Probabilistic Analysis of Soil-Structure Interaction

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ABSTRACT: This paper studies the effect of soil-structure interaction on the seismic performance of structures taking into account the prevailing uncertainties. The motivation for this study stems from the significant uncertainty in the earthquake ground motion and in the properties of the soil and the nonlinear behavior of the structure. The soil-structure system is modeled by the sub-structure method. The uncertainty in the properties of the system is described by random variables that are input to this model. Monte Carlo sampling is employed to compute the probability distribution of responses that describe the seismic performance of the structure, such as the ductility demand. In each sample, a randomly generated soil-structure system is subjected to a randomly selected and scaled ground motion. The selection of the ground motion follows an "adaptive" procedure. An extensive parametric study is conducted to cover a wide range of systems. The results reveal the probability that soil-structure interaction increases the ductility demand on a structure designed according to fixed-base provisions of seismic design codes. For instance, certain structures designed as fixed-base but built on soil may experience higher levels of ductility demand. Furthermore, it is found that practicing the soil-structure interaction provisions of modern seismic codes increases the likelihood of significantly high outcomes of the ductility demand.

1. INTRODUCTION

This paper revisits the phenomenon of dynamic soil-structure interaction (SSI) with a probabilistic approach. The overarching objective is to determine the effect of this phenomenon on the performance of the structure under seismic excitations. The need for a probabilistic framework in this study stems from recognition of three prevailing sources of uncertainty in this phenomenon. These are the uncertainty in the ground motion, in the soil properties, and in the nonlinear behaviour of the structure. The presented probabilistic approach is in line with recent and inevitable trends towards probabilistic performance–based earthquake engineering, which aim at quantifying the seismic performance of the structure with due account of uncertainties.

Many past studies in this field employed a deterministic framework. These studies range from the pioneering work of Veletsos and Meek (1974) to more recent studies by Aviles and Perez-Rocha (2003) and Ghannad and Ahmadnia (2006), among others. The latter study investigated the effect of SSI on the structural response through simplified deterministic methods. Few recent studies take into account the uncertainty in the soil and the structure. Mehanny and Ayou (2008), Barcena and Esteva (2007), Jin *et al*. (2000), and Raychowdhury (2009) showed that uncertainties in the structural and geotechnical properties of a system, and those in the ground motion characteristics, affect the performance of the structure markedly. Moreover, a statistical study by the Moghaddasi *et al*. (2010) showed that for structures with shallow foundations, inertial SSI effects can either decrease or increase the structural response depending on the particular site-structure-earthquake scenario and the level of probabilities.

This study accounts for all three major sources of uncertainty enumerated earlier. Reliability methods are employed to compute the probability distribution of parameters that describe the seismic performance of the structure, such as the maximum inelastic displacement and the ductility demand. Amongst reliability methods, Monte Carlo sampling is opted for in this first step. This is a straight-forward and powerful approach, yet with a high computational cost. In this analysis, random variables describe the uncertainty, and are input to probabilistic models. A novelty of this paper lies in quantifying the properties of these random variables to best describe the uncertainty in the soil and the structure. To model the soil-structure system, the sub-structure method is employed. A simplified model is used to represent structures with embedded foundation. Both inertial and kinematic interaction effects are taken into account.

To model the ground motion, natural records are utilized due to the lack of artificial ground motion models for soft soil. A suite of 60 accelerograms recorded on soft soil are employed. Details of selecting and scaling ground motions in the Monte Carlo sampling are explained in the Analysis Methodology section.

The models and analysis methods are implemented in Rt, which is a computer program for multi-model reliability and risk analysis (Mahsuli and Haukaas 2013). Rt is freely available online at inrisk.ubc.ca. Rt has an object-oriented design that facilitates the steady growth of its library of probabilistic models and analysis methods. It has a user-friendly, graphical interface that can be utilized to, say, visualize the probability distribution of results.

The primary results of this analysis are the probability distribution of the structural responses, *e.g.*, the ductility factor. Analyses cover a range of structures with different natural periods, "dimensionless frequencies," "aspect ratios," and "embedment ratios," all of which are described in the next section. Discussion of the results demonstrates the influence of the soil-structure interaction on the structural performance considering the full range of possible outcomes. That is, contrary to the past deterministic studies, the analysis shows how likely it is that SSI increases the ductility demand on a structure designed according to fixed-base provisions of design codes. In addition, the variation of this probability with respect to the period of the structure and other aforesaid parameters is studied. Furthermore, the results of practicing the SSI provisions from ASCE (2010) is investigated probabilistically. Prior deterministic studies by Ghannad and Ahmadnia (2006) have shown that using the provisions from previous versions of this seismic code generally leads to higher ductility demands. This research extends the performance-based earthquake engineering to account for the effect of soil-structure interaction, and exemplifies the utilization of probabilities in engineering practice.

2. PROBABILISTIC MODEL

2.1. Soil-Structure Model

The soil-structure system in this study is modeled by the sub-structure. Figure 1 illustrates this system. According to this figure, the soil beneath the structure is considered as a homogeneous half-space. The super-structure is modeled as a single degree of freedom system with height h, mass m and mass moment of inertia I. These can be considered as the effective values for the first mode of vibration of a multi-degree of freedom system. The foundation is considered to be rigid with embedment depth e, mass m_f, and mass moment of inertia I_f. The basic model for such a soil–structure system is shown in Figure 2 in which the structure is replaced by an elastoplastic spring of stiffness k and yield strength F_y. The damping of the structure is modeled by a dashpot with a coefficient of c. The foundation is considered to be in full contact with the surrounding soil. Thus, the foundation-soil dynamic stiffness is represented by a discrete model as proposed by Wolf (1994), and shown underneath the structure in Figure 2. Two degrees of freedom are introduced in this model for the foundation, namely, sway, u_f, and rocking, φ.

Consequently, by considering an additional internal degree of freedom for the soil model, φ_1, a four-degree-of-freedom model is formed for the whole soil–structure system. φ_1 allows the frequency dependency of the soil stiffness to be taken into account while all the coefficients in the model are frequency independent. These coefficients, *i.e.*, k_{0h}, c_{0h}, k_{0r}, c_{0r}, c_{1r}, I_{1r}, f_k, and f_c, are defined in terms of the soil density, ρ, Poisson ratio, v, shear wave velocity, υ_s, radius of the cylindrical foundation, r, and embedment depth, e (Wolf 1994). These properties are computed in terms of several dimensionless random variables, described shortly.

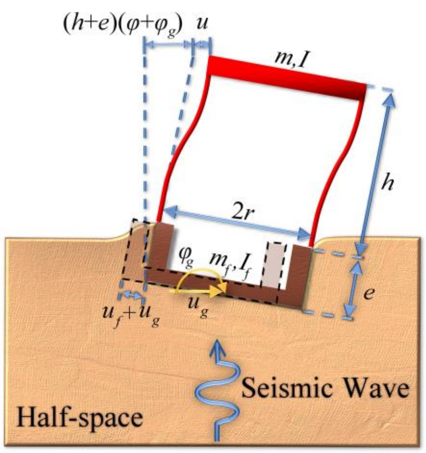

Figure 1:Soil–structure system.

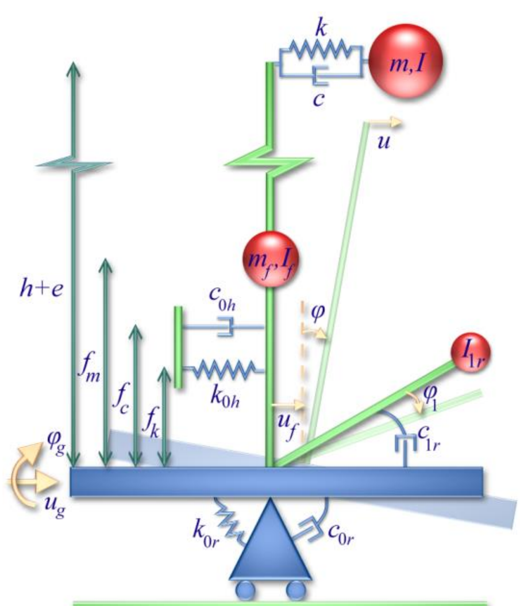

Figure 2:Basic soil–structure model.

Using the basic soil–structure model in Figure 2, a more complex model is generated to consider the soil material damping. Every spring and dashpot in the basic model is augmented with a dashpot and a mass, respectively. It is assumed in the model that the soil representative springs remain elastic. The effect of soil nonlinearity, however, is approximately introduced in the model through an equivalent linear approach: A degraded shear wave velocity, consistent with the estimated strain level in soil, is used for the soil medium. This is the same approach as in ASCE (2010) where the strain level in soil is implicitly related to the peak ground acceleration (PGA).

2.2. Kinematic Interaction

When an embedded foundation is subjected to a dynamic excitation, referred to as free-field motion, it experiences an average of the movements that are exerted on its interface with the soil. As a result, a single horizontal component of the dynamic excitation turns into a two-component excitations, referred to as the foundation input motion. The two components are a horizontal motion, u_g, and a rocking motion, φ_g. This phenomenon is a part of SSI and is called kinematic interaction. The amplitude of u_g is in general smaller than that of the free-field motion, and it decreases as the embedment ratio increases. On the contrary, the amplitude of φ_g increases with the embedment ratio. These two components are in fact the input motion to the soil–structure model of Figure 2. Hence, they are computed for any given free-field motion and applied to the model. In this paper, the method of Wolf (1994) is adopted to compute u_g and φ_g.

The model in Figure 2 is directly employed in a time domain analysis under the two excitation components u_g and φ_g. The response of the soil-structure system is computed using a step-by-step integration by Newmark (1959) β method. Various outputs that represent the performance of the structure are recorded, which include the maximum elastic and inelastic displacements, maximum elastic and inelastic strength demands, and ductility demand of the structure. This model is implemented in Rt.

2.3. Uncertainties

This paper aims at carrying out a comprehensive parametric study. To this end, a number of dimensionless parameters are defined that describe various aspects of the soil-structure system. The following parameters are deemed to best describe the properties and the behavior of the soil-structure system (Ghannad *et al*. 1998):

- "Dimensionless frequency" as an indicator of the relative stiffness of the structure to that of the soil, which reads

$$a_0 = \frac{\omega h}{\upsilon_S} \tag{1}$$

  where ω=natural circular frequency of the fixed-base structure. The practical range of a_0 for ordinary building-type structures is between 0 for a fixed-base structure to about 2 for cases with a predominant SSI effect (Ghannad and Ahmadnia 2006). Therefore, analyses are conducted for three mean a_0-values of 0, 1, and 2. In each analysis, a_0 is considered as a lognormal random variable with a 5% coefficient of variation (CoV). The uncertainty in a_0 reflects the uncertainty in ω and in υ_s.
- Aspect ratio of the building, h/r, which takes the deterministic values of 0.5, 1, and 2 to represent a range of conventional buildings.
- Embedment ratio of the foundation, e/r, which takes the deterministic values of 0 and 0.5 to represent a range of conventional foundations. The value of 0.5 is only considered for systems with h/r=2.
- Ductility demand of the structure, μ, defined as the ratio of the maximum inelastic displacement and the yield deformation of the structure, as a part of the soil-structure system.
- Structure to soil mass ratio index defined as

$$\bar{m} = \frac{m}{\rho r^2 h} \tag{2}$$

  which is modeled as a lognormal random variable with a 0.61 mean and 21% CoV.
- Ratio of the mass of the foundation to that of the structure, m_f/m, modeled as a lognormal random variable with 0.2 mean and 30% CoV.
- Poisson's ratio of the soil, ν, modeled as a uniform random variable between 0.2 and 0.5.
- Material damping ratio of the soil, ξ_0, modeled as a lognormal random variable with a 0.05 mean and 26% CoV.
- Material damping ratio of the structure, ξ_s, modeled as a lognormal random variable with 0.04 mean and 33% CoV.
- Ratio of the height of the foundation centroid to the embedment depth, f_m/e, modeled as a uniform random variable between 0 and 0.5. f_m is shown in Figure 2.

a_0, h/r, and e/r are the key parameters that define the principal SSI effect including both kinematic and inertial interaction. In contrast, μ controls the level of nonlinearity in the structure.

To obtain the probability distribution of the abovementioned parameters, they are derived and written in terms of the basic physical variables that define the geometry and material properties of the soil and the structure. Thereafter, a probability distribution is assigned to these basic variables based on expert opinion. Finally, the probability distribution of the desired parameters is obtained by carrying out a sampling analysis.

In addition, the natural period of the fixed-base structure, T_n=2π/ω, is modeled as a lognormal random variable with a 5% CoV. For each aspect ratio, a reasonable range for the mean value of the period is determined. In particular, for h/r=0.5 the mean T_n varies between 0.1s and 0.7s, for h/r=1.0 it varies between 0.3s and 1.0s, and for h/r=2.0 it varies between 0.7s and 1.5s. In all ranges, mean T_n-values at increments of 0.1s are selected. This random variable accounts for the uncertainty in the estimation of the stiffness and the mass of the structure.

3. ANALYSIS METHODOLOGY

A mean-centered Monte Carlo sampling analysis is carried out to compute the probability distribution of responses. In each sample, random realizations of the input variables is generated according to their probability distribution. These realizations enter the soil-structure model as input, hence establishing a randomly generated soil-structure system.

This system is then subjected to a randomly selected and scaled ground motion record. An adaptive method for the selection and scaling of records is employed. The PGA is represented by a random variable whose probability distribution is extracted from a "hazard curve" at the building location. In each sample, a random realization is generated for the PGA as well. The use of a hazard curve as the basis for the probability distribution of PGA ensures that the frequency of each PGA-value in the sampling analysis is realistic. Upon generation of the PGA realization, the records whose peak acceleration is close to that realization are identified. Thereafter, one of those records is randomly selected according to the realization of a uniform random variable. This record is then scaled to the PGA realization. The scaled record is employed as the free-field motion to compute the two sway and rocking components of the foundation input motion. The soil-structure model is then analyzed under these two components and the responses of the system are evaluated.

The sampling analysis continues for 20,000 samples. Hence, 20,000 random realization of structural response is produced. From these realization, the probability density function and cumulative distribution function (CDF) of these responses are evaluated. The number of samples are chosen so that the analysis yields sufficiently accurate probability distributions for responses. The metric for this accuracy is the CoV of the computed probabilities. At 20,000 samples, the CoV lies below a satisfactory amount of 2% for the responses in the range of 10^{th} to 90^{th} percentiles.

To conduct a comprehensive parametric analysis, 243 soil-structure systems are analyzed. Each soil-structure system has a unique combination of mean T_n, mean a_0, h/r, e/r, and μ, in the range that was presented in the preceding section. Considering 20,000 samples for each soil-structure system in three analysis types, described shortly, about 15 million nonlinear dynamic time history analyses on multi-degree-of-freedom systems are conducted to produce the results in this paper. This massive study is conducted by parallel processing on the cluster of the high-performance computing center at Sharif University of Technology.

4. RESULTS

Three types of analysis are conducted in this study. The difference between these types is within each sample. Each type elucidates a certain aspect of the SSI effect on structural performance. For each type, first the analysis procedure is explained, and then its results are presented. In the following, the term "flexible-base structure" refers to the structure in a soil-structure system. In turn, the term "fixed-base structure" denotes a structure with the same elastic properties as the flexible-base structure, *i.e.*, the same realization of period and damping ratio, but with a rigid base.

4.1. Analysis Type I

This analysis type assumes the same strength reduction factor, R_μ, for flexible-base and fixed-base structures. This is the assumption inherent in the SSI provisions in seismic codes, such as ASCE (2010). In each sample, first the elastic strength demand of both structures subject to a ground motion is computed. The yield strength of each structure is then computed by dividing its elastic strength demand by a prescribed R_μ. Thereafter, both structures are subjected to the same ground motion, and the responses are computed. The difference between the probability distribution of responses in both systems reveals the suitability of the assumption in seismic codes.

Figure 3 shows the CDF of the ductility demand for structures with a period of T_n=0.3s, a surface foundation, and R_μ=6. The black solid curve represents a fixed-base structure. Other curves represent flexible-base structures with varying degrees of SSI severity as indicated by a_0, and different aspect ratios, as indicated by h/r. This figure clearly shows that SSI increases the ductility exceedance probabilities at all realizations. The more the a_0, *i.e.*, the more severe the presence of SSI, the higher is the ductility exceedance probabilities. This is seen by the shift of the CDF curves to the right as a_0 increases. This result implies that the assumption of the same R_μ for fixed-base and flexible-base structures yields much higher ductility levels than anticipated.

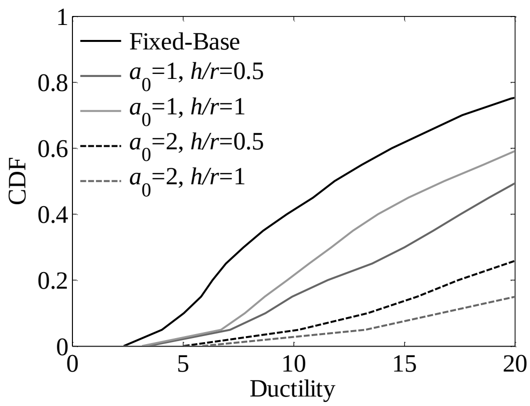

Figure 3: Ductility CDF curves for strcutures with T_n=0.3s, R_μ=6, and e/r=0 in Analysis Type I.

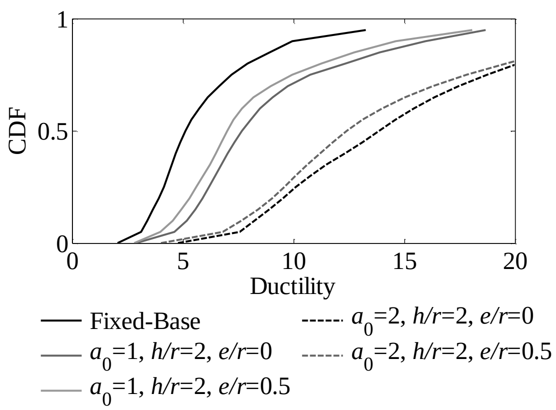

Figure 4: Ductility CDF curves for strcutures with R_μ=6 and T_n=1.5s in Analysis Type I.

Figure 4 demonstrates the same trend for structures with a longer period, *i.e.*, T_n=1.5s. It is inferred from Figures 3 and 4 that this trend is intensified as h/r increases. In other words, using a fixed-base R_μ to design a slender flexible-base structure yields even more non-conservative outcomes. Figure 4 features flexible-base structures with both surface and embedded foundations, denoted by e/r=0 and e/r=0.5, respectively. This figure indicates that foundation embedment alleviates this situation to some extent. That is, using a fixed-base R_μ to design a flexible-base structure with embedded foundation does not increase the ductility demand as much. This is because the horizontal component of the foundation input motion, u_g, decreases due to kinematic interaction but the embedment ratio is not large enough to produce a considerable rocking component, φ_g. For larger embedment ratios, φ_g increases significantly, which increases the demands (Mahsuli and Ghannad 2009).

4.2. Analysis Type II

This analysis type, in each sample, first determines the yield strength required by a fixed-base structure to achieve a target ductility, μ_{fix}, subject to a ground motion. Thereafter, that yield strength is assigned to the flexible-base structure. Finally, the flexible-base structure is subjected to the same ground motion and the resulting ductility demand, μ_{flex}, is computed. This analysis type reveals what would happen to a structure that is designed as fixed-base but is actually built on soil.

Figure 5 displays the results of this analysis type in the form of "probability spectra." This figure condenses the results of a total of 1,500,000 samples for 81 soil-structure systems in a single plot. That is, the curves in this figure are extracted from 81 probability distributions of ductility. Similar to a typical spectrum, the abscissa is the period. The ordinate, however, is the probability that the ductility of the flexible-base structure, μ_{flex}, exceeds that of the fixed-base structure, μ_{fix}, *i.e.*, $P(\mu_{\text{flex}}>\mu_{\text{fix}})$. This shows how likely it is that SSI increases the ductility demand on a structure designed according to fixed-base provisions. Figure 5 includes several curves for different a_0, h/r, and e/r values to properly show the variation of that probability with these parameters. Note that μ_{fix}=6 for all curves. This figure indicates that the chance of an increase in ductility demand due to SSI is less than 50% for most flexible-base structures. In other words, it is more likely that SSI decreases the ductility demand for most systems and thus, has a beneficial effect on the structure. However, there are a noticeable number of systems on which SSI has an adverse effect, and may increase their ductility demands. Those systems are the ones for which the spectra lie above the probability of 0.5. In particular, systems with a_0=1 are susceptible to this phenomenon, which are shown by black curves in the figure.

Systems with a severe SSI (a_0=2), shown by gray curves, are more likely to experience a lower ductility demand due to SSI. These are systems that are located on very soft soils.

Figure 5 also shows that higher h/r-values increase the probability. That is, it is more likely for slender structures to experience a higher ductility demand due to SSI. For a_0=1, this trend is alleviated when the slender structure has an embedded foundation rather than a surface foundation. However, for a_0=2, the effect of foundation embedment is both insignificant and non-uniform for different periods.

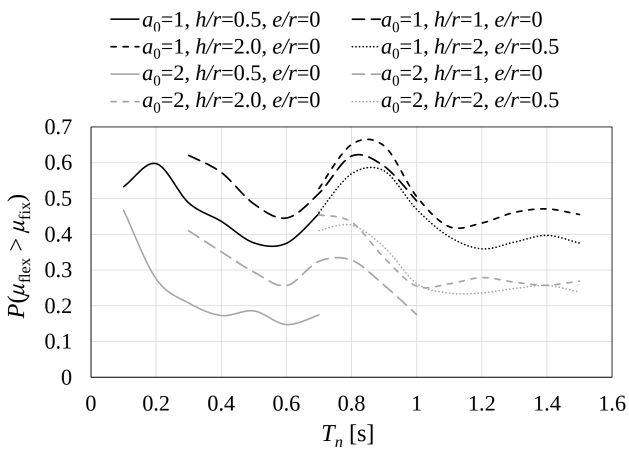

Figure 5: Probability spectra for μ_{fix}=6.

4.3. Analysis Type III

This analysis type computes the yield strength of the flexible-base structure in accordance with the SSI provisions in ASCE (2010). These provisions suggest a reduction in the base shear of the flexible-structure due to SSI effects. That is, they inherently assume that SSI always reduces the seismic demands on the structure. In each sample, the flexible-base structure with the reduced yield strength and the fixed-base structure with the original, unreduced yield strength are subjected to a ground motion. Comparing the response probability distributions of the two structures reveals whether the practice of SSI provisions in ASCE (2010) increases the structural demands.

Note that the yield strength of the structure in this analysis type is computed by the equivalent statics force procedure in the seismic code. That is, the yield strength is the base shear computed according to the strength reduction factor, R, provided by the seismic code. As a result, it is possible for a structure to remain elastic under a ground motion in this analysis type. In contrast, the strength reduction factor in Analysis Type I, R_μ, is one that reduces the elastic strength demand of the structure to obtain the yield strength. Hence, the structure always yields in Analysis Type I.

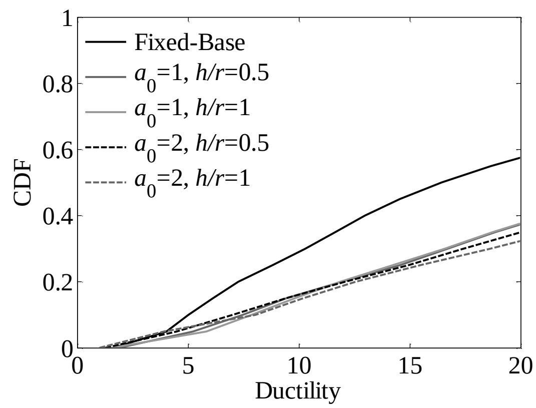

Figure 6: Ductility CDF curves for strcutures with T_n=0.3s, R=6, and e/r=0 in Analysis Type III.

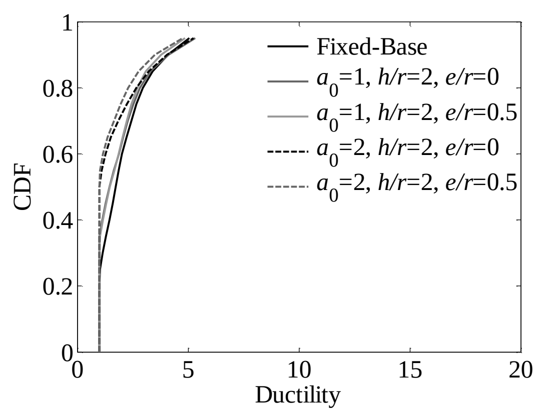

Figure 7: Ductility CDF curves for strcutures with T_n=1.5s, R=6, and h/r=2 in Analysis Type III.

Figure 6 shows the CDF of ductility for short-period structures with T_n=0.3s and a surface foundation, designed with R=6. The figure clearly indicates that the flexible-base structures with a reduced base shear have higher ductility exceedance probabilities. That is, practicing the SSI provisions in ASCE (2010) increases the likelihood of experiencing higher ductility demands. This observation is independent of the dimensionless frequency and aspect ratio.

Figure 7 shows the contrary for structures with a longer period, *i.e.*, T_n=1.5s. To be able to arrive at comprehensive conclusions, probability spectrum plots for many systems, such as the one in Figure 5, is needed. Such a plot would display the probability that the ductility of the flexible-base structure with a reduced base shear exceeds that of the fixed-base structure. Ongoing research by the authors will address this in near future.

5. CONCLUSIONS

This paper puts forward a probabilistic framework to assess the structural performance under the influence of soil-structure interaction. Probabilities prove to be an effective means in this study to uncover the significant variability of this performance. The major findings are as follows:

- A structure in a soil-structure system that is designed with the strength reduction factor of a fixed-base system experiences ductility demands well beyond the anticipated levels in the fixed-base system.
- The probability that soil-structure interaction increases the ductility demand of a structure designed with fixed-base provisions but built on soil is computed. It is concluded that this probability is strongly dependent on the period and slenderness of the structure and the relative stiffness of the soil and the structure. For most soil-structure systems, soil-structure interaction has a beneficial effect on the structure and for some an adverse effect.
- Practicing the soil-structure interaction provisions of modern seismic codes increases the likelihood of imposing higher ductility demands on the structure. For some soil-structure systems, these provisions cause a marked shift in the probability distribution of ductility to higher outcomes. This is in most part due to the fact that these provisions are based on the effect of soil-structure interaction on elastic structures.

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