Physically-based Seismic Reliability Evaluation of Water Distribution Networks

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ABSTRACT: In this paper, a physically-based seismic reliability evaluation program is proposed to evaluate the performance of water distribution networks (WDNs) subjected to earthquakes. Firstly, a hydraulic equation which considers the leakage of WDNs after earthquakes is introduced. Then, combining with seismic analysis for buried pipeline systems, the hydraulic equation can be solved and the nodal heads of WDNs can be given. Finally, employing a probability density evolution method and a physically-based random function model for ground motions, the probability density function of the extreme values of nodal heads can be obtained, and the seismic functional reliability of WDNs can be given readily.

1. INTRODUCTION

As an major component of lifeline systems, water distribution networks (WDNS) play an important role on sustaining the serviceability of a modern city. Experiences during many previous strong earthquakes, such as the Kobe earthquake(OERT, 1997), and the Wenchuan earthquake(SLDRCE, 2008), have demonstrated that WDNs are vulnerable to earthquakes. The damages of WDNs not only affect the residential and industrial activities, and result in huge loss, but also lead to catastrophic secondary disasters, such as fire disaster due to the lack of water. Because of the importance of WDNs, it is necessary to assess the performance of WDNs subjected to earthquakes.

In general, there are two approaches to analyze the seismic reliability of WDNs. One is a Monte Carlo simulation (MCS) method. Using this method, Shinozuka et al. (1981) introduced a flow analysis to assess the serviceability of WDNs subjected to earthquakes. Although the MCS method can give the result readily and conveniently, its deficiency is time-consuming and results being stochastic convergent. The other is a mean first-order second-moment (MFOSM) method. In 2004, Chen and Li presented a point leakage model and established a hydraulic equation to describe the post-disaster WDNs. After solving the equation, a performance function was established and the MFOSM method was employed to give the seismic functional reliability of WDNs. However, only the probability characteristics of means and deviation are used.

In this paper, a new approach for seismic functional reliability analysis of WDNs is proposed based on a probability density evolution method (PDEM) (Li and Chen, 2009). At the beginning, the flow analysis method for WDNs with leakages is introduced. Then, considering the randomness of ground motions and WDNs, a probability density evolution equation (PDEE) is established to give probability density functions (PDFs) of the extreme values of nodal heads. Thus, the seismic functional reliability of WDNs are given. Finally, in order to validate the feasibility of the present method, a small WDN is investigated.

2. HYDRAULIC ANALYSIS FOR NETWORS WITH LEAKAGES

The nodal balance formulation of a WDN with leakages can be written as

$$\mathbf{DQ_P} = \mathbf{Q_N} + \mathbf{Q_L} \qquad (1)$$

where $\mathbf{D}$=[d_{ij}] denotes the connectivity matrix of WDN (d_{ij} =1 in case the water flow in the *i-j* pipeline moves from *i* to *j*, and d_{ij}=-1 in case the water flow moves from *j* to *i*); $\mathbf{Q_P}$ (m^3/sec) represents the pipeline flow; $\mathbf{Q_N}$ represents the nodal flow vector, i.e. the demand flow of the consumers, and $\mathbf{Q_L}$ (m^3/sec) is the leakage flow vector.

For simplification, the leakage flow of a pipeline is assumed to be equally added to the two incident nodes. This treatment is advantageous in that it does not change the network topology and no new equation for hydraulic analysis is needed. Moreover, an example indicates that this treatment does not lead large error (Liu et al, 2014). In order to obtain $\mathbf{Q_L}$, the leakage flow of a joint between two pipe segments need to be given.

$$q_{L1} = \alpha A_{L1}\sqrt{2gH_{L1}} \qquad (2)$$

where H_{L1} is the head of the leakage point; α is the leakage coefficient and usually takes the value of 0.1 to 0.3; q_{L1} is the leakage flow of a pipeline joint; A_{L1} is the leakage area of a joint and can be written as follows (Chen and Li, 2004):

$$A_{L1} = \begin{cases} 0 & S_1 < R_1 \\ \pi d\left(S_1 - R_1\right) & R_1 \le S_1 \le R_1 + A/\pi d \\ A & S_1 > R_1 + A/\pi d \end{cases} \qquad (3)$$

where R_1 is the leakage displacement of the joint, i.e., the joint begins to leak when its displacement arrives at R_1. A is the pipeline section area corresponding to the joint, and S_1 is the joint displacement during earthquakes and can be given using a finite element method (Sun, 2009). Apparently, a component of $\mathbf{Q_L}$, q_{Li}, represents the leakage flow of node *i* and equals half of the leakage flow of all pipelines incident to this node.

3. SIMULATION OF GROUND MOTION FIELD

Recently, a physically based random function model was proposed (Wang and Li, 2011; 2012) to simulate ground motion fields. According to the model, the ground motion field can be described as follows (Wang and Li, 2011; 2012):

$$a(r,r_l,t) = -\frac{1}{2\pi}\cdot\int_{-\infty}^{+\infty} A(\boldsymbol{\eta},\omega,r,r_l)\cdot\cos[\omega t + \Phi(\boldsymbol{\eta},\omega,r,r_l)]d\omega \qquad (4)$$

where a represents the ground motion acceleration history in the engineering site; r is the distance from the hypocenter to the geometric center of the local site; r_l is the location of the ground motion to be simulated in the local site; A and Φ are the amplitude spectrum and phase spectrum of the ground motion, respectively; ω is the circular frequency of the ground motion; $\boldsymbol{\eta}$ = $[K,a',b',c',d']^T$ is a vector of deterministic parameters that reflects the effects of path when the seismic waves propagates from the source to the engineering site.

By using the physical random function model and the superposition method of narrow-band harmonic wave groups, the ground motion field samples can be synthetized by using the following formula (Wang and Li, 2011):

$$\begin{aligned} a(r,r_l,t) = -A_0\cdot\sum_i A_i(r,r_l) \\ \cdot F_i(r,r_l,t)\cdot\cos[\omega_i t + \Phi_i(r,r_l)] \end{aligned} \qquad (5)$$

$$\begin{aligned} A_i(r,r_l) = \frac{2}{\pi}\cdot\frac{\omega_i e^{-K\omega_i r}}{\sqrt{\omega_i^2 + (\frac{1}{T})^2}} \\ \cdot\sqrt{\frac{1+4\zeta_g^2(\frac{\omega_i}{\omega_g})^2}{[1-(\frac{\omega_i}{\omega_g})^2]^2 + 4\zeta_g^2(\frac{\omega_i}{\omega_g})^2}}\cdot e^{-\frac{\alpha_0\omega_i r_l}{2}} \end{aligned} \qquad (6)$$

$$F_i(r,r_l,t)=\frac{\sin[(t-\frac{r}{c_i}-\frac{r_l}{c_g})\cdot\Delta\omega_j]}{t-\frac{r}{c_i}-\frac{r_l}{c_g}} \tag{7}$$

$$\Phi_i(r,r_l)=\arctan(\frac{1}{T\omega_i})- \\ r\cdot d'\ln[(a'+0.5)\omega_i+b'+\frac{1}{4c'}\cdot\sin(2c'\omega_i)]-r_l\cdot\frac{\omega_i}{c_g} \tag{8}$$

$$c_i=\left.\frac{d\omega}{dk}\right|_{\omega=\omega_i}=\frac{(a'+0.5)\omega_i+b'+\frac{1}{4c'}\cdot\sin(2c'\omega_i)}{d'\cdot[a'+\cos^2(c'\omega_i)]} \tag{9}$$

where A_0 and T are the basic random parameters of the source; ω_g and ζ_g are the equivalent predominant circular frequency and equivalent damping ratio of the soil at the local site, respectively; α_0 and c_g are the attenuation parameter of the local site and the apparent seismic wave velocity.

4. SEISMIC RELIABILITY BASED ON EXTREME-VALUE DISTRIBUTION

4.1. The extreme-value distribution evaluation based on the PDEM

In practical engineering, the extreme value of a time history is very important, especially in reliability evaluation and engineering risk analysis. For WDNs, the mostly concerned thing by engineers is the minimal nodal heads, which indicate whether the serviceability is satisfied or not. Thus, the extreme value of nodal head can be expressed as

$$H_{i,\mathrm{ext}}=\min_{t\in[0,T]}|H_i(t)|\quad(i=1,2,\cdots,n) \tag{10}$$

where $\min_{t\in[0,T]}|\cdot|$ means the minimal value of $H_i(t)$ during time interval $[0,T]$, which can be given by solving Eq. (1) after substituting the ground motion.

In general, the nodal head is affected by many factors, such as ground motions, soil conditions, leak displacements of joints. Apparently, the nodal head is a random variable due to the randomness of ground motions and pipelines. Herein, the random variables of ground motions and pipelines can be described as $\mathbf{\Theta}=(\mathbf{\eta},\zeta)=(\eta_1,\eta_2,\cdots,\eta_{s1},\zeta_1,\zeta_2,\cdots,\zeta_{s2})=(\theta_1,\theta_2,\cdots,\theta_s)$, where s_1 and s_2 are the number of basic random variables of ground motions and pipelines; $s=s_1+s_2$ is the total number of basic random variables.

In recent years, a new approach for stochastic dynamical systems analysis, named probability density evolution method (PDEM), has been developed (Li and Chen, 2009). Using this method, the extreme-value distributions of nodal heads can also be obtained by constructing a virtual stochastic process.

For the nodal head of WDNs, the extreme value of node head $H_i(t)$, is unique and depends on the random parameter vector $\mathbf{\Theta}$. Thus, Eq. (10) can be further taken the form as

$$H_{i,\mathrm{ext}}(\mathbf{\Theta})=\min_{t\in[0,T]}(H_i(\mathbf{\Theta},t))=W_i(\mathbf{\Theta}) \tag{11}$$

Define a virtual stochastic process $Y_i(\tau)$ as

$$Y_i(\tau)=\phi(W_i(\mathbf{\Theta}),\tau) \tag{12}$$

which satisfies the conditions that

$$Y_i(\tau)\big|_{\tau=0}=\phi(W_i(\mathbf{\Theta}),\tau=0)=0 \tag{13}$$

and

$$Y_i(\tau)\big|_{\tau=\tau_c}=\phi(W_i(\mathbf{\Theta}),\tau=\tau_c)=W_i(\mathbf{\Theta}) \tag{14}$$

where τ is the "time" of the virtual stochastic process; τ_c represents the moment when $Y_i(\tau)$ reaches extreme value.

Then, a probability density evolution equation (PDEE) for the extreme values of nodal heads is

$$\frac{\partial p_{Y_i\mathbf{\Theta}}(y_i,\mathbf{\theta},\tau)}{\partial\tau}+\dot{\phi}(W_i(\mathbf{\Theta}),\tau)\frac{\partial p_{Y_i\mathbf{\Theta}}(y_i,\mathbf{\theta},\tau)}{\partial y_i}=0 \tag{15}$$

Correspondingly, the initial condition is

$$p_{Y_i\mathbf{\Theta}}(y_i,\mathbf{\theta},\tau)\big|_{\tau=0}=\delta(y_i)p_{\mathbf{\Theta}}(\mathbf{\theta}) \tag{16}$$

where $p_{\mathbf{Y}_i\mathbf{\Theta}}(y_i,\mathbf{\theta},\tau)$ is the joint PDF of Y_i and $\mathbf{\Theta}$; $\dot{\phi}(W_i(\mathbf{\Theta}),\tau)=\partial\phi(W_i(\mathbf{\Theta}),\tau)/\partial\tau$.

Considering the initial condition, Eq. (16), Eq. (15) can be solved. Then, the PDF of $Y_i(\tau)$ is

$$p_{Y_i}(y_i,\tau)=\int p_{Y_i\Theta}(y_i,\boldsymbol{\theta},\tau)d\boldsymbol{\theta} \tag{17}$$

Then, the PDF of the extreme value of nodal head is

$$p_{H_{i,\mathrm{ext}}}(h_i)=p_{Y_i}(y_i=h_i,\tau)\Big|_{\tau=\tau_c} \tag{18}$$

Herein, a sine function is constructed as follows

$$\phi(W_i(\boldsymbol{\Theta}),\tau)=W_i(\boldsymbol{\Theta})\sin(\omega\tau) \tag{19}$$

in which ω=2.5π, τ_c=1.

4.2 Seismic reliability evaluation of WDNs

In general, the reliable criteria of WDNs is whether the nodal demand head is satisfied or not. Thus, the performance function of node i can be written as:

$$Z_i=H_i\text{-}H_{i\mathrm{lim}} \tag{20}$$

in which H_i is the head of node i, and $H_{i\mathrm{lim}}$ is the demand head of node i.

Therefore, the functional reliability of the node i is expressed as:

$$P_{ri}=P_r(Z_i>0) \tag{21}$$

It can be further described as:

$$P_{ri}=\Pr\{H_{i,\mathrm{ext}}\geq H_{i\mathrm{lim}}\} \tag{22}$$

When the PDF of $H_{i,\mathrm{ext}}$ is obtained, it is convenient to evaluate the seismic functional reliability of the node i by

$$P_{ri}=\int_{H_{i\mathrm{lim}}}^{+\infty}p_{H_{i,\mathrm{ext}}}(h)dh \tag{23}$$

Thus, the steps of numerical calculation of seismic functional reliability of WDNs can be summarized as follows.

① Select representative point sets $\boldsymbol{\theta}_q$ (q=1,2,…,N_{sel}, where N_{sel} is the number of selected points) with the corresponding assigned probability P_q in the random parameter space Ω_Θ. For the case of the multi-dimensional random parameters, a strategy of selecting representative points with number theoretical method is recommended (Li and Chen, 2007).

②For each sample, a deterministic seismic response analysis is performed to give the responses of WDNs. Solving Eq. (1), the history of each node head is derived and the extreme-value of each nodal head can be obtained.

③ Construct a virtual stochastic process for the extreme values of the nodal heads, and solve the PDEE to give the PDFs of extreme-values of nodal heads.

④ Use Eq. (23) to give the seismic functional reliability of WDNs.

5. NUMERICAL EXAMPLE

To investigate the applicability of the proposed approach, a small WDN consisting of 17 nodes and 24 pipelines is studied. Figure 1 shows the layout of the network. In this network, the source is 17 and its nodal head is 41m. The other nodes are customer nodes with demand head of 10m. All the pipelines are cast iron pipes with joints filled with asbestos cement. Using the number theoretical method, 398 representative points with assigned probability are generated.

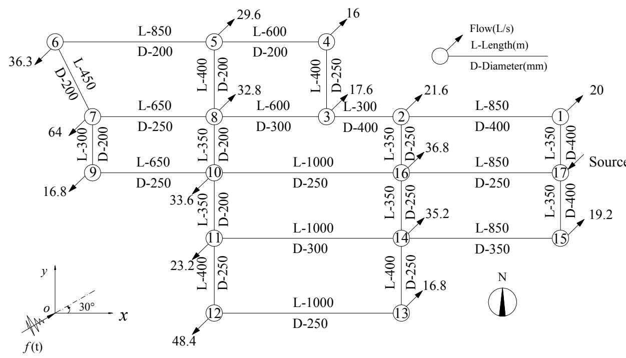
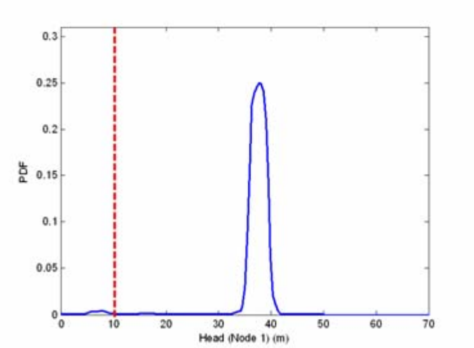
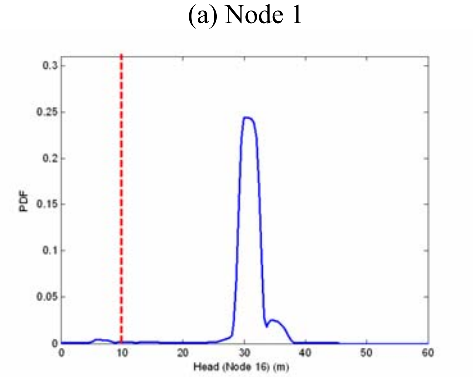


Figure 1 Layout of WDN

Supposing the seismic wave propagates from southwest to northeast with an angle of 30° (shown in figure 1) and the peak ground acceleration is 0.2g. Using the proposed method, the PDFs of the extreme-value of node heads are obtained. Figure 2 shows the PDFs of the head extreme-values of node 1 and 16. If the demand head is 10m, the functional reliabilities of these two nodes are 0.9832 and 0.9837, respectively. Figure 3 shows the functional reliability of all nodes. It is found in figure 3 that the nodal reliability is closely related with node location. For example, the functional reliabilities of the nodes close to the source, such as nodes 1, 2, 15 and 16, are higher while the reliabilities of the nodes far away from the source, such as nodes 6, 7 and 9, are lower.



(a) Node 1



(a) Node 16

Figure 2 PDF of head extreme-value of two nodes

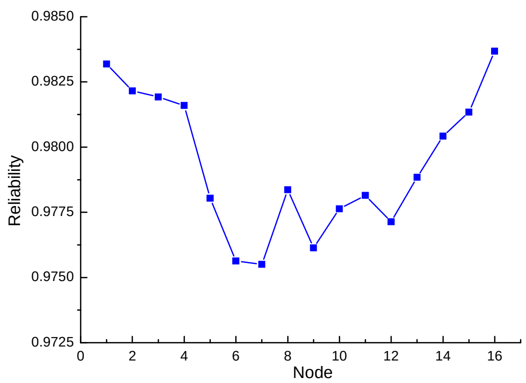


Figure 3. Reliabilities of customer nodes

6. CONCLUSIONS

In this paper, a new approach for assessing seismic functional reliability of WDNs has been developed. The seismic functional reliability of WDNs is evaluated in terms of the procedures of ground motion field generation based on physical random function model, seismic response analysis of WDNs, hydraulic analysis of WDNs with leakages, nodal head extreme-value probability distribution evaluation using PDEM, and seismic functional reliability analysis of WDNs. To validate the proposed approach, a small water distribution system with 17 nodes and 24 pipelines is studied. The PDFs of the head extreme values of nodes are obtained, and the seismic functional reliability of each node is obtained. It is found that the nodal reliability is closely related with the location.

Acknowledgments

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