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Development of Stochastic Heterogeneous Slip Distribution Model for Simulation of Earthquake Ground Motion

Hiroyasu Abe
Graduate Student, Graduate School of Engineering, The University of Tokyo

Naoto Sekimura
Professor, Graduate School of Engineering, The University of Tokyo

Tatsuya Itoi
Associate professor, Graduate School of Engineering, The University of Tokyo

ABSTRACT: Ground motion simulation is an important tool for analyzing seismic risk in engineering systems. Recently, ground motion simulations using fault model are being widely applied. Characterized fault models are conveniently used to model the heterogeneous slip distribution on fault plane, which divides the fault into two areas, i.e., asperity area and background area. The model, however, is too simplified to model the complex characteristics of actual slip. In this paper, a model to simulate the slip distribution of the fault plane is proposed.

1. INTRODUCTION

Ground motion simulation is an important tool for analyzing seismic risk for engineering systems. Recently, ground motion simulations using so-called fault model are being widely used for this purpose. A simplified characterized fault model was proposed and used for that purpose. The method, however, was proposed to calculate the average characteristics of ground motion. More detailed model is needed to conduct probabilistic seismic risk assessment, which would incorporate uncertainty in ground motion prediction. In some studies, the spatial coherence is investigated by considering zero offset and nonzero offset[2)], or stochastic characterization of the spatial complexity of slip was proposed[3)]. Most of the studies, however, focus on the correlation characteristics between source parameters and assume that the slip is normally distributed. In this study, a probabilistic modeling of slip distribution focusing on crustal earthquake is discussed and proposed.

2. STOCHASTIC MODEL FOR HETEROGENEOUS SLIP DISTRIBUTION ON FAULT PLANE

2.1 Probabilistic distribution of slip

In this study, the 2005 West Off Fukuoka Earthquake occurred in Japan is used as an example of crustal earthquakes.

Figure 1 shows the spatial distribution of slip in the fault plane of the 2005 West Off Fukuoka Earthquake[4),5)]. In a conventional method[1)], the fault plane is divided into two areas, i.e., asperity area and background area. In this study, a more detailed model for slip distribution is proposed.

The length in the slip point at Figure 1 is denoted as $y=\{y_1,\cdot\cdot,y_n\}$, where n is the number of divided areas. First, probabilistic distribution for the slip displacement is investigated. Figure 2 shows the probability paper plot of the actual slip. The log normal and gamma distributions are also shown for comparison. These distribution are obtained by maximum likelihood estimation. The log-normal distribution fits the actual data than gamma distribution. Therefore, the log-normal distribution is used hereafter. Table 1 shows the average and standard deviation of log y in Figure 1.

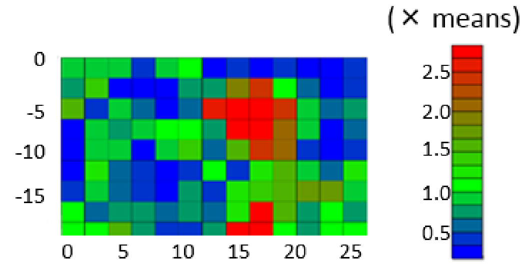

Figure 1: Fault plane of 2005 West Off Fukuoka Earthquake[4)5)]

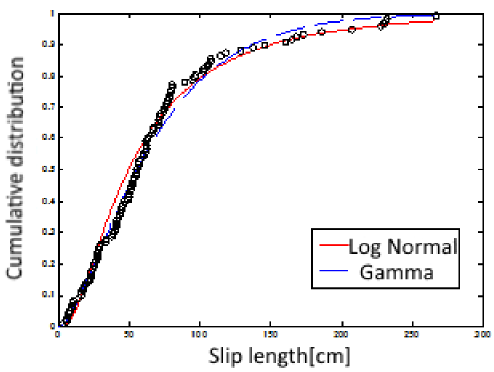

Figure 2: Cumulative distribution of slip at each element of 2005 West Off Fukuoka Earthquake

Table 1: Average and standard deviation of logarithmic slip of 2005 West Off Fukuoka Earthquake

Average	*Standard deviation*
3.90	*0.87*

2.2 Spatial correlation of slip

Next, the correlation structure from slip distribution during actual earthquake is analyzed. Semivariogram 'r' is used for that purpose. $r(Y_i,Y_j)$ is defined as follows:

$$r(Y_i,Y_j)=\frac{1}{2}\mathrm{E}[(Y_i-Y_j)^2] \qquad (1)$$

where, Y_i and Y_j are the lengths of slip at the i-th and j-th element respectively.

And, semivariogram '$r(Y_i,Y_j)$' is defined as follows:

$$r(Y_i,Y_j)=\sigma_Y^2(1-\rho_{YiYj}) \qquad (2)$$

where, σ_Y is the standard deviation of the fault plane and ρ_{YiYj} is the correlation coefficient between Y_i and Y_j .

Semivariogram and correlation coefficient have a relationship that correlation coefficient is closer to 1 when $r(h)$ is smaller, and it is closer to 0 when $r(h)$ is larger.

In this study, this correlation is assumed to depend on h, which is the distance between two slip points. The correlation coefficient $r(h)$ is defined as follows:

$$r(h)=\frac{1}{2N(h)}\sum_{i=1}^{N(h)}(Y(\mathbb{X}_{1i})-Y(\mathbb{X}_{2i}))^2 \qquad (3)$$

Provided that $N(h)$ is the number of pairs that fulfill equation (4) include $(\mathbb{X}_{1i},\mathbb{X}_{2i})$.

$$\mathrm{h}-\frac{\Delta h}{2}\leqq|\mathbb{X}_{1i}-\mathbb{X}_{2i}|\leqq \mathrm{h}+\frac{\Delta h}{2} \qquad (4)$$

where, $\varDelta h$ is decided to be 1.34km by trial and error. Figure 3(a) shows $r(h)$ obtained from Equation (3). Figure 3(b) shows $N(h)$ for each bin. In Figure 3(a), the fitted curve (Equation (5)) that is determined from the least –squares method is also shown.

$$r(h)=b(1-\exp(-ah^2)) \qquad (5)$$

where, b is equal to the dispersion $\sigma_{Y'}^2$ that is determined from Y. a is 0.0947 km^{-2}, and b is 3.4819×10^3.

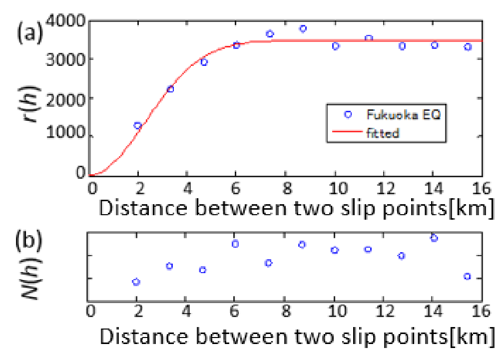

Figure 3: Semivariogram showing spatial correlation of slip obtained from slip distribution during 2005 West Off Fukuoka Earthquake

2.3 Stochastic simulation of slip distribution

Simulation of slip distribution in fault plane is conducted by Monte Carlo Simulation. The slip of the fault plane in i-th element is Y_i. Y=[Y_1,Y_2,… Y_N]'. Y follows the log-normal distribution. W is the normal random variable that is transformed from Y by the Rosenblatt transformation as follows:

$$W = \Phi^{-1}(F(Y)) \tag{6}$$

where, $\Phi^{-1}(\ )$ is the cumulative function of a standard normal distribution, and $F(Y)$ is the cumulative distribution function of Y.

W is determined from Equation (7) as follows:

$$W = \Phi_w Z \tag{7}$$

where, Z is the vector of random variables that fulfill the normal distribution, with zero mean and unit standard deviation. Φ_W is the eigenvector of covariance C_{WW} as follows:

$$C_{WW}\Phi_W = \Phi_W \Lambda_W \tag{8}$$

Λ_W is the square matrix, where diagonal element is the eigenvalues and the others are 0. C_{WW} is expressed as follows:

$$C_{WW} = \sigma_W{}^2 \begin{bmatrix} \rho_{W_1W_1} & \cdots & \rho_{W_1W_N} \\ \vdots & \ddots & \vdots \\ \rho_{W_NW_1} & \cdots & \rho_{W_NW_N} \end{bmatrix} \tag{9}$$

where, σ_W is the standard deviation of W. In this equation, $\rho_{W_iW_j}$ is the correlation coefficient between W_i and W_j. In this study, it is assumed that $\rho_{W_iW_j}$ is equal to $\rho_{Y_iY_j}$, and fulfills Equation (5). Therefore, it is assumed that $\rho_{W_iW_j}$ is a function of only h_{ij} which is the distance between i and j.

$$\rho_{W_iW_j} = exp\left(-ah_{ij}{}^2\right) \tag{10}$$

Distribution in fault is simulated under the condition of M_W 6.6 (M_0=9.0×10^{18}N・m) that is same as the 2005 West Off Fukuoka Earthquake. M_0 is seismic moment, and M_W is moment magnitude. Fault area S and seismic moment M_0 have a relation as follows:

$$S = 4.24 \cdot 10^{-11} \cdot (M_0 \cdot 10^7)^{1/2} (M_0 > 7.5 \bullet 10^{18}) \tag{11}$$

Fault area is 420km^2 and it is presumed that the depth of the area is 15km and the width is 28km. The fault area is divided into 140 elements.

The example of simulated slip distribution is shown in Figure 4. Real slip distribution of 2005 West Off Fukuoka Earthquake is shown in Figure 5.

On one hand, large slip area (red color in Figure 5) is concentrated on upper center area and below center area (Figure 1). On the other hand, large slip area is not so much concentrated in Figure 4. Though heterogeneous feature of fault slip can be modeled, the large slip area is scattered, compared with Figure 1.

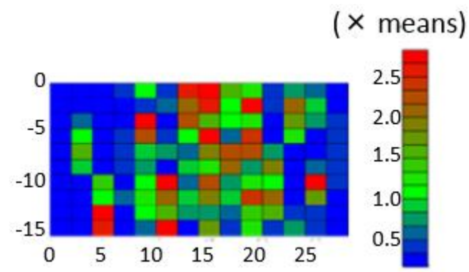

Figure 4: A sample of simulated distribution

3. VALIDATION OF PROPOSED MODEL USING GROUND MOTION

In this section, the proposed model is validated using ground motion simulation. Stochastic Green's function[7)] method is used for ground motion simulation.

The fault geometry and receiver location are showed in Figure 5. S-wave velocity on surface is assumed to be 400m/s in this study.

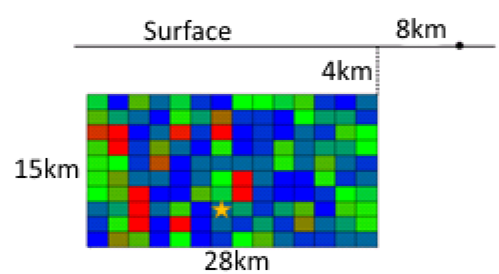

Figure 5: The receiver location of calculating point and fault

The seismic moment of all fault area is distributed at each element. The seismic moment at each element depends on the slip length at each element. Therefore, seismic moment M_{0i} at each element is given as follows:

$$M_{0i} = M_0 \bullet y_i \Big/ \sum y_i \tag{12}$$

where, y_i is the slip length at the i-th element .

Stress drop $\Delta\sigma_i$ is assumed to be given as follows:

$$\Delta\sigma_i = \frac{7\pi^{3/2} \cdot M_{0i}}{16 \cdot S_i^{3/2}} \tag{13}$$

where, S_i is the area of each element.

Simulated acceleration time history is shown in Figure 6. Ground motion is simulated for slip distribution shown in Figure 4. According to Figure 6, S-wave arrives at 3 seconds since earthquake occurrence and continues for about 17 seconds. The maximum acceleration is 17m/s^2 at t = 7.0s. In Figure 7, velocity history is shown. The maximum velocity is 0.9m/s at t = 7.5(s).

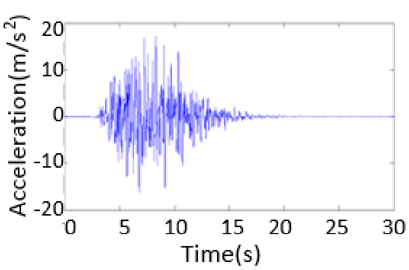

Figure 6: The acceleration for the case of Figure 4.

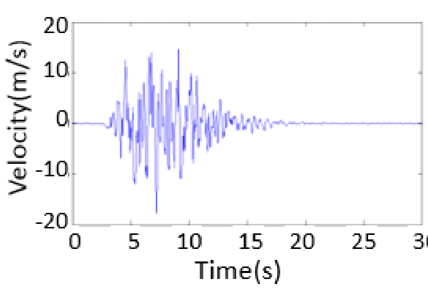

Figure 7: The velocity for the case of Figure 4

Monte Carlo simulation is conducted to simulate different slip distribution under the same condition as 2.3.

In Figure 8(a), examples of slip distribution are shown, while simulated ground motion for respective case are shown in Figure 8(b). As shown in the figure, the temporal characteristics of velocity history are different between samples. Maximum velocity increases if large slip appears between the hypocenter and the receiver as shown in the bottom figure in Figure 8.

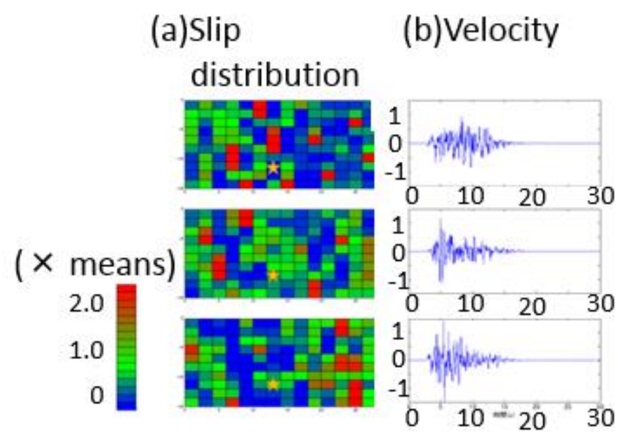

Figure 8: Distribution slip and velocity at the ground point

To verify the proposed method, maximum acceleration and maximum velocity are compared with existing attenuation model[10]. Figure 10 and Figure 11 show the comparison for maximum acceleration and velocity respectively. Simulated ground motions are larger than that by the attenuation models.

In these figures, logarithmic standard deviation are also shown. The logarithmic standard deviation by our proposed method is little smaller than that by the attenuation model. So, variation in other parameters such as hypocenter location is required to be considered. For example, stress drop for each element obtained by Equation (13) is required to be modified to simulate more realistic ground motion.

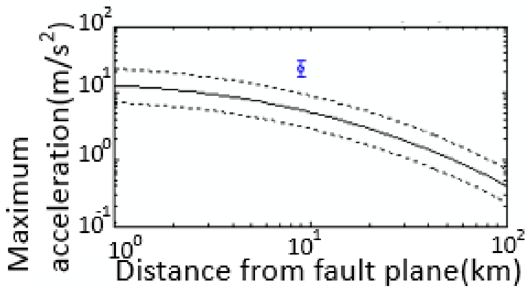

Figure 9: Comparison between maximum acceleration simulated and attenuation model
(○: *median*, **I**: *logarithmic standard deviation*)

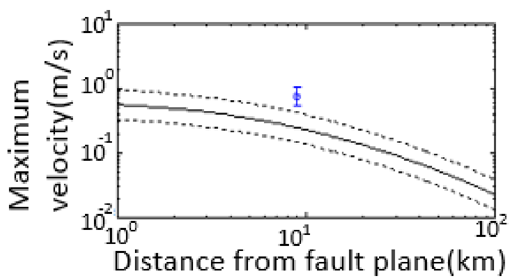

Figure 10: Comparison between maximum velocity simulated and attenuation model
(○: *median*, **I**: *logarithmic standard deviation*)

4. CONCLUSIONS

In this paper, a model to simulate the stochastic slip distribution of fault plane in crustal earthquake was discussed. It was demonstrated that the slip of the 2005 West Off Fukuoka Earthquake is log-normally distributed, and spatial correlation can be modeled using semivariogram. The seismic ground motion was simulated from a slip distribution of fault plane calculated by our proposed model. The simulated slip distribution was closer to real distribution than the characterized fault model. It needs, however, to be improved in the further study. One is that large slip area simulated by the proposed method scatters compared with the real slip distribution. Another issue to be solved is ground motion simulation method, e.g. how to determine stress drop for each element.

5. REFERENCES

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