# Topology Optimization for Buildings in Seismic Zones within a PBEE Framework

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ABSTRACT: This paper presents a probabilistic performance-based topology optimization framework for the conceptual design of uncertain building systems in seismic zones. The stochastic nature of the ground motions is rigorously considered in a simulation-based probabilistic performance assessment framework that allows for the definition of a novel decoupling technique that efficiently separates the probabilistic analysis from the optimization loop. In particular, the methodology is based on the construction of a series of approximate sub-problems with simplified governing equations, which conveniently allows their evaluation using established techniques for static, deterministic topology optimization problems. The applicability and efficiency of the method is demonstrated on a case study.

## 1. INTRODUCTION

Topology optimization techniques have recently been recognized as a powerful tool for conceptual building design (Kareem et al., 2013; Bobby et al., 2014). Although originally developed in a deterministic setting, it is necessary to rigorously consider the effect of the uncertainties affecting the problem (e.g. stochastic loading conditions, uncertainties in structural parameters, model idealization) to ensure satisfactory structural performance as these techniques essentially obtain efficient designs by eliminating redundancies and pushing designs to their limiting capacities. Furthermore, it is of interest to formulate the problem within a setting that is already acceptable to the building design community for straightforward integration into the structural design process. Probabilistic Performance-Based Design (PPBD) techniques have recently gained momentum in the structural engineering community for the design of structures within a rigorous probabilistic framework, and therefore it is of interest to frame the topology optimization methodology within the setting of PPBD.

Within the context of topology optimization, Reliability-Based Topology Optimization (RBTO) methods are capable of explicitly accounting for the effect of uncertainties during the optimization process using constraints written in terms of acceptable failure probabilities. The majority of these methods have focused on including time-invariant uncertainties in the topology optimization problem in an efficient manner, while few methods have thus far been developed for the incorporation of stochastic loading. Chun et al. (2013) accounts for stochastic seismic excitation using the instantaneous failure probability as opposed to the more complex first excursion probability, which is typically used to describe the reliability of structures. Although the method described in Bobby et al. (2014) does consider the first excursion probability, the structural performance is described using fragility constraints (conditional failure probabilities), which are less complete than a full probabilistic analysis. Furthermore, only stationary stochastic excitation, i.e. wind loading, was considered. Thus there is a need for a framework able to consider non-stationary excitation developed in a full PPBD setting.

This research will present a framework for the topology optimization of buildings in seismically active zones that rigorously accounts for the stochastic nature of the ground accelerations and the dynamic nature of the structural response within a PPBD setting. An efficient decoupling method is presented in which the topology optimization and probabilistic performance-based analysis are decoupled through the definition of a number of approximate sub-problems. Examples are presented in order to illustrate the proposed methodology.

## 2. PERFORMANCE-BASED DESIGN

The framework presented in this paper performs topology optimization within the PPBD setting proposed by the PEER Center (Porter, 2003). The particular focus of this research is the estimate and control of non-structural damage. It is therefore of interest to use the PEER methodology to estimate the mean annual rate of exceedance of a damage measure as follows:

$$\lambda(dm) = \int_{edp}\int_{im} G(dm|edp)\cdot|\mathrm{d}G(edp|im)| \cdot|\mathrm{d}\lambda(im)| \tag{1}$$

where $\lambda(a)$ is the mean annual rate of exceedance of event $A=a$, where capital letters indicate variables and lower case letters indicate their realizations, $G(a|b)$ denotes the complementary cumulative distribution function of random variable $A$ given $B=b$, $dm$ denotes a damage measure indicating damage to non-structural components, $edp$ is an engineering demand parameter characterizing a structural response, $im$ is the measure of the intensity of the event, and $|\mathrm{d}\lambda(im)|$ can be interpreted as $\nu|\mathrm{d}G(im)| = \nu\cdot f(im)\mathrm{d}im$ where $f(a)$ is the probability density function of $a$ and $\nu$ is the mean annual rate of arrival of significant events (Der Kiureghian, 2005). The various terms that appear in Eq. (1) can be obtained through separate analyses: hazard analysis for $\lambda(im)$, structural analysis for $G(edp|im)$, and damage analysis for $G(dm|edp)$.

## 3. THE PERFORMANCE-BASED TOPOLOGY OPTIMIZATION PROBLEM

The probabilistic performance-based topology optimization problem of interest considers probabilistic performance constraints written in terms of $\lambda(dm)$. Assuming a design domain that is discretized using finite elements, the topology optimization problem of interest may be written as:

$$\begin{aligned} \min_{\rho}\ & V(\rho) = \sum_{e=1}^{n}\int_{\Omega_e}\rho_e \mathrm{d}\Omega \\ \text{s.t.}\ & \lambda(dm_j) - \lambda_{0_j} \le 0,\ j=1,\ldots,N \\ & \mathbf{M}(\rho,\mathbf{U})\ddot{\mathbf{z}}(t) + \mathbf{C}(\rho,\mathbf{U})\dot{\mathbf{z}}(t) \\ & \quad + \mathbf{K}(\rho,\mathbf{U})\mathbf{z}(t) = \mathbf{f}(t;\mathbf{U},\mathbf{W}) \\ & 0 \le \rho_e \le 1,\ e=1,...,n \end{aligned} \tag{2}$$

where $\rho = \{\rho_1,\ldots,\rho_n\}^T$ is the element-wise normalized material density design variable vector, $n$ is the total number of elements in the discretized structure, $\Omega_e$ denotes the domain of element $e$, $V$ is the volume of material in the design domain, $dm_j$ and $\lambda_{0_j}$ are the damage threshold and target mean annual exceedance rate for the $j$th probabilistic constraint, respectively, $N$ is the total number of performance constraints, $\mathbf{U}$ is a vector containing the uncertain model parameters, $\mathbf{z}$ denotes the displacement vector, $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ denote the mass, damping, and stiffness matrices, respectively, and $\mathbf{f}$ is the seismic loading vector that depends on $\mathbf{U}$ as well as a white-noise stochastic process, $\mathbf{W}$.

## 4. PROBABILISTIC PERFORMANCE ASSESSMENT FRAMEWORK

This section describes the uncertainty models used during the assessment of the performance constraints for the subtasks of hazard analysis, structural analysis, and damage analysis and additionally defines the associated random variables, contained in $\mathbf{U} = \{\mathbf{U}_h^T, \mathbf{U}_s^T, \mathbf{U}_d^T\}^T$.

### 4.1. *Hazard analysis*

A stochastic ground motion model, specifically the point-source model described in Boore (2003), will be used to obtain a complete and detailed probabilistic description of the seismic hazard. In particular, the ground acceleration is defined using a radiation spectrum, $A(f,\mathbf{U}_h)$, describing the ground motion's frequency content, $f$, and an envelope function, $e_t(t;\mathbf{U}_h)$, describing the ground motion's variation in time $t$. To generate a ground motion acceleration time history, $\ddot{z}_g(t)$, first a white noise sequence, $\mathbf{w}$ (lower case indicating a realization), is generated for the time duration of interest and multiplied by $e_t(t;\mathbf{u}_h)$. The resulting sequence is transformed into the frequency domain and normalized by the square root of the mean of the amplitude spectrum. The normalized spectrum is then multiplied by $A(f,\mathbf{u}_h)$ and transformed back into the time domain, resulting in a realization of a ground acceleration time history.

The radiation spectrum is described as a product of the source spectrum $E(f;\mathbf{U}_h)$, path effect $P(f;\mathbf{U}_h)$, and site effect $G(f;\mathbf{U}_h)$ as follows:

$$A(f;\mathbf{U}_h) = (2\pi f)^2 E(f;\mathbf{U}_h) P(f;\mathbf{U}_h) G(f;\mathbf{U}_h) \quad (3)$$

The source spectrum is given by:

$$E(f;\mathbf{U}_h) = c M_w S(f;\mathbf{U}_h) \quad (4)$$

where $c$ is a constant, $M_w$ is the seismic moment and is related to the moment magnitude $M$ by the relationship $\log_{10} M_w = 1.5(M+10.7)$ and $S(f;\mathbf{U}_h)$ is the displacement source spectrum. The two-corner point-source displacement source spectrum developed by Atkinson and Silva (2000) for the region of California is used for this research:

$$S(f;\mathbf{U}_h) = \left[\frac{1-e}{1+(f/f_a)^2} + \frac{e}{1+(f/f_b)^2}\right] \quad (5)$$

where the lower corner frequency $f_a$, higher corner frequency $f_b$, and weighting parameter $e$ are related to $M$ by the relationships $\log_{10} f_a = 2.181 - 0.496M$, $\log_{10} f_b = 2.41 - 0.408M$, and $\log_{10} e = 0.605 - 0.255M$ respectively.

The path effect is defined as (Boore, 2003):

$$P(f;\mathbf{U}_h) = Z(R_r)\exp\left[-\pi f R_r/(Q(f)c_Q)\right] \quad (6)$$

where $Z(R_r)$ and $Q(f)$ are the geometrical spreading and regional attenuation functions, respectively, $c_Q$ is the seismic velocity, and $R_r = \sqrt{h^2+r^2}$ is the radial distance from the source to the site, where $r$ is the closest distance to the fault plane and $h$ is a moment-dependent equivalent point-source depth and is related to $M$ by the relationship $\log_{10} h = -0.05 + 0.15M$ (Atkinson and Silva, 2000).

The site effect may be given as follows:

$$G(f;\mathbf{U}_h) = D(f;\mathbf{U}_h) A_m(f) \quad (7)$$

where $A_m(f)$ and $D(f;\mathbf{U}_h)$ are the amplification and diminution functions, respectively. The diminution may be accounted for using the $\kappa_0$ filter:

$$D_1(f;\mathbf{U}_h) = \exp[-\pi\kappa_0 f] \quad (8)$$

or the $f_{max}$ filter:

$$D_2(f;\mathbf{U}_h) = \left[1 + (f/f_{max})^8\right]^{-1/2} \quad (9)$$

or a combination of both filters (Boore, 2003).

The envelope function is given as follows:

$$e_t(t;\mathbf{U}_h) = a_t\,(t/t_n)^{b_t}\exp\left[-c_t t/t_n\right] \quad (10)$$

where the parameters $a_t$, $b_t$, and $c_t$ are given by:

$$b_t = -\lambda_t \ln(\eta_t)/\left(1 + \lambda_t[\ln(\lambda_t) - 1]\right) \quad (11)$$

$$c_t = b_t/\lambda_t \quad (12)$$

$$a_t = (\exp[1]/\lambda_t)^{b_t} \quad (13)$$

where $t_n = 2T_w$ is the time duration parameter, $T_w = \frac{1}{2f_a} + 0.05R_r$ is the duration of strong ground motion (Boore, 2003; Atkinson and Silva, 2000) while $\lambda_t$ and $\eta_t$ are parameters defining the temporal envelope. The parameters $\mathbf{U}_h = \{M, r, f_a, f_b, e, h, \kappa_0, f_{max}, \lambda_t, \eta_t, T_w\}^T$ are modeled as uncertain using the distributions in Table 1.

### 4.2. *Structural analysis*

The stochastic nature of the structural response must be considered during the structural analysis. This may be achieved by solving the first excursion of the engineering demand parameter response process, $R_{EDP}$, over the threshold $edp$ during time period $T$, which is taken as the earthquake duration. In other words, the $EDP$ of interest is the largest value of $R_{EDP}$ occurring during $T$. It is, therefore,

*Table 1: Suggested distributions for parameters in **U**.*

| Variable | P1* | P2** | Distribution | Ref. |
|---|---|---|---|---|
| $M$ | $5^{lb}$ | $8^{ub}$ | Trunc. Exp. | [1] |
| $r$ | 30 km | 0.40 | Lognormal | [1] |
| $f_a$ | $\star$ | 0.25 | Lognormal | [2] |
| $f_b$ | $\star$ | 0.25 | Lognormal | [2] |
| $e$ | $\star$ | 0.25 | Lognormal | [2] |
| $h$ | $\star$ | 0.25 | Lognormal | [2] |
| $\kappa_0$ | $0.02^{lb}$ | $0.04^{ub}$ | Uniform | [1] |
| $f_{max}$ | 25 Hz | 0.25 | Lognormal | [2] |
| $\lambda_t$ | 0.2 | 0.40 | Lognormal | [2] |
| $\eta_t$ | 0.05 | 0.40 | Lognormal | [2] |
| $T_w$ | $\star$ | 0.40 | Lognormal | [2] |
| $s$ | 1 | 0.025 | Normal | [3] |
| $\omega_i$ | § | 0.01 | Lognormal | [4] |
| $\xi_i$ | § | 0.30 | Lognormal | [4] |
| $C_n$ | $0^{\dagger}$ | $1^{\ddagger}$ | Lognormal | - |

* Median unless otherwise stated
** C.o.V. unless otherwise stated
$^{lb}$ Lower bound
$^{ub}$ Upper bound
$\star$ Given by respective equation in text
§ Mean value, given by structural model
† Mean value
‡ Standard deviation
[1] Vetter and Taflanidis (2012)
[2] Vetter and Taflanidis (2014)
[3] Minciarelli et al. (2001)
[4] Bashor et al. (2005)

the estimate of this distribution that is of interest and although there is no closed-form solution for this quantity it may be estimated using simulation techniques.

Assuming linear structural behavior during the earthquake event, $R_{EDP}$ may be described by a load effect model using a vector of influence functions and quasi-static loads. For a displacement-based response this may be written as:

$$R_{EDP}(t;\rho,\mathbf{U}_s,\mathbf{U}_h,\mathbf{W}) = s\Gamma_R^T(\rho)\mathbf{F}(t;\rho,\mathbf{U}_s,\mathbf{U}_h,\mathbf{W}) \quad (14)$$

where $s$ models the uncertainty in the transformation of seismic effects into structural effects, $\Gamma_R$ is a vector of influence functions that, when multiplied by the quasi-static loading vector, gives the response of interest, while $\mathbf{F}$ is the following vector of quasi-static loads:

$$\mathbf{F}(t;\rho,\mathbf{U}_s,\mathbf{U}_s,\mathbf{W}) = \mathbf{K}(\rho)\Phi_k(\rho)\mathbf{q}_k(t;\rho,\mathbf{U}_s,\mathbf{U}_h,\mathbf{w}) \quad (15)$$

where $\Phi_k = [\phi_1,\ldots,\phi_k]$ is a matrix containing the structure's first $k$ mode shape vectors and $\mathbf{q}_k(t) = \{q_1(t),\ldots,q_k(t)\}^T$ is a vector containing the structure's first $k$ modal responses at time $t$. The displacement of the $i$th mode may be determined by solving the following equation:

$$\ddot{q}_i(t) + 2\xi_i\omega_i\dot{q}_i(t) + \omega_i^2 q_i(t) = \phi_i^T\mathbf{f}(t;\rho,\mathbf{U}_s,\mathbf{U}_h,\mathbf{W}) \quad (16)$$

where the damping ratio, $\xi_i$, and natural frequency, $\omega_i$, of the $i$th mode, respectively, are modeled as uncertain, and the external loading vector for the seismic hazard is given by:

$$\mathbf{f}(t;\rho,\mathbf{U}_s,\mathbf{U}_h,\mathbf{W}) = \frac{-\mathbf{M}(\rho)\boldsymbol{i}\ddot{z}_g(t;\mathbf{U}_s,\mathbf{U}_h,\mathbf{W})}{m_i} \quad (17)$$

where $\ddot{z}_g$ is the ground motion acceleration time history, $\boldsymbol{i}$ is the vector of earthquake influence coefficients, and $m_i$ is the $i$th modal mass. Suggested distributions for the components of the uncertain vector $\mathbf{U}_s = \{s,\omega_1,...,\omega_k,\xi_1,...,\xi_k\}^T$ are given in Table 1. The distribution of $EDP$ may then be estimated through simulation by taking, for each realization of $\mathbf{U}_s$, $\mathbf{U}_h$ and $\mathbf{W}$ fed through the system, the largest value of $R_{EDP}$ to occur during the duration of the simulated event.

### *4.3. Damage analysis*

For the damage model, an intuitive damage measure $DM$ defined by the following demand-to-capacity ratio is considered (Jalayer et al., 2007):

$$DM = EDP/C(\mathbf{U}_d) \quad (18)$$

where $C$ is the capacity corresponding to the particular EDP under consideration. A simple model for the uncertain capacity is $C = \bar{C}(1+\delta_C C_n)$ where $\bar{C}$ is the nominal (mean) capacity associated with the response of interest, $\delta_C$ is the coefficient of variation, and $C_n$ is the normalized capacity distribution. A suggested distribution for $\mathbf{U}_d = \{C_n\}$ is given in Table 1.

### 4.4. *Solution strategy*

In order to estimate the integral of Eq. (1) while considering the hazard, structural and damage models defined in the previous sections, Monte Carlo simulation is used while writing the integral in the following form (Spence and Kareem, 2014):

$$\begin{aligned}\lambda(dm) = \nu \int_{\mathbf{w}} \int_{\mathbf{u}_h} \int_{\mathbf{u}_s} \int_{edp} & G(dm|edp) \\ & \times p(edp|\mathbf{u}_s, \mathbf{u}_h, \mathbf{w}) p(\mathbf{u}_s) p(\mathbf{u}_h) \\ & \times p(\mathbf{w}) \mathrm{d}edp \; \mathrm{d}\mathbf{u}_s \; \mathrm{d}\mathbf{u}_h \; \mathrm{d}\mathbf{w}\end{aligned} \tag{19}$$

where the components of the vectors $\mathbf{U}_s$, $\mathbf{U}_h$ and $\mathbf{W}$ are independent therefore allowing their joint probability functions, $p$, to be simply written as the products of their marginal distributions.

## 5. THE AVV SOLUTION STRATEGY

The solution of Eq. (2) is not trivial due to the presence of the probabilistic performance constraints. As the design variable vector is typically of high dimension it is advantageous to use efficient gradient-based algorithms in the solution strategy. However, the implicit dependence of the probabilistic constraints on $\rho$ and the large size of vector $\mathbf{W}$ makes the solution of the problem, as written in Eq. (2), computationally impractical. Thus a simulation-based decoupling technique has been developed to efficiently solve the performance-based topology optimization problem of interest. To define the approximate sub-problem it is convenient to re-write the probabilistic constraints in Eq. (2) in their inverse form:

$$dm_j^{(\lambda_{0_j})}(\rho) - dm_j \leq 0 \tag{20}$$

where $dm_j^{(\lambda_{0_j})}$ is the damage measure with target mean annual exceedance rate $\lambda_{0_j}$.

### 5.1. *Model description*

A fine finite element discretization, though inefficient during the reliability analysis, is necessary for a detailed topology design. In order to increase computational efficiency of the reliability analysis, a "reduced system" comprised of a limited number of "master" degrees of freedom (DOFs) is used to evaluate the dynamic response of the structure. The master DOFs are located on the floor systems, which are part of the underlying secondary system of the structure, which is classified as a "non-designable" domain and thus is not altered during the optimization. The mass of the system is modeled as lumped at the master DOFs and thus is located at the floor levels, which is a typical assumption for buildings. The loading vector $\mathbf{f}$, defined in Eq. (17), thus has nonzero loads located only at these master DOFs. The "complete" system is obtained as the superposition of the design domain and secondary system. Using this classification, the dynamic response may be estimated in the master DOFs as:

$$\begin{aligned}\tilde{\mathbf{M}}(\rho, \mathbf{U})\ddot{\tilde{\mathbf{z}}}(t) + \tilde{\mathbf{C}}(\rho, \mathbf{U})\dot{\tilde{\mathbf{z}}}(t) \\ + \tilde{\mathbf{K}}(\rho, \mathbf{U})\tilde{\mathbf{z}}(t) = \tilde{\mathbf{f}}(t; \mathbf{U}, \mathbf{W})\end{aligned} \tag{21}$$

where $\tilde{\mathbf{M}}$ is the reduced mass matrix and is obtained by eliminating the rows and columns of $\mathbf{M}$ corresponding to the DOFs with zero mass, $\tilde{\mathbf{C}}$ is the reduced order damping matrix, $\tilde{\mathbf{z}}$ is the displacement vector describing the displacement of the complete system in the master DOFs, $\tilde{\mathbf{f}}$ is the reduced external load vector obtained by eliminating the components of $\mathbf{f}$ corresponding to the DOFs with zero mass (which are, by definition, zero-valued), and $\tilde{\mathbf{K}}$ is the reduced order stiffness matrix and can be estimated using static condensation.

#### 5.1.1. *Approximate sub-problem*

To formulate the approximate sub-problem, first consider the system defined in the current design point $\rho_0$ and conditioned on a particular realization of the random vectors $\mathbf{U} = \mathbf{u}$ and $\mathbf{W} = \mathbf{w}$. The *EDP* for one event may be represented as follows:

$$\begin{aligned}edp_j(\rho_0, \mathbf{u}, \mathbf{w}) &= R_{EDP}(\hat{t}; \rho_0, \mathbf{u}, \mathbf{w}) \\ &= s\Gamma_R^T(\rho_0)\tilde{\mathbf{F}}(\hat{t}; \rho_0, \mathbf{u}, \mathbf{w}) \\ &= \tilde{\Gamma}_{R_j}^T(\rho_0)\tilde{\Upsilon}_j(\rho_0, \mathbf{u}, \mathbf{w})\end{aligned} \tag{22}$$

where $\hat{t}$ is the time instant at which the largest response of interest occurs and $\tilde{\Upsilon}_j = s\tilde{\mathbf{F}}(\hat{t}; \rho_0, \mathbf{u}, \mathbf{w})$. The value of the damage measure associated with this $edp$ may be obtained as follows:

$$dm_j(\rho_0, \mathbf{u}, \mathbf{w}) = \tilde{\Gamma}_{R_j}^T(\rho_0)\tilde{\psi}_j(\rho_0, \mathbf{u}, \mathbf{w})/\bar{C}_j \tag{23}$$

where $\tilde{\psi}_j = \tilde{\Upsilon}_j(\rho_0, \mathbf{u}, \mathbf{w})/(1+\delta_{C_j} C_{n,j})$ is a vector that, when statically applied to the reduced system, will produce a response level that if divided by $\bar{C}_j$ yields the damage measure $dm_j$.

Considering the system unconditioned, the damage parameter with exceedance rate $\lambda_{0_j}$ may be represented as:

$$dm_j^{(\lambda_{0_j})}(\rho_0) = \mu_{dm_j}(\rho_0) + \eta_{dm_j}(\lambda_{0_j}, \rho_0)\sigma_{dm_j}(\rho_0) \quad (24)$$

where $\eta_{dm_j}$ is the reduced variate giving the number of standard deviations, $\sigma_{dm_j}$, by which the damage parameter with mean annual exceedance rate $\lambda_{0_j}$ exceeds the mean value, $\mu_{dm_j}$, of the damage measure. By observing that the mean value and the variance, $\sigma^2_{dm_j}$, of the damage measure may be expressed as:

$$\mu_{dm_j}(\rho_0) = \tilde{\Gamma}^T_{R_j}(\rho_0)\overline{\tilde{\psi}}_j(\rho_0)/\bar{C}_j \quad (25)$$

$$\sigma^2_{dm_j}(\rho_0) = \tilde{\Gamma}^T_{R_j}(\rho_0)\mathbf{C}_{\tilde{\psi}_j}(\rho_0)\tilde{\Gamma}_{R_j}(\rho_0)/\bar{C}^2_j \quad (26)$$

where $\overline{\tilde{\psi}}_j$ and $\mathbf{C}_{\tilde{\psi}_j}$ are the mean and covariance matrix of the vectors $\tilde{\psi}_j$ generated during the simulation, the damage measure with mean annual exceedance rate $\lambda_{0_j}$ may be written as:

$$dm_j^{(\lambda_{0_j})}(\rho_0) = \tilde{\Gamma}^T_{R_j}(\rho_0)\tilde{\Psi}_j(\lambda_{0_j}, \rho_0)/\bar{C}_j \quad (27)$$

where $\tilde{\Psi}_j$ is the AVV for the $j^{th}$ constraint and is given by:

$$\begin{aligned}\tilde{\Psi}_j(\lambda_{0_j}, \rho_0) &= \overline{\tilde{\psi}}_j(\rho_0) \\ &+ \eta_{dm_j}(\lambda_{0_j}, \rho_0)\frac{\mathbf{C}_{\tilde{\psi}_j}(\rho_0)\tilde{\Gamma}_{R_j}(\rho_0)}{\sigma_{dm_j}(\rho_0)\bar{C}_j}\end{aligned} \quad (28)$$

The significance of Eq. (27) is that it allows $dm_j^{(\lambda_{0_j})}$ to be calculated through the static application of the AVV to the master degrees of freedom of the nominal/mean system. In particular, the effects of the uncertainties considered in the problem are contained in the AVV, therefore the AVV provides a static and deterministic relationship between the nominal system and the damage measure with target mean annual exceedance rate.

The definition of the AVV for the reduced system may be used to define the following AVV for the complete system as follows:

$$\Psi^i_j(\lambda_{0_j}, \rho_0) = \begin{cases} \tilde{\Psi}^k_j(\lambda_{0_j}, \rho_0), & i = k \in \Xi \\ 0, & i \neq k \in \Xi \end{cases} \quad (29)$$

where $\Xi$ is the set of degrees of freedom defining the master nodes while $i = 1, \ldots, D_{dof}$ where $D_{dof}$ indicates the total number of degrees of freedom of the complete system. The inverse reliability constraints given in Eq. (20) may be written using the AVV for the complete system as follows:

$$\Gamma^T_{R_j}(\rho_0)\Psi_j(\lambda_{0_j}, \rho_0)/\bar{C}_j - dm_j \leq 0 \quad (30)$$

or using the following equivalent formulation:

$$\begin{aligned} \Lambda^T_j \mathbf{z}_j(\rho_0)/\bar{C}_j - dm_j \leq 0 \\ \mathbf{K}(\rho_0)\mathbf{z}(\rho_0) = \Psi_j(\lambda_{0_j}, \rho_0) \end{aligned} \quad (31)$$

where $\Lambda_j = \mathbf{K}\Gamma_{R_j}$ is a vector of constants extracting the displacement-based response to be constrained.

The relationship given by Eq. (31) is exact for the current design point. If, however, it is assumed that the vectors $\Psi_j$ are weakly dependent on the design variables, the following approximate topology sub-problem may be defined:

$$\begin{aligned} \min_{\rho} \quad & V(\rho) = \sum_{e=1}^{n} \int_{\Omega_e} \rho_e \mathrm{d}\Omega \\ \text{s.t.} \quad & \Lambda^T_j \mathbf{z}(\rho)/\bar{C}_j - dm_j \leq 0, \quad j = 1, \ldots, N \\ & \mathbf{K}(\rho)\mathbf{z}(\rho) = \Psi_j, \quad j = 1, \ldots, N \\ & 0 \leq \rho_e \leq 1 \end{aligned} \quad (32)$$

The definition of the approximate sub-problem not only decouples the probabilistic analysis from the optimization but also allows the use of optimization algorithms typically used to solve static, deterministic topology optimization problems due to the static nature of the governing equations. As this problem is approximate for changes in the design point, upon the convergence of the sub-problem the AVV must be updated for all constraints and the sub-problem resolved. This sequential solution procedure until convergence of the AVVs and structural design guarantees an optimum structure conforming to the reliability constraints defined in the original optimization problem of Eq. (2). The efficiency of the procedure may be estimated by the number of design cycles, or updates and solutions of the sub-problem, that are required for convergence.

## 6. CASE STUDY

The case study presented in this paper considers the optimum topology design of the bracing for a planar lateral load resisting frame of a 3-story building subject to earthquake excitation. The columns and floor beams of the secondary system are modeled using W8x10 steel members and are indicated by vertical and horizontal lines, respectively, in Fig. 1(a). Q4 elements (dimensions 0.1m x 0.1m x 0.1m with material properties of concrete) are used to model the design domain as indicated by the shaded gray area in Fig. 1(a). The initial structure had a uniform design variable vector with $\rho = 0.8$. A linear density filter with a radius of 0.3 m was used and the Solid Isotropic Material with Penalization (SIMP) method was implemented with a penalty $p = 5$ (Bendsøe, 1989). The Method of Moving Asymptotes (Svanberg, 1987) was used as the optimization algorithm for this research.

Ground accelerations were generated using the method described in Sec. 4.1 where the annual rate of arrival of significant events was taken as $\nu = 2.927$ for events of magnitude $5 \leq M \leq 8$, as determined using data from Caltech (2014). Time-invariant uncertain parameters were modeled using the distributions given in Table 1. The first three modes were used to calculate the structural response, where the mean value of the $i$th modal damping ratio was taken as $\xi_i = 0.01$ for $i = 1, \ldots, 3$. The response processes of interest $R_{EDP}$ were the inter-story drifts for each floor of the structure. The nominal capacity indicating damage was given as $\bar{C} = h_{floor}/400$ and $\delta_C = 0.01$. The target mean annual exceedance rates for all probabilistic constraints were taken as $\lambda_{0_j} = 0.01$ for a damage measure $dm_j = 1$.

The final bracing scheme for the case study is shown in Fig. 1(b). The volume history, given in Fig. 2, shows rapid and steady convergence and attests to the efficiency of the method. The original constraints on the target mean annual failure rates for all floors were achieved for the final design, as indicated in Fig. 3. Finally, the AVV shows rapid and steady convergence, as shown in Fig. 4 for a sample AVV on the 3rd floor of the structure. Other AVVs showed similar convergence properties.

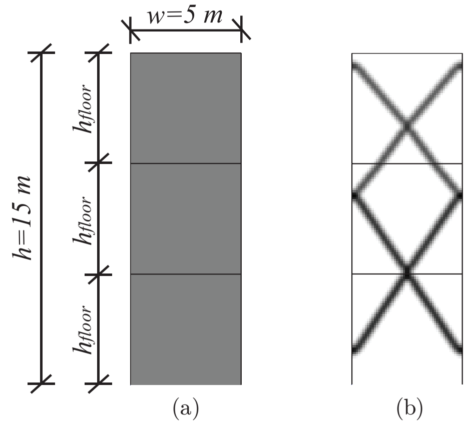


*Figure 1: (a) Design domain; (b) Final bracing scheme*

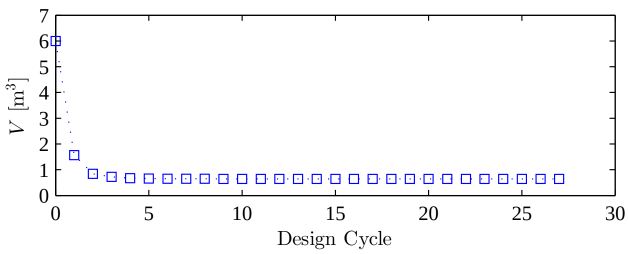


*Figure 2: Volume History*

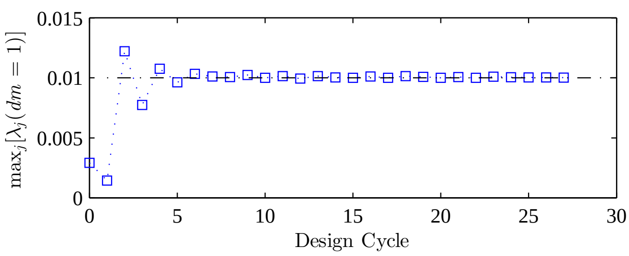


*Figure 3: Maximum annual exceedance rate for $dm = 1$ over all constraints for each design cycle*

## 7. CONCLUSIONS

This paper presented a framework for the topology optimization of uncertain and stochastic build-

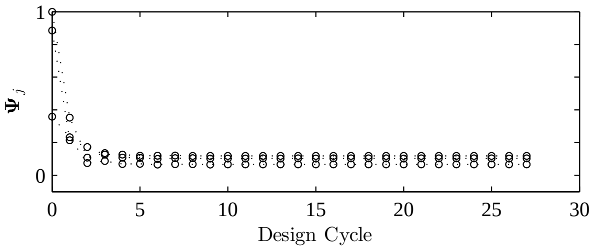


*Figure 4: Convergence history of sample AVV: 3rd floor*

ings within a probabilistic performance-based engineering setting. The stochastic nature of the seismic ground accelerations as well as additional time-invariant uncertainties describing the knowledge/state of the system were rigorously included using probabilistic performance constraints written in terms of the mean annual rate of exceedance of a damage measure. A novel decoupling approach was presented for the efficient solution of the probabilistic performance-based topology optimization problem in which the optimization loop and probabilistic analysis were decoupled using a series of sequential approximate sub-problems defined using a number of AVVs. The applicability and efficiency of the proposed framework was illustrated for the bracing design of a 3-story building.

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