VARIABLE-COMPLIANCE-TYPE CONSTITUTIVE MODEL FOR METHANE HYDRATE BEARING SEDIMENT

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ABSTRACT
In order to evaluate a methane gas productivity of methane hydrate reservoirs, it is necessary to develop a numeric simulator predicting gas production behavior. For precise assessment of long-term gas productivity, it is important to develop a mathematical model which describes mechanical behaviors of methane hydrate reservoirs in consideration of their time-dependent properties and to introduce it into the numeric simulator. In this study, based on previous experimental results of triaxial compression tests of Toyoura sand containing synthetic methane hydrate, stress-strain relationships were formulated by variable-compliance-type constitutive model. The suggested model takes into account the time-dependent property obtained from laboratory investigation that time dependency of methane hydrate bearing sediment is influenced by methane hydrate saturation and effective confining pressure. Validity of the suggested model should be verified by other laboratory experiments on time-dependent behaviors of methane hydrate bearing sediment.

Keywords: methane hydrate, triaxial compression test, stress-strain curve, strength, elastic modulus, strain rate, time dependency, constitutive equation

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INTRODUCTION
Methane hydrate is anticipated to be a promising energy resource of natural gas, since a large amount of reservoir exists in marine sediments or in permafrost regions worldwide [1-3]. For the

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purpose of efficient extraction of natural gas from the methane hydrate reservoirs, some methods to dissociate hydrate in-situ have been proposed: depressurization, thermal stimulation and inhibitor injection.

In order to evaluate methane gas productivity from the reservoirs, it is essential to develop a numeric simulator including formation and dissociation behavior of methane hydrate, thermal properties of the reservoirs, permeability and mechanical behaviors of the reservoirs and so on. For reliable simulation of long-term behavior, it is important to predict mechanical behaviors of the reservoirs such as consolidation and deformation in consideration of their time-dependent properties. Therefore, it is necessary to develop a mathematical model which describes constitutive relationship (stress-strain relationship) of methane hydrate bearing sediment including their time-dependent behaviors and to introduce it into the numeric simulator predicting gas production behavior.

In this study, based on previous experimental results of triaxial compression tests of Toyoura sand containing synthetic methane hydrate [4,5], stress-strain relationships were formulated by variable-compliance-type constitutive model which has been applied to various time-dependent behaviors of rock [6,7].

**EXPERIMENTAL METHOD**
Experimental method published previously can be summarized as follows [4,5,8].

**Preparation of host specimen**
A host specimen, in which synthetic methane hydrate was formed afterward, was prepared by compacting water-saturated Toyoura sand densely in a mold on a vibration table. The initial water content, which had a great influence on methane hydrate saturation of the specimen, was adjusted by draining excess water with a syringe pump. Then unsaturated sand was frozen in a freezing chamber so as to make it easy to handle the host specimen. The size of frozen host specimen was 50 mm in diameter and 100 mm in length and its porosity ranged from 37 % to 39 %.

**Experimental apparatus**
Experimental apparatus illustrated in Figure 1 was used in each process of triaxial compression test such as hydrate formation, water substitution, axial compression and hydrate dissociation. The apparatus is a digital servo-controlled testing machine with a capacity of 200 kN for axial load, 20 MPa for confining pressure and 20 MPa for pore pressure. The temperature in the pressure vessel can be controlled at the range of 243 K to 293 K with an accuracy of 0.5 K by circulating refrigerant liquid from a cooling tank. Experimental data, such as axial load measured by a strain gauge-type load cell and axial displacement measured by a linear voltage differential transformer, were recorded by the data acquisition system at every second during the experiments.

**Hydrate formation and water substitution**
Two kinds of specimen were tested: one is hereafter called “saturated-sand specimen” and the other “hydrate-sand specimen.” The former contained no methane hydrate and the latter contained synthetic methane hydrate in a variety of degrees. After a host specimen was set with top cap, rubber sleeve and pedestal as shown in Figure 1, methane gas was percolated into a host specimen to replace air existing in pore space at temperature of 278 K. Then pore pressure was increased up to the induction pressure of 8.0 MPa at the rate of 0.5 MPa to 1.0 MPa per minute,
while confining pressure was increased at the same rate. Successively, pore pressure, confining pressure and temperature were kept constant during 24 hours of induction period. After that, water substitution was conducted by injecting water into the specimen to replace methane gas remaining in pore space in the specimen, pedestal, top cap, pipes and so on. The volume of injected water was almost twice as much as that of specimen.

**Axial compression and hydrate dissociation**
Axial compression was carried out with a servo-controlled strain rate. Two control modes were conducted. In one mode, strain rate was kept constant 0.1 % per minute. In the other mode, strain rate was changed alternately between two speeds \( C_1 \) and \( C_2 \) at a constant strain interval 0.5 %. Hereafter, axial compression test conducted in the former and latter mode are called “constant strain rate test” and “alternating strain rate test” respectively. Since relatively sufficient data had been already obtained at constant strain rate of 0.1 % per minute, it was adopted as the higher strain rate \( C_1 \) in alternating strain rate test. With reference to previous study [9], the lower strain rate \( C_2 \) was decided to be 0.01 % per minute for saturated-sand specimen or 0.05 % per minute for hydrate-sand specimen. Axial compression was conducted at a temperature around 278 K in drained condition maintaining pore pressure of 8 MPa and confining pressure of 8.5 MPa, 9 MPa, 10 MPa or 11 MPa, and thus effective confining pressure was maintained at 0.5 MPa, 1 MPa, 2 MPa or 3 MPa during the axial compression process.

After the triaxial compression test was carried out, methane hydrate formed in the specimen was dissociated by depressurizing pore space and the amount of released methane gas was measured so that the initial methane hydrate volume in the specimen would be calculated.

**EXPERIMENTAL RESULTS**
**Constant strain rate test**
Stress-strain curves shown in Figure 2 were obtained from constant strain rate tests. As shown in Figure 2 (a), in the case of saturated-sand specimen of methane hydrate saturation 0 \%, stress increased and slope of the curve decreased until stress reached the peak strength at more than 5 \% of strain. Both of the stress and strain at the
peak strength increased with effective confining pressure. In the region after the peak strength, the stress gradually decreased with strain. Figure 2 (b) and (c) shows stress-strain curves of hydrate-sand specimens of methane hydrate saturation 25 % and 35 % respectively. As shown in these figures, the stress-strain curves of hydrate-sand specimen were generally similar to those of saturated-sand specimen in terms of shape, though peak strength, elastic modulus and strain softening tendency varied with methane hydrate saturation.

Peak strength was plotted against methane hydrate saturation in Figure 3. From the figure, peak strength increased with methane hydrate saturation. For example, in case of effective confining pressure 1 MPa, peak strength increased approximately two times with increase in methane hydrate saturation from 0 % to 55 %. Peak strength $\sigma_c$ can be approximately expressed as the following function of methane hydrate saturation $S_h$ and effective confining pressure $\sigma_3'$:
$$\sigma_c = 3.67 \sigma_3'^{0.754} + 0.00249 S_h^{1.86}. \quad (1)$$

Elastic modulus, or secant elastic modulus at 50 % failure, was plotted against methane hydrate saturation in Figure 4. From the figure, elastic modulus increased with methane hydrate saturation. Elastic modulus $E_{50}$ can be approximately expressed as the following function of methane hydrate saturation $S_h$ and effective confining pressure $\sigma_3'$:
$$E_{50} = 246 \sigma_3'^{0.482} + 10.8 S_h. \quad (2)$$

Masui et al. noted that the increases in peak strength and elastic modulus with methane hydrate saturation were due to cementation between sand particles by methane hydrate [4, 5].

**Alternating strain rate test**

In order to predict long-term mechanical behavior, it is essential to investigate time-dependent property. Loading rate dependency is one of important time-dependent behaviors. Loading rate dependency of hydrate-sand specimen was found to be significantly apparent, while that of saturated-sand specimen was negligible small [8, 9]. This result indicates that time-dependency of methane hydrate bearing sediment is strongly influenced by the methane hydrate saturation, which is likely because methane hydrate in pore space of sediment relates closely to deformation mechanism of time-dependent behaviors. In this study, loading rate dependency of saturated-sand specimen and hydrate-sand specimen of a variety of methane hydrate saturations was experimentally examined under conditions of effective confining pressure 0.5 MPa, 1 MPa, 2 MPa and 3MPa.

As shown schematically in Figure 5, stress-strain curves obtained from alternating strain rate tests showed an increase/decrease of stress at an increase/decrease of strain rate. Curve 1 in Figure 5 is the spline curve connecting the points marked with ◆ when strain rate was switched from $C_1$ to $C_2$ and can be considered as the stress-strain curve at constant strain rate $C_1$. Likewise, Curve 2 is the...
spline curve connecting the points marked with ● when strain rate was switched from \( C_2 \) to \( C_1 \) and can be considered as the stress-strain curve at constant strain rate \( C_2 \). In this way, two stress-strain curves and thus two peak strengths \( \sigma_{c1} \) and \( \sigma_{c2} \) corresponding to different strain rates \( C_1 \) and \( C_2 \) were obtained from a single specimen.

As shown in Figure 6, stress-strain curves obtained from alternating strain rate tests under effective confining pressure 1 MPa were generally similar to those from constant strain rate tests, though they undulated with switching strain rate. The amplitude of undulating curves seems to depend on methane hydrate saturation.

Strengths obtained from alternating strain rate tests of saturated-sand and hydrate-sand specimens under various effective confining pressures were shown in Figure 7. In this figure, two coupled marks represent a pair of strengths obtained from one specimen as schematically shown in the inset. Relations between strength \( \sigma_{c1} \) and methane hydrate saturation \( S_h \) under each effective confining pressures \( \sigma_3' \) were approximately expressed by Equation (1) displayed in Figure 7. From the figure, the difference between \( \sigma_{c1} \) and \( \sigma_{c2} \) widened as methane hydrate saturation increased.

For hydrate-sand specimen, the increasing rate of strength \( R_{c2} \) can be calculated by the following expression:

\[
R_{c2} = (\frac{\sigma_{c2}}{\sigma_{c1}}) - 1. \tag{3}
\]

Because the ratio of strain rate \( C_2 / C_1 \) was set to be 2 for hydrate-sand specimen, Equation (3) is regarded as giving the increasing rate of strength when strain rate is doubled. Relations between increasing rate of strength \( R_{c2} \) and methane hydrate saturation under various effective confining pressures are shown in Figure 7.
pressures were shown in Figure 8. They were linearly related and the degree of effect of methane hydrate saturation on increasing rate of strength depended on effective confining pressure. Considering that the increasing rate of strength of saturated-sand specimen varied little with confining pressure as described later, increasing rate of strength \( R_{a2} \) can be approximately expressed as the following function of methane hydrate saturation \( S_h \) and effective confining pressure \( \sigma_0^* \):

\[
R_{a2} = 0.00189 \sigma_0^{-0.333} S_h + 0.0072. \quad (4)
\]

The constant term in right side of Equation (4) was decided on experimental results of the increasing rate of strength of saturated-sand specimen as described later.

**CONSTITUTIVE MODEL**

Based on the experimental results above, applicability of variable-compliance-type constitutive model, which focuses on time-dependent behaviors such as strain rate dependency of strength, to methane hydrate bearing sediment was examined.

**Variable-compliance-type constitutive model**

In this study, variable-compliance-type constitutive model was adopted to simulate stress-strain relationships of saturated-sand and hydrate-sand specimen in constant strain rate tests. The model is applicable to various time-dependent behaviors of rock including strain rate dependency of peak strength. It was previously reported that cohesion or adhesive characteristics of sand was enhanced in the presence of methane hydrate by its cementing effect between sand particles [4,5]. Therefore, assuming that methane hydrate bearing sediment takes on somewhat similar properties with rock from a mechanical point of view, it is expected that time-dependent behaviors of methane hydrate reservoirs can be expressed by the model. This is a key reason why the model was decided to be examined in this study.

Variable-compliance-type constitutive model is expressed as follows:

\[
\frac{d\lambda^*}{dt} = a_1 (\lambda^* - 1)^{-\eta_1} \sigma^{\eta_1} + a_2 \lambda^m \sigma^{\eta_2}, \quad (5)
\]

\[\lambda^* = \frac{\lambda}{\lambda_0}, \sigma = \frac{\varepsilon}{E_i}, \sigma = \frac{\varepsilon}{\sigma_0}, \]

where \( t \) is time, \( \lambda \) is compliance, \( \lambda^* \) is normalized compliance, \( \lambda_0 \) is initial compliance or inverse of initial elastic modulus, \( \varepsilon \) is strain, \( \sigma \) is principal stress difference, \( E_i \) is initial elastic modulus, \( \sigma^* \) is stress severity or inverse of local safety factor, \( \sigma_{\varepsilon_0} \) is peak strength not at strain rate 0.1 % per minute but at a certain strain rate given later and \( a_1, m_1, n_1, a_2, n_2, m_3 \) and \( n_3 \) are model parameters. Among parameters in Equation (5), \( n_1 \) and \( n_3 \) indicate time-dependency or viscoelasticity. However, the difference between \( n_1 \) and \( n_3 \) has not been well understood actually. In this study, it was assumed that there is no difference between \( n_1 \) and \( n_3 \) [6,10]:

\[
n = n_1 = n_3. \quad (6)
\]

Given Equation (6), the differential equation in Equation (5) yields:

\[
\frac{d\lambda^*}{dt} = \{a_1 (\lambda^* - 1)^{-\eta_1} + a_2 \lambda^m \} \cdot \sigma^{\eta_2}. \quad (7)
\]

From the solution of Equation (7) under constant strain rate \( C \), it turns out that peak strength \( \sigma_\varepsilon \) is proportional to the \( 1 / (n+1) \) power of \( C \) [7]. Thus, if peak strengths \( \sigma_{\varepsilon_1} \) and \( \sigma_{\varepsilon_2} \) corresponding to different strain rates \( C_1 \) and \( C_2 \) are known, the value of parameter \( n \) can be obtained from the following equation:

\[
n = \frac{\log( C_2 / C_1)}{\log( \sigma_{\varepsilon_2} / \sigma_{\varepsilon_1})} - 1. \quad (8)
\]

For example, the values of \( (\sigma_{\varepsilon_2} / \sigma_{\varepsilon_1}) \) for saturated-sand specimens were independent of effective confining pressure \( \sigma_0^* \) and the average value was 1.0241 from results of alternating strain rate tests in which the ratio of strain rate \( (C_2 / C_1) \) was set to be 10. When 1.0241 and 10 are assigned to \( (\sigma_{\varepsilon_2} / \sigma_{\varepsilon_1}) \) and \( (C_2 / C_1) \) respectively in Equation (8), 95.6 is obtained as the value of \( n \) for saturated-sand
specimen. Therefore 1.0072 can be obtained as the value of \((\sigma_2 / \sigma_1)\) by solving Equation (8) in which 95.6 and 2 are assigned to \(n\) and \((C_2 / C_1)\) respectively. The constant term 0.0072 in right side of Equation (4), or the increasing rate of strength \(R_2\) for saturated-sand specimen when strain rate was doubled, was obtained by assigning this value 1.0072 to the \((\sigma_2 / \sigma_1)\) in Equation (3).

**Decision of parameters**
Given Equation (6), seven parameters, \(E_i, \sigma_0, a_1, m_1, a_3, m_3\) and \(n\), should be determined according to experimental results. In this study, these parameters were determined in the way hereinafter described.

Secant elastic modulus at 50 \% failure \(E_{50}\) was adopted as initial elastic modulus \(E_i\) in Equation (5):

\[
E_i = E_{50} = 246 \sigma_0^{0.482} + 10.8 S_h.
\]

The parameter \(n\) can be calculated by Equation (8). Firstly Equation (3) yields the following expression:

\[
(\sigma_2 / \sigma_1) = R_2 + 1.
\]

Then the following equation can be derived by assigning 2 and Equation (10) to \((C_2 / C_1)\) and \((\sigma_2 / \sigma_1)\) respectively:

\[
0.333 - \log 32 S_{\sigma} n = \frac{1}{\log \left(\frac{R_2 + 1}{1}\right)} - 1.
\]

Thus the following equation can be derived from Equation (4) and (11):

\[
n = \frac{1}{\log \left(0.00189 \sigma_0^{0.3333} S_h + 1.0072\right)} - 1.
\]

Because peak strength depends on strain rate, in order to determine the value of \(\sigma_0\), it is necessary to decide the strain rate \(C_0\) corresponding to \(\sigma_0\). In this study, \(C_0\) is determined to be the strain rate which meets the requirement that the time \(t_e\) from the start of loading until strain reaches \((\sigma_c / E_i)\) equals a given constant \(t_{e0}\). In this instance, the following equation can be derived from the solution of Equation (7) under constant strain rate 

\[
C:
\]

\[
\sigma_{e0} = \left(\frac{t_e}{t_{e0}}\right)^n \sigma_c = \left(\frac{\sigma_c / E_i / C}{t_{e0}}\right)^n \sigma_c.
\]

The time \(t_{e0}\) can be determined arbitrarily. In this study, \(t_{e0}\) was determined to be 120 seconds. The peak strength \(\sigma_{e0}\) can be calculated from Equation

Figure 9 Variation of stress-strain curve with the value of \(a_1\)

\[
\text{Figure 9: Variation of stress-strain curve with the value of } a_1.
\]

(13) with Equation (1), (9), (12) and assignment of the following expressions:

\[
t_{e0} = 120 \text{ seconds},
\]

(14)

\[
C = 0.1 \text{ % per minute}.
\]

(15)

The other parameters, \(a_1, m_1, a_3\) and \(m_3\) were determined as bellow so that the numerical solution fitted the experimental results of constant strain rate tests shown in Figure 2.

The term which includes \(a_1\) and \(m_3\) in Equation (7) is dominant mainly in the region after the peak strength. So the determination of \(a_1\) and \(m_3\) started with an assumption that \(a_3\) equaled 0, in reference to the stress-strain curves in the region before the peak strength:

\[
\frac{d\sigma_s}{dt} = a_1 (\lambda_{1} * -1)^{m_1} \sigma_s^{m_3}.
\]

(16)

The parameter \(m_1\) indicates mainly the shape of stress-strain curve in the region before the peak strength. In accordance with all of the experimental results in this study, it was decided that the value of \(m_1\) can be expressed as follows:

\[
m_1 = \frac{n+1}{4.90} - 1.
\]

(17)

For example, experimental and calculated stress-strain curves of hydrate-sand specimen of methane hydrate saturation 40 \% under effective confining pressure 1 MPa were shown in Figure 9. Calculated curves in Figure 9 were obtained with a variety of \(a_1\) in Equation (16). The value of \(a_1\) was
determined so that the calculated curve passed slightly above the experimental curve:  
\[ a_1 = 0.01. \]  \hspace{1cm} (18)

In case that \( a_3 \) equals 0, the calculated result does not sufficiently express the experimental residual strength or stress-strain relationship in the region after peak strength. To return to Equation (7), \( a_3 \) and \( m_3 \) were determined in reference to stress-strain curves in the region after the peak strength.

The slope of stress-strain curve after the peak strength depends on \( m_3 \). In this study, the value of \( m_3 \) was determined as follows:

\[ m_3 = 2.5. \]  \hspace{1cm} (19)

The parameter \( a_3 \) decides the peak strength. In this study, the value of \( a_3 \) was determined as follows:

\[ a_3 = 0.0001. \]  \hspace{1cm} (20)

Equation (18), (19) and (20), which were derived from in accordance with all of the experimental results in this study, suggested that \( a_1, a_3 \) and \( m_3 \) were almost independent of methane hydrate saturation or effective confining pressure.

As above, all of the parameters in Equation (5) corresponding to various methane hydrate saturations and effective confining pressures were determined with Equation (1), (6), (9), (12-15) and (17-20). The stress-strain relationships under constant strain rate 0.1 % per minute can be numerically calculated by these equations. Calculated curves (with marks) and experimental curves (without marks) were shown in Figure 10. It was found that experimental stress-strain curves under various methane hydrate saturations and effective confining pressures were approximately expressed by the model under corresponding conditions. This result indicates that variable-compliance-type constitutive model can be considered to be applicable to mechanical behavior of methane hydrate bearing sediment under triaxial stress state to a considerable extent.

It can be said that variable-compliance-type constitutive model is appreciably simple, because compliance \( \lambda \), or inverse of elastic modulus, is the only variable which varies with time \( t \) following Equation (5). Since one has only to incorporate a subroutine for variation of elastic modulus into a simulation code for elastic analysis, the model is easy to introduce into a numerical simulation of

![Figure 10](image-url)

Figure 10 Experimental and calculated stress-strain curves
mechanical behavior. Therefore the model is considered sufficiently promising because it can assess long-term behaviors of methane hydrate bearing sediments. The validity of the model will be appreciated by creep tests or other laboratory experiments concerning time-dependency.

CONCLUSIONS
In this study, applicability of variable-compliance-type constitutive model based on the results of constant and alternating strain rate tests to mechanical behavior of methane hydrate bearing sediment was examined. As a result, it was found that the model was sufficiently applicable to stress-strain relationships of saturated-sand and hydrate-sand specimens under triaxial stress state. One of important issues toward putting the model to practical use is to verify its validity by other mechanical experiments on time-dependent behaviors of methane hydrate bearing sediment.

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