Proceedings Tailings and Mine Waste 2011
Vancouver, BC, November 6 to 9, 2011

Modeling Consolidation of Tailings Impoundments in One and Two Dimensions

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Abstract

Estimating spatial variability of material produced by tailings consolidation represents a relatively unknown area of practice. With the ability to model tailings impoundments in two and three dimensions, SVOffice's large strain consolidation modelling package represents a new way of approaching these problems. Such modeling allows for the inclusion of flux and stress boundary conditions, drainage restrictions caused by liners, the effects of permeable zones on consolidation behaviour, etc. Benchmarking of this software is an ongoing process. This paper builds upon previous papers in evaluating 1 and 2 dimensional consolidation of previously published scenarios. It also includes the modeling of tailings deposits composed of several materials. The results show that multidimensional consolidation analyses can provide new insights into the consolidation behaviour of tailings.

Introduction

The consolidation of soil/liquid slurries plays a large role in the design of tailings impoundments and dredged fills. It is necessary to be able to predict the settlement of the material over time, as well as changes in the strength and hydraulic characteristics of the material.

This paper is intended to present some of the practical aspects which can be visualized when problems are considered in 2 dimensions. The SVOffice Consolidation software is used for the analyses unless otherwise stated. A tailings deposition scenario from the literature previously analyzed with a 1D approach is first validated using a 1D analysis, with several important modeling considerations discussed, such as the choice of compressibility and permeability formulations, as well as the effects of the Poisson's ratio. Following this, the model is extrapolated into a 2 dimensional tailings impoundment with sloped sides. The presence of an underdrain is also considered. Finally, a second material is added to show how this feature can be used to evaluate alternative deposition schemes.

Theoretical Background

In 1967, the late Professor Gibson and his co-workers, refined the Terzaghi formulation to allow for large strain consolidation where the layers are able to move and where hydraulic conductivity is not assumed to be constant (Gibson, English and Hussey 1967). Their one dimensional formulation (Equation 1) is also written in terms of void ratio and uses the Lagrangian coordinate system. The use of an Eulerian coordinate system becomes difficult when considering the large deformations in the domain or in the moving upper boundary. Therefore the reduced Lagrangian height coordinate (z) is used instead (Schiffman, Vick and Gibson 1988).

$$\pm\left(\frac{\rho_s}{\rho_f}-1\right)\frac{d}{de}\left[\frac{k(e)}{1+e}\right]\frac{\delta e}{\delta z}+\frac{\delta}{\delta z}\left[\frac{k(e)}{\rho_f(1+e)}\frac{\delta\sigma'}{\delta e}\frac{\delta e}{\delta z}\right]+\frac{\delta e}{\delta t}=0 \qquad \textbf{(1)}$$

Where ρ_s is the density of the solid phase, ρ_f is the density of water, e is the void ratio, k(e) is the hydraulic conductivity written in terms of void ratio (e), σ' is the effective stress and t is time. Before the general use of personal computers, numerical solutions were devised to solve the equation, however, with increased computing power, the ease of finite element solutions, as well as the wide variety of flux and deformation boundary conditions, has allowed for more complex scenarios to be considered.

Variations in Compressibility and Permeability

In order to obtain solutions to the formulation in equation 1, it becomes necessary to generate functions for the compressibility and permeability that will accurately model the soil behaviour as the void ratio changes. A commonly used compressibility function is the extended power function shown in equation 2.

$$e = A(\sigma'+Z)^B \qquad \textbf{(2)}$$

The *A* and *B* parameters are unitless and are determined by experimental results. The *Z* term has a value of the effective stress calculated at the initial void ratio. This acts as a limit to the minimum stress and is used to prevent numerical instability arising from low effective stresses where the void ratio increases exponentially (Liu and Znidarcic 1991). Permeability is widely modeled using a power function as shown in equation 3 (Townsend and McVay 1990; Jeeravipoolvarn, Scott and Chalaturnyk 2008 and Yao and Znidarcic 1997).

$$k = Ce^D \qquad \textbf{(3)}$$

C and *D* are unitless coefficients, which are determined experimentally and will be unique to each tailings material.

SVOffice Consolidation Software

SVOffice uses a coupled deformation flux analysis to solve large strain problems. By coupling their deformation package (SVSolid) with the flux package (SVFlux), the program is able to solve for large strain applications. The software makes use of a moving finite element mesh to update calculations. The improvement over previous works is that the stress analysis part of the solution is formulated to include i) large-strain deformations, and ii) it is consistent with legacy generalized stress/deformation formulations. The program requires much the same input data as is required for the Gibson, England and Hussey formulation, with the exception of Poisson's Ratio, but performs the calculations using a stress-strain formulation rather than a void ratio term representing the volume (Fredlund, Donaldson and Gitirana 2009).

In using a stress-strain formulation it is required to specify a value of Poisson's ratio as additional input to the analyses. Poisson's ratio is the ratio between vertical and horizontal deformations. This parameter is not considered in other consolidation formulations, with the exception of Cryer (1963). The effect of varying Poisson's Ratios is discussed later in this paper.

Previous Benchmarking of Large Strain Consolidation

Benchmarking of this new software is a critical part in confirming that the software can be relied upon for engineering decision making. A number of literature and theoretical models have been evaluated to

benchmark the formulation available within the software. A manual containing these benchmarks will be released with the software. Fredlund et al. (2009) and Priestley et al. (2010) present several preliminary analyses and benchmarks of multidimensional modeling including:

- Comparison between coupled and uncoupled formulations
- Townsend et al.'s (1990) Scenario A and B (Townsend and McVay 1990)
- Mandel Cryer effect in 2-D (Cryer 1963)
- An idealised 2-D model of tailings deposited into a pit
- Comparison to CONDES0 (Yao and Znidarcic 1997)
- Benchmarking of Column Tests performed by Jeeravipoolvarn et al. 2008 (Jeeravipoolvarn, Scott and Chalaturnyk 2008)

Note: the references refer to the source of the original data used in the benchmarking

One Dimensional Verification

To verify the preliminary results, a benchmark model was chosen from Case 1 of the Caldwell et al. (1984) paper on calculating consolidation in a number of different tailings impoundment using large strain finite element analyses. Caldwell et al (1984) evaluated the case where 2 different types of tailings are produced, and stored in different impoundments. The main impoundment stores sulfide tailings, while a smaller impoundment in the upper catchment stores gossan tailings. The verification model is depicted in Figure 1 and pertains to the main impoundment and sulfide tailings. The material parameters for the gossan tailings will be used when evaluating possible 2D scenarios.

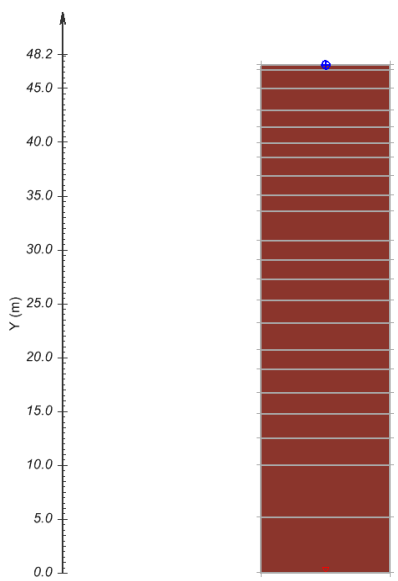

Figure 1: One Dimensional Column

Geometry and Boundary Conditions

The model consists of a 47.15m column of sulfide tailings. The main impoundment is filled over 20 years, therefore staged filling needs to be applied. This is a more realistic long term management simulation as opposed to instantaneous filling. SVOffice uses a phased approach whereby regions are added at specified time increments. Therefore, the filling curve provided from Caldwell et al (1984)

was used to determine a number of irregular increases in tailings height. Each step was phased at the halfway point between the start and end time of the respective phase, thereby resulting in a stepwise filling scheme.

Deformation is fixed along the bottom boundary. There are no restrictions with respect to deformation applied to the top boundary. The bottom boundary is set to be impermeable, while the water level on top of the pool is held constant at 47.15 m throughout filling resulting in a completely saturated system. Caldwell et al. (1984) also states that the tailings were deposited below water.

Material Parameters

Caldwell et al (1984) provided the effective stress – void ratio – hydraulic conductivity relations used in their calculations. These were digitized and subsequently extended power functions were fit to the data. These are shown in Figure 2. The sulfide tailings are stated to have a specific gravity of 4.2 and an initial void ratio of 1.1, while the gossan tailings have a specific gravity of 3.2 and an initial void ratio of 2.0.

The constitutive equations for the compressibility and permeability of the sulfide tailings are given in equations (4) and (5) respectively, while those of the gossan tailings are given in equations (6) and (7).

$$e = 1.29(\sigma' + 9.402)^{-0.071} \qquad \textbf{(4)}$$

$$k = 1.6572e^{6.7026} \qquad \textbf{(5)}$$

$$e = 1.97(\sigma' + 0.86)^{-0.099} \qquad \textbf{(6)}$$

$$k = 0.154e^{4.8205} \qquad \textbf{(7)}$$

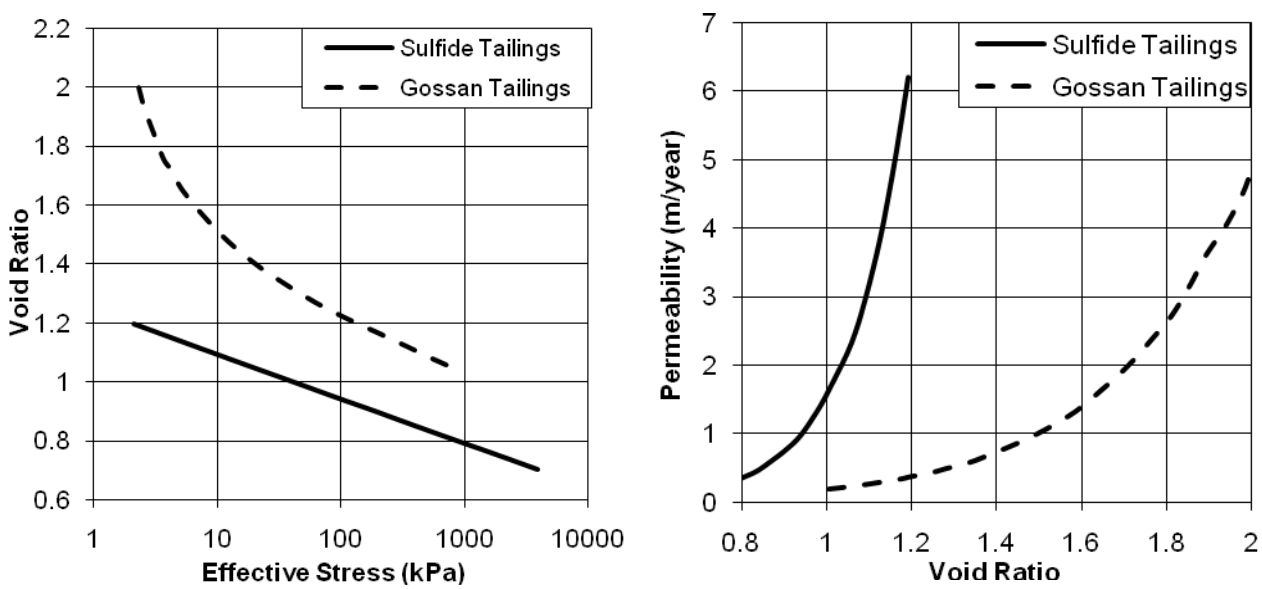

Figure 2: Compressibility and permeability relationships of sulfide and gossan tailings

A Poisson's ratio of 0.30 was assumed although there was no value provided in the report. Consoli (1991) used a value of 0.30, which is stated to be a typical value for cohesion-less materials (Consoli and Sills 2000). This value appears reasonable for loose sands, however, both types of tailings investigated here were finely ground (80% passing the number 400 sieve) (Caldwell, et al. 1984),

therefore higher values of Poisson's Ratio may be required. A sensitivity analysis is performed to determine the effect of the Poisson's Ratio on the results.

Results and Discussion

The results from Figure 3 illustrate that SVOffice agrees well with the 1D results obtained from Caldwell et al. (1984). The largest variations occur early in the filling of impoundment, with the SVOffice results showing slightly less consolidation than those recorded in the literature. Figure 4 shows the void ratio and permeability profiles of the tailings at various stages as the impoundment is filled. While there are some variations in the profiles, the values are generally in line with the previously obtained values. The Caldwell et al. (1984) values are plotted as points while SVOffice values are plotted as lines.

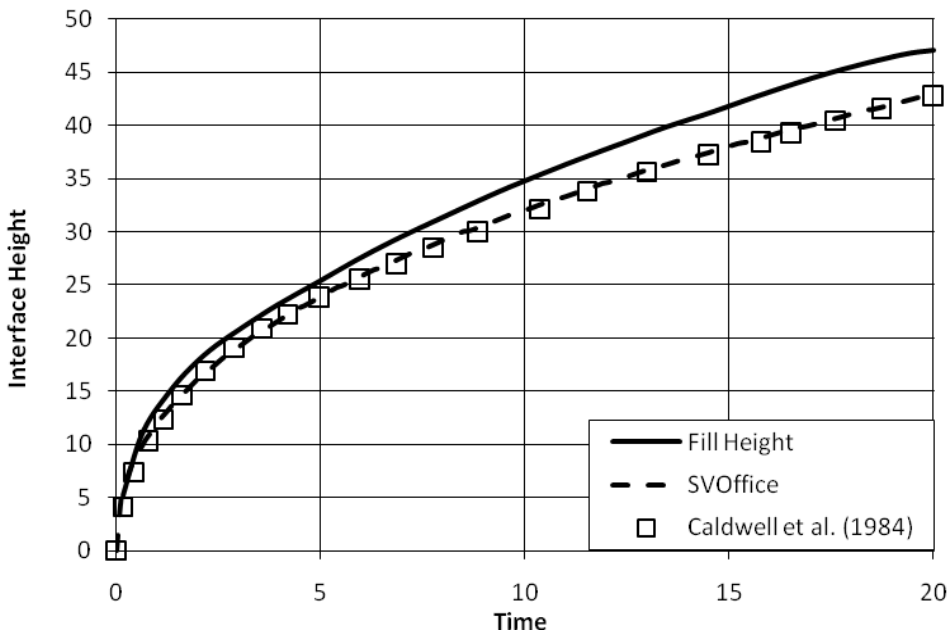

Figure 3: Comparison of filling curve and Caldwell et al.'s predictions against SVOffice

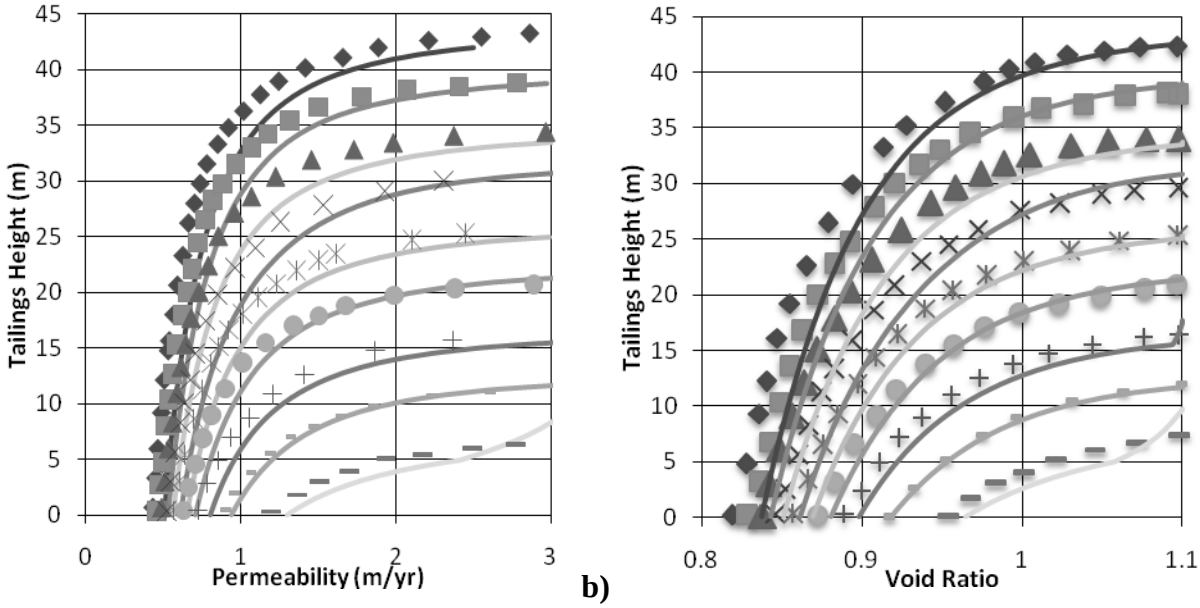

Figure 4: Comparison of predicted permeability (a) and void ratio (b) profiles at various time increments. Note: Points represent values from Caldwell, et al (1984), while lines represent the SVOffice predictions.

A simple sensitivity analysis was created by varying the Poisson's ratio and comparing the predictions for the final height of the tailings deposit. Poisson's ratio has a significant effect on the results observed, as is presented in Figure 5. The lowest values of Poisson's Ratio produce the least amount of consolidation, while the most consolidation is observed at the highest Poisson's Ratio. The Caldwell et al. (1984) results appear to correspond well with a Poisson's Ratio between 0.3 and 0.4, which appears reasonable for such finely ground tailings.

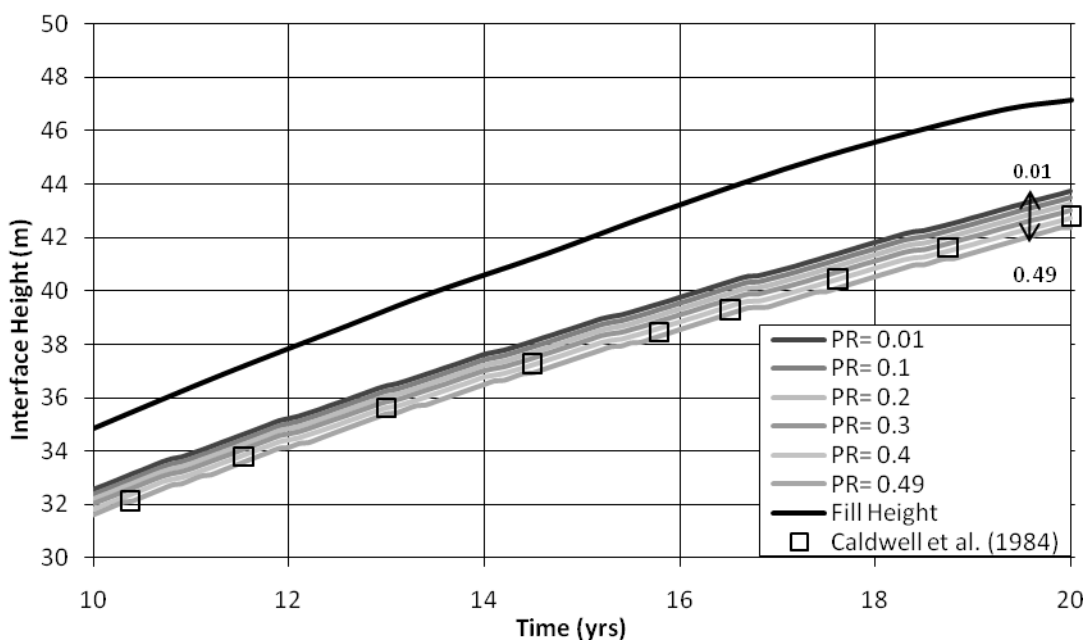

Figure 5: Changes in the final surface height with Poisson's Ratio

2D Tailings Deposits

While one dimensional modelling provides useful insight as to how the material will behave further insight is gained when the modeling is carried out in two and three dimensions. To demonstrate this, an extrapolation of the one dimensional model was created using the same filling curve and material parameters as the one dimensional model. The ability to successfully model 2D systems has been previously demonstrated (Fredlund, Donaldson and Gitirana 2009), therefore a 1H:2V sloped side geometry has been assumed (Figure 6). The sidewalls remain impermeable, although the use of different boundary conditions could be used to simulate the presence of low permeability embankments or various hydraulic conditions.

Sloping Boundary Conditions

Consolidation along a sloping boundary is an area that has not been fully investigated. Jeeravipoolvarn (2010) used fixed side boundaries to simulate consolidation along the inside of a conical containment vessel. It is acknowledged that this is a simplification. An alternative approximation is to allow the boundary to deform within the bounds of the system.

Figure 7 and Figure 8 demonstrate the deformed mesh after 20 years of the fixed and sliding boundary conditions respectively. The void ratio profile is also shown. The boundary conditions prove to have a significant impact upon the observed behaviour. The fixed condition results in a concave surface, with very little consolidation observed at the sloped boundaries, as would be expected when they are not allowed to deform. When the sliding boundary condition is introduced, the opposite is observed, there is a convex surface, with greater consolidation occurring near the side slopes.

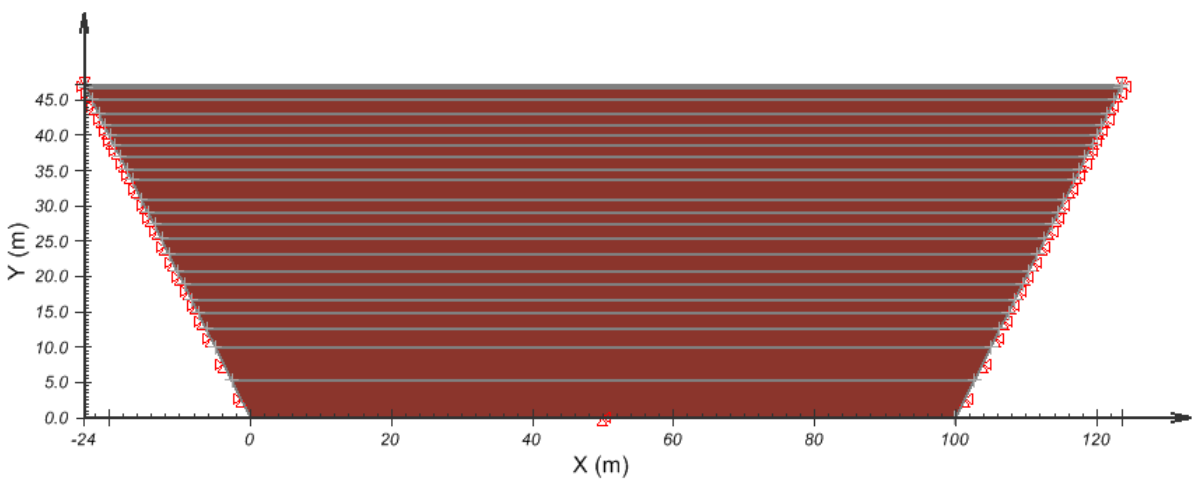

Figure 6: Geometry of 2-D tailings impoundment model

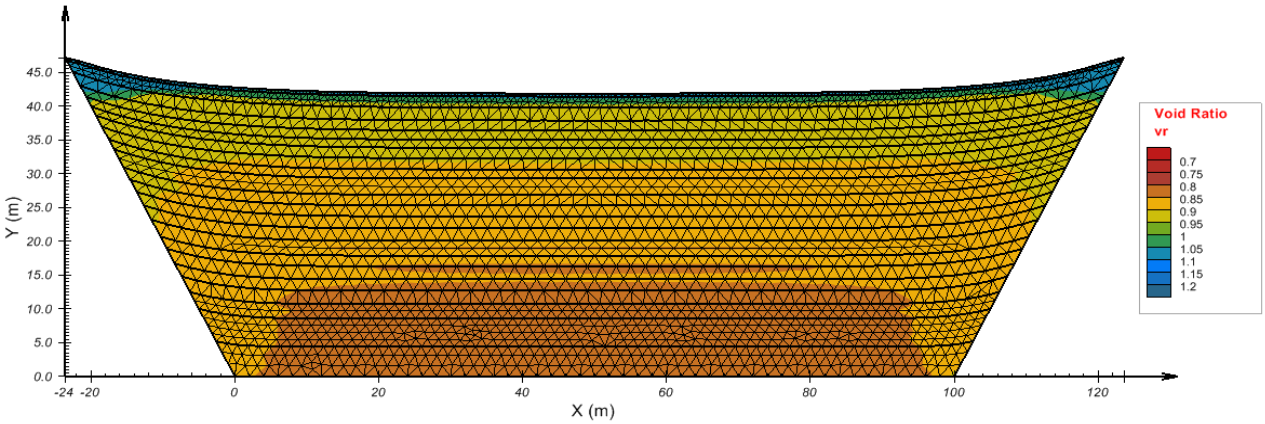

Figure 7: Fixed geometry deformation after 20 years with void ratio profile

Figure 9 compares the surface height predictions at the center of the 2D impoundment for the fixed and sliding sloped boundaries to those of the 1D column. Both show good correlation to the 1D prediction of Caldwell et al (1984). The fixed model predicts slightly more consolidation at the center than the sliding boundary condition. This could be caused by the fact that when a fixed boundary is applied, less consolidation occurs along the sides, meaning that the excess pore water pressure is able to dissipate to a lesser extent horizontally as well as vertically. The flow vectors are depicted in Figure 10 and show that there is drainage occurring towards the edges.

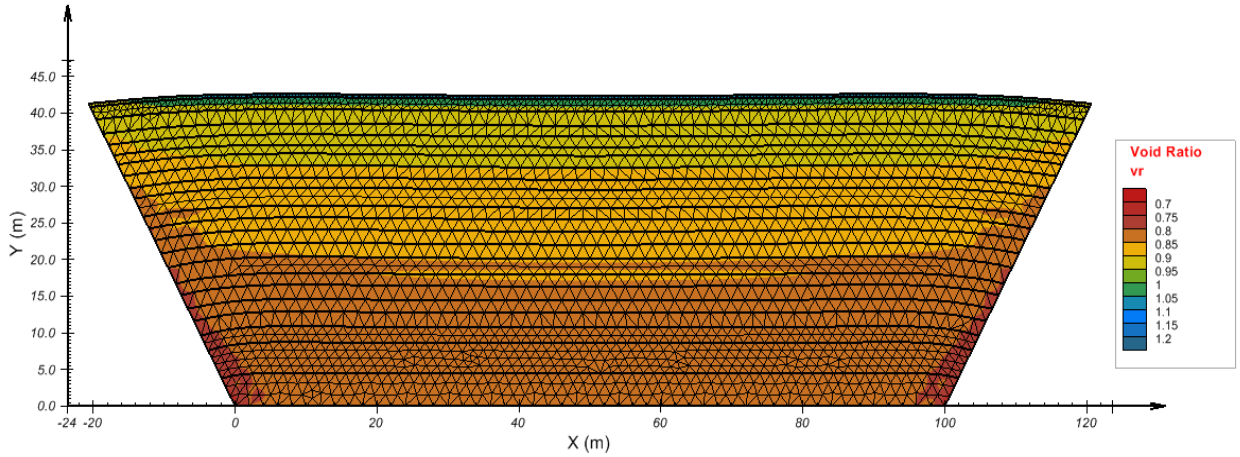

Figure 8: Sliding geometry deformation after 20 years with void ratio profile

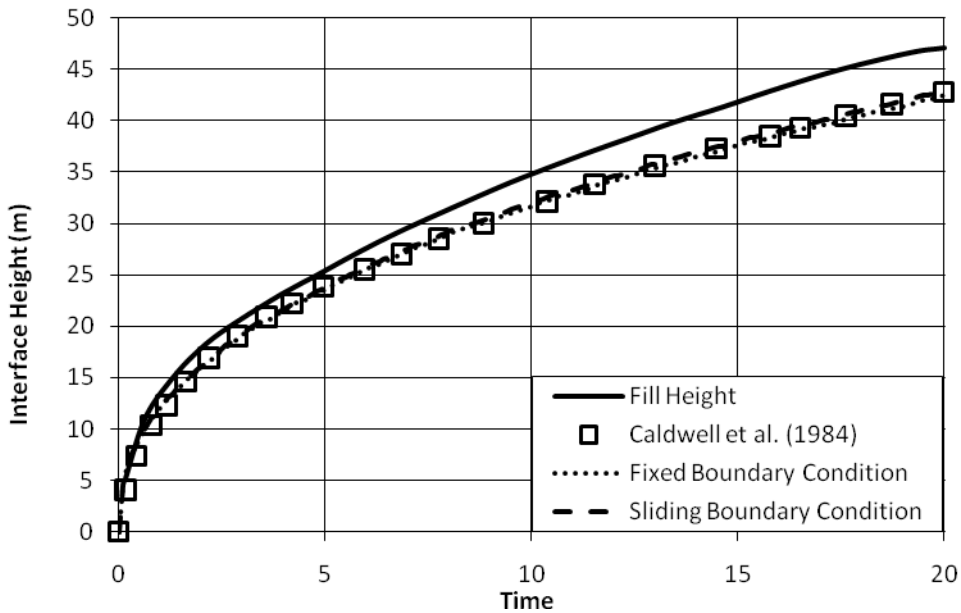

Figure 9: Comparison of 2D impoundment heights at center of deposit for fixed and sliding boundary conditions against the filling curve and Caldwell et al. (1984) prediction

Effect of an Underdrain

To further extend the two dimensional model, the effect of a drain layer along the bottom of the impoundment is investigated. To simulate a free draining surface, the bottom boundary condition is defined as having a water head of 0 meters. The sloping sides remain impermeable, and the fixed boundary condition is used.

Figure 11 shows the deformed mesh and void ratio profile after 20 years when an underdrain is included. The void ratio profile is more uniform, than when the lower boundary remains impermeable. For this geometry and material parameters, the presence of the underdrain was found to have a very small impact. While an underdrain is expected to increase consolidation over time, Anstey and Williams (2007) also found that for accumulating metals tailings, the underdrain did not significantly affect consolidation, but rather that the permeability of the tailings is the determining factor.

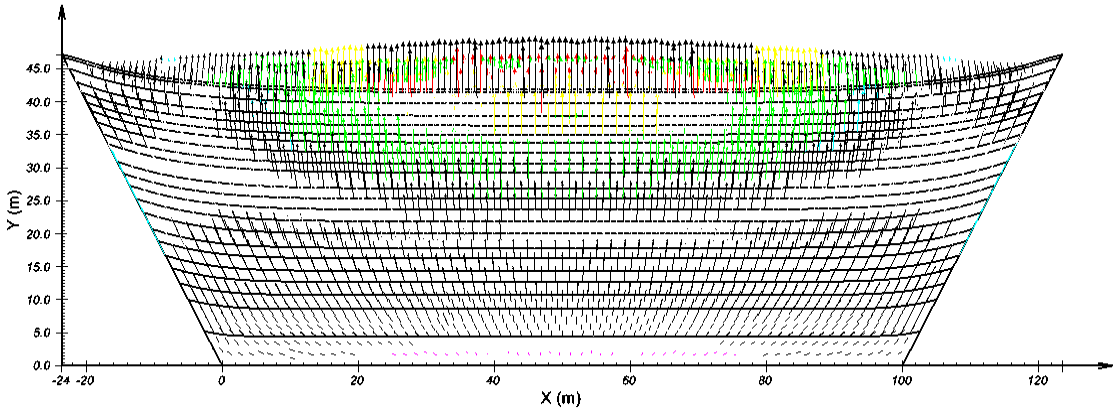

Figure 10: Flux vectors in the fixed boundary condition scenario

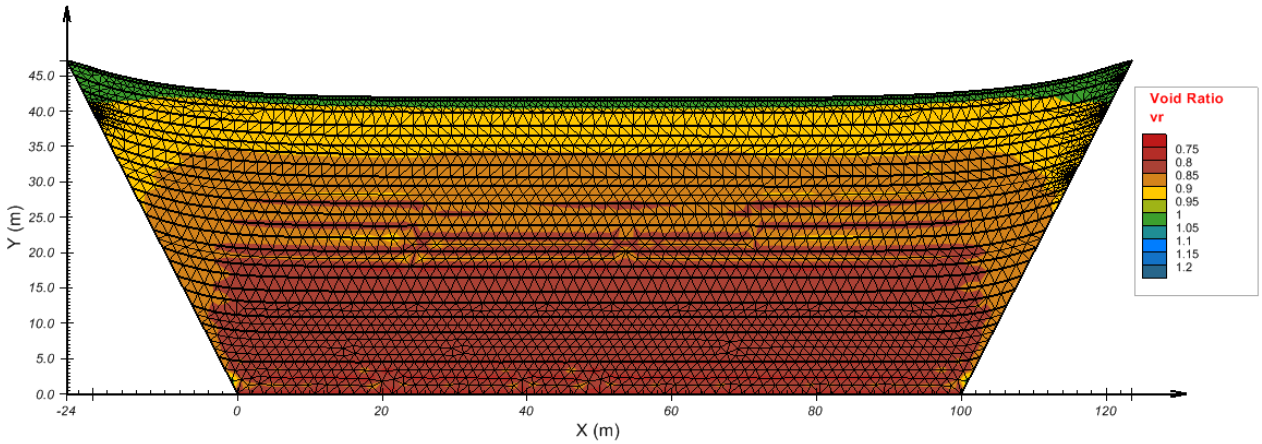

Figure 11: 2D deformation with under drain after 20 years with void ratio profile

Spatial Variability

As was previously mentioned there were both gossan and sulfide tailings produced at this location, with the gossan tailings being produced earlier in the mine life. One disposal scenario, which was initially considered, was to deposit the gossan tailings underneath the sulphides. As the gossan tailings have a much lower specific gravity and a completely different set of index and consolidation parameters, these tailings would behave differently than the sulfides. Although this scenario was questioned by reviewers at the time due to potential overturning of the tailings due to the gossan tailings having lower specific gravity, it does present an interesting finite strain model that can be evaluated.

It is assumed that the first two lifts, totalling 10 meters, are made up of the gossan material. The same filling curve was used from the main impoundment. A diagram of the model after 20 years is shown in Figure 12, while the tailings height predictions are shown in Figure 13. The two material model shows significantly more consolidation than the base fixed boundary scenario.

Conclusions

Determining the geotechnical characteristics and behaviour of tailings deposits is a difficult task. With the ability to model these deposits in two and three dimensions, the new SVOffice Consolidation Software is able to add to the practitioner's ability to evaluate these problems more completely. Through the ongoing process of verification, as well as applications to practical situations, greater confidence is gained in the results. 2D analysis opens new avenues of analysis with more precise representation of lateral flow. In assuming that the side boundaries remain fixed, preferential flow paths are introduced near the edges. An alternative sliding boundary condition is also presented. The validity of these methods will depend upon a number of factors, including the method of deposition, particle segregation and impoundment geometry. Through the inclusion of different material types and boundary conditions, tailings impoundment designs can be evaluated more realistically.

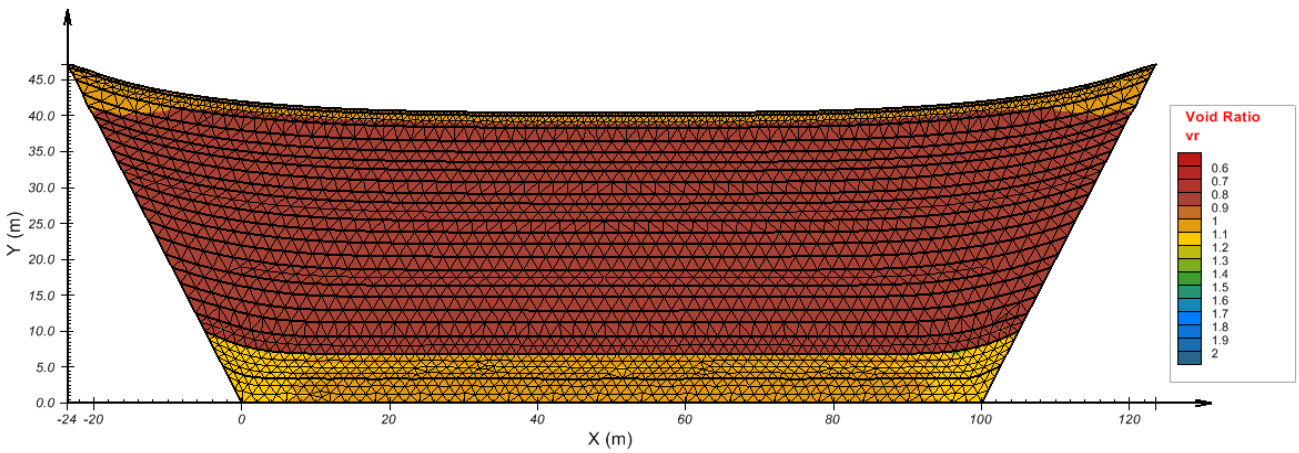

Figure 12: Two material disposal scenario after 20 years with void ratio profile

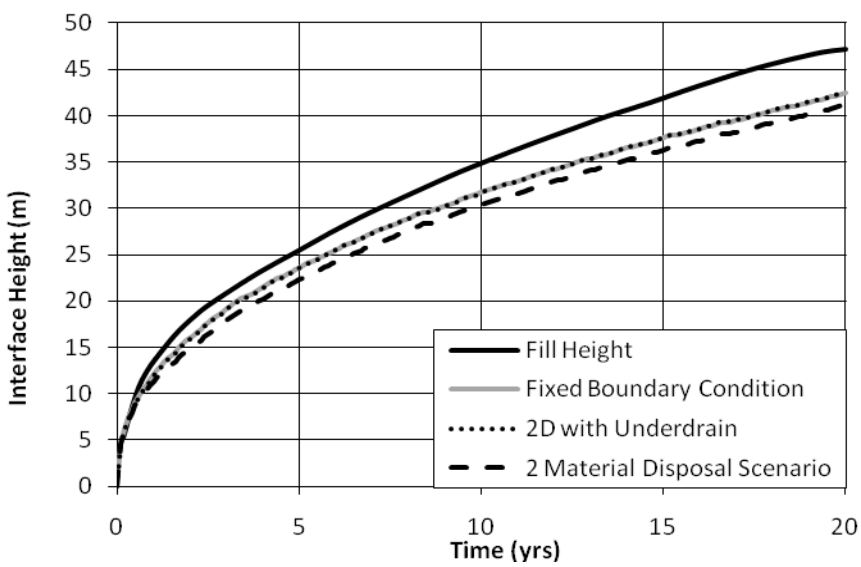

Figure 13: Filling height comparison of the effect of the under drain and the 2 material disposal scenario

Acknowledgement

The authors would like to acknowledge the financial support provided by SoilVision Systems Inc. as well as the National Science and Engineering Research Council of Canada

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