12th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP12
Vancouver, Canada, July 12-15, 2015

Prediction of Creep and Shrinkage based on Gamma Process Models

Alfred Strauss,
Assoc. Professor, Institute of Structural Engineering, Dept. of Civil Engineering and Natural Hazards, University of Natural Resources and Life Sciences, Vienna, Austria

Roman Wendner
Post Doc., Institute of Structural Engineering, Dept. of Civil Engineering and Natural Hazards, University of Natural Resources and Life Sciences, Vienna, Austria

Anja Vidovic
Ph.D candidate. Institute of Structural Engineering, Dept. of Civil Engineering and Natural Hazards, University of Natural Resources and Life Sciences, Vienna, Austria

Ivan Zambon
Ph.D candidate. Institute of Structural Engineering, Dept. of Civil Engineering and Natural Hazards, University of Natural Resources and Life Sciences, Vienna, Austria

Dan M. Frangopol
Fazlur Rahman Khan Endowed Chair of Structural Engineering and Architecture, Department of Civil and Environmental Engineering, ATLSS Engineering Research Center, Lehigh University, Bethlehem, PA, USA

ABSTRACT: In order to systematically study the main influence factors on bridge deformation probability prediction methods can be used. Gamma process has shown to be suitable for capturing evolving structural response quantities and deterioration mechanism like crack propagation, corrosion, creep and shrinkage. The main aim of this paper is to investigate the applicability of gamma process approaches for the prediction of the creep and shrinkage performance of pre-stressed concrete bridges. For this purpose the Colle Isarco Viaduct Bridge was used.

1. INTRODUCTION

In many countries, well–defined specifications for the design and prediction of the structural behavior of infrastructure systems such as highway bridges are in use. These specifications need to ensure an adequate degree of reliability. The wide range of processes which affect any infrastructure system in the course of its life–time leads to the requirement of periodic assessments based on monitoring data and inspections. When necessary, the performance of these infrastructure systems and their components is adjusted through carefully optimized maintenance measures. In recent years, new technologies were developed for the accurate description of physical processes such as corrosion, creep and shrinkage. In addition, the availability of data for calibration and validation, which can be gained e.g. through monitoring (Strauss et al., 2011, 2012; Wendner and Strauss, A., 2014), is increasing rapidly. This also creates a solid foundation for reviewing the effectiveness of long established inspection procedures for infrastructure systems, which are paramount for the predicted performance and safety of structural systems. The systematic collection of a wide range of site-specific data, if made available to the engineering community, is an essential element for the verification and improvement of current models and design concepts. This contribution focuses on problems caused by the creep and shrinkage processes in prestressed concrete structures. In particular, the possibilities for their prediction based on stochastic processes as an efficient alternative to computationally expensive mechanical models will be investigated.

2. RATE-TYPE FORMULATION FOR CREEP STRUCTURAL ANALYSIS

Considering the size scale and time span required in creep structural analysis for the long-term performance assessment of creep sensitive structures as well as the complex interactions with other phenomena such as steel relaxation or damage evolution it is necessary to abandon the traditional approach to creep analysis – the integral-type formulation based directly on the principle of superposition. Instead the more efficient and versatile rate-type formulation is used. It is well known that any realistic creep compliance function following ageing linear viscoelasticity, which is the case for concrete (at least under service load), can be approximated at any desired accuracy by a rheological model represented by Kelvin units, Maxwell units or hybrid units. In addition to accuracy, the rate-type formulation brings great benefits in computational efficiency and compatibility, which allows the coupling with other phenomena (e.g., cracking, fatigue, and steel relaxation).

An improved rate-type formulation using Kelvin units (Yu et al., 2012) is employed to simulate the long-term creep-shrinkage behavior of complex structural systems. By taking advantage of the Laplace transformation inversion and Widder's approximation (Widder, 1971), the ageing spectrum of each Kelvin unit at each time step can be uniquely and efficiently identified based on the given compliance function of the creep model used in the analysis (Bažant et al., 2012a, 2012b; Yu et al., 2012). With the aid of the exponential algorithm (Jirásek and Bažant, 2002), the rate-type formulation can be further extended to 3D quasi-elastic stress-strain incremental relation, which can be programmed in a user subroutine and then imported in general FEM software like ABAQUS for structural modeling. The detailed derivation and implementation of this improved rate-type formulation can be found in a recent investigation (Yu et al., 2012).

3. LONG-TERM PREDICTION OF CREEP EFFECTS

Degradation and aging processes of a structure can be described by non-negative continuous functions. These functions can be characterized by non-negative increments with independent path and variable uncertainty. Furthermore, the period of time until the occurrence of an observed undesirable incident is typically associated with considerable uncertainty and dependent on structural behavior. Currently established creep and shrinkage prediction models are deterministic and provide no quantification of uncertainty. Yet, the creep and shrinkage response clearly follows a non-negative and continuous function with independent path and variable uncertainty content. Creep and shrinkage processes are subject to numerous uncertainties. This is in fact the reason why we present stochastic processes as an approximate approach for capturing the macroscopic creep and shrinkage behavior of structural components and systems. Already in the 1970s Bažant and his coworkers explored stochastic process for the extrapolation of uncertain creep processes (Cinlar et al., 1977). Within the framework of life cycle modeling stochastic process predictions and probabilistic modeling have become key elements as acknowledged by Frangopol et al. (2004) and van Noortwijk (2009). The lack of failure data for buildings and bridges constitutes a major problem for the application of reliability methods as no calibration or validation is possible. Consequently, decisions based on lifetime distribution and/or very low structural failure rates are inconsistent and not particularly rigorous. Time dependent uncertain structural properties, as for example the deterioration rates of structures or structural components, are frequently calculated using random variables. To describe the stochastic processes Markov process representations which require predefined status categories are used frequently. Regarding stochastic processes, van Noortwijk (2009) distinguishes between discrete Markov processes, as for example Markov chains, continuous Markov processes such as Brownian movement, as well as Levy and Gamma processes. The physical processes behind structural deterioration clearly are not independent of previous time steps and thus cannot be realistically described by non-correlated incremental stochastic processes. According to Pandey et al. (2009), gamma processes are suitable for capturing such deterioration processes. In particular, van Noortwijk (2009) suggests using continuous stochastic gamma processes with independent non-negative increments for the description of a gradually developing deterioration processes. These are, for example, corrosion of reinforcement bars, creep and shrinkage of concrete, fatigue of concrete and reinforcement, and crack propagation.

4. CASE STUDY ON THE COLLE ISARCO VIADUCT

The presented framework for gamma prediction processes will be applied to multi-decade creep and shrinkage predictions, calculated for an existing 167.5 m long fully post-tensioned box girder bridge. Each of the four identical main structural elements of the Colle Isarco Viaduct, located in Northern Italy, consists of a main span of 91.0 m, a long cantilever of 59.0 m, and a short cantilever of 17.5 m, see also Strauss et al. (2009). The cross-section height varies between 10.8 m at the main pier and 2.85 m at the tip of the cantilever. The box girder itself has a width of 6.0 m whereas the top slab is 10.6 m wide. The bridge is erected sequentially out of 44 segments, 497 pre-stressing tendons in total and 149 tendons above the main support.

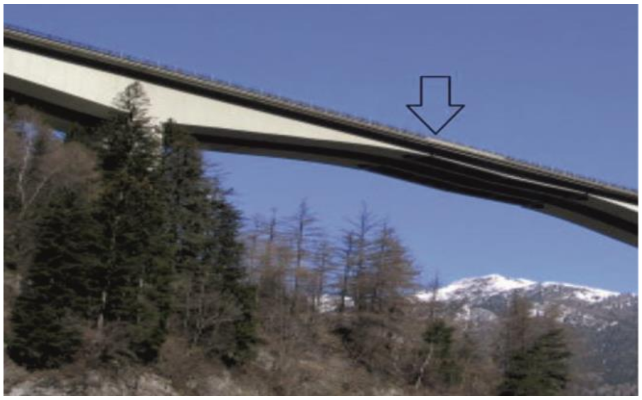

Figure 1: Colle Isarco Viaduct – bottom view on the minor and cantilever beam.

4.1. Deterministic analysis

Wendner et al. (2014d) recently investigated the most widely used creep and shrinkage models, namely ACI92 (ACI Committee 209, 2008), B3 (Bažant, 1995), B4 (Bažant et al., 2014), CEB-FIP90 (CEB-FIP, 1993), fib2010 (fib, 2013) based on the Colle Isarco's design specifications. This full discussion of the different models' characteristics and the respective consequences for long term predictions is based on most realistic deterministic and probabilistic structural analyses and provides important insights into the creep behavior that could not be derived based on laboratory data and theoretical considerations as presented in Hubler et al. (2014). Exploiting symmetry, the bridge was modeled in ABAQUS using solid elements for the concrete members and truss elements for the discretely modeled pre-stressing tendons.

4.2. Probabilistic study

The underlying stochastic models for all input variables were used in terms of mean value (corresponding to the inputs of the deterministic investigation) and coefficient of variation COV. With the exception of the concrete properties all input variables are considered to be statistically independent. The correlation matrix for the concrete properties, described by pairwise linear correlation coefficients according to Pearson (1895) and Stigler (1989), is given in Table 1. A low yet statistically representative number of samples was generated utilizing the well-known Latin hypercube sampling scheme (Iman and Conover, 1982) combined with simulated annealing (Vořechovský and Novák, 2009) with a population size of 45 samples.

The resulting scattering response not only allows the derivation of sensitivity factors between model inputs and structural response quantities (Bergmeister et al., 2007; Strauss et al., 2013) but also serve as input for the performance prediction using the concept of gamma processes as introduced in the subsequent section.

Table 1: Correlation matrix of concrete properties (Strauss, 2003)

	E_{28}	f_c	c	w/c	a/c
E_{28}	1	0.19	0.06	-0.07	0.05
f_c	0.19	1	0.5	-0.52	0.36
c	0.06	0.5	1	-0.86	-0.86
w/c	-0.07	-0.52	-0.86	1	0.8
a/c	0.05	0.36	-0.86	0.8	1

Simulated deformations are available in 100 day increments starting with the end of construction up to an age of 32 years. In Fig. 2 the scattering predictions for the shortening of the main girder of the Colle Isarco Viaduct are plotted.

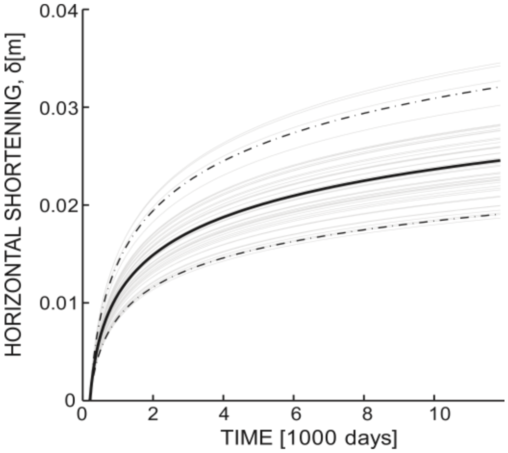

Figure 2: Numerical computed creep-shrinkage responses of the Colle Isarco Viaduct [adopted from Strauss et. al, 2014].

4.3. Alternative gamma process formulation

Gamma process formulation and calculation models are comprehensively described by van Noortwijk (2009) and again by Ohadi and Micic (2011). This approach is labeled as strategy I. In this paper, alternative approaches to the one proposed by them are investigated. In particular, a 2nd and 3rd order polynomial fitting procedure for determining the shape parameter $\alpha(t)$ and scale parameter β of the gamma distribution are explored.

Two alternative fitting strategies are investigated: In strategy IIa a continuous adaptation is only performed for the shape parameter $\alpha(t)$ while in strategy IIb the shape parameter $\alpha(t)$ is updated jointly with the scale parameter $\beta(t)$ (on the basis of the Maximum Likelihood Method). In each case the shape and scale parameters are fitted to the available information t_k where $k=1,\dots,n$ denotes the evaluation or assessment point and n = the total number of observations i.e. k = assessment time and i = prediction horizon.

Independently of the fitting strategy, two formulations for the prediction of future evolution of the gamma process parameters are investigated – a polynomial 2nd and 3rd order. In consequence, the parameters $\alpha(t)$ and β of the gamma process can be predicted up to the assessment horizon k and extrapolated to the prediction horizon $i = k+1,\ldots,n$.

The main goal of this phase of the presented investigation is the direct comparison of creep and shrinkage predictions between the van Noortwijk Moment Method on one side and the alternative strategy IIa and IIb on the other side. The basic features of the different fitting strategies and prediction approaches are discussed and compared with respect to their ability to predict the accurately simulated creep and shrinkage response. Investigation is based on the horizontal shortening data of the main girder as presented in Fig. 2. It is assumed that only n = 10 observations at times $t_{1\ldots n}$ = 212, 512, 773, 1533, 3033, 4533, 6033, 7533, 9033, 10533 days are available.

4.4. Investigation of gamma process properties

For the 2nd order polynomial fitting approach the development of the shape parameter is investigated in Fig. 3, the stationarity of the scaling parameter $\beta(t)=\beta$ is checked in Fig. 4. In general, for every prediction time horizon i the shape parameter α and scale parameter β are estimated using data up to the evaluation or assessment time horizon k. The corresponding gamma process predictions are labeled Ga_k^i. The approach in Fig. 3 was carried out for prediction time horizons $i = 7, \ldots, 10$ equivalent to t_i = 6033, …,10533 days based on data up to the assessment points $k = 6, \ldots, 10$ equivalent to t_k = 4533, …,10533 days. Figures 5 and 6 present the corresponding analyses utilizing a 3rd order polynomial fitting approach.

4.4.1. Discussion of the 2nd order polynomial fitting approach

In case of the 2nd order polynomial prediction $\alpha(t)$ increases from 1.5 to 4.0 with increasing k, see Fig. 3. It is interesting to note that the curvature of the parabola decreases with increasing data basis and the parameters of the gamma distribution approach their values for perfect information $k = n$ equivalent to data between day 0 and day 10533.

According to theory, both the shape and the scaling parameter should be positive and continuously increasing. These conditions are not automatically satisfied by a polynomial fitting function. Nonetheless, the 2nd order polynomial prediction represents a simple and flexible approximation of, arguably, sufficient accuracy, especially for modeling a well–known process. However, this approach should not be used lightly for predictions far beyond the information basis.

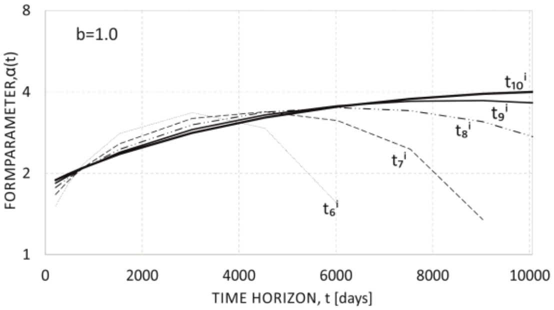

Figure 3: Development of shape parameter $\alpha(t)$ for the fitting strategy IIb in combination with a 2nd order polynomial prediction approach.

The alternative to updating both the shape parameter and scale parameter (strategy IIb) is the assumption of a stationary scale parameter $\beta(t)$ = β (fitting strategy IIa). Fig. 4 compares both fitting strategies utilizing the shortening data of the main girder for an assessment time horizon k = 7 equivalent to t_k = 6033 days. The predictions for strategy IIa (stationary β) are plotted as dashed lines; those for strategy IIb as solid lines. The differences between both strategies are marginal for prediction times i = 8, 10 equivalent to t_i = 7533,…,10533 days. Consequently, the simplification of assuming a stationary scale parameter seems justified.

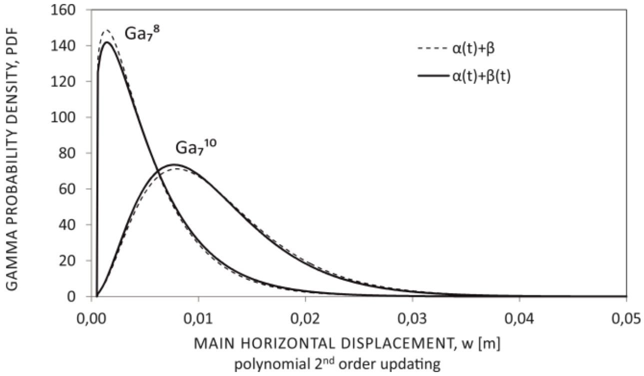

Figure 4: Gamma PDF; comparison between a constant scale parameter ß, and time-continuous adjustment of the scale parameter ß(t).

4.4.2. Discussion of the 3rd order polynomial fitting approach

All previously discussed analyses are repeated on the basis of a third order polynomial fitting approach. The time – dependent behavior of the shape parameter α(*t*) is presented in the Fig. 5. When compared to Fig. 3, in which the second order polynomial fitting procedure of the parameters is portrayed, it becomes clear that the third order approach provides more consistent results, since the 3rd order approximation of the parameters' development better approaches the theoretical requirement of non-decreasing quantities.

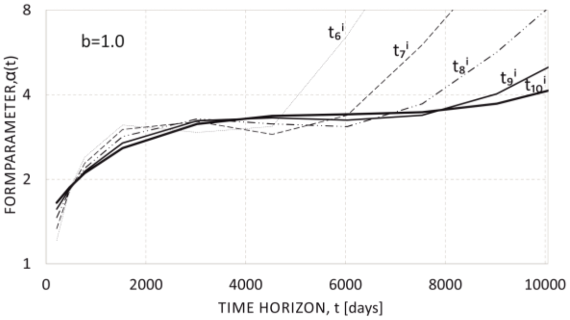

Figure 5: Development of shape parameter α(t) for the fitting strategy IIb in combination with a 3rd order polynomial prediction approach.

As concluded for the 2nd order polynomial, the stationarity of the scale parameter is confirmed also for the 3rd order approach, see Fig. 6. It is shown again that only minor improvements can be expected from updating the scale parameter.

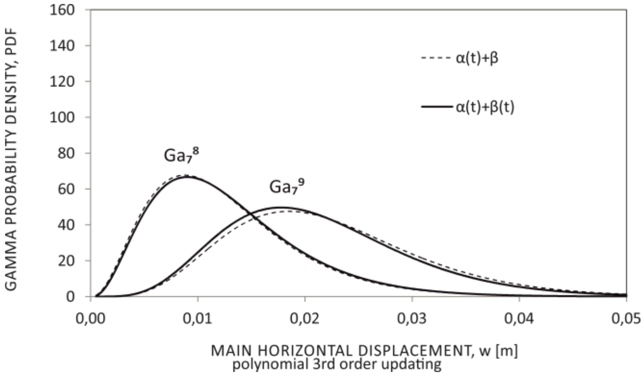

Figure 6: Gamma PDF; comparison between a constant scale parameter ß, and time-continuous adjustment of the scale parameter ß(t).

4.5. Strategy I - van Noortwijk approach

In further investigations the gamma prediction process was carried out according to the procedure recommended by van Noortwijk (2009) and Ohadi and Micic (2011), with a time dependent shape parameter following the power law $\alpha(t) = c \cdot t^b$ a constant scale parameter β. As in strategy IIa and IIb, a power law exponent *b* equal to 1.0 was taken (for detailed investigation of corresponding value of exponent b for creep and shrinkage phenomena see Strauss et. al, 2014). Fig. 7 portrays the predicted expected values of the axial shortening of the main girder w based on the van Noortwijk approach. It is evident that the bias in the prediction decreases with increasing prior information (increasing k) and concurrent reduction in the distance to the prediction horizon i. The properties shown in Fig. 7 for the van Noortwijk method are also valid for the previously described alternative strategies IIa and IIb. The information contained within Fig. 7 is essential for evaluating the required observation time for a given prediction quality or conversely for assessing the prediction quality for a given set of observations.

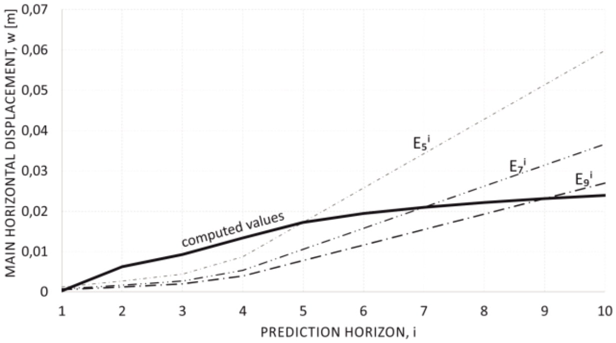

Figure 7: Expected values of the time periods for prediction time horizons ti, of the gamma process prediction.

5. CONCLUSIONS

The primary goal of the present study is the investigation of the suitability of the gamma process approach for the prediction of creep processes in prestressed concrete structures.

Two fitting strategies – the van Noortwijk´s approach based on statistical moments (strategy I) and individual fitting of observations (strategy II) in combination with a polynomial prediction approach of 2nd or 3rd order – are investigated. Time dependence of the scale parameter β has only a minor influence on the quality of the gamma process prediction and thus can be neglected. A 2nd order polynomial prediction approach provides a decent approximation of the gamma process but is unsuitable for predictions into the distant future. A 3rd order polynomial prediction approach better approximates the evolution of the gamma parameters and provides decent (and generally conservative) extrapolations. Alternative prediction equations with the correct asymptotic properties (ensuring e.g. a monotonous increase of the parameters α and β) provide better and more robust extrapolations.

6. ACKNOWLEDGEMENT

The authors would like to acknowledge the invaluable contributions to the research made by their many multidisciplinary colleagues at the Institute of Structural Engineering, Department of Natural Hazards and Life Sciences and the ATLASS Center of Lehigh University. The support of the ZIT Center for Innovation and Technology within the project "Lifetime assessment of fatigue endangered concrete structures, MSLeB, Call From Science to Products 2011; ID 699219 is gratefully appreciated. The financial support by the Austrian Federal Ministry of Economy, Family and Youth and the National Foundation for Research, Technology and Development is gratefully acknowledged.

7. REFERENCES

ACI Committee 209, 2008. Guide for Modeling and Calculating Shrinkage and Creep in Hardened Concrete (No. 209.2R-08). Farmington Hills.

Bažant, Z.P., 1995. Creep and shrinkage prediction model for analysis and design of concrete structures - model B3. Matér. Constr. 28, 357–365.

Bažant, Z.P., Hubler, M.H., Wendner, R., 2014. RILEM draft recommendation: Model B4 for Creep, Drying Shrinkage and Autogenous Shrinkage of Normal and High-Strength Concretes with Multi-Decade Applicability. Mater. Struct. RILEM (accepted).

Bažant, Z.P., Yu, Q., Li, G.-H., 2012a. Excessive long-term deflections of prestressed box girders. I Record-Span Bridge in Palau and Other Paradigms. J. Struct. Eng. 138, 676–686.

Bažant, Z.P., Yu, Q., Li, G.-H., 2012b. Excessive long-term deflections of prestressed box girders. II. Numerical Analysis and Lessons Learned. J. Struct. Eng. 138, 686–696.

Bergmeister, K., Strauss, A., D., N., Pukl, R., 2007. Structural analysis and reliability assessment: SARA part I, in: Bakht, B., Mufti, A.A. (Eds.), Presented at the Structural Health Monitoring & Intelligent Infrastructure, SHMII-3.

CEB-FIP, 1993. CEB-FIP MODEL CODE 1990. Committee Euro-International du Béton.

Cinlar, E., Bazant, Z.P., Osman, E., 1977. Stochastic process for extrapolating concrete creep. ASCE J Eng Mech Div 103, 1069–1088.

fib, 2013. fib Model Code for Concrete Structures 2010. Wilhelm Ernst & Sohn.

Frangopol, D.M., Kallen, M.J., van Noortwijk, J.M., 2004. Probabilistic models for life–cycle performance of deteriorating structures: review and future directions. Prog. Struct. Eng. Mater. 6, 197–212.

Hubler, M.H., Wendner, R., Bažant, Z.P., 2014. Statistical Justification of Model B4 for Drying and Autogenous Shrinkage of Concrete and Comparisons to Other Models. Mater. Struct. RILEM (accepted).

Iman, R.L., Conover, W.J., 1982. A distribution–free approach to inducing rank correlation among input variables. Commun. Stat. Comput. 11, 311–334.

Jirásek, M., Bažant, Z.P., 2002. Inelastic Analysis of Structures. J. Wiley, London.

Kirkpatrick, S., Gelatt, C., Vecchi, M., 1983. Optimization by simulated annealing. Science 220, 671–680.

Ohadi, A., Micic, T., 2011. Stochastic process deterioration modelling for adaptive inspections, in: Applications of Statistics and Probability in Civil Engineering. Presented at the ICASP 11, Taylor & Francis Group, London, pp. 1085–1091.

Pandey, M.D., Yuan, X.X., van Noortwijk, J.M., 2009. The influence of temporal uncertainty of deterioration on life–cycle management of structures. Struct. Infrastruct. Eng. 5, 145–156.

Pearson, K., 1895. Notes on regression and inheritance in the case of two parents. Proc. R. Soc. Lond. 58, 240–242.

Stigler, S.M., 1989. Francis Galton’s Account of the Invention of Correlation. Stat. Sci. 4, 73–79.

Strauss, A., 2003. Stochastische Modellierung und Zuverlässigkeit von Betonkonstruktionen (PhD). University BOKU, Vienna

Strauss, A., Hoffmann, S., Wendner, R., Bergmeister, K., 2009. Structural assessment and reliability analysis for existing engineering structures, applications for real structures. Struct. Infrastruct. Eng. 5, 277–286. doi:10.1080/15732470601185638

Strauss, A., Wendner, R., Bergmeister, K., Adley, M., Horvatits, J., 2011. Monitoring and influence lines based performance indicators. Beton- Stahlbetonbau 106, 231–240. doi:10.1002/best.201100003

Strauss, A., Wendner, R., Bergmeister, K., Costa, C., 2013. Numerically and experimentally based reliability assessment of a concrete bridge subjected to chloride–induced deterioration. J. Infrastruct. Syst. 19, 166–175.

Strauss, A., Wendner, R., Frangopol, D.M., Bergmeister, K., 2012. Influence line-model correction approach for the assessment of engineering structures using novel monitoring techniques. Smart Struct. Syst. 9, 1–20.

Strauss, A., Wendner, R., Vidovic, A., Zambon, I., Yu, Q., Frangopol, D., Bergmeister, K., 2014. Gamma Prediction models for Long - Term Creep Deformations of PC Bridges, Tehnopress (submitted)

van Noortwijk, J.M., 2009. A survey of the application of gamma processes in maintenance. Reliab. Eng. Syst. Saf. 94, 2 – 21.

Vořechovský, M., Novák, D., 2009. Correlation control in small–sample Monte Carlo type simulations I: A simulated annealing approach. Probabilistic Eng. Mech. 24, 452–462.

Wendner, R., Hubler, R., Bažant, Z.P., 2014c. Statistical Justification of Model B4 for Multi-Decade Concrete Creep and Comparisons to Other Models Using Laboratory and Bridge Databases. Mater. Struct. RILEM Be Submitt.

Wendner, R., Strauss, A., Guggenberger, T., Bergmeister, K., Teplý, B., 2010. Ansatz zur Beurteilung von chloridbelasteten Stahlbetonbauwerken mit Bewertung der Restlebensdauer. Beton- Stahlbetonbau 105, 778–786. doi:10.1002/best.201000049

Wendner, R., Strauss, A., 2014. Inclined Approach Slab Solution for Jointless Bridges: Performance Assessment of the Soil–Structure Interaction. J Perform Constr Facil (in press). doi:10.1061/(ASCE)CF.1943-5509.0000522

Wendner, R., Tong, T., Strauss, A., Yu, Q., 2014d. Long-Term Deformation Asymptotes of Segmentally-Erected Prestressed Concrete Box Girders and Their Correlation with Concrete Creep. Struct. Infrastruct. Eng. (in press).

Widder, D.V., 1971. An Introduction to Transform Theory. Academic Press, New York.

Yu, Q., Bažant, Z.P., Wendner, R., 2012. Improved algorithm for efficient and realistic creep analysis of large creep-sensitive concrete structures. ACI Struct. J. 109, 665–676.