12th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP12
Vancouver, Canada, July 12-15, 2015

Message-passing Sequential Detection of Multiple Structural Damages

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ABSTRACT: This paper introduces a multiple structural damage detection algorithm for structural health monitoring. We propose a sequential damage detection algorithm that uses the Bayesian inferences to diagnose multiple damages and their relationships. The proposed algorithm is purely data driven and does not require the prior knowledge of the structural properties. The sequential detectors are implemented by a computationally efficient message passing protocol that enables distributed and simultaneous damage detection. Also, the detectors achieve minimum detection delay with a desired false alarm rate. The performances of our algorithm are validated on the ASCE benchmark structure.

1. Introduction

Structural health monitoring (SHM) involves using hardware and software to ensure the safety of civil structures by diagnosing the damage efficiently and reliably. In a typical application of SHM, a sensor network is deployed for real-time monitoring, processing new data samples as they arrive, and making decisions about whether damages have occurred. The monitoring process contains three subcomponents: (i) collecting structural responses via the sensor network; (ii) extracting the damage sensitive features; and (iii) detecting damages via statistical inferences. There have been various successful efforts to the structural data acquisition, such as Lynch et al. (2002), Wang et al. (2007), and Nagayama and Spencer Jr (2007). The reliable and robust algorithms for damage detection should be applicable for complex structures with various materials and complicated geometry. Moreover, the algorithms are expected to be insensitive to the uncertainties caused by the environmental and loading conditions but to be sensitive to the damages. Furthermore, for life-threatening damages, the detectors are required to have to high accuracy and low latency.

There have been many existing works on the statistical detection of structural damages. The algorithms in Nair et al. (2006) and Noh et al. (2009) fit the structural responses with the autoregressive moving average (ARMA) model and then perform hypothesis testing on the model coefficients. Nair and Kiremidjian (2007) models the ARMA coefficients as a Gaussian mixture. The damage is diagnosed by measuring the distance between the Gaussian mixtures fitted by the undamaged and damaged data. Peter Carden and Brownjohn (2008) also uses the ARMA coefficients as the features. The damages are detected by an unsupervised classifier. All of the works above are centralized batch processing, which has large detection delay and is inefficient for a large-scale monitoring system. There have been several papers using the Bayesian inferences to detect damages, such as Vanik et al. (2000) and Ching and Beck (2004). However, these works require to have prior knowledge of the model parameters. Recently works, i.e. Noh et al. (2013) and its extension Liao et al.

(2014), use the Bayesian models to detect the damage sequentially and do not require the model parameters. A disadvantage of these algorithms is that only the samples from a specific sensor are utilized for damage detection.

In this paper, we propose a damage detection algorithm that uses all time-series based features in a network to detect damages in a Bayesian setting. The algorithm purely relays on the data and no prior knowledge of structures are needed. The sequential detectors can detect multiple damages with minimum detection delay for a desired false alarm rate simultaneously. Also, the detectors can be implemented in a message passing and distributed fashion. Moreover, the proposed detectors identify not only single damage but also multiple multi-stage damages and their relationships, such as when the first damage occurs and when all the possible damages have happened. This algorithm is motivated by Amini and Nguyen (2013), which only focuses on singe-stage damage and the earliest damage detection.

The paper is organized as follows. Section 2 defines the damage sensitive features and formulates the damage detection problem in the Bayesian statistics. In addition, the detectors and the message-passing algorithm are discussed. Section 3 validates the proposed algorithm on the ASCE benchmark structure and discusses the performances. Section 4 draws summaries and conclusions.

2. ALGORITHM

The proposed damage detection algorithm includes three steps: (i) collecting structural responses; (ii) extracting damage sensitive features (DSFs) from structural responses; and (iii) detecting damages via message passing protocol. For (i), the structural responses are sequentially obtained from multiple sensors and are normalized. In the second step, the autoregressive (AR) model is applied to extract the DSFs. The first three AR coefficients are sensitive to damages (Noh et al. (2009),Noh et al. (2011)) and follow a Gaussian mixture model (Nair and Kiremidjian (2007), Noh et al. (2013)). In (iii), the sequential detectors identify the damages based on the posterior probabilities of damages, which is computed based on the prior distribution and the distributions of DSF before and after the occurrence of damage. Usually, the computation of the posterior probability requires to centralize the DSFs from all the sensors. In the proposed algorithm, we introduce the message passing (MP) method to compute the posterior probability in a distributed approach.

2.1. *Feature Extraction*

The DSF extraction consists of two steps: (i) normalization and (ii) AR model fitting. The discrete time acceleration signal from sensor j, $Y_j(n)$, is divided into chunks with a size N. Let $Y_j^i(n)$ denote the the ith chunk of the signal $Y_j(n)$. The normalized signal $\tilde{Y}_j^i(n)$ is obtained as follows:

$$\tilde{Y}_j^i(n) = \frac{Y_j^i(n) - \mu_j^i}{\sigma_j^i},$$

where μ_j^i and σ_j^i denote the mean and the standard deviation of the ith chunk. For notation convenience, we will use $Y_j^i(n)$ as $\tilde{Y}_j^i(n)$ in the following text. In addition, we will use the ith time index as an alternative term for the ith chunk.

After normalizing the signal, the chunk data are fitted with a single-variant AR model of order p,

$$Y_j^i(n) = \sum_{k=1}^{p} \theta_k Y_j^i(n-k) + \varepsilon_j^i(n), \qquad (1)$$

where θ_k is the kth AR coefficient and $\varepsilon_j^i(n)$ is the residual. The selection of AR model includes removing the trends, choosing the optimal model order p, and checking the assumptions of the residuals ε. The model selection process will be discussed with more details in Section 3.

2.2. *Damage Detection*

For the monitoring structure, we assume there are d potential damage locations. Each damage location is assigned with a random variable $\lambda_i \in \mathbb{N}$, for $i \in [d] := \{1,2,\ldots,d\}$, independently. The damage variable λ_i represents the time at when the damage occurs and follows a prior distribution $\pi_i(\lambda_i)$. The examples of damage locations include floors, walls, braces, balls and etc.. We will use the notation $\lambda_\star = (\lambda_i, i \in [d])$ as a collection of all damages.

Let the sequence of DSFs up to the time n be represented as $\mathbf{X}_j^n := (X_j^1, X_j^2, \ldots, X_j^n)$ where X_j^i denote the ith DSF of sensor j. Then, given M sensors in the network, $\mathbf{X}_\star^n := (\mathbf{X}_j^n, j \in [M])$ denotes the DSFs of all sensors up to the time n.

Depending on the installation location, the signal acquired by a sensor is affected by one or more damages. Let index set $S_j = \{i_1, i_2, \ldots, i_r\} \subseteq [d]$ contains the indices of all damage variables that affect the measurement of sensor j. Then, λ_{S_j} is the set of the damage variables that are associated with sensor j. For example, if sensor 1 measures the response affected by λ_2 and λ_3, then $S_1 = \{2,3\}$ and $\lambda_{S_1} = \{\lambda_2, \lambda_3\}$. By the definition of $\lambda_\star$, we know $\lambda_\star \equiv \left(\bigcup \lambda_{S_j}, j \in [M]\right)$. Therefore, in the Bayesian formulation, the joint distribution of $\lambda_\star$ and $\mathbf{X}_\star^n$ is

$$P(\lambda_\star, \mathbf{X}_\star^n) = \prod_{i \in [d]} \pi_i(\lambda_i) \prod_{j \in [M]} P(\mathbf{X}_j^n | \lambda_{S_j}), \quad (2)$$

where $P(\mathbf{X}_j^n | \lambda_{S_j})$ is the likelihood probability and depends on the damage variables. The distributions of DSFs are affected by the damages. However, conditioning on the damage variable λ_{S_j}, the DSF X_j^n is independently and identically distributed (i.i.d) with the same distribution. How to find the conditional distribution will be discussed in details later.

In the damage detection problem, our primary interest is the estimation of the damages $\lambda_\star$. There are two groups of detectors, as shown below:

$$\phi_{\min} := \phi_{\min}(\lambda_\star) := \lambda_{\mathbb{S}}^{\min} := \min_{i \in \mathbb{S}} \lambda_i, \quad (3)$$

$$\phi_{\max} := \phi_{\max}(\lambda_\star) := \lambda_{\mathbb{S}}^{\max} := \max_{i \in \mathbb{S}} \lambda_i, \quad (4)$$

for some subset $\mathbb{S} \subseteq [d]$. The first category, which is referred as the minimum detector, focuses on the detection of the earliest damage. Examples include the detection of a single damage $\mathbb{S} = \{j\}$, the earliest among two damages $\mathbb{S} = \{i, j\}$, and the earliest among the entire network $\mathbb{S} = [d]$. For the second category, which is referred as the maximum detector, we are interested in how many damages have occurred or whether all the damages in $\mathbb{S}$ have occurred or not. Examples include the detection of two damages $\mathbb{S} = \{i, j\}$ that have occurred and all the damages in the network $\mathbb{S} = [d]$ have occurred.

We assume τ to be the damage detection rule for ϕ. Based on $\mathbf{X}_\star^n$, the rule sets $\tau = n$ if one or more damages are detected. Thus the random variable τ is a stopping time (Rajagopal et al. (2008), Durrett (2010)). τ is chosen based on two metrics: probability of false alarm $P(\tau \leq \phi)$ and the detection delay $E(\tau - \phi)_+$. Here, we use the Neyman-Pearson criteria to choose the stop rules set that has false alarm at most α:

$$\Delta_\phi(\alpha) := \{\tau : P(\tau \leq \phi) \leq \alpha\}. \quad (5)$$

Among $\Delta_\phi(\alpha)$, we want to consider the rule that has the minimum detection delay. We use $\tau_{\mathbb{S}}$ to denote the stopping rule associated with $\lambda_{\mathbb{S}}$. We propose a stop rule that stops at the first time the posterior probability is larger than a given threshold,

$$\tau_{\mathbb{S}}^{\min} = \inf\left\{n : P(\lambda_{\mathbb{S}}^{\min} \leq n | \mathbf{X}_\star^n) \geq 1 - \alpha\right\}, \quad (6)$$

$$\tau_{\mathbb{S}}^{\max} = \inf\left\{n : P(\lambda_{\mathbb{S}}^{\max} \leq n | \mathbf{X}_\star^n) \geq 1 - \alpha\right\}, \quad (7)$$

where $n \in \mathbb{N}$ and α is the maximum allowable false alarm rate. As proven in Amini and Nguyen (2013), $\tau_{\mathbb{S}}$ is in $\Delta_\phi(\alpha)$ when $\phi = \lambda_{\mathbb{S}}$. Thus, $\tau_{\mathbb{S}}$ has the minimum detection delay.

Let A_i denote the domain set of λ_i and A_S denote the product $A_{i_1} \times A_{i_2} \times \cdots \times A_{i_r}$. For each sensor, there is a function $\alpha_j(\lambda_{S_j}) : A_{S_j} \to \mathbb{R}^+$. Let's call the variable set λ_{S_j} the local domain and the function α_j the local kernel. In our setup, we want the products of the local kernels, $\prod_{j=1}^M \alpha_j(\lambda_{S_j})$, to be the joint distribution in (2). Therefore, α_j is either $P(\mathbf{X}_j^n | \lambda_{S_j})$ or $P(\mathbf{X}_j^n | \lambda_{S_j}) \prod_{\lambda_i \in \lambda_S} \pi_i(\lambda_i)$ with $S \subseteq S_j$. To compute the posterior probability of the local domain, we want to marginalize the joint distribution and eliminate the irrelative random variables, as follows:

$$\beta_j(\lambda_{S_j}) := P(\lambda_{S_j}, \mathbf{X}_\star^n) = \sum_{\lambda_{S_j^c} \in A_{S_j^c}} P(\lambda_\star, \mathbf{X}_\star^n), \quad (8)$$

where S_j^c denotes the complement of S_j relative to the entire set $[d]$. We call the function $\beta_j(\lambda_{S_j})$ the objective function of sensor j. Based on the Bayes' theorem, the posterior probability $P(\lambda_{S_j} | \mathbf{X}_\star^n) \propto \beta_j(\lambda_{S_j})$.

As discussed in Aji and McEliece (2000) and Amini and Nguyen (2013), if a sensor network can be formulated as a polytree, then we can use the generalized distributive law (GDL) or the belief propagation algorithm to produce exact values of the objective function $\beta(\lambda_S)$. In the polytree, if there exists any edge (probabilistic linkage) between sensor i and sensor j, then the message from i to j, at time n, is

$$m_{ij}^n(\lambda_{S_i\cap S_j}) = \sum_{\lambda_{S_i\setminus S_j}\in A_{S_i\setminus S_j}} \alpha_i(\lambda_{S_i}) \prod_{r\in\partial i\setminus\{j\}} m_{rj}^n(\lambda_{S_r\cap S_i}), \tag{9}$$

for $\lambda_{S_i\cap S_j} \in A_{S_i\cap S_j}$, where ∂i denotes the index set of all the neighbors that have an edge with sensor i. At time n, the dimension of the message is $(n+1)^{|S_i\cap S_j|}$. The message sent from sensor i is the product of its local kernel with all messages it has received from its neighbors other than sensor j with filtering out the irrelative information (by marginalization). Therefore, by passing the messages, we send the inferences of one sensor to all other in the network. Once the message passing is completed, on each sensor, the objective function $\beta(\lambda_S)$ can be computed as

$$\beta_j(\lambda_{S_j}) = \alpha_j(\lambda_{S_j}) \prod_{r\in\partial j} m_{rj}^n(\lambda_{S_r\cap S_j}) \quad \forall j\in[M]. \tag{10}$$

Then, the posterior probability $P(\lambda_{S_j}|\mathbf{X}_\star^n)$, which relay on all the DSFs that have been collected across the network, can be computed directly.

In SHM, we usually define the prior $\pi_i(\lambda_i)$ as a geometric distribution with parameter $\rho_i\in(0,1)$, i.e. $\pi_i(k) := (1-\rho_i)^{k-1}\rho_i$. Therefore, at time n, the domain of the local kernel is $A_i \equiv [n+1]$ for all $i\in[d]$. When $\lambda_i \le n$, it means that the damage has occurred. When $\lambda_i = n+1$, it means that the damage will happen in the future. We define $\pi_i(n+1) = \sum_{k=n+1}^{\infty}\pi_i(k)$. Therefore, for the MP setup, the prior is defined as

$$\tilde{\pi}_i^n(k) := \begin{cases} \pi_i(k) & \text{for } k\in[n] \\ \pi_i[n]^c = \sum_{k=n+1}^{\infty}\pi_i(k) & \text{for } k=n+1. \end{cases}$$

where $[n]^c := \mathbb{N}\setminus[n] = \{n+1, n+2, \dots\}$.

Assume that the network can be organized as a polytree, we propose the MP algorithm at each time step n as follows:

1. Choose one sensor in the network as the root of the tree
2. Initialize messages $m_{ij}^n \in \mathbb{R}^{(n+1)^{|S_i\cap S_j|}}$ to the all ones tensor for all edges. Compute $\tilde{\pi}_i^n(k)$ for $k\in[n+1]$, $i\in[d]$.
3. Compute and pass messages m_{ij}^n from sensor i to sensor j according to (9). Pass messages from the leaves to their parents. Continue the process from the bottom of the tree to the top till the root sensor is reached.
4. Repeat Step 3 but start from the root. Pass messages from the root to its children. Continue the process from the top of the tree to the bottom till all the leaves are reached. When compute m_{ij}^n based on (9), use the messages received in Step 3.
5. Compute $\beta_j(\lambda_{S_j})$ based on (10) for $\lambda_{S_j} \in A_1\times A_2\times\cdots\times A_{|S_j|}$ and $A_i \equiv [n+1]$ for all j. Then normalize $\beta_j(\lambda_{S_j})$ such that $\sum_{\lambda_{S_j}\in A_1\times A_2\times\cdots\times A_{|S_j|}}\beta_j(\lambda_{S_j}) = 1$.

After the normalization, $\beta_j(\lambda_{S_j})$ becomes to the posterior probability, i.e. $P(\lambda_{S_j}|\mathbf{X}_\star^n) = \beta_j(\lambda_{S_j})$. This process is repeated as a new DSF is available.

If S_j contains the indicates of the damage variables of sensor j and the subset $\mathbb{S}\subseteq S_j$ contains the indices of the damage variables that we want to detect, then at time n, the posterior probability of the minimum detector can be computed as

$$\begin{aligned} & P(\min(\lambda_{\mathbb{S}}) \le n|\mathbf{X}_\star^n) \\ = \;& 1 - \sum_{\lambda_{S_j\setminus\mathbb{S}}\in A_{S_j\setminus\mathbb{S}}} \beta_j(\lambda_{\mathbb{S}} = n+1, \lambda_{S_j\setminus\mathbb{S}}), \end{aligned} \tag{11}$$

where $\lambda_{\mathbb{S}} = n+1$ means that $\lambda_i = n+1$ for every $i\in\mathbb{S}$ and $A_s = [n+1]$ for every $s\in S_j\setminus\mathbb{S}$. The posterior probability of the maximum detector can be computed as

$$P(\max(\lambda_{\mathbb{S}}) \le n|\mathbf{X}_\star^n) = \sum_{\lambda_{\mathbb{S}}\in B_{\mathbb{S}}} \sum_{\lambda_{S_j\setminus\mathbb{S}}\in A_{S_j\setminus\mathbb{S}}} \beta_j(\lambda_{S_j}), \tag{12}$$

where $B_s = [n]$ for every $s\in\mathbb{S}$. For the single damage detection, the maximum detector and the minimum detector are equivalent. Since the computation of $\beta_j(\lambda_{S_j})$ only depends on the local kernel and the received messages, the detectors on each sensor can be computed independently. This allows multiple damages to be diagnosed simultaneously and in a distributed fashion.

3. SIMULATIONS AND RESULTS

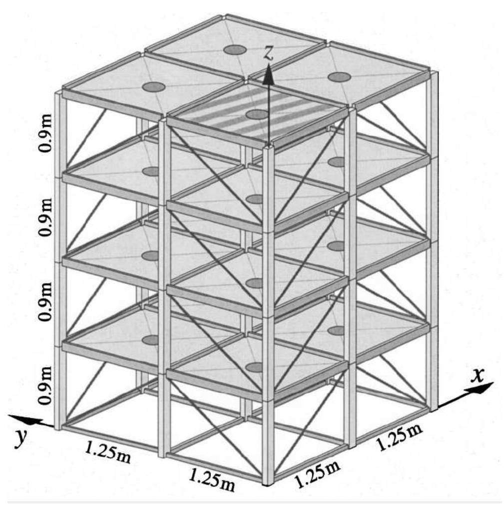

Figure 1: Diagram of the ASCE benchmark structure (Johnson et al. (2004))

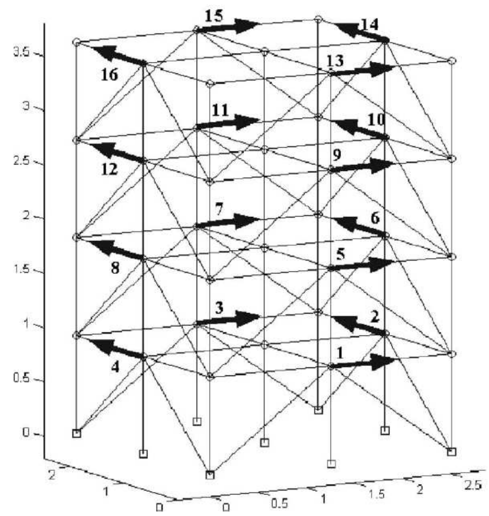

Figure 2: Sensor location and direction of acceleration measurements (Nair and Kiremidjian (2007); Johnson et al. (2000))

To validate the proposed damage detection algorithm, we apply it to the ASCE benchmark structure (Johnson et al. (2004)). The benchmark structure is a four story, two bay by two bay steel braced frame, as shown in Fig. 1. 16 sensors are installed to measure the accelerations. The locations are shown in Fig. 2. All the sensors with odd series number are measured the acceleration signals on the x-axis and the rest sensors measure the responses on the y-axis. The ASCE benchmark structure provides a numerical Matlab simulator which allows us to collect the acceleration signals with different degrees of freedom, mass distribution and excitations. In this paper, we use a 12 degrees of freedom structure with symmetric mass on each floor. In addition, we collect the acceleration signals of ambient vibrations.

In this paper, we use the Burg algorithm to estimate the AR coefficients. The 1st AR coefficient, θ_1, is picked as the DSF. Nair and Kiremidjian (2007) suggests that the AR coefficients are sensitive to the chunk size N. To choose the optimal chunk size, we analyze the mean and the standard deviation of the first three AR coefficients with the size grows from 3000 to 7000. We find that when $N = 7000$, the AR coefficients have the minimum standard deviation. Therefore, we use $N = 7000$ as the chunk size.

A widely used criteria for choosing the optimal model order is the Akaike information criteria (AIC) (Brockwell and Davis (2002),Nair et al. (2006),Noh et al. (2013)). Fig. 3 shows the variation of the AIC values with the AR model order for the undamaged signals from different sensors. We can observe that an AR model with order $p = 8$ is appropriate for this data set. The residuals are validated to be i.i.d with normal distribution and have no trend.

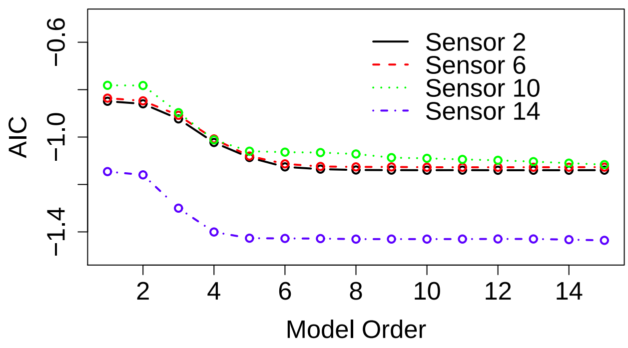

Figure 3: AIC value against AR model order for the undamaged signals

The ASCE simulator includes non-damage pattern and six pre-defined damage patterns, which include both major damages and minor damages. However, these patterns cannot demonstrate simultaneous damages. In order to get the responses associated with multiple simultaneous damages, we modify the original simulator and introduce the following damage patterns (DPs):

- DP0: no damage
- DP1: all the braces of the 1st floor are removed
- DP2: all the braces of the 3rd floor are removed
- DP3: all the braces of the 1st and 3rd floors are removed.

In the ASCE benchmark structure, all the masses are loaded symmetrically. In addition, after removing the braces, the structure remains symmetric. Therefore, rather than using the data collected by all the sensors, in this study, we only use the responses collected by sensor $2, 6, 10$ and 14. Because the damage patterns are introduced to each floor, we assign one damage variable to each floor, i.e. λ_i for the ith floor. As the damages only happen on the 1st and 3rd floors, we assume that only λ_1 and λ_3 will be the active damage variables in this test.

After assigning the damage variables, we need to investigate how the damage variables affect the sensors. In other word, we need to decide the local domain and the local kernel of each sensor. Fig. 4 shows the box plots of the DSFs with different damage patterns. From this figure, we can observe that λ_1 has significant effects on sensor 2 and sensor 6. λ_3 affects the DSFs of all the sensors. Although λ_2 and λ_4 are not triggered by the defined DPs, we still assume it may affect the sensors installed one floor above and below. Based on these observations, we form the local domains and kernels in Table. 1. The polytree graph is shown in Fig. 5. The edge between sensors is both the probabilistic linkage and the communication link.

As discussed above, the likelihood probability depends on the damage variables. Since only λ_1 and λ_3 are active, given $\lambda_1 = n_1, \lambda_3 = n_2$, we define the likelihood probability as follows:

$$P(\mathbf{X}_j^n|\lambda_1,\lambda_3) = \prod_{k=1}^{k_1-1} g_j(X_j^k) \prod_{k=k_1}^{k_2-1} f_j(X_j^k) \prod_{k=k_2}^{n} f_j^3(X_j^k) \tag{13}$$

where k_1 is $\min(n_1,n_2)$, k_2 is $\max(n_1,n_2)$, $f_j(X_j^k)$ is $f_j^1(X_j^k)\mathbb{I}(n_1 < n_2) + f_j^2(X_j^k)\mathbb{I}(n_2 < n_1)$, and $\mathbb{I}(.)$ is the indicator function. For sensor j, the function g_j is the probability density function (PDF) of DP0 and f_j^1, f_j^2, and f_j^3 are the PDFs of DP1-DP3 respectively. The DSF X_j^n is i.i.d with one of them. We assume that all the densities are Gaussian densities with different known parameters (Nair and Kiremidjian (2007)). In field applications, the distribution parameters can be estimated empirically by utilizing historical or simulation data. (13) shows that the likelihood probability depends on not only the damage patterns but also the order of the damage patterns. Therefore, our proposed detectors can apply to multi-stage damages. To keep the summation of the density over the local domain to be one, we need to spread the density function over the inactive variables, i.e. $P(\mathbf{X}_j^n|\lambda_1,\lambda_2=k,\lambda_3) = \frac{1}{n+1}P(\mathbf{X}_j^n|\lambda_1,\lambda_3)$ for all $k \in [n+1]$.

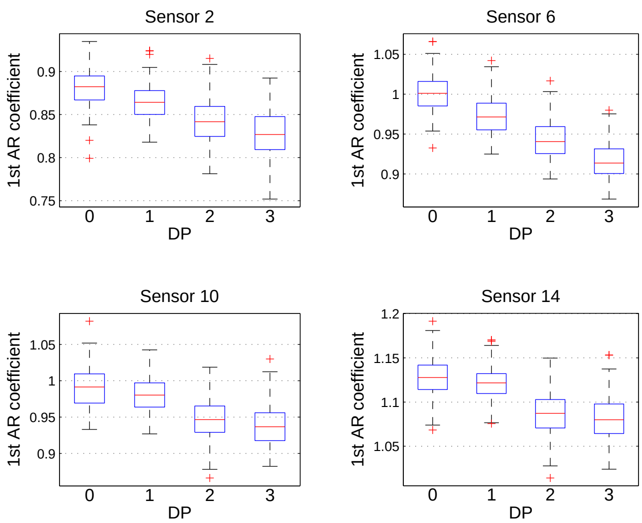

Figure 4: Box plot of the 1st AR coefficient, θ_1, for different damage patterns

Table 1: Local domains and local kernels of the ASCE benchmark structure

sensor	local domain	local kernel
2	$\{\lambda_1,\lambda_2,\lambda_3\}$	$\pi_1(\lambda_1)P(\mathbf{X}_2^n|\lambda_1,\lambda_2,\lambda_3)$
6	$\{\lambda_1,\lambda_2,\lambda_3,\lambda_4\}$	$\pi_2(\lambda_2)P(\mathbf{X}_6^n|\lambda_1,\lambda_2,\lambda_3,\lambda_4)$
10	$\{\lambda_2,\lambda_3,\lambda_4\}$	$\pi_3(\lambda_3)P(\mathbf{X}_{10}^n|\lambda_2,\lambda_3,\lambda_4)$
14	$\{\lambda_3,\lambda_4\}$	$\pi_4(\lambda_4)P(\mathbf{X}_{14}^n|\lambda_3,\lambda_4)$

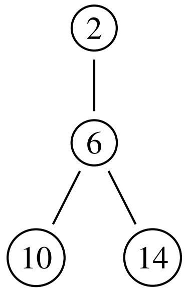

Figure 5: The polytree graph for the sensor network installed on the benchmark structure. The local domain associated with each sensor is shown in Table. 1.

Fig. 6 shows the plots of the expected delay against the maximum allowable probability of false alarm. The plots are generated by Monte Carlo simulation over 500 replications. All prior distributions are geometric distributions with $\rho_j = 0.1$. Table 2 summarizes the detection rules we test on each sensor.

Table 2: Detectors on each sensors

sensor	detector
2	$\min(\lambda_1,\lambda_2,\lambda_3)$, $\min(\lambda_1,\lambda_3)$, $\max(\lambda_1,\lambda_3)$, $\max(\lambda_1)$, $\max(\lambda_3)$
6	$\min(\lambda_1,\lambda_2,\lambda_3,\lambda_4)$, $\min(\lambda_1,\lambda_3)$, $\max(\lambda_1,\lambda_3)$, $\max(\lambda_1)$, $\max(\lambda_3)$
10	$\min(\lambda_2,\lambda_3,\lambda_4)$, $\max(\lambda_3)$
14	$\min(\lambda_3,\lambda_4)$, $\max(\lambda_3)$

To compare the performances of the MP algorithm, we introduce another method that uses the same detection rules but does not exchange messages with the neighbors, i.e. $\tau_{\mathbb{S}}^{\text{LOCAL}} := \inf\{n \in \mathbb{N} : P(\lambda_{\mathbb{S}} \leq n|\mathbf{X}_j^n) \geq 1-\alpha\}$. We call this method the LOCAL method. In. Fig. 6, we can observe that the MP algorithm is generally similar or better than the LOCAL algorithm. Specifically, for sensor 2 and sensor 6, the minimum detectors of both algorithms have similar performances. The LOCAL method is better than the MP algorithm when α is large. Then α is small, the MP method is better than the LCOAL method. For sensor 2, the detection delays of the LOCAL maximum detectors are much larger than those of the MP maximum detectors. For sensor 10, the LOCAL maximum detector fails when $\alpha < 10^{-4}$. However, the MP maximum

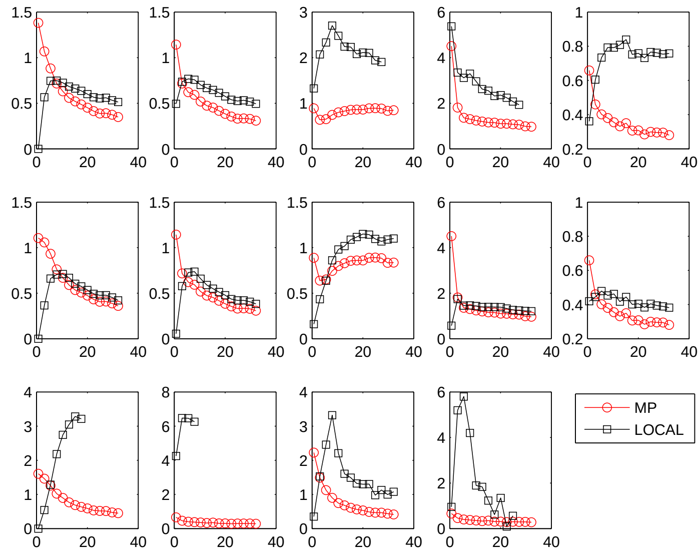

Figure 6: Plots of the slope $\frac{1}{-\log\alpha}E\left[\tau_{\mathbb{S}} - \phi|\tau_{\mathbb{S}} \geq \phi\right]$ against $-\log\alpha$. The plots in the 1st row are the detectors of sensor 2. The plots in the 2nd row are the detectors of sensor 6. The first two plots in the 3rd row are the detectors of sensor 10 and the last two plots are the detectors of sensor 14. The plots from left to right follow the order of the detectors in Table. 2

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detector has consistent performance. For sensor 14, the LOCAL detectors have very high delay when α is large. Meanwhile, the detection delay of MP decreases as $\alpha \to 0$. In summary, the MP algorithm has reliable performance compared with the LOCAL detectors for different detectors. It means the global information helps to reduce the detection delay. Also, the MP detectors are more robust to the false alarm.

4. CONCLUSION

In this paper, we propose a sequential damage detection algorithm that detects the damages based on the AR model coefficients. The message passing protocol computes the posterior probabilities conditioning on all the observed DSFs in the network efficiently and allows the detectors to identify multiple damages simultaneously and in a distributed fashion. In addition, the proposed detectors can achieve minimum detection delay with the desired false alarm rate. The numerical validation on the ASCE benchmark structure shows the robustness to various false alarm rates and the consistence with different types of detectors. Moreover, the numerical results indicate that the global information improves the performance of the damage detection.

To further improve this algorithm, it can be applied to other types of damages, such as removing one brace of each floor. In Fig. 6, we can observe that when $\alpha \to 0$, the MP detectors converge to some constants. It is worth exploring the asymptotic property of the detectors. Last, we need to apply the algorithm to more field data and different structures for further validation.

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