Reliability Based Design Optimization of Insulation Systems Considering Climate Change and Workmanship Uncertainties

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ABSTRACT: Today, the political and regulatory framework related to the building sector is more demanding as it remains the principal source of air pollution leading to global warming. Therefore, it is important to have a better control on the energetic performance of the building envelope in order to control the consumption. This paper presents a methodology allowing us to obtain optimal designs of the envelope considering different kinds of uncertainties mainly those related to climate changes, material deterioration and man-made defects. The time-dependent uncertainties are considered as random processes while the time-invariant variability related to defects is considered as random variable. The results show the impact of considering these uncertainties on the long term reliability of the system and give different optimal designs according to the considered climatic scenario.

The European Union has identified the building sector as a key area for achieving its obligations for greenhouse gas (GHGs) emission reductions as specified in the Kyoto protocol. This is addressed in the EU directives on the energy performance of buildings (EPBD) which form the basis for national regulations to be implemented in the member states (Jentsch 2008). The main objectives of these regulations are to set annual maximum energy consumptions per square meter that should be respected for all buildings to have a better control on the gas emissions and to reach the energetic independence of the country. However, the actual behavior of buildings does not always go in the way of the predictions. Mainly because all the uncertainties related to the environmental context are not considered.

The energetic performance of buildings is based on different simulation models that correspond to a simplification of reality (Manfren 2013). These models depend on three groups of data as detailed in the flowchart presented in Figure 1.

The first group of data is related to the physical properties of the materials constituting the building envelope. The second group concerns the data associated to the external random environment. Whereas the third group of data is related to the internal environment of the building depending on the behavior of the residents towards their use of the Building Combined Heating and Power systems (BCHP). This use differs from a person to another according to their proper comfort sensation.

In this framework, many studies used uncertainty analyses to estimate the accuracy of the energetic performance estimations, according to the accuracy of these three input data groups but also on the basis of the accuracy of the used simulation models (Spitz 2012, Hopfe 2011). For these reasons, we believe that the use of probabilistic approaches should be used for energetic simulations in order to take account of...
all the uncertainties related to the environmental and structural contexts.

Regarding the insulation design, the failure mode considered for reliability analysis can be defined by its non-conformity to standards. In other words, we look at the situation where the insulation system does not ensure anymore the standards in terms of energy consumption.

The following equation represents the limit state function that defines the failure mode:

\[ G(X,t) = E_{C_{\text{ref}}} - E_C(X,t) \]  

where \( X \) is the vector of random input variables, \( t \) describes the time, \( E_{C_{\text{ref}}} \) is the maximum energy consumption allowed by the regulations and \( E_C \) the annual energy consumption of the system at \( t \). The system is then considered non-conforming (failing) when \( G(x,t) \leq 0 \) whereas it complies to regulations when \( G(x,t) > 0 \).

In this paper, we mainly focus on the materials and climate change uncertainties. A time-variant reliability analysis is carried out to show the impact of these uncertainties on the conformity of the system during the operating time. Then, the insulation design that ensures the conformity of the system all over the operating time is set as optimal when it generates the minimum cost of the structure. Therefore, the organization of this paper will follow three principal sections. After a brief presentation of the different uncertainties related to climate changes and material properties, the results of the reliability analysis will be presented for an application on exterior walls and finally the results of the optimization analysis will be presented.

1. MODEL UNCERTAINTIES

1.1. Climate changes

In order to design the building services, the weather files used for the prediction of the building average energy requirements are generated from long-term measured data (Fatichi et al. 2011, Jentsch 2008). However, these files represent ‘typical years’ and deliberately exclude peak year conditions (Jentsch 2008).

In the literature, many authors express their unconfidence about these climate predictions for energy simulation purposes. Most of them pointed out the fact that the large uncertainties associated to the future performance of buildings are due to the changes of climate (Lisø 2006, Tian and De Wilde 2012). However, the different studies conducted to assess the future energy consumptions do not take into account this climatic variability (Yau and Hasbi 2013).

In its Fourth Assessments Report, the Intergovernmental Panel on Climate Change-IPCC (Parry 2007) speaks about four principal scenarios of possible climate changes as shown in Figure 2. These scenarios are associated to different variations of social, demographic, environmental, technological and economic possible progresses.

To provide robust information for decision-makers and risk managers, Dessai in his review, provides some examples on the necessity of considering the probabilities of climate change to determine the likelihood of drying and wetting conditions, which would better fit a risk assessment framework (Dessai 2004). In this case, we propose to model the variability of climate changes by providing a mathematical
idealization of the process governing its evolution by generating random processes.

![Figure 2: Different trends of the global average warming according to the IPCC (Parry 2007).](image)

Generally speaking, a stochastic or random process can be defined as a family of random variables indexed to time $t$. Actually, a random variable $X$ will associate to each $\omega \in \Omega$ a realization $X(\omega)$, whereas a stochastic process $\{X_t\}_{t \in T}$ associates to each $\omega$ a function or a time-dependent path $\{X_t(\omega)\}_{t \in T}$. Thereby, moving from random variables to stochastic processes returns to switch between a point analysis to a function analysis. The process autocorrelation is therefore an important property to be considered in time-variant reliability analysis.

The relevance of using stochastic processes in structural and environmental contexts comes from the fact that the natural phenomena depends on time (deterioration, seismic loads, climatic loads, etc.). In the literature, various studies can be found for stochastic processes when dealing with time-variant uncertainties (Bahn et al. 2008, Lin and Beck 2012), as it allows the consideration of the statistical properties and correlations among variables.

1.2. Envelope

Depending on the climatic conditions of the geographic region, a substantial share of energy goes to heat. These loads can be reduced by increasing the thermal performance of the building envelope.

The principal works found in the literature concerning the building envelope focus on determining the optimum insulation thicknesses (Al-Sanea et al. 2011, Bollaturk 2006), on the layers sequencing (Eben Saleh 1990, Bojic et al. 1997) and on the best insulation materials to use (Dominguez-Muñoz et al. 2010). Most of the works are based on deterministic assumptions and data from survey and experimental measurements. However, materials are subjected to different external sources that affect their internal properties. These sources affect more or less seriously according to: 1) the defects in the materials produced during the implementation process, 2) the different ways the materials have been conveyed and stored before use, and 3) the unavoidable measurements errors, random errors and the non-representativeness of sample data, etc. (Lu et al. 2013). In this paper, two groups of uncertainties are considered: the time dependent uncertainties related to aging and the uncertainty related to initial defects.

1.2.1. Physical properties of materials

The principal function of insulation is to provide high resistance to heat flows. In the building sector, the materials should be as low conductive as possible and the innovation race has already started. Indeed, more and more efficient insulation materials are proposed regularly in the construction market.

However, in reality the physical properties given by the manufacturers do not always fit the real values of those properties. This is related to the external factors that affect the material between $t_0$ related to production and $t_f$ related to the time of use.

Over time, these properties are also brought to deterioration mainly because of the water content that can be retained in the wall. Figure 3 shows the impact of water content on the thermal conductivity of different materials, while Figure 4 presents the different increasing uncertainties expected on the thermal conductivity for polystyrene and mineral wool over time.

Actually, the deterioration rates of the thermal conductivity and the associated...
variability depend on the type of materials and its ability of not retaining water over time. For instance, the expected variability of mineral wool is higher than the variability of polystyrene.

Figure 3: Impact of water content on the thermal conductivity of different materials (Cammerer, 1984).

Figure 4: Degradation of thermal conductivity of polystyrene and mineral wool.

1.2.2. Workmanship errors
This type of uncertainty is actually very difficult to estimate and is associated to the quality of the workers and their accuracy during the implementation stage. They are mostly due to the passage of wires and ducts routing and are of high importance as they can have a very bad impact on the thermal performance of the material.

In this way, experimental measurements have been performed to determine the variation on the thermal conductivity of defective samples. These experiments were done on samples of 30x30cm using the guarded hot plate apparatus. Thermography analyses permitted to detect and measure the differences of temperature in the different zones of the defective samples. Figure 5 displays these differences of temperatures in the case of an excavated sample of Polystyrene.

Figure 5: Impact of an excavation on the outdoor temperature obtained for a polystyrene specimen.

These observations allowed us to deduce the deviation of the thermal conductivity of defective samples compared to healthy samples. A statistical study highlighted then a measure of relative dispersion of the thermal conductivity to consider according to the type of defect.

For modeling purposes, we propose to consider the variability associated to this kind of uncertainty as constant all over the lifetime as the presence of the defect is permanent all over the lifetime of the insulation.

2. RELIABILITY ANALYSIS
As an application, a conventional sequencing of an external wall (Figure 6) is considered. This wall is affected by two excavations occurred
while trying to run wires in the polystyrene (Figure 5). The local climate considered is related to a French region with very hot summers and very cold winters (Clermont-Ferrand).

Figure 6. Conventional sequencing of an external insulated wall.

To evaluate the energetic performance of the multilayered wall, it is necessary to compute the heat flow that crosses the wall (Dombayci 2007):

\[
Q = 86400 \times DD \times \frac{1}{R}\]

(2)

where \(Q\) is the total heat losses, \(DD\) represents the degree-days (Kelvin) and \(R\) is the thermal resistance of the wall calculated with:

\[
R = \left( \frac{1}{h_i} + \frac{1}{h_c} + \frac{x_1}{k_1} + \frac{x_2}{k_2} + \cdots + \frac{x_n}{k_n} \right)
\]

(3)

with \(x_i\) the thicknesses associated to layers of the wall and \(k_j\) their relative thermal conductivities, whereas \(h_i\) and \(h_c\) are the convective and radiative heat transfer coefficients of the indoor and the outdoor surfaces respectively.

First, all the different uncertainties related to the insulation material are considered. Both variabilities related to deterioration and mankind errors are considered at the same time. The time variant variability related to the deterioration is defined by generating random processes. For each path, the initial thermal conductivity is a random variable according to the considered mankind errors, then all over the lifetime a family of random variables are generated using the Karhunen-Loève decomposition to consider the autocorrelation of the process, as the deterioration at \(t_{i+1}\) depends on the degree of deterioration at \(t_i\).

Figure 7 shows for 10 simulations, the evolution of the thermal conductivities paths of polystyrene all over the lifetime. We can clearly notice the large variation in initial conductivities related to the dispersion of the thermal conductivity due to mankind errors.

Figure 7: Different simulations of the deterioration path of polystyrene.

In case of data related to climate, an annual variability is considered associated to the degree-days used in the models. These degree-days are generally used to simplify calculations when estimating the annual energy consumption (Kaynakli 2012). We can talk about two kinds of degree days: those associated to heating and those associated to cooling. Actually, a base temperature is set according to the indoor comfort. When the outdoor temperature is lower than this base temperature, heating loads would be needed, whereas if the outside temperature is higher than the base temperature, cooling loads would be needed. The total number of annual heating and cooling degree-days is calculated as follows (Guan 2009):

\[
HDD = \sum_{i=1}^{n_i} (T_{base} - T_o(i))^+ \\
CDD = \sum_{i=1}^{n_i} (T_o(i) - T_{base})^+
\]

(4)

where \(T_{base}\) is the base temperature of the heated or cooled space environment (e.g. 18°C in France), \(T_o\) is the average outdoor temperature of the day \(i\), \(HS\) is the average duration of the heating, \(DS\) the average duration of cooling and \((\bullet)^+\) indicates that we account only for positive values.
In the present work, the climate change evolution is defined by a random process. Each path of the random process considers the changing degree-days over the lifetime, where each $DD$ at $t_{i+1}$ depends on the $DD$ at $t_i$. Figures 8 and 9 give different paths of the Heating-DD and the Cooling-DD of climate changes according to the variabilities associated to the pessimistic scenario A2. The random variables are defined by the statistical parameters presented in Table 2.

![Figure 8. Different evolutions of the Heating degree days over time by using random processes.](image)

![Figure 9. Different evolutions of the Cooling degreee days over time by using random processes.](image)

As indicated previously, the different climatic scenarios given by the IPCC describe the predictions of global warming with different intensities. In the case of the pessimistic scenario A2, the average world temperature is expected to increase highly, resulting in an increasing need of cooling loads and less heating loads.

The yearly energy consumption depends on all the input variables presented and generated earlier. The total energy consumption will be the sum of the consumption related to heating $E_{C,H}$ and to cooling system $E_{C,C}$ and are given as follows (Dombayci 2007):

$$E_{C,H} = \frac{86400 \ HDD}{R \ \eta_s}$$

$$E_{C,C} = \frac{86400 \ CDD}{R \ \text{COP}}$$

where $\eta_s$ represents the efficiency of the heating system and COP is the coefficient of performance of the cooling system. Table 1 summarizes the input data used in calculations and Table 2 presents the considered random variables. The values in these tables correspond to the average data in the French market.

### Table 1. Input parameters used for calculations.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials</td>
<td>Thicknessess $k_i$ (W/mK)</td>
</tr>
<tr>
<td>Hollow blocks</td>
<td>0.200 m 1.60</td>
</tr>
<tr>
<td>Internal plaster</td>
<td>0.014 m 0.32</td>
</tr>
<tr>
<td>External plaster</td>
<td>0.015 m 0.25</td>
</tr>
<tr>
<td>Expanded polystyrene</td>
<td>0.080 m 0.04</td>
</tr>
<tr>
<td>Max Consumption Ec ref</td>
<td>40 kWh/m²/year</td>
</tr>
<tr>
<td>Fuel</td>
<td>Electricity</td>
</tr>
<tr>
<td>$\eta_s$</td>
<td>0.6</td>
</tr>
<tr>
<td>COP</td>
<td>2.5</td>
</tr>
<tr>
<td>Life time ($L_T$)</td>
<td>40 years</td>
</tr>
<tr>
<td>Investment cost $C_{ins}$</td>
<td>10 €/m² for 10cm</td>
</tr>
<tr>
<td>Energy cost $C_e$</td>
<td>0, 1202 €/kWh</td>
</tr>
</tbody>
</table>

### Table 2. Statistical variables of the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>COV</th>
<th>Distribution law</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>Mankind: 12%</td>
<td>Normal</td>
</tr>
<tr>
<td></td>
<td>Deterioration:[1,20%]</td>
<td>Normal</td>
</tr>
<tr>
<td>HDD</td>
<td>A2: [5%, 44%]</td>
<td>Lognormal</td>
</tr>
<tr>
<td></td>
<td>B1: [5%, 38%]</td>
<td></td>
</tr>
<tr>
<td>CDD</td>
<td>A2:[5, 44%]</td>
<td>Lognormal</td>
</tr>
<tr>
<td></td>
<td>B1:[5, 38%]</td>
<td></td>
</tr>
</tbody>
</table>

3. RELIABILITY-BASED OPTIMIZATION
The reliability analysis allows the estimation of the reliability of the system along its lifetime. As said before, it will consist in verifying the conformity of the insulation system to standards
all over the operating time of the system, according to equation 1.

Figure 10 shows the evolution of the failure probability along the lifetime of the insulation in the case of pessimistic climatic scenarios A2. Monte-Carlo simulations were used to determine the failure probabilities when considering the input random variables. At $t_1$, it can be noted that the failure probability of the system is not null due to the uncertainty related to the workmanship errors. This failure probability $P_f$ increases significantly over the years according to the time-variant uncertainties.

When studying the other climatic scenarios, it is noted that the failure probability at $t_1$ are higher than $P_f$ at A2. This is mainly due to the fact that other scenarios are colder than A2 thus the thermal losses obtained because of the defects affect more the reliability of the insulation in a cold climate.

It is now proposed to use the failure probability as a constraint for optimization purpose. A Reliability-Based Design Optimization (RBDO) mode is defined as following (Aïssani 2014):

$$\begin{align*}
\min C_{tot}(x) &= C_{inv}(x) + \sum_{t=t_1}^{L_T} C_{energ}(x,t) \\
\text{under} \quad \sum_{t=t_1}^{L_T} P_f(x,t) &\leq P_{f0}
\end{align*} \quad (6)$$

where $C_{tot}$ is the total cost of the external wall considered as the investment cost $C_{inv}(x)$ and the sum of all the annual energy costs $C_{energ}(x,t)$.

The obtained thickness should verify the reliability constraint over the lifetime ($L_T$) by getting each time a failure probability that should be lower than a failure probability threshold ($P_{f0}$). $P_{f0}$ is set according to the building function and according to the designer judgment.

In conclusion, only the thicknesses that verify this constraint will be admissible and the one corresponding to the minimal total cost is considered as optimal. Table 3 presents the optimal thicknesses obtained for the climatic scenarios at two failure probability thresholds.

![Figure 10. Evolution of the failure probability of the wall (scenario A2).](image)

As can be seen, the optimal thicknesses depend not only on the failure probability threshold but also on the aggressiveness of the climate. It is to note that the optimal thicknesses obtained for B1 are higher than those obtained for A2 as the failure probability obtained for B1 is higher than for A2.

<table>
<thead>
<tr>
<th>Climatic scenario</th>
<th>Optimal thickness</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2 $P_{f0}=10^{-0.5}$</td>
<td>4.5 cm</td>
<td>1313.84 €/m²</td>
</tr>
<tr>
<td>A2 $P_{f0}=10^{-2}$</td>
<td>5.5 cm</td>
<td>1159.00 €/m²</td>
</tr>
<tr>
<td>B1 $P_{f0}=10^{-0.5}$</td>
<td>5.0 cm</td>
<td>1240.29 €/m²</td>
</tr>
<tr>
<td>B1 $P_{f0}=10^{-2}$</td>
<td>6.23 cm</td>
<td>1079.51 €/m²</td>
</tr>
</tbody>
</table>

This study shows also that the total cost of the system is higher when low reliabilities are required. An important reliability imposed during design provides then more benefits. This is an important result as it is easier to convince an owner to invest in better insulations if it ensures more benefits.

4. CONCLUSION

In energy performance simulations, the consideration of the uncertainties related to the performance model is mandatory for better control of the system over the long term.

In this paper, it has been proposed to consider the different kinds of uncertainties
which impact the optimal design according to the considered climatic scenario.

The main results show that the uncertainties related to workmanship errors can lead to the appearance of a initial failure probability that can be more or less important according to the considered climatic scenario. The study can be extended to uncertainties related to comfort providing that sufficient amount of data can be collected.

5. REFERENCES