An Improved Approach for Aftershock Hazard Assessment

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ABSTRACT: Estimation of aftershock hazard is one of the most critical issues in evaluation of the post-earthquake safety of damaged structures. Unfortunately, misevaluation of this risk has claimed many lives in the past. In this proceeding, an improved probabilistic aftershock hazard analysis (IPAHA) approach is presented. Proposed method has two contributions to existing approaches that are aimed at achieving more reliable estimation of aftershock hazard. First contribution is related to the spatial distribution model for aftershock clusters. In order to develop a reliable model, spatial distribution of aftershock focal points is investigated statistically for set of earthquake sequences. Specifically, positions of aftershock focal points relative to the mainshock rupture plane are investigated. Second contribution of the proposed method is related to quantification of the correlation of “epsilon” values for pairs of mainshock-aftershock ground motion intensities. Epsilon refers to the standard random error parameter that is defined as the difference between the actual and the estimated median ground motion intensity divided by standard deviation representing the total variability. Using the correlation between mainshock-aftershock epsilons, an improved aftershock hazard estimation model has been established. These developed models establish the basis of the proposed IPAHA procedure. As an example application of proposed method, the aftershock hazard is assessed for a site in Istanbul, Turkey that is assumed to be affected by a scenario mainshock earthquake.

1. INTRODUCTION
Aftershock probabilistic seismic hazard analysis (APSHA) provides the expected exceedance rate of a given level of peak motion intensity at a site due aftershock activity that follows a mainshock. The conventional probabilistic framework for evaluating aftershock hazard was developed by Yeo and Cornell (2005). In principle, aftershock hazard is evaluated in similar way to the conventional probabilistic seismic hazard analysis (PSHA) (Cornell, 1968). The most important difference between the conventional APSHA and the common PSHA is related to models utilized for capturing the rate of occurrence of seismic events. Unlike the common PSHA in which the rate is assumed to be constant over time, in the APSHA the occurrence rate of events is considered to decay exponentially with the increasing time that has passed since the mainshock earthquake. This decaying character known as Omori’s law is explicitly taken into account in the APSHA method developed by Yeo and Cornell (2005). Still however, apart from this decaying behavior there are other distinctive features of aftershock hazard that are not directly considered in the existing APSHA methods.

One of the distinctive features of APSHA compared to PSHA is the non-uniform spatial distribution aftershock epicenters within the source zone. Comprehensive investigation of aftershock clusters by Utsu (1970) have shown that aftershock epicenters are often not spreaded over the entire mainshock rupture area but tend
to cluster around particular regions. More recently, Marsan and Lengline (2010) investigated aftershock clusters and developed a simple predictive model for estimating the radius of the zone around the mainshock epicenter where the aftershocks occur. Still however in the conventional APSHA, uniform likelihood of occurrence is assumed throughout the entire aftershock source zone.

Impact of assuming higher occurrence likelihoods for the end regions of the source zone was investigated by Yeo and Cornell (2005). Results showed that assuming a non-uniform distribution leads to significant changes in the estimated hazard. However, apart from this hypothetical investigation, the issue of assumed likelihood distribution for aftershock epicenters in APSHA had not received so far. Currently, there is a need for a probabilistic spatial distribution model that would enable taking into account the observed spatial clustering of aftershocks in the assessment of aftershock hazard.

One of the primary objectives of this study is to develop a reliable spatial distribution model for aftershock epicenters in APSHA. Moreover, a strategy is presented here for implementing the developed model into the proposed IPAHA framework.

Another distinctive feature of APSHA and compared to PSHA arises when the mainshock strong ground motion recorded at the site of interest is available. The mainshock ground motion intensity measured a site -to some extent- reveals the amplification or attenuation characteristics of the path of the seismic waves emitted from the source and reaching to the site. Since the aftershock induced waves are expected to follow a similar path to those induced by the mainshock, the resulting attenuation or amplification of the aftershock ground motion intensity at the site is expected to be similar to that of the mainshock event. This expectation is only applicable when the site response is not significantly nonlinear. The similarity noted above is generally referred to as the “common path effect”. Improved predictive models for aftershock ground motion intensities can be developed using the noted similarity and the peak motions recorded during mainshock earthquake. The mathematical formulation for exploiting this potential was laid out by Yeo and Cornell (2005). On the other hand, the actual level of correlation between the mainshock-induced and the aftershock-induced ground motion intensities has not been investigated numerically using recorded mainshock-aftershock strong motion sequences.

One of the primary objectives of this study is to quantify the correlation explained above by using pairs of recorded mainshock-aftershock ground motion sequences. The correlation characteristics of the registered peak intensities are investigated by considering response of single-degree of freedom systems with different periods of vibration. Findings of this investigation provide the missing tool that is needed for putting the idea of estimating aftershock hazard conditioned on the measured mainshock ground motion intensity proposed by Yeo and Cornell (2005) into practice.

Reliable estimation of aftershock hazard plays a crucial role in the assessment of the post-earthquake safety structures. The IPAHA framework proposed here is expected to be useful in the improvement of the estimated probabilities in the practice of aftershock hazard estimation and post-earthquake risk evaluation.

2. PROPOSED IMPROVED PROBABILISTIC AFTERSHOCK HAZARD ANALYSIS (IPAHA) METHOD

The IPAHA method proposed in this proceeding contributes to the existing aftershock hazard assessment practice through development of two models: a model for spatial aftershock distribution and a correlation model for the pairs of mainshock-aftershock epsilons.

In the conventional APSHA by Yeo and Cornell (2005), mean rate $\mu$ of aftershock ground motion intensity $Y$ exceeding a given level $\gamma$ at a site, in the time interval of $T$ days that
starts \( t \) days after the mainshock, is evaluated as follows:

\[
\tilde{\mu}(y,t,T;m_m) = \mu^*(t,T;m_m)...
\]

where \( \mu^*(t,T;m_m) \) is the mean number of aftershocks with magnitudes in the interval \([m_l;m_m]\) that occur in time interval \([t; t+T]\), \( m_m \) is

The models that are proposed in this study are aimed to provide improved estimates of the terms \( f_{RM}(r|m) \) and \( P(Y>y|m,r) \) in Eq. (1).

2.1. Spatial aftershock distribution model

Definition of aftershock source zone and the estimation of the spatial distribution of likelihoods across the source zone plays an important role in the estimation of \( f_{RM}(r|m) \) that appears in Eq.(1).

Several alternative assumptions have been adopted in APSHA for modeling the spatial distribution characteristics of aftershocks (Greenspan, 2013). Some of these are: assuming equal likelihood of occurring over the entire mainshock rupture plane, assuming higher likelihoods for the end regions of the mainshock rupture plane compared to its center, or assuming the density of aftershocks decaying with the increasing distance from the epicenter of the mainshock (e.g. Yeo and Cornell, 2005; Marsan and Lengliné, 2010).

A new spatial distribution model is proposed here that results in assignment of likelihoods over aftershock source zone which captures the observed clustering tendency of the actual aftershock sequences. For this purpose, statistical characteristics of spatial distribution of aftershock focal points are analyzed for a set of aftershock sequences that have occurred in Turkey.

2.1.1. Considered aftershock sequences

Actual observed aftershock sequences were established by querying the Turkish earthquake catalog provided by the Kandilli Observatory and Research Institute. First step in compiling aftershock sequences from earthquake catalogs is the identification of specific events that form an aftershock cluster. In this study, Reasenberg (1985) algorithm has been used for identifying the aftershock sequences. In total, 8 aftershock sequences listed in Table 1 are considered in analysis.

<table>
<thead>
<tr>
<th>#</th>
<th>Year</th>
<th>EQ Name</th>
<th>( M_w )</th>
<th>( M_c^* )</th>
<th>( N_a^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1995</td>
<td>Dinar.</td>
<td>5.9</td>
<td>3.7</td>
<td>491</td>
</tr>
<tr>
<td>2</td>
<td>1998</td>
<td>Ceyhan.</td>
<td>6.4</td>
<td>3.1</td>
<td>36</td>
</tr>
<tr>
<td>3</td>
<td>1999</td>
<td>Duzce</td>
<td>7.3</td>
<td>3.3</td>
<td>94</td>
</tr>
<tr>
<td>4</td>
<td>2002</td>
<td>Sultandagi</td>
<td>6.3</td>
<td>3.1</td>
<td>391</td>
</tr>
<tr>
<td>5</td>
<td>2003</td>
<td>Bingol</td>
<td>6.4</td>
<td>3.4</td>
<td>544</td>
</tr>
<tr>
<td>6</td>
<td>2010</td>
<td>Elazig</td>
<td>6.1</td>
<td>3.0</td>
<td>320</td>
</tr>
<tr>
<td>7</td>
<td>2011</td>
<td>Simav</td>
<td>6.0</td>
<td>2.8</td>
<td>4699</td>
</tr>
<tr>
<td>8</td>
<td>2011</td>
<td>Van</td>
<td>7.2</td>
<td>2.9</td>
<td>4815</td>
</tr>
</tbody>
</table>

\( M_w \): magnitude of completeness / \( N_a \): number of aftershocks in the cluster.
epicenter of the aftershock with respect to half of the mainshock rupture plane length $L_m$. Accordingly, the dimensionless location parameter $"a"$ (i.e. $a = 2x_{\text{epi}}/L_m$) is obtained for each aftershock. Here, the mainshock focus is assumed to occur in the midpoint of the rupture. In the analysis here, $L_m$ lengths are estimated using the empirical relationships by Wells and Coppersmith (1994).

In the proposed IPAHA approach, the $F_{\mathcal{A}}(a)$ distribution (in Figure 2) is implemented into the evaluation of $f_{RM}(r | m)$ in Eq.(1). This is achieved by modifying the usual APSHA computation procedure. Developed procedure employs the $F_{\mathcal{A}}(a)$ model above in the stochastic simulation of random aftershock ruptures. For each stochastic realization of $A$, the corresponding aftershock rupture is modelled as a segment centered at the midpoint coordinate, $x_{\text{mid}}$ with the total length $L_a$. An illustration of the considered distance parameters is presented in Figure 2. Coordinate of $x_{\text{mid}}$ is determined as follows:

$$x_{\text{mid}} = \frac{L_a}{2} + a(L_m - L_a)$$

(2)

where $L_m$ and $L_a$ are the rupture lengths of the mainshock and the aftershock, respectively and $a$ is the stochastically simulated value of the random variable $A$. Note that, the $x_{\text{mid}}$ values are assumed to extend symmetrically towards the two sides of mainshock focal point.

2.2. Aftershock ground motion intensity estimation

The conditional probability $P(Y > y | m, r)$ of aftershock ground motion intensity $Y$ exceeding a given ground motion level $y$ forms one of the most important components of the conventional APSHA (Eq. 1). The ground motion intensity $Y$ is often represented by a lognormally distributed random variable with mean $\mathbb{E}[\ln Y]$ and standard deviation $\sigma_{\ln Y}$. Typically, ground motion prediction equations (GMPEs) are used for estimating $\mathbb{E}[\ln Y]$ and $\sigma_{\ln Y}$ for a particular site affected by a particular earthquake. Using the resulting $\mathbb{E}[\ln Y]$ and $\sigma_{\ln Y}$, $P(Y > y | m, r)$ is evaluated as follows:

$$P(Y > y | m, r) = P(E_a > \varepsilon'_a | m, r)$$

$$P(E_a > \varepsilon'_a | m, r) = \int_{\varepsilon'_a}^{\infty} f_{E_a} (\varepsilon) d\varepsilon$$

(3)

where $\varepsilon'_a = \frac{\ln(y) - \mathbb{E}[\ln Y]}{\sigma_{\ln Y}}$.
where $E_a$ is the random variable (i.e. standard error) that is referred as the “aftershock epsilon” in the following, $\varepsilon_a'$ is the specific value of $E_a$ that corresponds to $Y = \gamma$, and $f_{E_a}(\varepsilon)$ is the probability density function of $E_a$. Note that $f_{E_a}(\varepsilon)$ in Eq. (3) is equal to the standard normal distribution function.

2.2.1. Aftershock epsilon
The distribution function $f_{E_a}(\varepsilon)$ represents the variability of aftershock ground motion intensity $Y$ occurring at the site during a particular earthquake. Due to existence of “common path effects” noted in the introduction section, for a given site the attenuation characteristics of the peak motion exhibited during the mainshock is expected to be related to those characteristics exhibited during the aftershock (Figure 3). This expected relationship implies the presence of some level of statistical correlation $\rho_{E_a,E_m}$ between the $E_a$ and the mainshock epsilon parameter $E_m$. This correlation represents the degree of consistency in the level of amplification or attenuation of seismic waves that are originating from the rupture plane and reaching the site. A high level of correlation $\rho_{E_a,E_m}$ would suggest a strong causal relationship between the common path effects and the peak motions that exhibited at the site during the mainshock and the aftershock earthquakes.

If the mainshock ground motion is recorded at the site, the observed value $\varepsilon_m^*$ of the mainshock epsilon $E_m$ can be identified using a suitable GMPE. Making use of the correlation $\rho_{E_a,E_m}$ and the observed value $\varepsilon_m^*$, the conditional distribution $f_{E_a|\varepsilon_m}(\varepsilon_a | \varepsilon_m^*)$ of $E_a$ can be expressed as follows:

$$f_{E_a|\varepsilon_m}(\varepsilon_a | \varepsilon_m^*, m_m, r_m) = \frac{f_{E_a,E_m}(\varepsilon_a, \varepsilon_m^*; m_m, r_m)}{f_{E_m}(\varepsilon_m^*; m_m, r_m)}$$

(4)

where $f_{E_a,E_m}(\varepsilon_a, \varepsilon_m^*; m_m, r_m)$ is the probability density function with chosen model of standard bivariate Gaussian distribution for $E_a$ and $E_m$. Since both $E_m$ and $E_a$ are standard random variables with zero mean and unit variance, $f_{E_a|\varepsilon_m}(\varepsilon_a | \varepsilon_m^*)$ above can be practically evaluated as:

$$f_{E_a|\varepsilon_m}(\varepsilon_a | \varepsilon_m^*, m_m, r_m) = \phi\left(\frac{\varepsilon_a - \rho_{E_a,E_m}\varepsilon_m^*}{\sqrt{1 - (\rho_{E_a,E_m})^2}}\right)$$

(5)

where $\phi(\cdot)$ is the standard normal distribution function. A formulation similar to Eq.s (4-5) was proposed by Yeo and Cornell (2005). However, to the author’s knowledge the level of correlation $\rho_{E_a,E_m}$ has not been investigated quantitatively in the existing literature.

Level of correlation $\rho_{E_a,E_m}$ influences the degree of impact of taking into account $\varepsilon_m^*$ in the estimation of probability distribution of $E_a$. If $\rho_{E_a,E_m}$ is close to one (i.e. perfect correlation), the conditional distribution $f_{E_a|\varepsilon_m}(\varepsilon_a | \varepsilon_m^*)$ of $E_a$ in Eq. (5) would indicate a dispersion smaller than that of the corresponding unconditional distribution $f_{E_a}(\varepsilon)$ in Eq. (3).

2.2.2. Correlation model for epsilons
In order to enable implementation of Eq. (6) into the proposed IPAHA framework, the level of correlation $\rho_{E_a,E_m}$ was investigated using pairs of mainshock-aftershock strong motion records from Turkey and California. In addition to
earthquake sequences given in Table 1, data set is enriched with 1994 Northridge EQ. records presented in Ruiz-Garcia and Negrete-Manriquez (2011) study and 1999 Izmit EQ. records. The peak ground acceleration (PGA) and the pseudo-spectral acceleration $S_a(T)$ are utilized as the ground motion intensity measure in this investigation. Estimated spectral accelerations at site are determined by using 2008 version of Boore and Atkinson GMPE (Douglas, 2011).

The set of resulting $\varepsilon_a$ and $\varepsilon_m$ values and the identified correlations $\rho_{E_a, E_m}$ are presented in Figures 4&5 obtained for $S_a(T)$’s corresponding to periods $T$ being equal to 0.5s., 0.9s, 1.4s., 1.8s, 2s and peak ground acceleration (PGA). The results indicate that, among the considered set of periods, highest correlation (i.e. 0.65) is obtained for $T$=0.9s and observed to be lowest for the cases when $T$=1.8s and $T$=2s.

Finally, the resulting set of correlations in Figure 5 can be substituted into Eq. (5). Subsequently, the conventional APSHA integral in Eq. (1) can be revised to take into account the observed mainshock epsilon $\varepsilon_m^*$, as follows:

$$\bar{\mu}(y, t, T; m_m) = \mu^*(t, T; m_m) \ldots$$

$$\int \int \int \int f_{R M}(r | m) f_m(m; m_m)f_{E_a}(e | m, r; \varepsilon_m^*) dm dr dc$$

The equation above and the models developed for capturing $P(A<a)$ and $\rho_{E_a, E_m}(T)$, form the basis of the proposed IPAHA approach. In the next section, an example application of the IPAHA is presented for a scenario event.

3. EXAMPLE APPLICATION

3.1. Scenario earthquake and the site
Recent research by Ergintav et. al. (2014) revealed a slip deficit accumulation in “Princes’ Islands Fault Segment (PIF)” of North Anatolian Fault (Figure 6). This segment which has a length of approximately 60 km is located very close to Istanbul and it is estimated to be capable of generating an $M$>7.1 earthquake. Accordingly, a rupture along the PIF segment extending between [40.92°N; 28.18°E; 40.68°N; 29.57°E] is assumed as the scenario event. The magnitude of the resulting mainshock earthquake is assumed to be equal to $M7.2$. The aftershock peak ground acceleration (PGA) hazard at the site (41°N; 28.88°E) shown in Figure 5, is evaluated in this application. This site corresponds to a very densely populated district of Istanbul.

![Figure 4: Mainshock-aftershock epsilon pairs ($\varepsilon_m, \varepsilon_a$) for recorded mainshock-aftershock motions](image)

![Figure 5: Correlation coefficients obtained for intensity measures corresponding to different periods](image)

![Figure 6: Assumed scenario earthquake.](image)
3.2. Analysis results

Aftershock hazard is evaluated here for the time interval that spans the period of 1 year (i.e. \( T=365 \)) starting one week (i.e. \( t=7 \)) after the mainshock earthquake. The aftershock source zones and the spatial distribution of aftershocks are estimated according to the approach presented in Section 2.1. The lengths of the mainshock and the aftershock ruptures are estimated using the relationship by Wells and Coppersmith (1994). The smallest aftershock magnitude \( m_l \) that is of engineering interest is assumed to be 5. Median PGA and its dispersion is estimated using the 1997 version of Abrahamson and Silva GMPE (Douglas, 2011).

In Figure 7, aftershock hazard curves estimated using conventional APSHA method and the proposed IPAHA approach are presented. Here, the correlation \( \rho_{E_r,E_m} \) in IPAHA is assumed to be equal to 0. Therefore, the difference between the two approaches arises only from the aftershock clustering that is considered in IPAHA (as discussed in Section 2.1). It is observed in Figure 7 that aftershock hazard estimated using IPAHA method is considerably higher than that obtained using APSHA.

![Figure 7: Aftershock hazards estimated using alternative approaches: conventional (APSHA) and proposed IPAHA.](image)

Figure 8 presents the mean number \( \bar{\mu} \) of aftershocks estimated using Eq. (6) by assuming different correlation \( \rho_{E_r,E_m} \) levels when \( \varepsilon_m^* = 1 \). For the PGAs in the range from 0.1g to 0.5g, the estimated mean number \( \bar{\mu} \) of aftershocks increases with the increasing correlation \( \rho_{E_r,E_m} \).

![Figure 8: Aftershock hazard curves corresponding to different \( \rho_{E_r,E_m} \) values when \( \varepsilon_m^* = 1 \).](image)

Figure 9 presents the mean number \( \bar{\mu} \) of aftershocks estimated using Eq. (6) assuming a set of different \( \varepsilon_m^* \) values while \( \rho_{E_r,E_m} = 0.5 \). For the increasing values of \( \varepsilon_m^* \), higher levels of aftershock hazard is estimated. The significant differences between the estimated hazard levels in Figure 9 shows the importance of taking into account \( \varepsilon_m^* \) for a reliable estimation of hazard.

![Figure 9: Aftershock hazards obtained for different \( \varepsilon_m^* \) values when \( \rho_{E_r,E_m} = 0.5 \).](image)
4. CONCLUSIONS
An improved probabilistic aftershock hazard assessment approach that considers the clustering of aftershocks and the correlation of the mainshock- aftershock epsilon pairs is presented. Application of the proposed method is illustrated for a scenario M7.2 earthquake near Istanbul, Turkey. The impact of the key variables on the estimated aftershock hazard is assessed by performing parametric analysis.

Following conclusions can be drawn based on the findings of this study:

- Pseudo-spectral acceleration epsilons for the aftershock and the mainshock earthquakes are noticeably correlated for the systems with period of vibration equal to 0.9s. For the shorter and the longer periods, lower correlations coefficients are found (Figure 5).
- For the scenario earthquake and the site considered in the example application, taking the clustering of aftershocks into account resulting in estimation of higher aftershock hazard levels (Figure 7).
- Likewise, higher aftershock hazard levels are estimated using the proposed approach when higher $\epsilon_{in}$ values are observed at the site during the mainshock.
- Results of the example application indicate that for the assumed scenario earthquake near Istanbul, the annual rate of aftershock PGA at the site exceeding 0.4g is estimated to be equal to 0.8 for $\rho=0$ (Figure 8). This PGA level matches the design PGA assumed in the design/assessment of structures located in the highest seismic hazard zone in Turkey.

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5. REFERENCES


