On the Value of SHM in the Context of Service Life Integrity Management

Jianjun Qin  
Assistant Professor, Shanghai Institute of Disaster Prevention and Relief, Tongji University, Shanghai, China.

Sebastian Thöns  
Associate Professor, Dept. of Civil Engineering, Technical University of Denmark, Lyngby, Denmark.

Michael H. Faber  
Professor, Dept. of Civil Engineering, Technical University of Denmark, Lyngby, Denmark.

ABSTRACT: This paper addresses the optimization of structural health monitoring (SHM) before its implementation on the basis of its Value of Information (VoI). The approach for the quantification of the value of SHM builds upon a service life cost assessment and generic structural performance model in conjunction with the observation, i.e. monitoring, of deterioration increments. The structural performance is described with a generic deterioration model to be calibrated to the relevant structural deterioration mechanism, such as e.g. fatigue and corrosion. The generic deterioration model allows for the incorporation of monitored damage increments and accounts for the precision of the data by considering the statistical uncertainties, i.e. the amount of monitoring data due to the monitoring period, and by considering the measurement uncertainty. The value of structural health monitoring is then quantified in the framework of the Bayesian pre-posterior decision theory as the difference between the expected service-life costs considering an optimal structural integrity management and the service life costs utilizing an optimal SHM system and structural integrity management. With an example the application of the approach is shown and the value of the monitoring period optimized SHM information is determined.

1. INTRODUCTION

The existence of uncertainties in the assessment of the system performance is one of the most important reasons for risk throughout the service life of engineered structures. Structural health monitoring (SHM) is one major means of collecting relevant information for the reduction of risks. Structural health monitoring has over the last 2-3 decades become a topic of significant interest within the structural engineering research community, but also in the broader areas of civil and mechanical engineering, see e.g. Doebling, et al., (1996), Staszewski, et al., (2004) and Providakis and Liarakos, (2014). Whereas the merits of health monitoring are generally appreciated in qualitative terms, and SHM as such forms a rather developed research area in itself, only more recently dedicated research on the quantification of the benefit of health monitoring – prior to its implementation - has been reported, see e.g. Pozzi and Kiureghian, (2011) and Thöns and Faber, (2013).

This paper addresses the optimization of SHM before its implementation for engineered structures on the basis of the Value of Information (VoI). The concept of VoI was introduced in 1960s, i.e. Raiffa and Schlaifer, (1961). Starting from this century, this concept is of great interest in the study of life-cycle decision making of engineered structures due to the rising concern of large-scale systems and the complex functional and statistical dependencies in the systems, see Straub and Faber (2005), Bayraktarli, et al., (2006) and Pozzi and Der Kiureghian, (2011) as examples. Straub and Faber, (2005) considers the risk based inspection (RBI) planning for engineering systems together with the discussion of various aspects of

In the following, the assessment of service-life costs in conjunction with the structural performance description is outlined (Section 2). Consecutively, the probabilistic models of structural performance considering its deterioration and the remedial actions, which is generalized for the convenience of discussion (Section 3), are integrated into the proposed theoretical framework of Vol from the Bayesian pre-posterior decision analysis (Section 4). A case study is presented to document the utilisation of this approach for the derivation of a monitoring time SHM system. Finally, conclusions in respect to the general model and the results of the case are drawn.

2. SERVICE-LIFE COST ASSESSMENT

An engineered structure with two states, i.e. “failure” and “no failure” is considered. It is assumed that one inspection can be planned within the service life of the structure, i.e. $T_s$. Depending on the inspection result at time $t_j$ (measured in number of years with $t_0 < t_j < T_s$, where $t_0$ is the starting time of the structure in use), the structure might be repaired or not. Note that actually the interval between inspection and the decision to repair could be any length, e.g. several days, months or years. For convenience and without loss of generality, the interval is set to one year in the following discussion.

The decision event tree utilized in the assessment of service-life cost is illustrated in Figure 1. In the figure, $C_{rep}$, $C_{insp}$ and $C_{fail}$ represent the cost of repair, inspection and failure (damage loss) at the end of the service life $T_s$, respectively. At the starting time of the structure in use, the expected service life costs with the plan that the inspection would be done at $t_j$ may be written as the function of $t_j$ as:

$$C_{SL}(t_j) = C_1 + C_2 + C_3 + C_4 + C_5 =$$

$$P_{S_j} C_{rep} \frac{1}{(1 + r)^{t_j}} + \sum_{i=1}^{T_s} P_{F_i} C_{fail} \frac{1}{(1 + r)^{t_i}} +$$

$$P_{IR_j} C_{rep} \frac{1}{(1 + r)^{t_j}} + \sum_{i=1}^{T_s} P_{F_i, IR_i} C_{fail} \frac{1}{(1 + r)^{t_i}} +$$

$$\sum_{i=1}^{T_s} P_{F_i, IR_i} C_{fail} \frac{1}{(1 + r)^{t_i}}$$

where $P_{S_j}$ and $P_{IR_j}$ are the probabilities of the event that the structure is in the state “no failure” and it needs repair at time $t_j$ respectively. $P_{F_i}$ is the probability of the event that the structure is in the state “failure” at time $t_i$ but in the “no failure” state at $t_i - 1$. $P_{F_i, IR_i}$ and $P_{F_i, IR_i}$ represent the probabilities of the event that the structure fails at $t_i$ given repair or given no repair at $t_j$. In Eq. (1), $r$ is the interest rate. The optimal inspection strategy could be defined as the strategy to minimize the expected service-life cost, which is identified by solving the following minimization problem:

$$C^* = \min_{t_j} C_{SL}(t_j)$$

Considering that we may have an SHM strategy, the Vol of monitoring can then be assessed as the difference between the expected costs defined in Eq.(2) and the expected costs taking SHM into account. This will be discussed later in detail.
3. PROBABILISTIC ANALYSIS OF STRUCTURAL PERFORMANCE

One key point in the optimization of the inspection strategy and the assessment of VoI of SHM is the assessment of structural performance for the calculation of the probabilities in the definition of the service-life costs, which are functions of the time \( t \). The performance of engineered structures can be represented through a time-dependent ultimate limit state function \( g(X,t) \) with the vector of random variables \( X \):

\[
g(X,t) = R_0 \theta_o (1 - D(t)) - z \theta_S S, \tag{3}
\]

where \( R_0 \) is the initial resistance and \( D(t) \) is a deterioration function. \( t \) is the time measured in number of years and \( z \) is a design parameter calibrated such that the probability of failure of the structure at time \( t = 1 \) is equal to some given value (e.g. \( 1 \times 10^{-5} \) for normal civil structures). The model uncertainties for deterioration and loading are denoted with \( \theta_o \) and \( \theta_S \), respectively. The model uncertainty for the resistance may be smaller than the model uncertainties for the loading and deterioration (see e.g. JCSS, (2006)) and is thus neglected for clarity.

The load process is represented through a vector of random variables \( (S_1, S_2, ..., S_{i-1}, S_i)^T \) representing the annual extreme loads during the service life \( T_s \) of the structure.

The deterioration function may be represented by a random process \( D(t) \) modelling the material deterioration during the service life. Various materials in conjunction with their exposures lead to different deterioration process models. For example, Qin and Faber, (2012) introduce the formulation of the probabilistic modeling of concrete chloride corrosion in the marine environment. Further works can be found in Schneider, et al., (2014); a detailed review of probabilistic modeling of concrete corrosion, chloride, and carbon dioxide corrosion is documented in DuraCrete, (2000). The probabilistic modeling of fatigue and corrosion degradation of steel structures can be found e.g. in Straub, (2004). Furthermore, soil liquefaction phenomena due to cyclic loading represent a degradation mechanism and has recently received much attention for wind turbine foundations, see e.g. Cuéllar, (2011). Despite the variety of the mechanisms, a general and generic formulation may be found. The deterioration could be regarded as an accumulation process with time:

\[
D(t) = \sum_{i=1}^{i} \Delta_{D,i}
\tag{4}
\]

with the annual increments \( \Delta_{D,i} \) having the same distribution with uncertain expected value \( M_{\mu_D} \) and constant standard deviation \( \sigma_{\Delta_D} \).

The event of failure \( F_i \) in year \( t_i \) is written as the following safety margin \( M_F(t_i) \):

\[
F_i = \{M_F(t_i) < 0\} = \left\{ R_0 \theta_o \left(1 - \sum_{i=1}^{i} \Delta_{D,i}\right) - z \theta_S S_i < 0 \right\} \tag{5}
\]

This hierarchical probabilistic model utilized in Eq. (5) is illustrated in Figure 2.

![Figure 2. Illustration of the probabilistic model utilized to model failure at time \( t \) without inspection and repair](image)

Further, it is assumed that one inspection can be planned within the service life. At the time of the inspection \( t_j \), a detection of damage and subsequent repair is undertaken if \( D(t_j) \) is not less than some critical value \( D_{CR} \). When the structure has been repaired, it performs as a new
one with the same probabilistic characteristics as originally but uncorrelated from these. The event of detection and repair at year $t_j$, i.e. $\text{IR}_{j}$, is written as:
\[ \text{IR}_{j} = \left\{ \sum_{k=1}^{j} \Delta_{D,k} \geq D_{IR} \right\} \]  
(6)

Then the event of failure at year $t_i$ after detection and repair at year $t_j$, i.e., $\text{IRF}_{i}$, is written as the following safety margin $M'_{F}$:
\[ F_{i, \text{IR}_{j}} = \{ M'_{F} < 0 \} = \left\{ R_{D} \theta_{D} \left( 1 - \sum_{k=1}^{t_i} \Delta_{D,k} \right) - z \theta_{S} S_{t} < 0 \right\} \]  
(7)

The hierarchical probabilistic model utilized in Eq.(7) is illustrated in Figure 3.

![Figure 3. Illustration of the probabilistic model utilized to model failure at $t_i$ after inspection and repair at $t_j$.](image)

In order to assess the expected value of the service-life costs, it is necessary to calculate the probabilities of failures, inspections and repairs in the situation before and after inspections as illustrated in Eq.(1). The five probabilities in the equation, namely the probability of survival up to the year $t_i$, $P_{S_i}$, the probability of survival up to and failure in year $t_i$, $P_{F_i}$, the probability of survival up to and detection and repair of damage in year $t_i$, $P_{IR_i}$, the probability of survival up to and detection and repair in year $t_j$ and subsequent survival up to and failure in year $t_i$, $P_{IR_j}$, and the probability of survival up to and no detection and repair in year $t_j$ and subsequent survival up to and failure in year $t_i$, $P_{IR_j}$, are written as:
\[ P_{S_i} = P \left( \bigcap_{k=1}^{t_i} F_{k} \right) \]  
(8)
\[ P_{F_i} = P \left( \bigcap_{k=1}^{t_i-1} F_{k} \bigcap F_{t_i} \right) \]  
(9)
\[ P_{IR_i} = P \left( \bigcap_{k=1}^{t_i-1} F_{k} \bigcap \text{IR}_{t_i} \bigcap F_{t_i} \bigcap \text{IR}_{t_j} \right) \]  
(10)
\[ P_{IR_j} = P \left( \bigcap_{k=1}^{t_j-1} F_{k} \bigcap \text{IR}_{t_j} \bigcap F_{t_j} \bigcap \text{IR}_{t_i} \right) \]  
(11)
\[ P_{IR_j} = P \left( \bigcap_{k=1}^{t_j-1} F_{k} \bigcap \text{IR}_{t_j} \bigcap F_{t_j} \bigcap \text{IR}_{t_i} \right) \]  
(12)

### 4. ASSESSMENT OF VOI FROM ANNUAL OBSERVATIONS OF THE DETERIORATION

Following the foregoing elaborations above, the optimal inspection time could be identified from the minimization of the service-life cost (in accordance with Eq.(2)). Now the next task concerns the assessment of the VoI which can be achieved from annual observations, i.e. SHM, of the deterioration.

It is assumed here that there is some possible choice to monitor, i.e. to observe or measure the annual deterioration increment $\Delta_{D,t}$, and the SHM results could be used as basis for updating the probability distribution function of the uncertain expected value of the annual deterioration increment $M_{\mu_0}$. It is further assumed that it is possible to monitor in any number of years (denoted with $t_{mon} = T_S - t_{mon,\alpha}$) starting from the beginning till the of the service life $T_S$. Then $t_{mon,\alpha}$ represents the starting time to monitor the annual deterioration increment and $t_{\alpha}$ is the time...
to inspect whether the total increments are over some critical value and repair is necessary or not. Each year after monitoring, the latest deterioration increment may be observed to update the probabilistic model for the annual deterioration increments (represented by \( M_{\mu_0} \)) and thereby to facilitate the identification of the optimal inspection and repair plan considering the residual service life.

The probability distribution function of \( M_{\mu_0} \) is updated using the monitoring results and remains normally distributed with posterior parameters:

\[
\mu_{M_{\mu_0}}(t_{\text{mon}}, \Delta_{D,\text{mon}}) = \left( \frac{1}{t_{\text{mon}}} + \frac{1}{n'} \right)^{-1} \left( \frac{1}{t_{\text{mon}}} \sum_{k=1}^{n} \Delta_{D,k} \right)
\]

\[
\sigma_{M_{\mu_0}}^2(t_{\text{mon}}, \Delta_{D,\text{mon}}) = \left( \frac{n'}{n'} + \frac{t_{\text{mon}}}{t_{\text{mon}}} \right)^{-1} \left( \frac{1}{t_{\text{mon}}} \sum_{k=1}^{n} \Delta_{D,k} \right)^2
\]

where \( \mu_{M_{\mu_0}} \) and \( \sigma_{M_{\mu_0}} \) are the updated (posterior) expected value and the standard deviation of \( M_{\mu_0} \), respectively, while \( \mu_{M_{\mu_0}} \) and \( \sigma_{M_{\mu_0}} \) are the original (prior) expected value and the standard deviation of deterioration. In the equations, \( t_{\text{mon}} \) is the number of samples or observations of annual deterioration increments made up until the chosen monitoring period \( t_{\text{mon}} \) and \( \Delta_{D,\text{mon}} = (\Delta_{D,1}, \Delta_{D,2}, \ldots, \Delta_{D,\text{mon}})^T \) are the corresponding observations. \( \Delta_{D,\text{mon}} \) is representing the uncertainty of the observations, i.e. the standard deviation of the \( t_{\text{mon}} \) samples caused by e.g. the measurement uncertainty.

Now, the service-life costs can be written as a function of the outcomes of the monitoring measurements \( \Delta_{D,\text{mon}} \), the chosen monitoring period \( t_{\text{mon}} \), and the time of the inspection \( t_j \):

\[
C_{SL,\text{mon}}(t_{\text{mon}}, \Delta_{D,\text{mon}}, t_j) = \sum_{i=1}^{n_{\text{mon}}} \sum_{i=1}^{t_{\text{mon}}} F_{\text{fail}} \frac{1}{(1+r)^j} + \left( 1 - \sum_{i=1}^{n_{\text{mon}}} F_{\text{fail}} \right) \times
\]

\[
\sum_{i=1}^{t_{\text{mon}}} \sum_{i=1}^{n_{\text{mon}}} F_{\text{fail}} \frac{1}{(1+r)^j} + \left( 1 - \sum_{i=1}^{t_{\text{mon}}} F_{\text{fail}} \right) \times
\]

\[
\sum_{i=1}^{t_{\text{mon}}} \sum_{i=1}^{n_{\text{mon}}} F_{\text{fail}} \frac{1}{(1+r)^j}
\]

where the \( P^* \) indicate the posterior probabilities together with the input of the updated uncertain expected value of the deterioration increments applied to the probabilistic modeling of all events subsequent to the end of the monitoring period \( t_{\text{mon}} \). These probabilities (corresponding to Eqs. (8)–(12)) are defined as follows:

\[
P_{\text{F}_{\mu_0}} = P \left( \bigcap_{k=1}^{n_{\text{mon}}} \bigcap_{i=1}^{t_{\text{mon}}} F_{\text{fail}}^k \bigcap_{j=1}^{t_{j}} F_j \right)
\]

\[
P_{\text{F}_{\mu_0}}^* = P \left( \bigcap_{k=1}^{n_{\text{mon}}} \bigcap_{i=1}^{t_{\text{mon}}} \bigcap_{j=1}^{t_{j}} F_j \right)
\]

\[
P_{\text{F}_{\mu_0}}^* = P \left( \bigcap_{j=1}^{t_{j}} F_j \right)
\]

\[
P_{\text{F}_{\mu_0}}^* = P \left( \bigcap_{k=1}^{n_{\text{mon}}} \bigcap_{i=1}^{t_{\text{mon}}} \bigcap_{j=1}^{t_{j}} F_j \right)
\]

In Eqs. (17)–(21), the events \( * \) correspond to the events of defined in Eqs.(5)–(7), but for which the random variable representing the uncertain expected value of the deterioration increments,
M_{\mu_{D}}^* (normal distribution with \mu_{M_{D}}^* and \sigma_{M_{D}}^* from Eqs. (13)-(14) as the mean value and the standard deviation respectively) is utilized.

The decision problem of optimizing the monitoring strategy is again defined as the minimization of the service-life cost, which may be formulated as:

\[
C^*_{\text{mon}} = \min_{t_{\text{mon}}} E_{M_{\mu_{D}}} \left[ \min_{t_j} C_{SL_{\text{mon}}}(t_{\text{mon}}, \Delta_{D_{\text{mon}}}, t_j) \right]
\]

(22)

where \(E_{M_{\mu_{D}}} \) represents the expected value in the bracket with the uncertain expected value of the deterioration increments \( M_{\mu_{D}} \). Now, the \( \text{VoI}_{\text{mon}} \), actually could be regarded as the expected benefit, can be defined by the difference between \( C^* \) and \( C^*_{\text{mon}} \):

\[
\text{VoI}_{\text{mon}} = C^* - C^*_{\text{mon}} = \min_{t_j} C_{SL}(t_j) - \min_{t_{\text{mon}}} E_{M_{\mu_{D}}} \left[ \min_{t_j} C_{SL_{\text{mon}}}(t_{\text{mon}}, \Delta_{D_{\text{mon}}}, t_j) \right]
\]

(23)

5. EXAMPLE
An illustrative application is presented to identify how SHM can be of value in a life cycle cost context. For the sake of a clear presentation, a simplistic case including most features of a real application is considered.

The structure has a service life of 50 years. The probabilistic characteristics of the random variables presented in the proposed approach are provided in Table 1. Note that the mean value and standard deviation of \( M_{\mu_{D}} \) listed in the table are adopted in the analysis of \( C^* \), while for the analysis of \( C^*_{\text{mon}} \), the mean value and standard deviation are calibrated with the input of the SHM results. The design parameter \( z \) is set to be 0.21 which results in a failure probability at the beginning of the service of 1.1x10^{-5}. The repair criterion parameter \( D_{R} \) (see Eq. (6)) is set to be 0.2. The values of the interest rate, the inspection cost and other cost relevant parameters are given in Table 2.

Table 1: probabilistic characteristics of the random variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_0 )</td>
<td>Lognormal</td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>( \theta_D )</td>
<td>Lognormal</td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>( \theta_S )</td>
<td>Lognormal</td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>( S_i )</td>
<td>Gumbel</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>( \Delta_{D_{\text{mon}}} )</td>
<td>Normal</td>
<td>( M_{\mu_{D}} )</td>
<td>0.1</td>
</tr>
<tr>
<td>( \mu_{\mu_{D}} )</td>
<td>Normal</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 2: values of the parameters in the cost calculation

<table>
<thead>
<tr>
<th>Variable</th>
<th>( R )</th>
<th>( C_{\text{rep}} )</th>
<th>( C_{\text{fail}} )</th>
<th>( C_{\text{rep}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.02</td>
<td>1</td>
<td>10</td>
<td>100</td>
</tr>
</tbody>
</table>

Monte Carlo simulations are adopted for the calculation of the service-life costs without annual observations of the deterioration, which are shown in Figure 4. From Figure 4, it can be found that \( C_{SL}(t_j) \) has minimal variation when the inspection time \( t_j \) is planned at the beginning of the service life when the structure is in the good state. However, when \( t_j \) becomes large, the cost first decreases and then increases with the increase of the inspection time. The value of \( C^* \) is 24.76 taking from the 19th year. The variation of different costs with the increase of \( t_j \) is different. The value of \( P_{t_j} C_{\text{rep}} / (1 + r)^t \) (expected inspection cost without any repair and failure at \( t_j \)) and \( \sum_{t_{j+1}}^{T_{f}} P_{t_j} C_{\text{rep}} / (1 + r)^t \) (expected damage loss in the remaining service life without repair at \( t_j \)) gradually decrease, while \( \sum_{t_{j+1}}^{T_{f}} P_{t_j} C_{\text{fail}} / (1 + r)^t \) (expected damage loss before the planned inspection at \( t_j \)) has the opposite trend. The value of \( \sum_{t_{j+1}}^{T_{f}} P_{t_j} C_{\text{rep}} / (1 + r)^t \) (expected damage loss in the remaining service life with repair at \( t_j \)) first increases and then decreases; again \( P_{t_j} C_{\text{rep}} / (1 + r)^t \) (expected repair cost at \( t_j \)) has the opposite trend.

Now, the annual deterioration increment is monitored and these SHM results are utilized to
update the probabilistic characteristics of $M_{\mu_0}$ and thus to modify the service-life costs. The value of $E_{M_{\mu_0}} \left[ \min_{t_i, t_j} C_{\text{SL,mon}}\left( t_{\text{mon},i}, \hat{D}_{\text{mon},i}, t_j \right) \right]$ in Eq. (22) with the variation of $t_{\text{mon}}$ is shown in Figure 5. It can be seen from the Figure 4 and Figure 5 that the variation of the two costs are similar. The value of $C^*_{\text{mon}} = 22.16$ is derived corresponding $t_{\text{mon}} = 31$ years ($t_{\text{mon},st}$ is 19). Then value of the SHM information is calculated to $\text{Vol}_{\text{mon}} = 2.6$ from Eq.(23).

6. CONCLUSIONS

This paper introduces an approach for the quantification of the value of SHM build upon a service-life cost assessment and a generic structural performance model in conjunction with SHM. The value of SHM is quantified in the framework of the Bayesian pre-posterior decision theory as the difference between the expected service-life costs considering an optimal structural integrity management and the expected service-life costs utilizing an optimal SHM strategy to support an optimal structural integrity management. It is demonstrated how the introduced approach can be applied to determine the optimal SHM operation period on the basis of the value of the information of the SHM strategy.

The developed generic deterioration model is has due to its generality the potential to be calibrated and applied to various structures exhibiting various degradation processes. It allows for monitoring of the damage increments and accounts for the precision of the data by considering the statistical uncertainties and the measurement uncertainty.

With the example, it is demonstrated that the value of the SHM information may vary significantly with the number of monitoring years as the costs for the structural integrity management vary significantly accounting for different monitoring periods.

7. ACKNOWLEDGEMENTS

The first author greatly acknowledges the support from the National Natural Science Foundation of China [grant number 51408438]; the Scientific Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry of the People’s Republic of China; and Shanghai Pujiang Program [grant number 14PJ1408300].

8. REFERENCES


