Reliability-based Seismic Hazard Analysis

Hossain Rahimi
Graduate Student, Dept. of Civil Engineering, Sharif University of Technology, Tehran, Iran

Mojtaba Mahsuli
Assistant Professor, Dept. of Civil Engineering, Sharif University of Technology, Tehran, Iran

Ali Bakhshi
Associate Professor, Dep. of Civil Engineering, Sharif University of Technology, Tehran, Iran

ABSTRACT: This paper presents a new approach to probabilistic seismic hazard analysis. Hazard in this context means the probability of exceeding a measure of earthquake intensity. The main components of the proposed approach are twofold: 1) Reliability methods compute the exceedance probabilities; 2) Multiple probabilistic models compute the earthquake intensity. These two components are presented in details. The proposed methodology is employed in a comprehensive seismic hazard analysis of Iran with an area of nearly 1.65 million square kilometers and 112 sources of seismicity. In this analysis, a grid of 2045 points is chosen to produce a high resolution hazard map. The analysis consists of 861 random variables and 57,615 model instances. The primary results are seismic zonation maps. The results are compared to other studies that employed the conventional seismic hazard analysis for this region. This paper concludes with a discussion on the advantages and practical differences between the results of the reliability-based approach and those of the conventional probabilistic seismic hazard analysis.

1. INTRODUCTION

This paper puts forward reliability methods as an alternative to conventional approaches in probabilistic seismic hazard analysis (PSHA). In the proposed methodology, reliability methods compute the exceedance probability curve of a ground motion intensity measure, hereafter referred to as "hazard curve." The advantage of this methodology over existing approaches is enumerated below and explained in more detail subsequently: 1) Comprehensive treatment of epistemic uncertainty; 2) Accounting for the spatial correlation of residuals in ground motion prediction equations; 3) Computing a "unified" exceedance probability of the intensity measure.

Firstly, comprehensive characterization and modeling of epistemic uncertainty facilitates targeted efforts to reduce this uncertainty over time by research and data-gathering efforts. Examples of prevailing epistemic uncertainty in a seismic hazard analysis are those in the rate of occurrence of seismic events, in the geometry of the sources of seismicity, in the parameters that define the maximum magnitude and the relative frequency of different magnitudes in each source, and in the regression parameters of the ground motion prediction equations. In the first step in this paper, the epistemic uncertainty in the maximum magnitude and the relative frequency of different magnitudes is accounted for. The rest is being addressed in ongoing research.

Secondly, it is usual in conventional PSHA to neglect the spatial correlation of the residuals in ground motion prediction equations. This is in part due to the nature of the analysis framework. The conventional PSHA computes the hazard curve at each site independently. In fact, the starting point in this approach is to select a single site for which the hazard analysis is carried out. In contrast, the reliability-based approach computes the hazard curves at a grid of many points simultaneously thus allowing to model any correlations between model parameters at different sites. In fact, accounting for correlations between different sources of uncertainty is an

inherent ability of reliability methods.

Finally, this paper postulates a discussion on the philosophical difference between the results of the reliability-based approach and those of the conventional PSHA. In the conventional PSHA, the probabilities that are expressed by the likes of "10% in 50 years" often include merely the aleatory uncertainty in the occurrence, magnitude, and location of the earthquake. The remaining uncertainty, e.g., the one in the intensity model error, is expressed by means of "median or 50th percentile" and "one-sigma or 84th percentile" intensities. Hence, the total uncertainty is divided between two different probability values. On the contrary, the reliability-based approach presents all uncertainty in one probability estimate.

The scope of the reliability methods that are employed in this paper is limited to mean-centered Monte Carlo sampling. Application of more efficient methods, such as first- and second-order reliability methods (FORM and SORM) is addressed by ongoing research. A specific implementation of Monte Carlo sampling, called "scenario sampling," is developed. Details are provided in the Methodology section. This approach was originally developed by Mahsuli and Haukaas (2013a) for large-scale seismic risk analysis. To support the proposed approach, a host of probabilistic models with a specific format are developed and/or adopted. More details on these models are provided in the "Models" section. The models are implemented in Rt, a computer program for multi-model reliability and risk analysis (Mahsuli and Haukaas 2013b). Rt is freely available online at inrisk.ubc.ca. Rt has an object-oriented design that facilitates the steady growth of its library of probabilistic models and analysis methods. It has a user-friendly, graphical interface that can be utilized to, say, visualize the sources of seismicity on Google Maps®.

The first efforts to carry out PSHA date back to the late 1960's. The seminal paper by Cornell (1968) pioneered the field. The results of this work were in terms of the peak ground acceleration (PGA) versus the mean return period. Since then, many researchers have contributed to this field. In the 1970's, hazard curves were developed directly for spectral ordinates (McGuire 1974). This development introduced an elastic measure of structural response, e.g., spectral acceleration, in the hazard analysis. McGuire (2008) presents an overview of the evolution of PSHA. The models in PSHA were in the form of conditional probabilities. In contrast, the models in the proposed reliability-based methodology make probabilistic prediction of physical phenomena. They receive random variables that describe the uncertainty as input, and produce physical measureable responses, such as the earthquake magnitude, location, or intensity, as output. Two studies employed Monte Carlo sampling for hazard analysis, however, in a framework that is entirely different from the proposed methodology. These studies include the work by Ebel and Kafka (1999) and Han and Choi (2008). Both studies characterize the hazard levels solely based on a seismic catalog in lieu of employing models that describe the geometry of seismic sources. In contrast, the present study proposes probabilistic location models that describe the geometry of the sources and produce random realizations of the earthquake location.

The developments in this paper are showcased by a comprehensive application: A nationwide seismic hazard analysis is carried out for Iran with an area of over 1.65 million km^2 and a population of nearly 78 million. Iran is a significantly seismically active region with 112 active sources of seismicity in this study. The resulting seismic zonation maps are presented for PGA and spectral acceleration, *Sa*, at various natural periods of vibration. This analysis yields uniform hazard spectra for all cities in Iran, one of which is presented as a case in point.

2. METHODOLOGY

Reliability methods have been developed over the last three decades. These methods are suited for hazard analysis because they are tailored to compute the probability of rare events. Such events are particularly important in hazard analysis applications because design-level events correspond to small probabilities of exceedance,

e.g., 2% or 10% in 50 years.

In a reliability analysis, random variables describe the uncertainty and a limit-state function defines the event for which the probability is sought. In classical structural reliability, the limit-state function is defined in terms of the demand and capacity of the structure. To extend the usage of reliability methods to hazard analysis, the present study expresses the limit-state function in terms of the earthquake intensity measure. In this case, the limit-state function, g, defines the event that the intensity, I, exceeds a prescribed threshold, I_o. Thus, the limit-state function reads

$$g(\mathbf{x}) = I_o - I(\mathbf{x}) \tag{1}$$

where $\mathbf{x}$=vector of random variables. Reliability analysis computes the probability that g takes on negative values, i.e., $p=P(g\leq 0)$. The pair of (I_o,p) constitutes a point on the hazard curve. The intensity, I, is computed by a model that depends on the earthquake location and magnitude. Each of these two is also represented by a model in this approach. All these models are probabilistic and they describe the uncertainty by random variables. In the course of a reliability analysis, the limit-state function is repeatedly evaluated. In each evaluation, the models receive the trial realization of the random variables as input, and output physical responses, e.g., earthquake magnitude or intensity. These responses enter another model "downstream" as input, or directly enter the limit-state function. In summary, hazard analysis with reliability methods requires a host of probabilistic models. The specific models that are employed in this paper are presented in the next section.

The use of methods like FORM and SORM requires repeating the abovementioned analysis at various I_o-values to obtain the entire hazard curve. Therefore, an approach based on Monte Carlo sampling is explored. This approach is called scenario sampling and employs mean-centered Monte Carlo sampling. In each sample, a random scenario with a duration of T years is generated. That is, for each seismic source, an occurrence model produces random occurrence times of seismic events within that source for a given time span, T. For each event, a location model and a magnitude model produce random realizations of the rupture location and magnitude. Once the location of the seismic event is known, the distance to any site is computed. The realizations of distance and magnitude enter the earthquake intensity model. The intensity model, in turn, computes the intensity at each given site.

At the end of the scenario, the maximum intensity across all events in the scenario is recorded. Random generation of scenarios continues until a sufficiently accurate probability distribution of the maximum intensity is obtained. That is, a target coefficient of variation of 2% is achieved for the probability point of interest.

3. MODELS

A probabilistic model in the proposed methodology outputs a unique response given the realization of random variables. This contrasts most models in the conventional PSHA, which have the form of a conditional probability, i.e., they produce a probability as output.

In this section, each probabilistic model that is employed in this paper is presented. This is followed by the specific model parameters that are used to carry out the hazard analysis for Iran. These parameters are extracted from the earthquake catalog compiled by the International Institute of Earthquake Engineering and Seismology (IIEES 2014). This catalog only includes the instrumental seismicity data.

3.1. Occurrence Models

An occurrence model for each seismic source simulates the random occurrence time of seismic events within that source over time. The well-known Poisson point process is adopted for this purpose. However, it is emphasized that the methodology as well as the software framework in Rt are not limited to Poisson models. For instance, the analyst may implement clustering models in this framework.

The Poisson process is based on the assumption of independence between the occurrence of events. As a result, the entire earthquake catalog is subjected to declustering

algorithms to remove any foreshocks and aftershocks, which are dependent on a mainshock. For this purpose, the method by Reasenberg (1985) is employed. For input parameters of this method, the suggestion of van Stiphout et al. (2012) is used. Thereafter, each event in the declustered catalog is assigned to the nearest seismic source to establish a catalog specific to that source. Finally, for each of the 112 sources of seismicity in Iran, described shortly, a Poisson process model is established. The rate of occurrence for each model is selected from the Gutenberg-Richter (1944) relationship of its source. This rate for each source is the rate of the minimum considered magnitude, $M_{\min}$=4.8.

3.2. Location Models

The literature provides probability distributions for the distance between the rupture location to site for various source geometries (Der Kiureghian and Ang 1977). Technically, these probability distributions can be employed directly in the reliability analysis, with the premise that the distance is input as a random variable. However, this approach has several disadvantages in a large-scale seismic zonation. To address all sites, a different random variable with a different distribution would be necessary for each site. Therefore, an alternative is adopted here, whereby location models are developed that take random variables as input and give rupture location as output. Each realization of the random variable(s) is associated with one rupture location and once the coordinates of this location are known, the distance to any and many sites is readily computed. The proposed approach is also advantageous if aftershocks are modeled, because their locations are dependent on the location of the main shock. This approach also make it easier to model non-uniform probability of occurrence within a source, although this is not relevant in this paper. These models are developed by the second author (Mahsuli 2012).

Figure 1 demonstrates the modeled seismic sources in Iran in the Google Maps® interface of Rt. Line sources are shown by gray lines and area sources with pink polygons. The geometry of most seismic sources is adopted from Hessami and Jamali (2006) and Tavakoli and Ghafory-Ashtiany (1999). As a result, a total of 112 line and area sources are modeled in this paper.

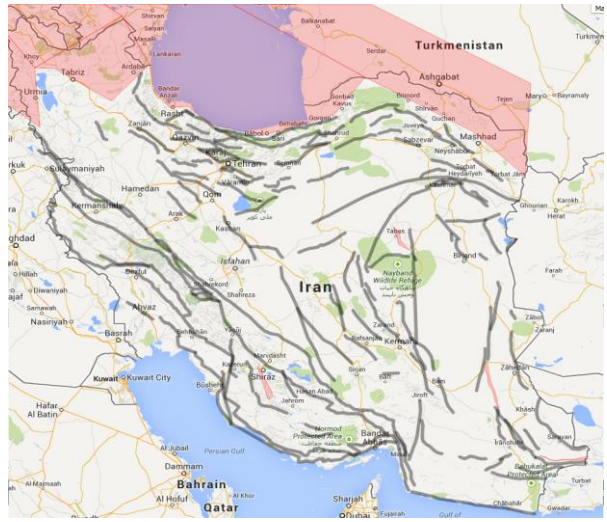

Figure 1: Map of 112 seismic sources in Iran visualized in the Google Maps® interface of Rt.

3.3. Magnitude Models

Several magnitude models that conform to the aforesaid criteria are available. A common model represents the magnitude by a bounded exponential random variable (McGuire 2004). The probability distribution of this random variable is based on the Gutenberg-Richter (1944) law and the probability density function is

$$f(m) = \frac{b' \cdot \exp\left[-b' \cdot \left(m - M_{\min}\right)\right]}{1 - \exp\left[-b' \cdot \left(M_{\max} - M_{\min}\right)\right]} \qquad (2)$$

where m=moment magnitude, b'=parameter that depends on the relative occurrence of different magnitudes, and $M_{\max}$=magnitude upper bound. Eq. (2) is valid for $M_{\min} \le m \le M_{\max}$. This study addresses the uncertainty in b' and $M_{\max}$ by modeling them as random variables. As a result, Eq. (2) represents a random variable whose parameters are also random. It is hence necessary to transform Eq. (2) into a model that yields a magnitude realization, m, given the realizations of b' and $M_{\max}$ as well as another random variable that describes the uncertainty in m. For this purpose, a standard normal random variable, x, is

introduced as input. Its cumulative distribution function is denoted by $\Phi(x)$ and it is employed as a surrogate measure of the earthquake magnitude. Specifically, a relationship between m and x is introduced by the well-known probability-preserving transformation (Ang and Tang 2007), which results in

$$m = -\frac{1}{b'}\ln\left[1-\Phi(x)\left(1-e^{-b'(M_{\max}-M_{\min})}\right)\right]+M_{\min} \quad (3)$$

In the conventional PSHA, b' is a parameter of the Gutenberg-Richter relationship, and is determined by a classical linear regression analysis on the earthquake catalog of each source. In this paper, a Bayesian linear regression analysis is carried out instead to determine not only the mean b', but also its standard deviation for each source in Iran. Thereafter, b' is modeled as a normal random variable the aforesaid mean and standard deviation. Furthermore, the uncertainty in $M_{\max}$ is quantified by the Kijko and Sellevoll (1992) approach. As a result, $M_{\max}$ is a Weibull random variable for each source whose mean and standard deviation are determined using the catalog of that source.

3.4. Intensity Models

An intensity model employs characteristics of the earthquake and the path of shock wave propagation to predict characteristics of the site-specific ground shaking. Specifically, models are sought that take earthquake location and magnitude as input and return the elastic 5%-damped Sa for given periods T_n at specific sites. The models employed in this study are of the generic form of

$$\ln(Sa_{ij}) = h\left(m_i, R_{ij}, V_S, T_n\right) + \eta_i + \varepsilon_{ij} \quad (4)$$

where subscript i indicates a value in the ith seismic event in the T-year scenario, subscript j indicates a value at the jth site, h=function or algorithm, R_{ij}=distance between site and the rupture location, V_S=shear-wave velocity of the ground, η_i=inter-event model error, and ε_{ij}=intra-event model error. Each η_i and ε_{ij} is a normal random variable with a zero mean and a given standard deviation. η_i and ε_{ij} are independent of each other. However, there is a perfect correlation between η_i variables at all sites for a given event. In contrast, ε_{ij} variables are only partially correlated between different sites for a given event. This correlation is a function of the distance between the sites. The shorter the distance, the higher is the correlation. The proposed hazard analysis methodology is capable of taking into account such correlations.

To model the earthquake intensity in Iran, four ground motion prediction equations are adopted and implemented in Rt. These include the relationships by Abrahamson and Silva (2008), Chiou and Youngs (2008), Boore and Atkinson (2008), and Campbell and Bozorgnia (2008). Each relationship produces a different Sa given the same realizations of the input random variables. This variability is due to model uncertainty. To properly account for this epistemic uncertainty, the Sa-values from the four relationships are combined using four randomly generated weights, u_1 to u_4, each of which is uniformly distributed between 0 and 1:

$$Sa = \sum_{k=1}^{4}\left(u_k \cdot Sa_k\right) / \sum_{k=1}^{4} u_k \quad (5)$$

Eq. (5) allows for any possible combination of the responses from the four ground motion prediction equations, thus properly accounting for the model uncertainty in the context of these four models.

The hazard analysis is carried out for the site class B of ASCE (2010). Therefore, V_s is represented by a random variable that is uniformly distributed between 760 and 1500 m/s.

4. RESULTS

Scenario sampling analysis is carried out for a grid of 2045 sites at distances of about 35 km, i.e., ⅓° in latitude and longitude. The analysis is conducted at seven periods of 0.01s, which essentially yields PGA, 0.1s, 0.3s, 1.0s, 2.0s, 4.0s and 10.0s. The analysis entails 861 random variables and 57,615 model instances. To ensure sufficient accuracy, 20,000 samples, i.e., 20,000 50-year scenarios, are generated that guarantee a coefficient of variation of 2% for the exceedance probability of 10% in 50 years. This massive

analysis is conducted by parallel processing on the high-performance computing center at Sharif University of Technology.

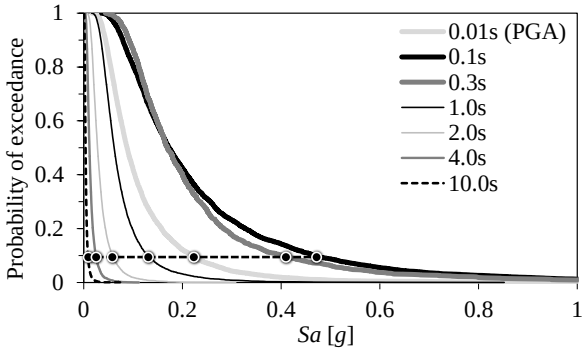

Figure 2: Sa exceedance probabilities for Tabriz.

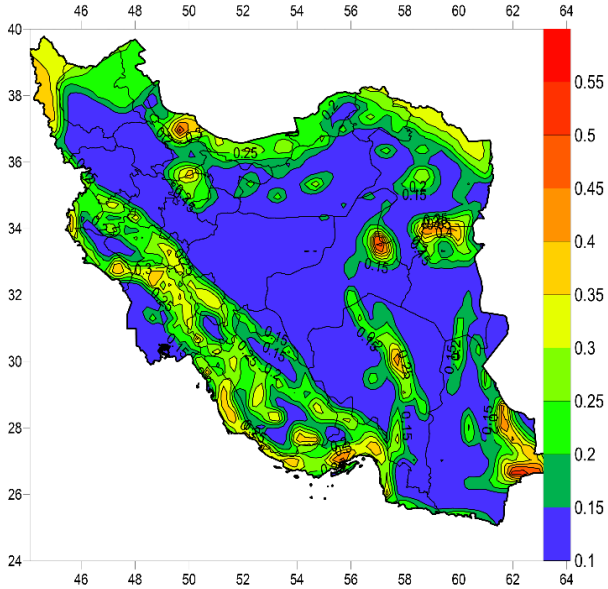

Figure 3: PGA iso-accelaration contours in g.

Scenario sampling analysis determines the entire probability distribution of *Sa* at the aforesaid periods and at each of the 2045 sites. Figure 2 shows an example of *Sa* exceedance probability curves at different periods for the City of Tabriz. The exceedance probability level of 10% in 50 years is shown in this figure by a thin horizontal dashed line. From such curves, the *Sa*-values that correspond to the exceedance probability of 10% in 50 years are read at all sites. Figure 2 identifies these values for Tabriz by solid dots. Such *Sa*-values at all sites are plotted in the form of iso-acceleration contours on the map of Iran to generate seismic zonation maps. Figure 3 shows the seismic zonation map for the 475-year PGA. Nearly half the area of Iran is characterized by a moderate PGA of around 0.15g. However, the PGA goes beyond a severe value of 0.55g in some small areas. The maps of the seismic activity in Figure 4 supports the variation seen in the PGA map of Figure 3. Regions with the highest frequency and magnitude of seismic activity in Figure 4 are associated with the highest PGA levels in Figure 3.

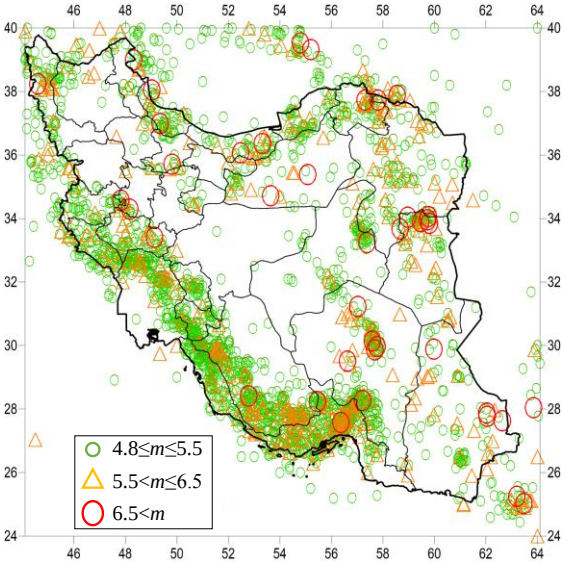

Figure 4: Map of seismic activity in Iran.

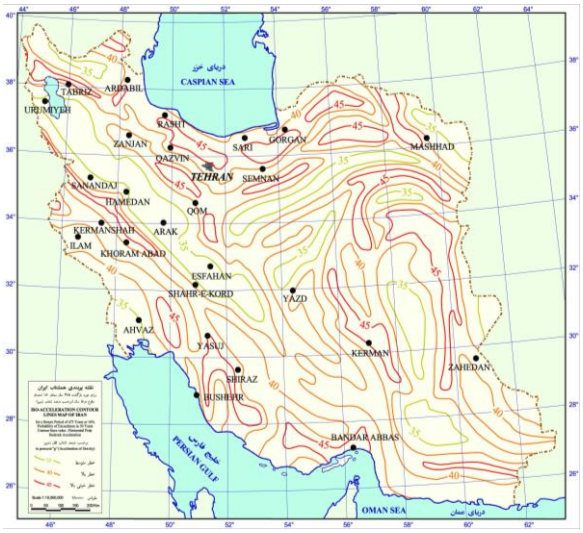

Figure 5: PGA contours by Tavakoli and Ghafory-Ashtiany (1999).

To compare the results with other studies,

Figure 5 demonstrates a 475-year PGA zonation map that is obtained by a conventional PSHA (Tavakoli and Ghafory-Ashtiany 1999). The general trend in this figure is similar to that of Figure 3. For instance, the highest seismic hazard in both studies are in the north-northwest at Dasht-e-Bayaz, in Tabas, and in Rudbar fault zones. In addition, the lowest seismic hazard in both studies are in central Iran, e.g., the Isfahan region. However, it is noted that the two figures are results of analyses that, in addition to methodology, differ in the catalog of earthquakes, geometry of seismic sources, and ground motion prediction equations. For instance, Tavakoli and Ghafory-Ashtiany (1999) employed earthquake catalogs and ground motion prediction equations from before some 15 years ago. They also divided the country into 20 "seismic provinces," and assumed constant magnitude model parameters for all fault lines within each province.

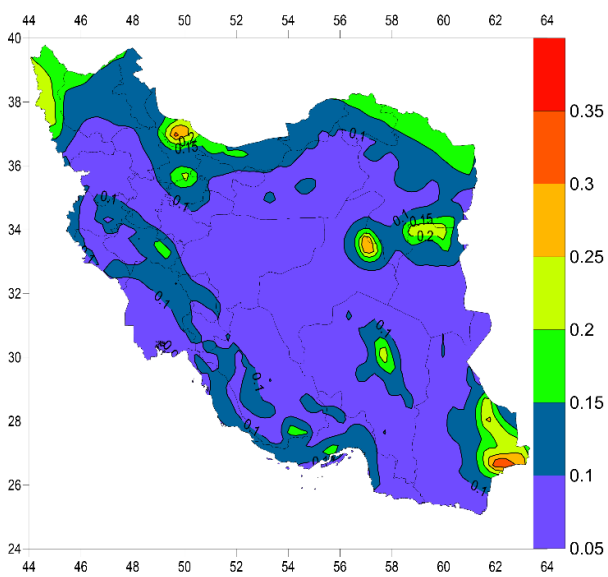

Figure 6: Sa contours for the period of 1.0s in g.

Figure 6 presents the iso-acceleration contours of 475-year *Sa* at the period of 1.0s. Maps for *Sa* at other periods are not shown for brevity. Almost the same areas with a high PGA possess high *Sa*-values, which is not surprising. From these maps, a uniform hazard spectrum is obtainable for any given site and city in Iran. An example is shown in Figure 7 for the City of Tabriz. The spectra are shown for two exceedance probabilities of 10% and 2% in 50 years that correspond to return periods of 475 and 2475 years, respectively. The spectra cover a wide range of structures within the period range of 0.01s to 10.0s. Any ordinate on, say, the 475-year spectrum is truly associated with an exceedance probability of 10% in 50 years. That is, there is no need to state mean and one-sigma spectral values because the 10% probability represents all the uncertainty in the problem. This contrasts several contemporary seismic design codes that employ conventional PSHA, e.g., the National Building Code of Canada (Adams and Halchuk 2003).

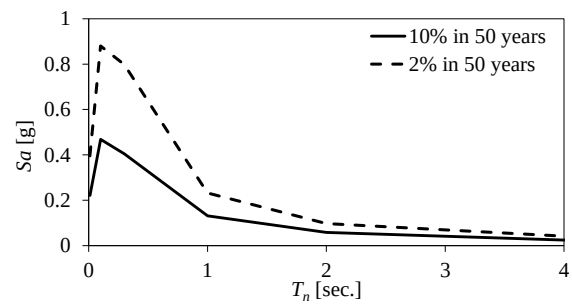

Figure 7: Uniform hazard spectra for Tabriz.

5. CONCLUDING REMARKS

This paper revisits the probabilistic seismic hazard analysis. In a novel approach, reliability methods are employed to compute exceedance probabilities of earthquake intensity measures. This approach requires a host of probabilistic models for the occurrence, location, magnitude, and intensity of seismic events. The modeling approach emphasizes on comprehensive characterization of epistemic uncertainty. This allows for targeted efforts to reduce this uncertainty with further research and data-gathering efforts. Ongoing research by second author addresses other sources of epistemic uncertainty, such as that in the geometry of seismic sources. The proposed methodology presents the hazard exceedance probabilities in a unified probability value contrary to many conventional seismic hazard analysis approaches. It also takes into account any correlations, e.g., between the intra-event residuals at different sites. The methodology is employed to carry out a

nationwide seismic hazard analysis in Iran, which represents a large-scale application of probabilities in practice. The primary results are seismic zonation maps for spectral acceleration at several periods. These maps will hopefully make their way to the Iranian seismic code, which currently suffers from a non-uniform hazard spectrum with a fixed shape for the entire country.

6. REFERENCES

ASCE (2010). Minimum design loads for buildings and other structures. ASCE/SEI 7-10, *American Society of Civil Engineers*, Reston, VA.

Abrahamson, N., and Silva, W. (2008). "Summary of the Abrahamson & Silva NGA ground-motion relations." *Earthquake Spectra*, 24(1), 67–97.

Adams, J., and Halchuk, S. (2003). "Fourth generation seismic hazard maps of Canada: Values for over 650 Canadian localities for the 2005 National Building Code of Canada." *Open File 4459*, Geological Survey of Canada, Ottawa, Canada.

Ang, A. H., and Tang, W. H. (2007). *Probability concepts in engineering.* Wiley, New York.

Boore, D. M., and Atkinson, G. M. (2008). "Ground-motion prediction equations for the average horizontal component of PGA, PGV, and 5%-damped PSA at spectral periods between 0.01 s and 10.0 s." *Earthquake Spectra*, 24(1), 99–138.

Chiou, B. S. J., and Youngs, R. R. (2008) "Chiou-Youngs NGA ground motion relations for the geometric mean horizontal component of peak and spectral ground motion parameters," *Earthquake Spectra* 24, 173–215.

Campbell, K. W., and Bozorgnia, Y. (2008). "NGA ground motion model for the geometric mean horizontal component of PGA, PGV, PGD and 5% damped linear elastic response spectra for periods ranging from 0.01 to 10 s." *Earthquake Spectra*, 24(1), 139–171.

Cornell, C. A. (1968). "Engineering seismic risk analysis." *Bulletin of the Seismological Society of America*, 58(5), 1583-1606.

Der Kiureghian, A., and Ang, A. H. (1977). "A fault-rupture model for seismic risk analysis." *Bulletin of the Seismological Society of America*, 67(4), 1173-1194.

Ebel, J. E., Kafka, A. L. (1999). "A Monte Carlo Approach to Seismic Hazard Analysis", *Bull. Seism. Soc. Am.*, 89(4), 854-866.

Han, S.-W., and Choi, Y.-S. (2008). "Seismic hazard analysis in low and moderate seismic region-Korean peninsula." *Structural Safety*, Elsevier, 30(6), 543–558.

Hessami, K. and Jamali, F. (2006). "Explanatory notes to the map of Major Active Faults of Iran" *Journal of Seismology and Earthquake Engineering* 8 (1), 1–11.

IIEES (2014). International Institute of Earthquake Engineering and Seismology Available at: http://www.iiees.ac.ir/iiees/EQsearch/%28rgiauxmiwbwmba55uqakes45%29/EventQuery.aspx.

Kijko, A., and Sellevoll, M. A. (1992). "Estimation of earthquake hazard parameters from incomplete data files. Part II. Incorporation of magnitude heterogeneity." *Bulletin of the Seismological Society of America*, Seismological Society of America, 82(1), 120–134.

Mahsuli, M., and Haukaas, T. (2013). "Seismic risk analysis with reliability methods, part I: Models." *Structural Safety*, Elsevier, 42, 54–62.

Mahsuli, M., Haukaas, T. (2013). ,"Seismic risk analysis with reliability methods, partII: Analysis*." Structural Safety*, 42, 54-62.

Mahsuli, M. (2012). "Probabilistic models, methods, and software for evaluating risk to civil infrastructure." University of British Columbia.

McGuire, R. K. (1974). "Seismic structural response risk analysis incorporating peak response regressions on earthquake magnitude and distance." *Rep. No. Publ. 399*, Department of Civil Engineering, Massachusetts Institute of Technology, Boston, MA.

McGuire, R. K. (2004). *Seismic hazard and risk analysis.* Earthquake Engineering Research Institute, Berkeley, CA.

McGuire, R. K. (2008). "Probabilistic seismic hazard analysis: Early history." *Earthquake Engineering and Structural Dynamics*, 37(3), 329-338.

Reasenberg, P. (1985). "Second-order moment of central California seismicity, 1969--1982." *Journal of Geophysical Research: Solid Earth (1978--2012)*, Wiley Online Library, 90(B7), 5479–5495.

Tavakoli, B., and Ghafory-Ashtiany, M. (1999). "Seismic hazard assessment of Iran." *Annals of Geophysics*, 42(6).

Van Stiphout, T., Zhuang, J., and Marsan, D. (2012). "Seismicity declustering." *Community Online Resource for Statistical Seismicity Analysis*, 10.