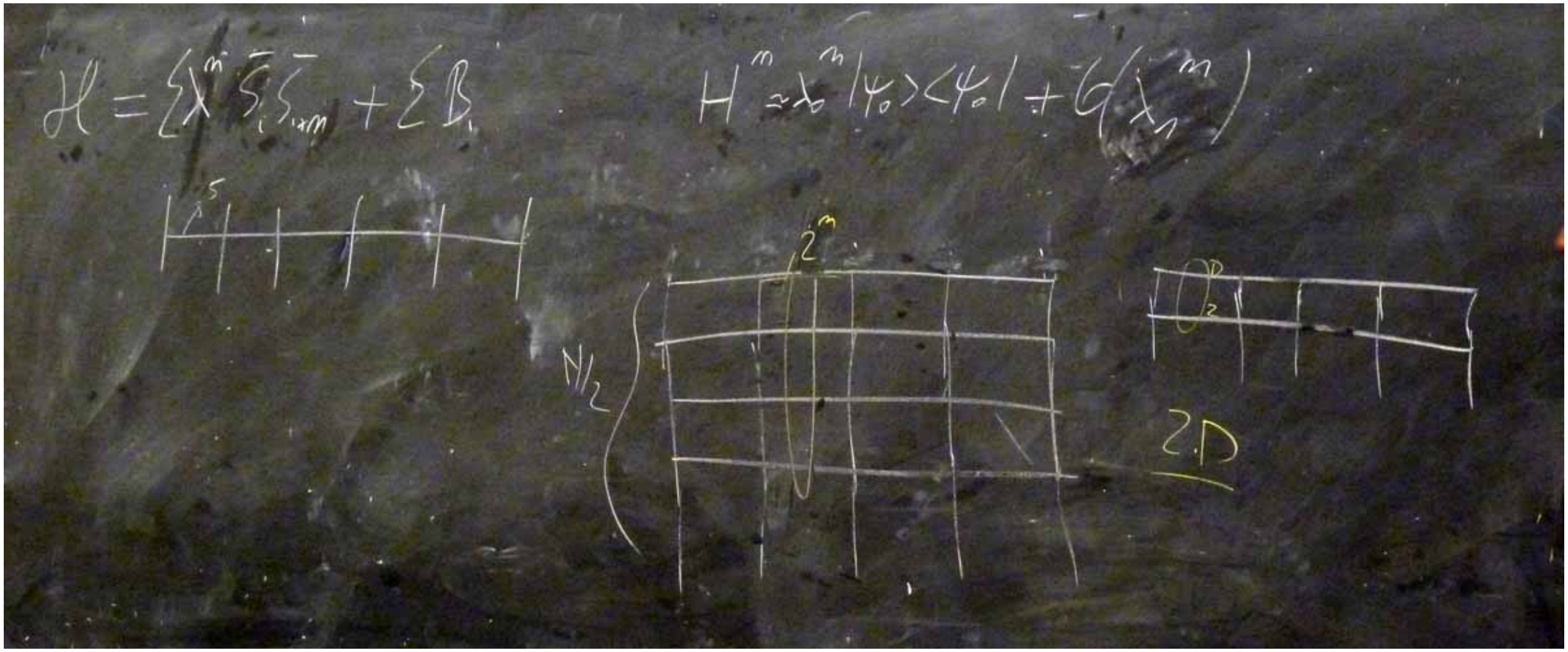
$$\rho_{AB} \approx \rho_A \otimes \rho_B \longrightarrow \underline{|\psi_{AA'}\rangle}\,\underline{|\psi_{BB'}\rangle} = U_{A'B'}\,|\psi_G\rangle$$

$$\rho_A \begin{cases} \to |\psi_{\alpha\beta}\rangle \\ \to |\chi_{AA'}\rangle \hookrightarrow Y \end{cases}$$



$Z = \sum_{\{s_i\}} \exp\left(\beta \sum s_i s_j\right)$
$X_{s\mu} = \begin{bmatrix} \sqrt{\cosh\beta} & \sqrt{\sinh\beta} \\ \sqrt{\cosh\beta} & -\sqrt{\sinh\beta} \end{bmatrix}$
$s_i \quad s_{i+1} \quad \mu_i \quad \mu_{i+1}$
$Z = \sum_{\{s_i\}} \sum_{\{\mu_i\}} \prod X_{s_i \mu_i} = \mathrm{Tr}\, Y^N$
$\rightarrow \lambda_{max}(Y)^N$
$\begin{pmatrix} e^{\beta} & e^{-\beta} \\ e^{-\beta} & e^{\beta} \end{pmatrix} = \sum_{\mu_i} X_{s_i \mu_i} X^T_{s_{i+1} \mu_i}$
$= \; s_i \quad \mu_{i} \quad s_{i+1} \quad \mu_{i+1}$
$\alpha \; \beta \equiv X_{\alpha\beta}$
$X^T X \equiv Y$


$e^{\beta s_i s_j} \rightarrow X_{s_i,\delta} X_{s_{i+1},\delta}$
$\sum |ii\rangle$
$T_{\alpha\beta\gamma\delta} = \sum_s X_{s,\alpha} X_{s,\beta} X_{s,\gamma} X_{s,\delta}$
Matrix
Product
Operator


$\mathcal{H} = \sum \lambda^m \vec{S}_i \vec{S}_{i+m} + \sum B_i$
$H^m \approx \lambda_0^m |\psi_0\rangle\langle\psi_0| + O(\lambda_1^m)$
2^m
$N/2$
2D