Stochastic Dynamic Analysis of a Marine Riser using the First-Order Reliability Method

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ABSTRACT: The dynamic analysis of a deepwater floating production systems has many complexities, such as the dynamic coupling between the vessel and the riser, the coupling between the first-order and second-order wave forces, several sources of nonlinearities. These complexities can be captured by fully coupled time domain analyses. Moreover, the sea state is random, hence the need of stochastic dynamic analysis. In this paper the non-Gaussian responses of the system are obtained through the well-known First-Order Reliability Method (FORM) of the structural reliability analysis. The application to a simplified 2 degrees-of-freedom model shows the accuracy and effectiveness of the presented procedure.

1. INTRODUCTION

Floating production systems (FPSs) have become an integral part of deepwater development in oil and gas exploration and production. Risers, mooring system and floater represent an integrated dynamic system responding to environmental loading due to waves, current and wind in a complex way. The general principles for a dynamic analysis of risers are provided in the design code (DNV 2001). Recent research efforts (Ward et al. 1999) have shown that the mooring lines and risers can have a significant dynamic influence on the platform. Therefore, in full rigor, a "coupled analysis" of the vessel and of all the collected lines in the time domain should be held to take into account all the dynamic interactions within the system.

Moreover, the environmental loads are random, see DNV (2007), hence the need of stochastic dynamic analysis. The aim of the stochastic dynamic analysis is the evaluation of the response statistics of dynamic systems subjected to stochastic excitations. Unfortunately, their evaluation is a task relatively easy to be accomplished only when the input is Gaussian and the dynamic system is linear. This is not the case in FPS, since some forces follow a non-Gaussian distribution and the dynamic system is inherently nonlinear. Moreover, the computational cost becomes more and more demanding, especially in the context of reliability analyses (Ditlevsen and Madsen 1996, Melchers 1999) of the FPS. Indeed, we are interested in the design of structures with high reliability, which implies very small failure probabilities.

Define the tail probability as the probability that the response of the system is greater than a chosen threshold. The most robust approach is represented by the Monte Carlo Simulation (MCS), but it is too demanding for practical engineering purposes. Actually, it is known that for estimating failure probabilities $P_f = 10^{-4}$, we need to run roughly 1,000,000 analyses, to obtain an estimate of P_f with a coefficient of variation $\nu = 10\%$. To reduce the computational cost, some techniques of smart sampling have been recently proposed (Au and Beck 2001, Pradlwarter et al. 2007, Bucher 2009).

The First Order Reliability Method (FORM) gives a good tradeoff between accuracy and efficiency. Most applications of FORM in stochastic dynamic analysis have been quite promising (Der Kiureghian 2000, Koo et al. 2005, Alibrandi and Der Kiureghian 2012), including some applications in offshore engineering, like jack-up units, ship responses, offshore wind turbines (Jensen & Kapul 2006, Jensen 2007, 2011, Jensen et al. 2011).

In this paper, FORM is applied to a simplified model of FPS with 2 degrees of freedom (Low & Langley 2008) able to collect the main aspects of the dynamic interactions existing between the vessel and the lines. The presented numerical example allows to detect the main features of the method. Although the method is applied in this paper to a simplified model, of course it can be applied easily to a system with any number of finite elements and degrees of freedom as well. The main difference between the simplified model here presented and a more sophisticated one is simply the computational cost of each analysis of the complicated model.

2. DESCRIPTION OF THE SIMPLIFIED MODEL

The simplified model, presented by Low and Langley (2008), has been previously described in Alibrandi et al. (2014). In this paper the formulation is modified to adopt the FORM approach. The FPS is modeled as a system with 2 degrees of freedom, shown in Figure 1.

It comprises two effective lumped masses of the vessel m_V and of the lines m_L, with generalized displacements $x_V(t)$ and $x_L(t)$, where the effective masses include added masses contributions. The model does not consider the coupling between different rigid body motions of the vessel. Without loss of generality, $x_V(t)$ and $x_L(t)$ can be seen as the surge motion of the vessel, and the first mode of vibration of the lines, respectively. The restoring force given by the lines is in general nonlinear due to the geometrical changes of the lines, and it is well approximated through a geometric nonlinearity $F_{NL}(x) = k_1 x + k_3 x^3$ (Roberts and Spanos 1990).

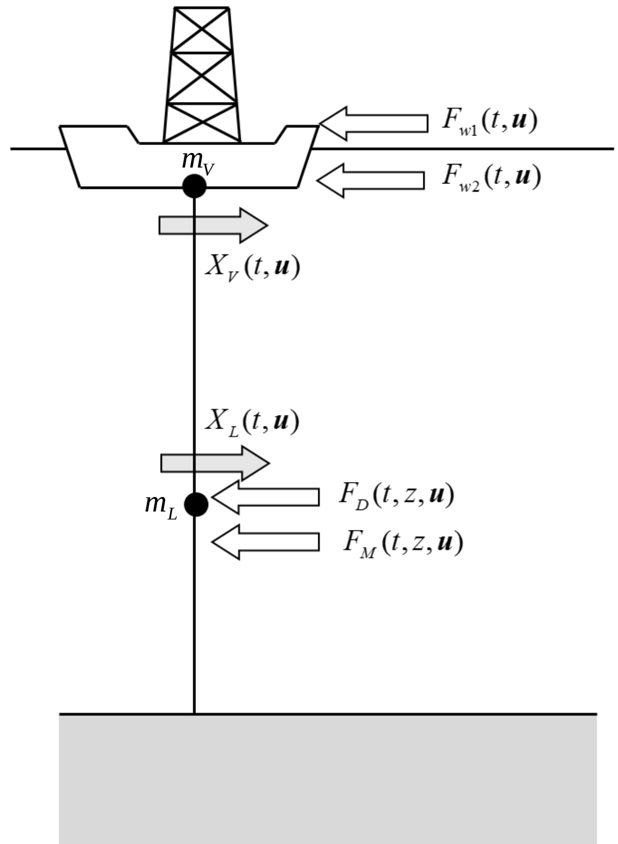

Figure 1: 2-dof simplified model of the marine riser

The lines are connected to a fixed boundary with an identical spring. The equations of motion of the system are:

$$\begin{cases} m_V \ddot{x}_V + c_V \dot{x}_V + F_{NL}(x_{VL}) = F_V \\ m_L \ddot{x}_L + c_L \dot{x}_L + F_{NL}(x_L) - F_{NL}(x_{VL}) = F_L \end{cases} \tag{1}$$

where $x_{VL} = x_V - x_L$, c_V and c_L are the damping of the vessel and lines respectively, k_1 and k_3 are the linear and non-linear contribution of the stiffnesses, respectively, $F_V(t)$ and $F_L(t)$ are the loads acting on the vessel and the lines, respectively. The structural damping of the lines is assumed equal to zero, $c_L = 0$ *N sec/m*, while the viscous damping is given by the drag term in the wave force, as described in section 2.2. The damping of the vessel c_V comes from several sources, such as viscous drag, radiation, aerodynamic, and so on. However, in presence of second-order surge motions, the wave drift damping is the most dominant and it can be evaluated using the formulation proposed by Aranha (1994). The other damping contributions are neglected because they are small.

2.1. Forces on the vessel

The time varying force on the vessel $F_V(t)$ is the sum of the following components:

$$F_V(t,\boldsymbol{u}) = F_w^{(1)}(t,\boldsymbol{u}) + F_w^{(2)}(t,\boldsymbol{u}) + F_{wind} + F_{curr} \quad (2)$$

where $\boldsymbol{u}$ is a vector of N normal standard random variables, $F_w^{(1)}(t,\boldsymbol{u})$ and $F_w^{(2)}(t,\boldsymbol{u})$ represent the first- and second-order wave force, respectively, F_{wind} is the wind force, and F_{curr} is the force of the currents. Wind and current are assumed to be constant and collinear.

2.1.1. Random seastate

The random sea state $\eta(t,\boldsymbol{u})$ is modelled as

$$\begin{aligned}\eta(t,\boldsymbol{u}) &= \sum_{i=1}^{n} \sigma_{\eta,i}\left[\cos(\omega_i t)u_i^c + \sin(\omega_i t)u_i^s\right] \\ &= \sum_{i=1}^{n} s_{\eta,i}^c(t)u_i^c + s_{\eta,i}^s(t)u_i^s = \boldsymbol{s}_\eta(t)\cdot\boldsymbol{u}\end{aligned} \quad (3)$$

where n is the number of harmonic components, $u_1^c, u_2^c, \ldots, u_n^c$ and $u_1^s, u_2^s, \ldots, u_n^s$ are normal standard random variables, while the correlation structure of $\eta(t,\boldsymbol{u})$ is given in terms of the underlying one-sided wave spectrum $G_{\eta\eta}(\omega)$, through the deterministic shape functions $s_{\eta,i}^c(t) = \sigma_{\eta,i}\cos(\omega_i t)$ and $s_{\eta,i}^s(t) = \sigma_{\eta,i}\sin(\omega_i t)$, being $\sigma_{\eta,i} = \sqrt{G_{\eta\eta}(\omega_i)\Delta\omega}$ and $\Delta\omega$ the frequency step. In the last term of Eq. (3) we have collected the $N = 2n$ normal standard random variables in the $2n$-vector $\boldsymbol{u} = \{\boldsymbol{u}^c \quad \boldsymbol{u}^s\}$, where $\boldsymbol{u}^c = \{u_1^c \quad \ldots \quad u_n^c\}$ and $\boldsymbol{u}^s = \{u_1^s \quad \ldots \quad u_n^s\}$ and the corresponding shape functions in the $2n$-vector $\boldsymbol{s}_\eta(t) = \{\boldsymbol{s}_\eta^c(t) \quad \boldsymbol{s}_\eta^s(t)\}$. Thus, the simulation of a time history of $\eta(t,\boldsymbol{u})$ requires simply the simulation of the $2n$ normal standard random variables collected in $\boldsymbol{u}$. Moreover, from Eq.(3) it follows that mean and variance of $\eta(t,\boldsymbol{u})$ are $\mu_\eta(t) = 0$ and $\sigma_\eta^2(t) = \left\|\boldsymbol{s}_\eta(t)\right\|^2$.

2.1.2. First-Order Wave Forces

The first-order wave forces $F_w^{(1)}(t,\boldsymbol{u})$ acting on the vessel may be expressed as

$$\begin{aligned}F_w^{(1)}(t,\boldsymbol{u}) &= \sum_{i=1}^{n} T^{(1)}(\omega_i)\eta(\omega_i,\boldsymbol{u}) = \\ &= \sum_{i=1}^{n} s_{F_1,i}^c(t)u_i^c + s_{F_1,i}^s(t)u_i^s = \boldsymbol{s}_{F_1}(t)\cdot\boldsymbol{u}\end{aligned} \quad (4)$$

where $T^{(1)}(\omega_i)$ are the first-order transfer functions and they are given by a linear diffraction analysis. From Eq.(4) it follows that the forces $F_w^{(1)}(t,\boldsymbol{u})$ are a Gaussian stochastic process, whose mean and variance are given as $\mu_{F_1}(t) = 0$ and $\sigma_{F_1}^2(t) = \left\|\boldsymbol{s}_{F_1}(t)\right\|^2$.

2.1.3. Second-Order Wave Forces

In addition to the first-order forces, the vessel is subjected to the second-order forces coming from nonlinear hydrodynamic effects (Faltinsen 1990). These forces are determined from a second order diffraction analysis, and typically only the slowly varying forces caused by difference frequencies are of interest

$$\begin{aligned}F_w^{(2)}(t,\boldsymbol{u}) &= \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{\eta,i}\sigma_{\eta,j} \times \\ &\times \left\{h_{ij}^c(\boldsymbol{u})\cos(\omega_i - \omega_j)t + h_{ij}^s(\boldsymbol{u})\sin(\omega_i - \omega_j)t\right\}\end{aligned} \quad (5)$$

where

$$\begin{cases} h_{ij}^c(\boldsymbol{u}) = T_{ij}^c\left(u_i^c u_j^c + u_i^s u_j^s\right) + T_{ij}^s\left(u_i^c u_j^s - u_i^s u_j^c\right) \\ h_{ij}^s(\boldsymbol{u}) = -T_{ij}^c\left(u_i^c u_j^s - u_i^s u_j^c\right) + T_{ij}^s\left(u_i^c u_j^c + u_i^s u_j^s\right) \end{cases} \quad (6)$$

being $T_{ij}^c = T^c(\omega_i, \omega_j)$ and $T_{ij}^s = T^s(\omega_i, \omega_j)$ the Quadratic Transfer Function (QTF). Typically they satisfy the symmetry conditions $T_{ij}^c = T_{ji}^c$ and $T_{ij}^s = -T_{ji}^s$. From (5) and (6) it is seen that the simulation of a sample of a slow-drift force require a very time-consuming double summation; in this work, the computational cost has been reduced adopting the Newman hypothesis (1974).

2.2. Forces on the lines

As proposed in Low and Langley (2008), the lines are modeled as N_L identical vertical cylinders, each of effective depth d_e and diameter D. Using the deep water approximation of classical linear wave theory (Faltinsen 1990), at a chosen depth z, the horizontal wave particle velocity $v_w(z,t,\boldsymbol{u})$ and acceleration $a_w(z,t,\boldsymbol{u})$ are given as

$$v_w(z,t,\boldsymbol{u}) = \sum_{i=1}^{n} s_{v,i}^c(z,t)u_i^c + s_{v,i}^s(z,t)u_i^s = \\ = \boldsymbol{s}_v(z,t)\cdot\boldsymbol{u} \tag{7}$$

$$a_w(z,t,\boldsymbol{u}) = \sum_{i=1}^{n} s_{a,i}^c(z,t)u_i^c + s_{a,i}^s(z,t)u_i^s = \\ = \boldsymbol{s}_a(z,t)\cdot\boldsymbol{u} \tag{8}$$

where the $2n-$ vectors $\boldsymbol{s}_v(z,t)$ and $\boldsymbol{s}_a(z,t)$ collect the deterministic shape functions $s_{v,i}^c(t)=\sigma_{v,i}\cos(\omega_i t)$ and $s_{v,i}^s(t)=\sigma_{v,i}\sin(\omega_i t)$, $s_{a,i}^c(t)=\sigma_{a,i}\cos(\omega_i t)$ and $s_{a,i}^s(t)=\sigma_{a,i}\sin(\omega_i t)$, being $\sigma_{v,i}=\omega_i e^{k_i z}$ and $\sigma_{a,i}=\omega_i^2 e^{k_i z}$ while $k_i=(\omega_i^2/g)$ is the $i-$th component of the wave number. A uniform current of velocity v_c is assumed to act at all the depths. The fluid force acting on the lines, $F_L(t,\boldsymbol{u})$, is then calculated using Morison's equation

$$F_L(t,\boldsymbol{u}) = F_D(t,\boldsymbol{u}) + F_M(t,\boldsymbol{u}) \tag{9}$$

where $F_D(t,\boldsymbol{u})$ and $F_M(t,\boldsymbol{u})$ are the drag and the inertia terms, respectively, and they are given as

$$F_D(t,\boldsymbol{u}) = \frac{1}{2}\rho_w C_D D N_L d_E\ r(t,\boldsymbol{u})\ \left|r(t,\boldsymbol{u})\right| \tag{10}$$

and

$$F_M(t,\boldsymbol{u}) = \rho_w \frac{\pi}{4} D^2 C_M N_L d_E\ a_w(z_L,t,\boldsymbol{u}) \tag{11}$$

with

$$r(t,\boldsymbol{u}) = v_c + v_w(z_L,t,\boldsymbol{u}) - \dot{x}_L(t,\boldsymbol{u}) \tag{12}$$

In (10) and (11) ρ_w is the density of the water, C_D and C_M are the drag and the inertia coefficient, respectively,while $v_w(z_L,t,\boldsymbol{u})$ and $a_w(z_L,t,\boldsymbol{u})$ are the velocity and the acceleration of the water particle at $z=z_L$, corresponding to the lumped mass of the lines. For further details about the choice of the mass m_L, the effective depth d_E and the depth z_L, the reader is referred to Low and Langley (2008).

3. FORM FOR STOCHASTIC DYNAMIC ANALYSIS

Consider the response of the FPS to the stochastic excitation. Owing to the random variables $\boldsymbol{u}$, the response is stochastic and we denote it as $X(t,\boldsymbol{u})$, where it can mean $X_V(t,\boldsymbol{u})$ or $X_L(t,\boldsymbol{u})$ for the vessel and the lines, respectively. For a specified threshold x_0 and time t, we define the tail probability as $P_f(t,x_0)=\text{Prob}[X(t,\boldsymbol{u})\geq x_0]$. To apply the tools of the structural reliability theory, we define the limit state function $g(t,x_0,\boldsymbol{u})=x_0-X(t,\boldsymbol{u})$ so that the failure probability with respect to the limit state $P_f(t,x_0)=\text{Prob}[g(t,x_0,\boldsymbol{u})\leq 0]$ is equal to the tail probability (Der Kiureghian 2000). For engineering design, we are interested in the first-passage probability over a duration $(0,T)$, defined as $\text{P}_f(x,T)$=Prob[$\max_{0\leq t\leq T} X(t,\boldsymbol{u})\geq x$]. A good approximation to the first passage probability is given by Prob[$\bigcup_{i=0}^{m}\{X(t_i,\boldsymbol{u})\geq x\}$], $t_i\in(0,T)$ (Au & Beck 2001, Koo & Der Kiureghian 2003, Fujimura & Der Kiureghian 2005). The union event represents a series-system reliability problem where the components of system are the tail probabilities evaluated for any set of discrete time points $t_1, t_2, \ldots, t_m$. The union event shows that the evaluation of the tail probability is a crucial step for obtaining other reliability measures of interest for stochastic dynamic analysis, and only the tail probability will be analyzed in detail in this paper.

In our case, we have a clearly nonlinear dynamic system: (*i*) the nonlinear behavior of the vessel and riser is modelled through a cubic geometric nonlinearity $F_{NL}(x)$, (*ii*) the second-order wave forces $F_w^{(2)}(t, \boldsymbol{u})$ are a non-Gaussian stochastic process, (*iii*) the drag forces $F_D(t, \boldsymbol{u})$ are a non-Gaussian stochastic process depending on the non-Gaussian response of the riser $\dot{x}_L(t, \boldsymbol{u})$ Consequently, the limit state is not linear, and the design point is given as the solution of the following constrained optimization problem

$$\boldsymbol{u}^*(t, x_0) = \arg\min\{\|\boldsymbol{u}\| : g(t, x_0, \boldsymbol{u}) = 0\} \quad (13)$$

being the design point the point belonging to the limit state closest to the origin of the standard normal space. The reliability index is given as $\beta(t, x_0) = \|\boldsymbol{u}^*(t, x_0)\|$ and the FORM solution $P_{f,FORM}(t, x_0) = \Phi[-\beta(t, x_0)]$ gives a first-order approximation of the tail probability.

The FORM solution has equation $g_{FORM}(t, x_0, \boldsymbol{u}) = x_0 - \boldsymbol{a}(t, x_0) \cdot \boldsymbol{u} = 0$, where $\boldsymbol{a}(t, x_0)$ gives the slope of the FORM hyperplane and it is related in correspondence one-to-one to the design point $\boldsymbol{u}^*(t, x_0)$ through the relationship $\boldsymbol{a}(t, x_0) = x_0 \cdot \boldsymbol{u}^*(t, x_0) \big/ \|\boldsymbol{u}^*(t, x_0)\|^2$ (Fujimura and Der Kiureghian 2007). It is also noted that with FORM the response is modeled for each threshold x_0 as Gaussian $X(t, x_0, \boldsymbol{u}) = \boldsymbol{a}(t, x_0) \cdot \boldsymbol{u}$ whose variance is $\|\boldsymbol{a}(t, x_0)\|^2$. Consequently, FORM gives a Non-Gaussian approximation of the system response, giving rise to an improved solution with respect to any Gaussian linearization technique, especially at the tails of the distribution.

4. NUMERICAL APPLICATION

The representative vessel used in this work is a large moored FPSO whose input parameters are represented in Table 1. The random sea state is described by a JONSWAP spectrum of parameters H_s, T_z and γ.

Table 1: Input parameters for the numerical application

Parameter	*Value*	*Parameter*	*Value*
Significant wave height H_s	*15.7 m*	*Drag coefficient C_D*	*2.0*
Average crossing period T_z	*13.5 s*	*Inertia Coefficient C_M*	*1.1*
Peakness parameter γ	*2*	*Depth z_L*	*-100 m*
F_{curr} + F_{wind}	*1.5×10^6 N*	*Mass of vessel m_V*	*1.17×10^8 kg*
Effective depth d_e	*667 m*	*Mass of lines m_L*	*5.00×10^6 kg*
Current velocity v_c	*0.5 m/s*	*k_1*	*2.8×10^6 N/m*
No. of cylinders N_L	*15*	*k_3*	*2,240 N/m*
Diameter of cylinder D	*0.13 m*	*Structural line damping B_L*	*0 N s/m*
Vessel damping B_V	*2.68×10^5 N s/m*		

It is seen that the system achieves the stationarity after $t = 1{,}400$ *sec*. The spectrum has been discretized from $\omega = 0$ *rad/sec* to $\omega = 0.8$ *rad/sec*, with a frequency step $\Delta\omega = (2\pi)/t = 0.00488$ *rad/sec*, leading to $n = 180$ harmonic components and $N = 2n = 360$ normal standard random variables.

At first we applied Monte Carlo Simulation (MCS) with 50,000 samples, which is adopted to benchmark the results of the FORM approach.

The main challenge with FORM is the evaluation of the design point. Indeed the gradient-based procedures (Liu and Der Kiureghian 1991) are not very efficient in high dimensions. Typically, the finite elements codes do not provide these gradients, which can be approximated by the finite differences method. This implies that each iteration of the iterative procedure require $N+1=361$ dynamic computations, so that the computational cost may be excessive. Moreover, in high-dimensional spaces the accuracy of the numerical response gradient is generally poor. These shortcomings may be overcome by using the Direct Differentiation Method (Zhang and Der Kiureghian 1993, Haukaas and Der Kiureghian 2004, 2006) which is implemented in some finite element codes such as OpenSees (McKenna *et al.* 2003). Here we determine the design point with a relatively low computational effort using a suitable free-derivative algorithm developed to this aim (Alibrandi 2014), which requires only 100 analyses per threshold. In Figures 2 and 3 we represent the tail probabilities of the vessel and the lines, respectively.

The response thresholds are normalized with respect to the standard deviation of the corresponding displacement. It is noted that the first threshold, corresponding to a reliability index $\beta=0$ and $P_f=0.5$, is greater than zero, because the system response is not zero-mean. In Figures 2 and 3 we represent the FORM solution (continuous line) together with MCS with 50,000 samples (circle markers) and a Gaussian approximation whose variance is evaluated through MCS (dotted line).

It is seen that the both displacement of the vessel and of the riser are markedly non-Gaussian; for the riser it is also noted that the Gaussian solution is from the unsafe side, so that for the example can be considered questionable the application of a classical equivalent linearization method for the highest thresholds.

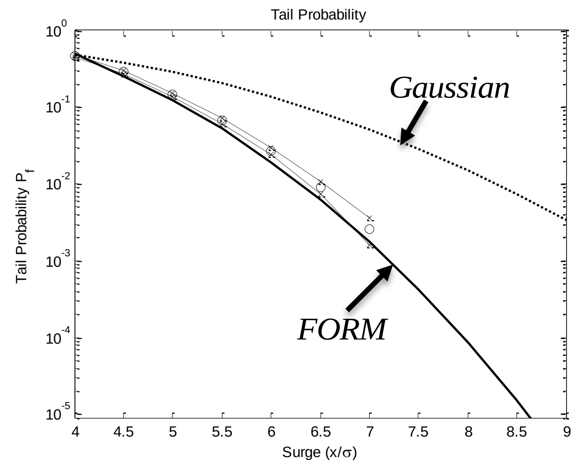

Figure 2: Tail Probability of the displacement of the vessel

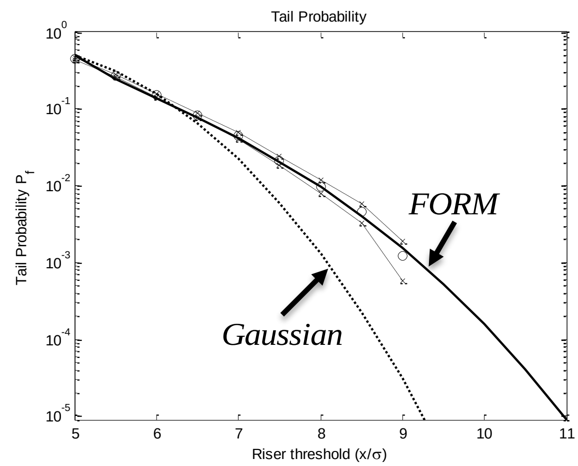

Figure 3: Tail Probability of the displacement of the riser

Conversely, FORM fits quite well, especially for the displacement of the lines. It is moreover underlined that FORM is very effective especially in the range of the very small small probabilities $P_f=10^{-4}\div10^{-5}$, which are the most crucial in a reliability analysis, where FORM requires 100 dynamic computations while MCS would require on average more than $10^6\div10^7$ analyses.

5. CONCLUSIONS

In this paper we applied the FORM approximation for Stochastic Dynamic Analysis of Floating Production Systems, including risers. All the stochastic quantities have been formulated in terms of normal standard random variables, so that the implementation of FORM is straightforward. The developed numerical application has shown that FORM allows to obtain with a reduced computational cost (of several orders of magnitude compared to MCS) very good approximations of the tail probability especially in the range of the very small probabilities, which are the most crucial for a reliability analysis. Although the method has been applied to a simplified 2 DOF model of the riser, it can be easily applied to an industry-size system modelled with finite elements or lumped masses as well. The only difference between the complicated model and the simplified one presented in this paper is the computational cost of each analysis. For design engineering purposes, a quantity of great interest is represented from the first passage probability, however starting from FORM solution it can be easily determined without no further dynamic computation (Fujimura & Der Kiureghian 2005, 2007).

However, FORM has some known drawbacks. As it is known, it does not good give good approximations in presence of multiple design points. Moreover, it has been recognized that in presence of a Duffing-type cubic geometric nonlinearity the failure domain may be disjoint (Katafygiotis & Zuev 2008, Valdebenito et al. 2010, Koduru & Der Kiureghian 2012), and in such cases the evaluation of the design point may be very challenging. This shortcoming has not been detected with the numerical application presented in this paper, where realistic data have been chosen. However, extended numerical experimentation applied to real-world engineering risers have to be developed before applying FORM as a black-box for this important class of systems.

6. ACKNOWLEDGMENTS

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