Proceedings Tailings and Mine Waste 2011
Vancouver, BC, November 6 to 9, 2011

Determination of Consolidation Properties, Selection of Computational Methods, and Estimation of Potential Error in Mine Tailings Settlement Calculations

David Geier

Golder Associates Inc., USA

Gordan Gjerapic, Ph.D., PE

Golder Associates Inc., USA

Kimberly Morrison, PE, RG

AMEC Environmental & Infrastructure, USA

Abstract

Accurate estimates of tailings density and settlement are critical for successful design, operation, and closure of tailings storage facilities (TSFs). Limitations of the computational methods employed to determine these quantities are often overlooked, leading to inaccurate results and potentially inadequate engineering designs. This paper provides a brief discussion of the methods that are commonly used to model tailings consolidation, discusses relative accuracy of these methods, and outlines common pitfalls encountered in their use. Engineering estimates of the commonly encountered errors are presented for selected methods. Applicability of various methods and specific recommendations are discussed with regard to TSF geometries, tailings properties, filling rates, and the calculation accuracy requirements. The present study compares traditional small strain and large-strain consolidation analyses, and evaluates differences between one-dimensional and three-dimensional calculation approaches. Also, a comparison between estimated (calculated) and actual tailings densities (based on mill production data and bathymetric surveys) is provided for an existing operational tailings facility.

Comparison between Small Strain and Large Strain Methods

Small Strain Method

A traditional small strain method for calculating settlements utilizes the approach that was originally developed by Terzaghi. This method often assumes that the compressibility is constant over the range of stresses used in the analysis. The amount of compression is typically determined based on the change in void ratio:

$$\Delta e = C_c log\left(\frac{\sigma_1}{\sigma_0}\right) \qquad \textbf{(1)}$$

where

Δe = void ratio

C_c = compression index of the soil

σ_1 = final vertical effective stress

σ_0 = initial vertical effective stress

C_c is the slope of the e vs. log σ curve for normally consolidated soils. A "typical" e vs. log σ curve for tailings material is shown in **Figure 1**. Generally, the slope between two arbitrary points on the e-log σ curve is not a constant value, which needs to be accounted for in the consolidation analysis. Typically, tailings are deposited at relatively high void ratios, corresponding to solid contents of 40% or less and effective stress magnitude close to zero. Over time, the tailings material will consolidate under its own weight with partial consolidation of tailings typically achieved while the height of the deposit is still increasing. Tailings heights in excess of 100 m and ultimate effective stresses at the base of the impoundment of over 1,500 kPa are not uncommon.

Typically, small strain consolidation analyses assume that the coefficient of consolidation (material parameter required to calculate the time rate of consolidation) remains constant. The coefficient of consolidation depends on the compressibility, void ratio, and permeability of a soil:

$$c_v = \frac{k}{\gamma_w m_v} = \frac{k}{\gamma_w \frac{a_v}{(1+e_0)}} \quad \textbf{(2)}$$

where

c_v = coefficient of consolidation

k = permeability

γ_w = unit weight of water

m_v = coefficient of volume compressibility

a_v = coefficient of compressibility (change in void ratio with respect to change in stress, which is assumed constant for small strains)

e_0 = initial void ratio

In reality, the coefficient of consolidation may display significant variation over the applicable stress range. Consequently, the small strain consolidation approach is likely to result in errors between the actual and predicted settlement behavior when applied to large strain problems.

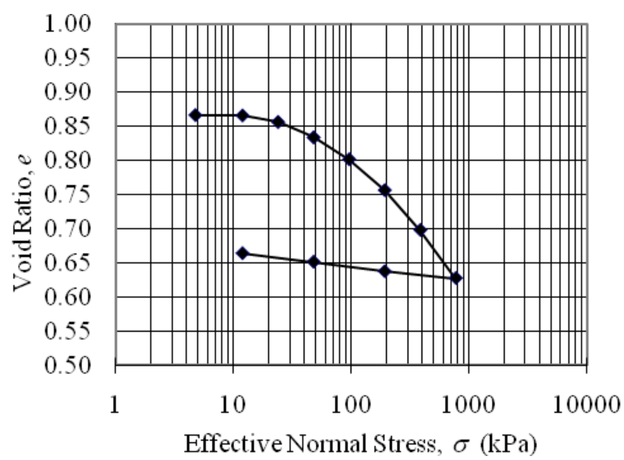

Figure 1: Example of a typical *e* vs. log *σ* curve

Large Strain Method

Both the compressibility and the permeability of tailings deposits are likely to exhibit significant changes when subjected to stress increases caused by continuous tailings deposition.

Large strain analyses presented in this study are based on the consolidation method proposed by Gibson et al. (1967). The large strain method removes the constraints imposed on the compressibility and permeability relationships by the small strain method. For large strain analyses, permeability and compressibility relationships are typically expressed as arbitrary functions of the void ratio.

The large strain equations are relatively complex, and are typically solved using computer programs such as CONDES (Yao and Znidarcic 1997) or FSCONSOL (GWP Software 1999). In these computer programs, the relationships between permeability, compressibility, void ratio, and effective stress are often expressed in closed form. For example, the relationships used in FSCONSOL are:

$$e = A\sigma'^{B} + M \qquad \textbf{(3)}$$

$$k = Ce^{D} \qquad \textbf{(4)}$$

where σ' is the vertical effective stress and A, B, M, C, and D are material parameters determined from appropriate laboratory tests.

Assigning and Interpreting Laboratory Tests

Consolidation Testing

For small strain analyses, a traditional oedometer test (ASTM D2435) is often used to determine material parameters. Oedometer tests are relatively inexpensive, and offered by most geotechnical soils laboratories. A standard oedometer test is performed by applying a vertical load at the surface of a confined sample and measuring deformation during the consolidation process. The applied load is increased (typically doubled) in every loading increment until reaching the maximum desired stress level. The oedometer test provides c_v value as a function of the applied vertical stress (although typically only one c_v value is selected for small strain analysis) and an e vs. log σ curve, from which a compression index, C_c value, can be derived.

For large strain analyses, sufficient laboratory data must be collected to fit material parameters A, B, M, C, and D in equations (3) and (4). Typically, this requires either a staged slurry consolidation test or a seepage induced consolidation test (SICTA). These tests are not standardized by ASTM, and may require shipping samples to a specialized laboratory. While providing sufficient data to model the non-linear tailings behavior, these tests also have an advantage over the standard oedometer test in determining tailings parameters at low stresses. A SICTA test is particularly well suited for determining soil parameters at effective stresses below 30 kPa.

A slurry consolidation test is typically performed on a loose/low density tailings sample. Ideally, the initial sample placed in the cell should be close to its initial in-situ (deposition) density, but at the same time prepared at a high enough density to avoid segregation. The first loading increment is typically between 15 and 30 kPa. Once the sample has consolidated, a permeability test is performed. This procedure is repeated, typically by doubling consolidation loads until reaching maximum desired vertical effective stress.

A SICTA test is conducted similarly to the slurry consolidation test for effective stresses larger than approximately 30 kPa. Consolidation parameters at lower effective stresses, however, are determined by inducing seepage flows in order to consolidate the sample. Material parameters are typically determined via inverse-solution modeling procedures. The SICTA test may be used to accurately determine consolidation parameters for effective stresses below 10 kPa.

Material Sampling

Regardless of the selected laboratory test, it is important to obtain a representative sample of the tailings material, and to prepare test specimens in accordance with the testing objectives. Consolidation properties are a function of grain size distribution, particle shape, mineralogy, etc. In addition, the initial tailings void ratio and the tendency of tailings to segregate depend on the selected depositional method. Traditional dilute slurry deposition often produces highly segregated tailings, resulting in sandy material with relatively high density near the discharge points (beach sands) and fine-grained, slowly consolidating, low density tailings in the center of the TSF (tailings slimes). Conversely, thickened tailings or paste have significantly lower tendency to segregate, and are more likely to result in uniform tailings properties over the entire TSF footprint.

If tailings are prone to segregation, it is important to determine the range of consolidation properties for different tailings fractions, from beach sands to tailings slimes, and estimate their distribution and relative proportion within the impoundment. Ideally, tailings samples required to characterize conditions at an existing facility are collected at different locations within the TSF (e.g. near beach, end of beach, tailings pool). For a proposed or relatively new facility, this type of geotechnical investigation may not be possible, while at an existing facility, the sampling program may be difficult to execute considering financial and logistical constraints and/or conflicts with production.

Flume Testing

A representative sample of the tailings feed (composite tailings sample) is often obtained from a pilot plant or from a tailings discharge point (for an existing facility). As noted previously, the segregation potential for thickened tailings or paste samples may be minimal. For conventional slurry, however, the segregation potential is often significant requiring further evaluation based on flume test results. A flume test is performed by discharging composite tailings at a pre-defined solid content and flow rate, based on the operating and design parameters. Following the test, segregated tailings are sampled from various points along the flume at different distances from the discharge point. These samples are then subjected to classification and consolidation testing to obtain the representative range of material parameters.

A flume test example displaying the depth of tailings along a flume for various initial solid contents is shown in **Figure 2**. In general, lower discharge densities (lower solid contents) typically yield greater tailings segregation. For example, a sample deposited at 35.5% solid content (**Figure 2**) displays a relatively large quantity of material at relatively high deposition angle near the discharge point, a clear indication of segregating tailings. Grain size distributions for the flume deposits sampled near the discharge point and at the distal end of the flume (**Figure 3**) demonstrate a relatively high segregation potential. In this particular case, a composite feed sample with 73% fines produced material containing 43% fines near the discharge point and material with 100% fines at the end of the flume.

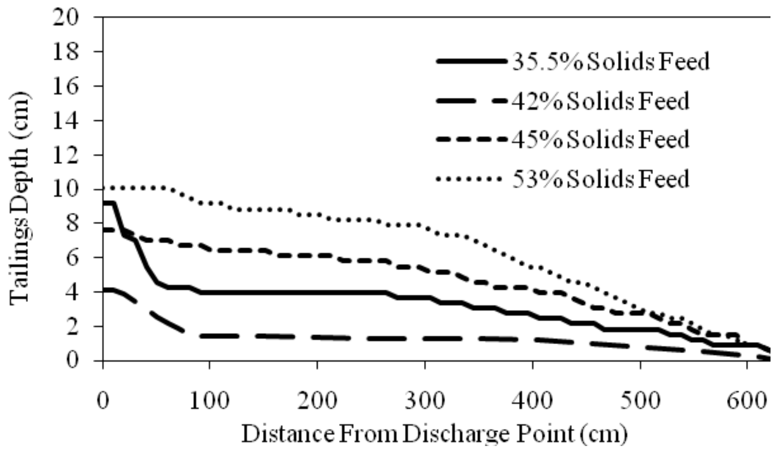

Figure 2: Sediment height vs distance from discharge for flume tests at varying solid contents

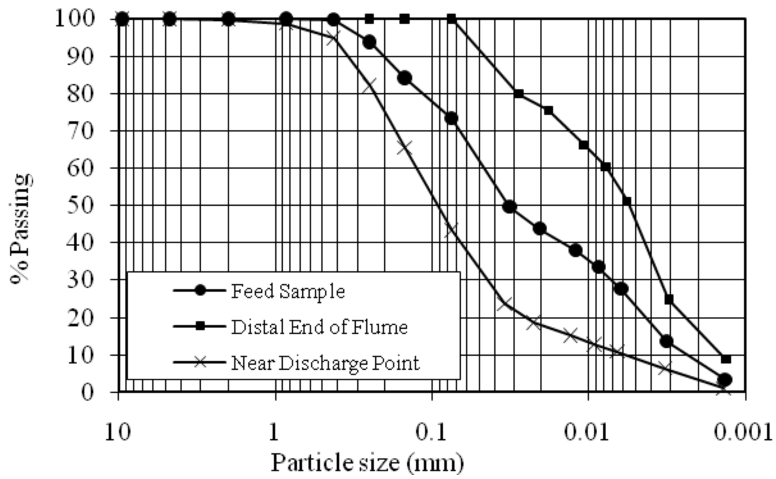

Figure 3: Grain size distribution curves for segregated tailings samples taken from flume test sediment. Flume test performed using a 35.5% solids feed.

A SICTA test was performed on the feed sample, with slurry consolidation tests conducted on the near-discharge and end-of-flume (distal) samples. Material parameters for these samples are summarized in **Table 1** with the corresponding compressibility and permeability curves shown in **Figures 4 and 5**, respectively.

Table 1: Material parameters for FSCONSOL input

Sample	A	B	M	C (m/s)	D
Discharge Point	0.85	-0.13	0.19	8.6E-07	4.7
End of Flume	2.42	-0.16	0.041	5.4E-09	3.1
Feed	2.78	-0.07	-1.15	6.7E-08	3.2

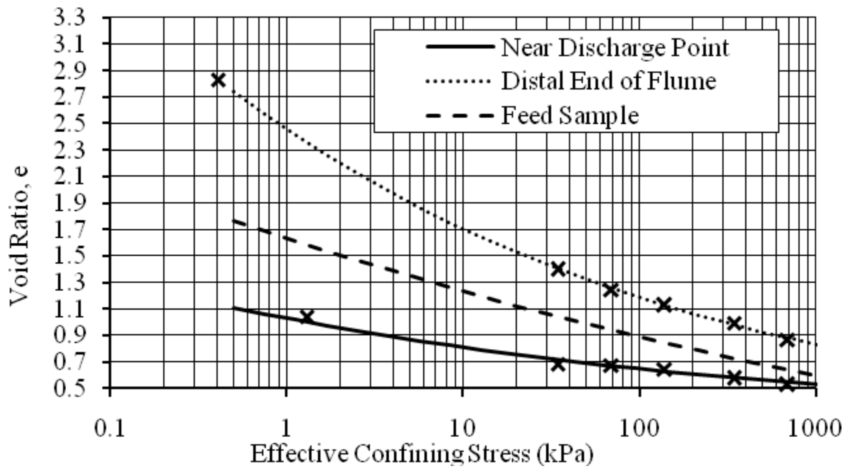

Figure 4: Best fit void ratio vs. effective stress curves from SICTA and slurry consolidation tests. "X" marks points from laboratory tests. Lines are best fit curves utilizing material parameters shown in Table 1

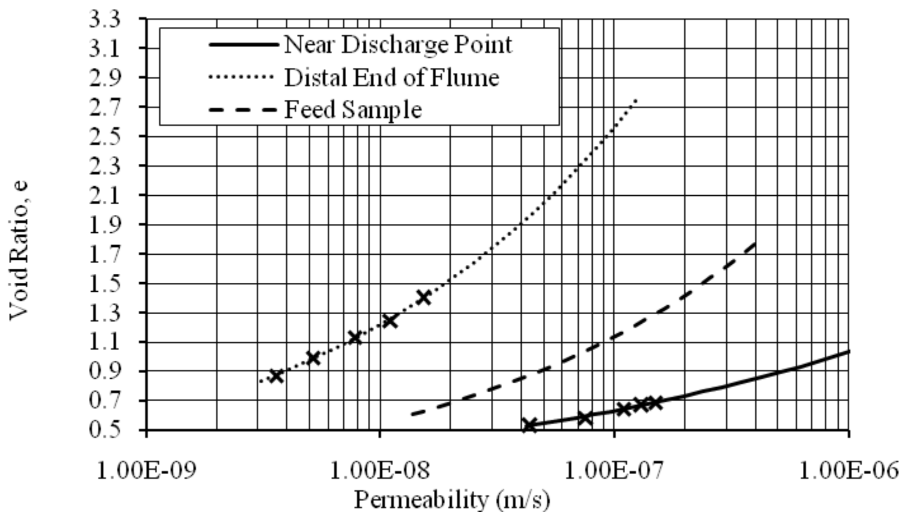

Figure 5: Best fit void ratio vs. permeability curves from SICTA and slurry consolidation tests. "X" marks points from laboratory tests. Lines are best fit curves utilizing material parameters shown in Table 1

Figures 4 and 5 indicate that tailings segregation may lead to large differences in material properties between beach (near-discharge) and distal (end-of-flume) tailings. The distal tailings void ratio at the vertical effective stress of 10 kPa is approximately two times larger than the beach tailings void ratio: e_{distal}=1.72, e_{beach}=0.82 (**Figure 4**). The difference in permeability between beach and distal tailings samples at the same stress level exceeds one order of magnitude (**Figure 5**). Finer grained distal tailings are likely to exhibit higher compressibility (higher settlements), higher void ratio (lower density), and lower permeability (slower consolidation rates).

Column Settling

At low solids contents, the tailings behavior is often influenced by sedimentation. Only after the tailings attain sufficient consistency, preventing relative movement between different size particles (i.e. preventing segregation), may tailings settlements be modeled using large strain consolidation theory.

Column settling tests are commonly performed to determine an appropriate initial void ratio for the consolidation analysis. A column settling test utilizes settlement of a dilute tailings sample poured in a graduated cylinder to estimate segregation potential and estimate consolidation characteristics at low effective stresses. The height of the column is monitored and recorded until the settlement is completed. A grain size distribution of the tailings samples collected from the top and the bottom of the settling column may be used as a rough estimate of the segregation potential.

Figure 6 shows the results of a column settling test. As before, different tailings types behave significantly different. The near-discharge (beach) sample reached a higher equilibrium density (void ratio of 1.2) than the end-of-flume (distal) sample (void ratio of 2.9). In addition, the consolidation of the near-discharge sample was significantly faster. Column settling tests for both samples were initiated with approximately the same initial height and using the same boundary conditions.

Comparison of Calculated Densities and Consolidation Time

The largest disadvantage of small strain methods is that it is relatively cumbersome to account for the non-constant material properties. To illustrate this point, a small strain model was developed to determine settlement of a 1 m thick layer of tailings when gradually loaded to 500 kPa.

The ranges of c_v and C_c values for the feed and distal (end-of-flume) samples were estimated from the SICTA and slurry consolidation tests, and are presented in **Table 2**. **Table 3** illustrates the variance in calculated settlements for the 1 m thick tailings layer at the base of the TSF.

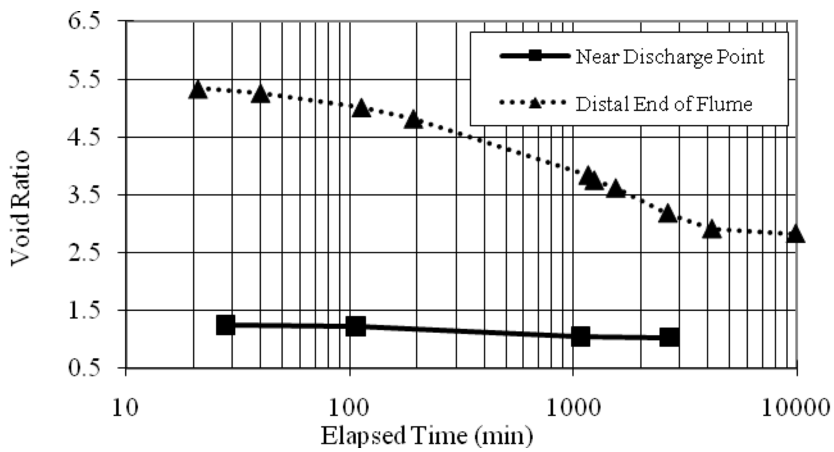

Figure 6: Example of column settling test results

Table 2: Range of c_v and C_c values derived from SICTA and slurry consolidation tests

Sample	Max C_c	Min C_c	Max c_v (cm^2/sec)	Min c_v (cm^2/sec)
Feed Sample	0.53	0.34	2.0E-02	9.5E-04
End of Flume	0.41	0.22	2.0E-01	5.1E-03

Table 3: Range of calculated settlements for 1 m thick layer using small strain calculations

Sample	Maximum Compression (m)	Minimum Compression (m)
Feed Sample	0.35	0.19
End of Flume	0.37	0.24

Comparison of 1D and 3D Methods

Theoretical Preliminaries

Programs used to model large strain consolidation typically provide solution to a non-linear second order partial differential equation (Gibson et al. 1967). These programs provide one-dimensional, time-dependent solutions of void ratio distribution (solid content distributions), layer thickness, pore pressures, and degree of consolidation. For 1D analyses, the TSF capacity can be calculated using the following procedure (e.g. Gjerapic et al. 2008):

- Input material parameters and TSF geometry into the modeling software.
- Discretize the TSF into several columns of varying height. Each column has a base area selected such that the sum of the base areas of each column multiplied by the height of each respective column will produce a volume closely approximating that of the actual TSF. A schematic showing a simplified TSF discretization is shown in **Figure 7**.
- Use mine planning data to determine the average tailings inflow (typically equal to production rate expressed in dry tonnes per day). Divide the production rate (in cubic meters per day) by the area of the first column, A_1, to determine the first filling rate, q_1, as illustrated in **Figure 7**. Fill the TSF at this rate until the elevation of tailings reaches the base of the second column, H_1.
- Increase the area to include the first and second columns, A_2. Recalculate filling rate using the larger area (i.e. determine the filling rate, q_2), and continue filling until reaching the base of the third column at elevation H_2. Repeat this step, increasing the area (i.e. apply filling rates q_3 and q_4 over areas A_3 and A_4) until the top of the TSF is reached.
- Calculate the TSF capacity by multiplying the tailings production rate with the filling time.
- Calculate the average density of tailings by dividing the TSF capacity determined in the previous step with the TSF volume.
- Other outputs (e.g. pore pressure, degree of consolidation) can typically be imported into a spreadsheet from the modeling software for further analysis.

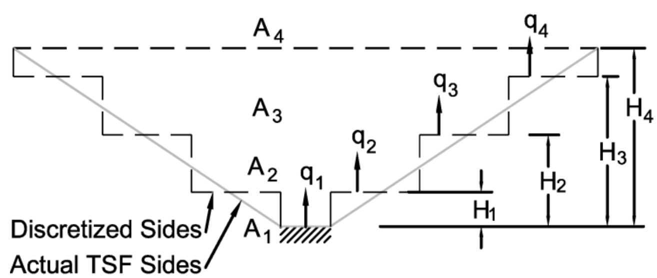

Figure 7: Example of discretized TSF

The 1D method to determine TSF capacity is also referred to as an upper bound method (Gjerapic et al. 2008) because it implicitly assumes that the sides of the TSF undergo the same deformation as the tailings material in the center of the TSF at the same elevation. Typically, foundation soils are much less compressible than the tailings. As a result, the 1D model typically over-estimates foundation soil settlements. Consequently, the time required to fill the TSF is also overestimated, resulting in potentially unrealistic estimates of TSF capacity and average tailings dry density. **Figure 8** illustrates compression of the TSF foundation soils introduced by the 1D method.

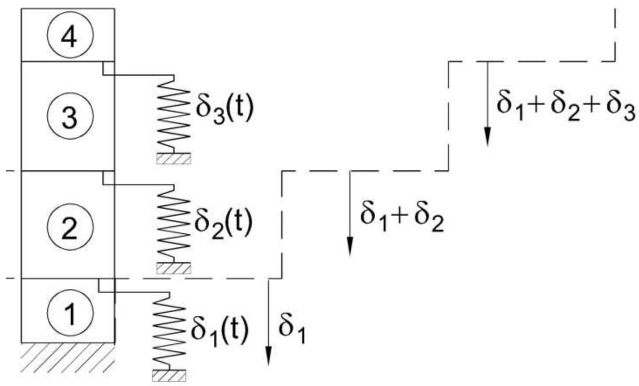

Figure 8: TSF foundation settlements in 1D model

To eliminate the error introduced by compressible TSF boundaries, a 3D approach can be implemented. An approach for eliminating calculation errors caused by compressible TSF boundaries has been developed by Gjerapic et al. (2008). In summary, a series of 1D large strain models is developed for individual columns (from deep to shallow TSF areas) enforcing incompressible boundaries at the base of each column. In addition, adjustments are made to filling rates and filling times of individual columns in order to compensate for settlements occurring during filling in adjacent columns.

The error caused by compressible boundaries is a function of the TSF geometry, material properties, boundary conditions, and filling rate. **Table 4** compares the results of analyses performed using both 1D and 3D methods for two different TSFs. **Figures 9 and 10** show filling curves for these two impoundments. **Figure 10** demonstrates that the error produced by using the 1D method (i.e. by implicitly assuming compressible boundaries) may be significant. The difference between 1D and 3D method results in an error of 94.6% for TSF "B" scenario.

Table 4: Error due to compressible boundaries (1D analysis)

TSF	1D Capacity (years)	3D Capacity (years)	% Error
A	4.8	4.4	8.6
B	30.0	15.4	94.6

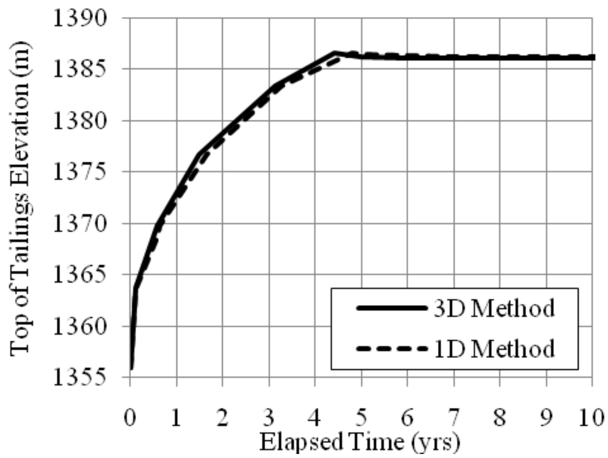

Figure 9: Filling curve for TSF "A" (see Table 4)

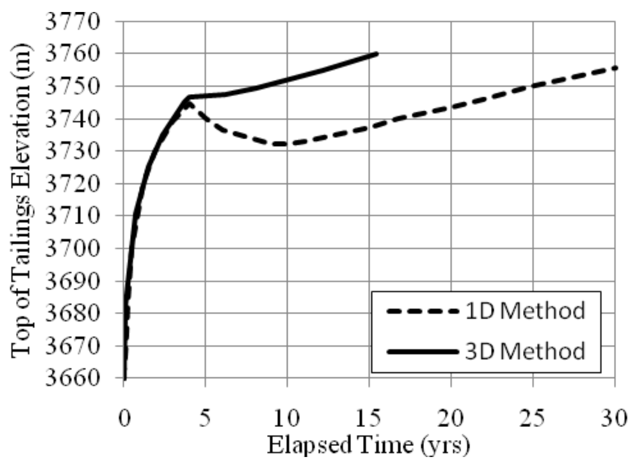

Figure 10: Filling curve for TSF "B" (see Table 4)

Errors in Mass Conservation

One source of errors that is often overlooked is the computer program itself. In some cases, the computer program may exhibit difficulties in converging to a correct solution potentially resulting in a mass conservation error, i.e. the mass conservation may not be maintained (e.g. mass may be lost) throughout the calculation process. The convergence and mass conservation errors appear to vary between different programs. Preliminary studies indicate that higher rates of rise, more compressible tailings, and slower consolidating may increase the magnitude of the mass conservation error.

To illustrate the magnitude of this error, a 1D large strain calculation was performed on two theoretical TSFs. One of the TSFs was relatively flat-bottomed, with an average rate of rise of 0.3 m/yr. The second TSF was cone-shaped, with an average rate of rise of 2.7 m/yr.

To estimate the mass conservation error, one may calculate the average dry density of tailings in two different ways.

Method 1: The average impoundment density is calculated using the 1D procedure described in the previous section. In summary, the time required to fill the impoundment is multiplied by the filling rate to determine total mass of solids residing in the impoundment. The average dry density of tailings is determined by dividing the total mass of solids with the TSF volume at the end of deposition.

Method 2: The second method is based on integrating the void ratio profile generated by the modeling software using the stage curve (elevation-area-volume relationship) for a given tailings impoundment. In effect, the TSF is divided into horizontal slices (e.g. 1 m high). Model output is then used to determine tailings density within each slice. Finally, the average density is determined by averaging the density over all slices, the average tailings density effectively calculated as a weighted average (weighted by volume of the individual slices).

If the compressible boundary effect discussed in the previous section is relatively negligible, both Method 1 and Method 2 should yield the same average density. The difference between two methods, however, is an indication that a 3D analysis may be necessary, or that there is a mass conservation error caused by mesh discretization (see e.g. Gjerapic and Znidarcic 2007), or some other error.

Table 5 compares mass conservation error estimates based on the above methods. As expected, the mass conservation error is smaller for the TSF exhibiting the lower rate of rise.

Generally, Method 2 produces more realistic results, e.g. one would expect lower tailings density for the TSF exhibiting the higher rate of rise because tailings have less time to consolidate (before the TSF capacity is reached). While the densities calculated based on void ratio profile integration (Method 2) are consistent with the expected trend, the densities calculated using filling times (Method 1) are not. In addition, Method 2 is more likely to result in conservative average density estimates.

Table 5: Difference in calculated tailings density due to software error

TSF Shape	Average Rate of Rise (m/yr)	Average Dry Density Method 1 (tonnes/m^3)	Average Dry Density Method 2 (tonnes/m^3)	% Error
Flat	0.3	1.44	1.34	7.6
Cone	2.7	1.63	1.23	27.7

Comparison between Calculated and Measured Densities – Field Study

In order to verify the accuracy of the consolidation analyses, the authors compared calculated and in-situ average densities for an operating TSF. The in-situ tailings volume was calculated in AutoCAD using the as-built topography for the empty TSF and a bathymetric survey defining the top of tailings. The mass of deposited tailings was estimated using the daily production rates provided by the TSF operator. The average TSF in-situ densities were then calculated by dividing the estimated mass with the in-situ tailings volume. Three comparisons were made at 5, 7, and 12 months after the TSF commenced operation.

Calculated densities based on a 3D large strain model were used for comparison. The 3D model incorporated two different tailings materials (segregated and un-segregated tailings). The volume distribution of the two selected tailings types was estimated using the flume tests (see **Figure 2**) and best-guess estimates for the operational percent solids of the tailings at the discharge points. A comparison between average densities based on in-situ measurements and calculated values based on the 3D consolidation model is shown in **Table 6**.

The calculated densities were lower than the measured densities by 11.9 to 12.5%. The consistency of the calculated difference indicates the analysis methods were well suited for the tailings type and TSF site under consideration. The difference between the in-situ data and values predicted by a numerical model is likely due to engineering conservatism applied to the analysis. Specifically, the division between segregated and un-segregated tailings was uncertain. The uncertainty was partially due to difficulty extrapolating flume tests to the full-scale TSF, and partially due to uncertainty regarding the solids content of the operational tailings discharge. Hence, the prediction in **Table 6** can be relatively easily corrected by assigning larger percentage of the impoundment to un-segregated (coarser) tailings.

Table 6: Comparison of calculated and measured average dry density of tailings

Cumulative TSF Filling Time (months)	Average In-Situ Dry Density (tonnes/m^3)	Calculated Average Dry Density (tonnes/m^3)	% Difference
5	1.22	1.08	-11.9
7	1.21	1.06	-12.5
12	1.17	1.03	-12.1

Conclusions and Recommendations

The basis of a defensible settlement model for a TSF starts with performing laboratory tests on representative tailings materials. Laboratory tests need to be designed and conducted in a manner to provide the range of the consolidation parameters governing the in-situ tailings behavior.

One of the primary purposes of a TSF settlement model is to estimate TSF capacity. The assumption that the tailings will not segregate may lead to unrealistic TSF capacity estimates. A justification for the assumption that tailings will not segregate should be confirmed by laboratory testing.

While non-segregating tailings are typically beneficial from a TSF capacity standpoint, such tailings may not be desirable if coarser material is required for constructing future raises of the TSF dam.

Small strain analyses may be cost-effective and reasonably accurate for old desiccated tailings deposits that are being subjected to additional loads (e.g. during closure). However, small strain analyses are not recommended for new, operational, or recently closed TSFs.

1D large strain analyses are often the most appropriate modeling tool for conceptual planning or trade-off studies. The 1D large strain analyses are more accurate than small strain methods, while requiring relatively small additional effort to perform.

3D large strain analyses are preferred for feasibility and detailed level engineering work due to potentially large errors caused by compressible foundation boundaries, the assumption that is implicitly applied when using 1D large strain analyses.

The mass conservation error should be evaluated for both 1D and 3D large strain consolidation analyses. If survey data is available, settlement models can be calibrated to provide better prediction of future tailings behavior and ultimate TSF capacity. By incorporating operational details into the modeling process, the predicted tailings behavior is likely to exhibit favorable agreement with field observations.

References

Gibson, R.E., England, G.L. & Hussey, M.J.L., 1967. The Theory of One-Dimensional Consolidation of Saturated Clays. *Geotechnique*, 17, 261-273.

Gjerapic, G., Johnson, J., Coffin, J. & Znidarcic, D., 2008. Determination of Tailings Impoundment Capacity via Finite-Strain Consolidation Models. *In:* Alshawabkeh, A.N., Reddy, K.R. & Khire, M.V. eds. *GeoCongress*

2008: Characterization, Monitoring, and Modeling of GeoSystems (GSP 179) conference proceedings. New Orleans, LA, 798-805.

Gjerapic, G. & Znidarcic, D., 2007. A Mass-Conservative Numerical Solution for Finite-Strain Consolidation during Continuous Soil Deposition. *In:* Siegel, T.C., Luna, R., Hueckel, T. & Laloui. L., eds. *Geo-Denver 2007: Computer Applications in Geotechnical Engineering: New Peaks in Geotechnics (GSP 157) conference proceedings*. Denver, CO, CD-ROM.

GWP Software 1999. FSConsol User's Manual.

Yao, D.T.C. & Znidarcic, D., 1997. Crust Formation and Desiccation Characteristics for Phosphatic Clays - Users' Manual for Computer Program CONDES, Florida Institute of Phosphate Research (FIPR) Publication No. 04-055-134.