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Reliability of Corroded Pipelines Accounting for System Effects

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ABSTRACT: The present paper addresses reliability analysis of corroding pipelines also with consideration of system reliability effects. The analysis is performed by utilization of an enhanced Monte-Carlo simulation method which has been found to be very efficient for quantification of system reliability in relation to multiple components with arbitrary correlation levels. Examples of application comprise systems with corrosion defects that are both independent and with quantification of the effect of correlation.

1. INTRODUCTION

Corrosion of ageing pipelines is found to be a significant problem for both onshore and subsea transportation systems. In order to determine the remaining time in service for such pipelines, it is required that both deterministic capacity models as well as associated probability models for the key parameters are established. Having obtained these models, a proper reliability analysis and risk assessment can be performed.

The importance of this issue is reflected by the large number of associated publications throughout the years. Some examples of probabilistic reliability analyses are found in the papers by Ahammed & Melchers (1996), Ahammed (1998), Caleyo et al. (2002), Lee et al (2006), Santosh et al (2006), Li et. al. (2009a), Li et al (2009b), Pandey (1998), Zhou (2011) and Hasan et al (2012). Examples of relevant historic and more recent design guidelines are respectively ANSI/ASME B31G (1980) and DNV (2010).

As discussed in more detail below, there are significant differences between the applied probabilistic models which are employed in the various reliability analyses that have been performed. This applies in particular to the statistical parameters which are employed for characterization of the internal pressure and the corrosion defects. Furthermore, in most cases a single corrosion defect is analyzed. Only brief comments are usually given with respect to the system modeling which would account for multiple defects.

In a few cases system analyses are performed by means of Ditlevsen bounds for estimation of failure probability for series systems (Melchers, 1999). In the present paper, system modeling is taken into account in a more direct way by means of efficient simulation methods.

2. METHOD FORSYSTEM RELIABILITY ANALYSIS

2.1. Basic formulation

The enhanced Monte Carlo (MC) based method for estimation of system reliability which is applied for the present study is further described in Naess et al. (2009). The aim of this method is to reduce computational cost while maintaining the advantages of crude MC simulation. This applies in particular when dealing with complex systems. The key idea is to exploit the regularity of the tail probabilities. Based on results obtained from small Monte-Carlo samples for moderate levels of reliability this regularity enables prediction of far tail failure probabilities.

The motivation behind this approach is that systems with multiple and complex failure modes or limit states are often exceedingly difficult to analyze using traditional methods of structural reliability. On the other hand, even if direct MC does not suffer from this problem, it is computationally heavy for small probabilities. This gives rise to the idea of sampling at relatively high probability levels and subsequently applying a statistical tail extrapolation. A related idea is presented by Bucher (2009).

The fundamentals of the Naess et al. (2009) method are as follows. A safety margin $M = G(X_1,.., X_n)$ expressed in terms of n basic variables, is extended to a parameterized class of safety margins using a scaling parameter λ ($0 \leq \lambda \leq 1$):

$$M(\lambda) = M - (1-\lambda)\mathrm{E}(M) \tag{1}$$

Under certain conditions related to the distributions of the basic random variables (Naess et al., 2009), the failure probability is then assumed to behave as follows:

$$p_f(\lambda) = \mathrm{Prob}\ (M(\lambda) \leq 0) \underset{\lambda\to 1}{\approx} q(\lambda)\exp\{-a(\lambda-b)^c\} \tag{2}$$

where the function $q(\lambda)$ is slowly varying compared with the exponential function $\exp\{-a(\lambda-b)^c\}$. In practice, $q(\lambda)$ is therefore replaced by a constant q for $\lambda > \lambda_0$ for a suitable choice of λ_0. Clearly, the relevant failure probability $p_f = p_f(1)$ can then be obtained from values of $p_f(\lambda)$ for $\lambda_0 < \lambda < 1$ if the parameters q, a, b, c have been determined. It is clearly easier to estimate the (larger) failure probabilities $p_f(\lambda)$ for $\lambda < 1$ than the target value itself, since they require fewer simulations to achieve the same level of accuracy. Fitting the parametric form $q\exp\{-a(\lambda-b)^c\}$ for $p_f(\lambda)$ to the estimated values would then provide an estimate of the target value by extrapolation. The viability of this approach is demonstrated by both analytical and numerical examples e.g. in Naess et al. (2009, 2010, 2012).

2.2. *System Reliability Analysis*

Using Monte Carlo methods for system reliability analysis has several attractive features, the most important being that the failure criterion is relatively easy to check almost irrespective of the complexity of the system. It is hence worthwhile to extend the above approach to systems reliability analysis.

Let $M_j = G_j(X_1,\dots,X_n)$, $j = 1,\dots,m,$ be a set of m given safety margins expressed in terms of n basic variables. Then the parameterized class of safety margins $M_j(\lambda) = M_j - (1-\lambda)E(M_j)$, $j = 1,\dots,m,$ is then introduced. For a given value of λ, the series system reliability expressed in terms of the failure probability can then be written as,

$$p_f(\lambda) = \mathrm{Prob}\big(\cup_{j=1}^{m}\{M_j(\lambda) \leq 0\}\big), \tag{3}$$

while for parallel systems,

$$p_f(\lambda) = \mathrm{Prob}\big(\cap_{j=1}^{m}\{M_j(\lambda) \leq 0\}\big). \tag{4}$$

In general, any system can be written as a series system of parallel subsystems. The failure probability would then be given as,

$$p_f(\lambda) = \mathrm{Prob}\left(\cup_{j=1}^{l}\ \cap_{i\,\epsilon C_j}\{M_i(\lambda) \leq 0\}\right) \tag{5}$$

Here, each C_j is a subset of $\{1,\dots,m\}$, for $j = 1,\dots,l$. The C_js denote the index sets defining the parallel subsystems. The assumption which is represented by Eq. (2) is then applied also for the system reliability analysis.

2.3. *Implementation*

The method to be described in this section is based on the assumption expressed by Eq. (2). For practical applications it is implemented in the following form:

$$p_f(\lambda) \approx q \cdot \exp\left\{-a(\lambda - b)^c\right\} \quad (6)$$

expressed in terms of four parameters q, a, b, c, for $\lambda_0 \leq \lambda \leq 1$, for a suitable value of λ_0. It is therefore necessary to identify a suitable range for λ so that the right hand side of Eq. (6) represents a fairly good approximation of $p_f(\lambda)$ for $\lambda \in [\lambda_0, 1]$.

For a sample of size N of the vector of basic random variables $\boldsymbol{X} = (X_1, \ldots, X_n)$, let $N_f(\lambda)$ denote the number of samples in the failure domain of the system. The corresponding estimate of the failure probability is then

$$\hat{p}_f(\lambda) = \frac{N_f(\lambda)}{N} \quad (7)$$

The coefficient of variation C_v of this estimator is

$$C_v\left(\hat{p}_f(\lambda)\right) = \sqrt{\frac{1-\hat{p}_f(\lambda)}{\hat{p}_f(\lambda)N}} \quad (8)$$

A fair approximation of the 95% confidence interval for the value $\hat{p}_f(\lambda)$ can hence also be obtained.

Assuming now that we have obtained empirical Monte Carlo estimates of the failure probability, the problem then becomes that of optimal use of the information available.

The problem of finding the optimal values of the parameters q, a, b, c is solved by optimizing the fit on the log level by minimizing the following mean square error function with respect to all four arguments (Naess and Gaidai, 2009),

$$\mathrm{F}(q,a,b,c) = \sum_{j=1}^{M} w_j(\log \hat{p}_f(\lambda_j) - \log q + a(\lambda_j - b)^c)^2 \quad (9)$$

where $0 < \lambda_1 < \cdots < \lambda_M < 1$ denotes the set of λ values where the failure probability is empirically estimated. The w_j denote weight factors that put more emphasis on the more reliable estimates.

A Levenberg–Marquardt least squares optimization method is applied for the present minimization. Although this method generally works well, it may be simplified by exploiting the structure of F. It is realized by scrutinizing Eq. (10) that if b and c are fixed, the optimization problem reduces to a standard weighted linear regression problem. That is, with both b and c fixed, the optimal values of a and $\log q$ are found using closed form weighted linear regression formulas in terms of w_j, $y_j = \log \hat{p}_f(\lambda_j)$ and $x_j = (\lambda_j - b)^c$, see Naess et. al. (2009) for further details.

3. EXAMPLES

3.1. General

An adequate reliability formulation for a corroding pipeline system requires that the time variation of both the internal pressure and the corrosion damage are taken into account. If the largest defect throughout the time period is applied in combination with the highest pressure during the same period, a conservative estimate of the failure probability for a single pipe cross-section will be the result, see e.g. Zhou (2011).

A convenient and quite accurate simplification is to apply the distribution of the annual maxima of the internal pressure (instead of the extreme pressure during the entire period). This distribution is then combined with the distribution of the corrosion damage during the same year (instead of the distribution of the largest defect during the period under consideration). This modeling can also be combined with the system reliability formulation to reflect the possibility that failure may occur at one specific out of a many different possible defects.

In this approach, the probability distribution of the total population of corrosion defects is applied for each of the cross-sections (instead of that for the single largest defect at a given point in time). Hence, the possibility of failure at a number of "competing" cross-sections is properly represented through the series system model. This formulation is applied as the basis for the numerical studies in the present paper.

In many of the reliability studies found in the literature, the probability distribution of the instantaneous (normal) operation pressure is applied instead of the distribution for the extreme pressure during a specified time interval. The reason for this is a bit unclear, and it also creates difficulties when trying to establish a reference time period for the calculated probability of failure (unless some characteristic short-term duration of the pressure is specified, i.e. of the order of minutes or hours)

In the present paper, the probability distribution of the operation pressure is taken to correspond to the annual extreme pressure. The corresponding failure probabilities will accordingly also refer to a one-year period. Representation of the annual maximum pressure should then be based on the maximum instantaneous pressure (MAIP) which in general is higher than the maximum allowable operation pressure (MAOP). The mean value of the former pressure is typically given by means of a bias factor times the latter, typically with the MAIP being expressed as 1.05 or 1.07 times the MAOP. The effect of taking the time-dependent variability of the internal pressure into account is considered e.g. by Zhou (2011).

The failure function which is presently applied for a pipeline cross-section where a corrosion defect is present is expressed as follows:

$$g(X) = \left\{ 2 \cdot m_f s_y \left(\frac{t}{D}\right) \left(\frac{1-\left(\frac{d}{t}\right)}{1-\left(\frac{d}{tF}\right)} \right) - p_a \right\} \qquad (10)$$

Here, s_y is the yield stress of the steel material and m_f is a multiplying factor which converts yield stress into flow stress. This expression is based on a rectangular or nearly rectangular defect shape, with a defect depth of d and a defect length of L. The pipe wall thickness is designated by t and the pipe diameter by D. The quantity p_a is the internal overpressure that is acting at the relevant cross-section.

The quantity F is the Folias factor (also known as the bulging factor). The expressions which are applied for the Folias factor are as follows, Kiefner and Vieth (1990):

$$F = \left(1 + 0.6275\frac{L^2}{Dt} - 0.003375\frac{L^4}{D^2 t^2}\right)^{\frac{1}{2}} \quad \text{for } \frac{L^2}{Dt} \le 50$$

$$F = \left(0.032\frac{L^2}{Dt} + 3.3\right) \quad \text{for } \frac{L^2}{Dt} > 50 \qquad (11)$$

For pipeline steels, the value of the conversion factor above, i.e. m_f, is usually taken to be between 1.10 and 1.15. This shows that there is also uncertainty in the value of m_f. In order to take this uncertainty into consideration, m_f, is represented as a random variable.

It is referred to Ahammed & Melchers (1996) as well as ANSI/ASME B31G (1984) and Kiefner & Vieth (1990) for further details associated with the present formulation. Clearly, it would be relevant to introduce an additional model uncertainty factor to reflect differences between observed versus calculated pressure levels at failure. However, this topic has not been pursued in the present study due the illustrative scope of the computed results.

A refined reliability analysis should distinguish between different types of failure modes with different consequences. This is discussed in more detail by Zhou (2011). Distinction is made between small leaks, large leaks and burst. Clearly, the latter will represent the failure mode with the worst consequence. In the present paper a more simplistic approach with only the single type of failure mode (which is defined by the failure function given above) is applied. Hence, the effect of a more refined subdivision will need to be studied as part of future investigations, which can also be accommodated in the proposed method.

3.2. Single corrosion defect.

In order to illustrate the above concepts, a pipeline with a known probability distribution for the corrosion defects is analysed. This reflects the situation where e.g. defect dimensions are known from measurements by 'intelligent' inspection tools. The random variables considered in this study are presented in Table 1 along with the typical statistical values and distribution functions for the variables. The distribution functions and statistical values are mainly based on those found in Ahammed and Melchers (1996).

In the present paper, the intention is primarily to illustrate how the present simulation procedure can be applied for the analysis of pipeline corrosion reliability. The applied probability distributions should in general hence be adapted to the available information for a specific application.

Table 1. Random variables and statistical properties

Symb.	Variable Desc.	Distr.*	Properties	
			Mean	C.o.V
d	Defect depth	N	5mm	0.10
D	Pipe diameter	N	610mm	0.03
L	Defect length	N	305mm	0.05
m_f	Multipl. factor	LN	1.1	0.05
p	Fluid pressure	N	5-8 MPa	0.05
s_y	Material yield str.	LN	425 MPa	0.056
t	Wall thickness	LN	12-5 mm	0.05

* N: Normal (Gaussian)
LN: Lognormal

In the present study, the effect of applying different types of distributions for the internal pressure is investigated. The base case analysis corresponds to the pressure having a Gaussian distribution as indicated by the letter N in Table 1. In addition, results for a pressure distribution corresponding to the Gumbel distribution are also computed as a sensitivity study below.

The case with a Gaussian pressure distribution and a single corrosion defect is first considered. Counting of the observations with negative values of the failure function is performed for each value of the parameter lambda.

The simulated results for different values of lambda together with extrapolation to the relevant probability level are shown in Figure 1. It is seen that the failure probability which corresponds to $\lambda = 1$ lies in the interval from 0.5 10^{-5} to 3.0 10^{-5} with the best estimate being 1.3 10^{-5}. It is noted that this probability is somewhat lower than the value which was obtained based on performing a Taylor series expansion of the failure function around the mean point, see Ahammed & Melchers (1996).

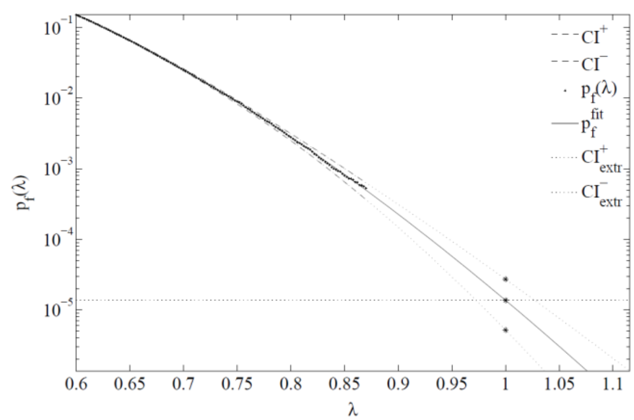

Figure 1. Extrapolation of failure probability from sampled values to relevant failure probability level. Mean value of internal pressure is 7 MPa.

The effects of varying the mean value of the pressure and the defect depth are next investigated. The results for different mean values of the pressure (from 5 MPa and upwards) and for different mean values of the defect depth are shown in Figure 2. Not unexpectedly, it is seen that the combination of low internal pressure and small defect depths give very high reliability levels (i.e. reliability indices of the order of 3 and above). A high internal pressure and large defects give very low reliability levels (with failure being an almost certain event).

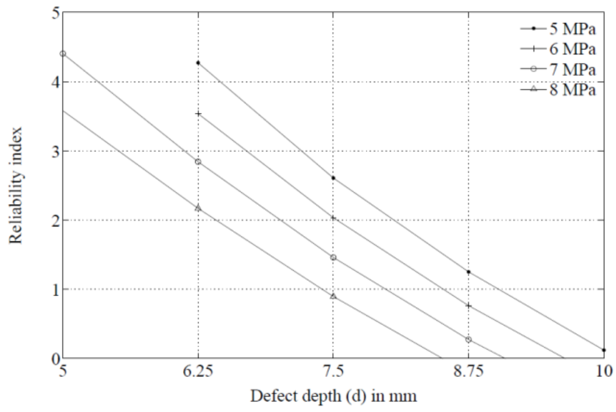

Figure 2. Reliability index, β , as a function of mean values of internal pressure and defect depth. Pressure distribution is Gaussian. (Failure probability can be obtained as Φ(-β))

3.3 Multiple corrosion defects. The effects of model uncertainty and of correlation.

3.3.1 Effect of number of independent defects

Instead of applying the extreme value distribution for the defect characteristics corresponding to a certain number of defects, it is more correct to apply a system representation where the defect depth and length instead are random variables for each component in the system.

First a series system is analysed which represents 100 pipe cross-sections for which the defect characteristics are independent. The obtained results (which are based on a simulation sample size of 1000 000) are shown in Table 2. It is seen that for the two highest levels of the failure probabilities the results are around twice the values in Table 4. The reason is that failure now has the potential to occur at several additional cross-sections (i.e. there is a number of competing failure components).

Table 2 Failure probability for system of 100 cross-sections where defects are present.

Mean pressure	Failure prob.	Unc. interval
6 MPa	1.5E-5	7.6E-6
7 MPa	9.6E-4	6.1E-5
8 Mpa	2.1E-2	2.8E-4

The series system effect is illustrated by the curves in Figure 3. The lowermost (red) curve corresponds to results for a single defect, while the uppermost (green) curve corresponds to 100 defects and with a mean value for the pressure of 7 MPa.

Since the coefficient of variation for the internal pressure is quite low (i.e. 0.05) the results for the case where the correlation between the failure events is taken into account are quite similar to the case with independent components. This is the reason for the almost identical uppermost curves (blue curve: correlated events, green curve: independent events).

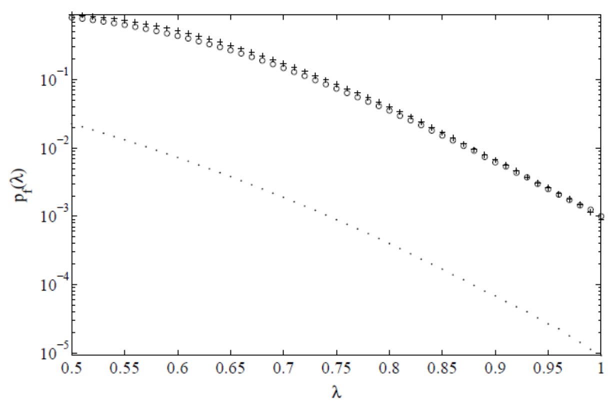

Figure 3. Failure probability of single defect (lowermost curve) versus 100 independent defects (uppermost curve with crossed tick-marks). (Mean value of internal pressure is 7 MPa.)

The number of cross-sections (and accordingly the number of independent defects) is subsequently increased from 100 to 1000 with the sample size being the same as before (i.e. 1000 000). The resulting failure probabilities are found in Table 3. Roughly, the failure probabilities are increased by a factor of 10 except for the highest probability level for which the increase is somewhat less (i.e. a factor of 7).

Table 3 Failure probability for system of 1000 cross-sections where defects are present.

Mean pressure	Failure prob.	Unc. interval
6 MPa	1.5E-4	2.4E-5
7 MPa	8.1E-3	1.8E-4
8 Mpa	1.4E-1	6.8E-4

3.3.2 Effect of including model uncertainty

A model uncertainty factor is next introduced for the capacity term (i.e. the first term) in Eq. (10). A Gaussian model with a mean value of 1.0 and a standard deviation of 0.10 is applied. The corresponding updated failure probabilities for 100 cross-sections are given in Table 4 as a function of the mean pressure.

Table 4 Failure probability for 100 cross-sections with model uncertainty factor included.

Mean pressure	Failure prob.	Unc. interval
6 MPa	1.2E-3	6.7E-5
7 MPa	2.2E-2	2.9E-4
8 MPa	1.8E-1	7.6E-4

Corresponding results for 1000 cross-sections are given in Table 5. As observed, the failure probabilities are increased significantly. This applies in particular to the lowest probability values. For a mean pressure level of 6 MPa, an increase by almost a factor of 100 is observed both for 100 and for 1000 cross-sections.

Table 5 Failure probability for 1000 cross-sections with model uncertainty factor included.

Mean pressure	Failure prob.	Unc. interval
6 MPa	1.1E-2	2.0E-4
7 MPa	1.7E-1	7.3E-4
8 MPa	7.4E-1	8.6E-4

3.3.3 Effect of correlation between defect dimensions

Increasing correlation between the defect dimensions is next introduced for the case with 1000 cross-sections with defects. The pairwise correlations between the defect length and the defect depth are both varied from 0 to 1 at a number of discrete intervals. Combinations of results for the three levels 0, 0.5 and 1.0 are given in Table 6 for a mean pressure level of 6 MPa.

It is observed that the failure probability is reduced by a factor of around 2 when moving from the upper left corner to the lower right corner for both tables. A rather low sensitivity of the computed failure probability is observed with respect to correlation between the defect lengths. The non-uniform trend observed from the tables (i.e. that the failure probability increases slightly for increasing correlation for some of the entries in the table) is assumed to be an artificial effect due to statistical sampling variability.

Table 6 Failure probability for system of 1000 cross-sections where defects are present which have correlated dimensions. Mean value of pressure is 6 MPa.

Failure prob.		Correlation for defect depth		
		0.	0.5	1.0
Defect length corr.	0.	1.5E-4	1.0E-5	1.8E-4
	0.5	1.8E-4	3.0E-5	2.0E-4
	1.0	1.6E-4	2.0E-5	8.0E-5

The effect of increasing the correlation between the individual failure events for the case with 100 defects is investigated next. This is now accounted for by instead increasing the coefficient of variation for the internal pressure. The results are shown in Figure 5 for a C.o.V. in the range from 0.05 to 0.2. The corresponding pairwise correlation between the "failure events" for the cross-sections ranges from 0.04 for the lowest value to 0.4 for the highest value of the pressure C.o.V.

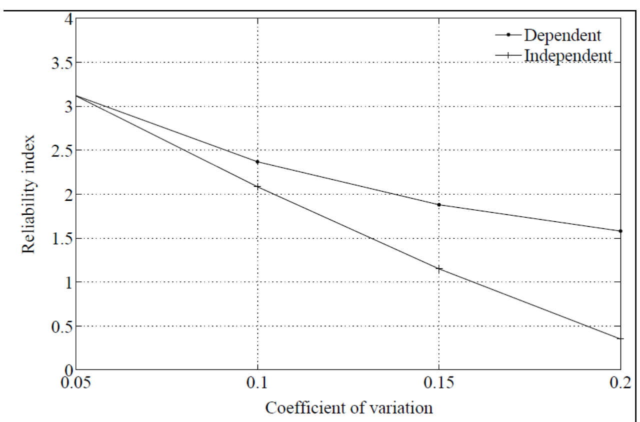

Figure 5. Reliability index for a system with 100 independent corrosion defects as function of coefficient of variation for (fully correlated) internal pressure. Independent (lowermost curve) versus dependent (uppermost curve) failure events. Correlation between "failure events" is equal to 0.04 for a pressure C.o.V. of 0.05 and to 0.42 for a pressure C.o.V of 0.2. (Mean value of internal pressure is 7 MPa)

7. CONCLUSIONS

The present paper deals with application of an enhanced Monte Carlo simulation method to reliability analysis of pipeline systems with multiple corrosion defects. The presence of such multiple defects requires that a structural system reliability analysis is performed.

The effect of correlation between the failure events is highlighted. As expected, the failure probability decreases strongly as the degree of correlation increases. However, the degree of correlation needs to be higher than around 0.2 before this effect starts to have any influence. For this type of correlation the failure probability can decrease by several orders of magnitude once the correlation starts to be significant (cfr. Figure 5).

Furthermore, it appears that for low values of the correlation coefficient the system failure probability can be obtained simply by multiplying the failure probability for a single defect (based on the ensemble probability distribution for the dimensions of the corrosion defects) with the total number of defects (cfr. the results in Table 5 and Table 6).

As mentioned, the present study is mainly intended as a basic illustration of how systems reliability effects need to accounted for in relation to corroded pipelines. Among multiple topics to be addressed by future work, computation of failure probability levels for different types of failure modes represents one particular example.

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