

Uncertainty Propagation in Seismic Reliability Evaluation of Aging Transportation Networks

Keivan Rokneddin

Senior Research Scientist, AIG, Philadelphia, USA

Jayadipta Ghosh

Assistant Professor, Indian Institute of Technology Bombay, Mumbai, India

Leonardo Dueñas-Osorio

Associate Professor, Rice University, Houston, TX, USA

Jamie E. Padgett

Associate Professor, Rice University, Houston, TX, USA

ABSTRACT: Uncertainty quantification is an integral part of many fields of science and engineering, but its application to seismic reliability and risk assessment in highway transportation networks is still in its infancy. This study identifies major known sources of uncertainties associated with seismic loss assessments in aging transportation networks, including hazards, structures, aging parameters, and network topology sources, while quantifying the impact of a subset of them on mean network-level reliability estimates. The uncertainty tracking process is illustrated with a case study network in South Carolina, USA. The source-to-response uncertainties are propagated and errors aggregated as they emerge with the adoption of surrogate seismic response models at both bridge and network levels. The observed range of uncertainties from the considered sources suggests that uncertainty quantification must become a standard procedure for reliability and risk assessment in transportation networks. Moreover, while the bridge surrogate models contribute significantly to overall uncertainties, network surrogate model's contribution is found to be the least of all considered variables. Future opportunities exist to further identify key sources that should be targeted for improved confidence in risk estimates.

1. INTRODUCTION

Uncertainty is inherent in many modeling and experimental processes in science and engineering (Dror et al. 2006). Whether aleatory or epistemic, the sources of uncertainty which affect the scientific evaluations must be identified and their impact quantified and reported, if possible, so as to avoid inducing the sense of certainty into uncertain estimates. In stochastic processes where the reliability of physical systems is evaluated, quantifying uncertainties provides the distribution of the possible error around an estimated output, which can impact practical decision making in scientific, engineering, and financial applications (Adhikari et al. 2009; Alpak et al. 2013).

Uncertainty quantification in engineering loss estimation is of particular interest to risk managers who maintain a portfolio of structures and are concerned about the risk posed by natural hazards. For risk assessment, it is customary to have a catalog of historical or synthetic hazard events with varying predicted rate of occurrences. Once the losses associated with each event are evaluated, one may aggregate the results in the form of an exceedance probability curve (also known as the risk curve), which presents the probability of exceeding a certain threshold of loss over a defined period of time (FEMA 2009). The uncertainties around the estimated losses contribute to the development of the risk curve, and can significantly influence the tail losses as

demonstrated in the literature [e.g., Aslani et al. (2012) and Bazzurro et al. (2008)]. Identifying the sources of uncertainty and quantifying their impact on the risk assessment in transportation networks is essential to study the seismic resilience of these systems, particularly at a time when the lack of adequate maintenance as the result of expense cuts has left many bridges in seismically active regions vulnerable against potential seismic excitations. Moreover, aging and deterioration has made bridges which may be lacking proper seismic detailing even more vulnerable to earthquake hazard (Choe et al. 2009; Ghosh and Padgett 2010). Accordingly, evaluating the effects of uncertainties from component to system in risk estimates should be considered an integral part of risk assessment studies.

Table 1 lists a few sources of uncertainty which are known to influence direct and indirect loss estimates in highway transportation networks. Although evaluating all listed uncertainties is out of the scope of this study, identifying the possible sources provides opportunities for future studies. This paper focuses on the structural sources for uncertainty quantification, and adopts the mean occurrence rates and USGS (Peterson et al. 2014) recommended Ground Motion Prediction Equations (GMPEs) to develop the seismic hazard catalog, as discussed later in the paper.

In addition to the listed sources, there are uncertainties originating from the computational methods employed for risk assessment (e.g., from the number of samples in Monte Carlo simulations). This study aggregates the impact of the computational uncertainties with the known structural sources and provides the uncertainty bounds for a sample simulated seismic event to demonstrate possible ranges and emphasize the significance of incorporating uncertainty quantification in risk assessments. Section 2 elaborates on the methods for uncertainty propagation and quantification. Section 3 introduces the adopted computational methods for bridge and network reliability evaluations, and

demonstrates the case study bridge network. The uncertainty bounds are discussed in the results section, and the paper concludes by listing key findings and future research opportunities.

Table 1. Example of known sources of uncertainty for seismic risk assessment in highway bridge networks

HAZARD
Geophysics of faulting systems
Maximum possible rupture length and maximum possible magnitude
Simulated or historical event rates of occurrence
Ground motion prediction equations
STRUCTURE
Bridge locations
Material properties
Geometrical properties
Deterioration parameters
Live load (Traffic)
NETWORK
Topology
Traffic assignment model
Loss correlations

2. UNCERTAINTY PROPAGATION AND QUANTIFICATION METHODS

This section elaborates on methods to propagate the uncertainty in input variables (select variables from Table 1) to establish the uncertainty bounds on the estimated output (for example, bridge fragility or network reliability). As the focus is on structural and computational uncertainties, the bridge locations and network topology are fixed and no uncertainty is associated with the hazard catalog either. The uncertainty propagation is explored at two stages: a) Bridge fragility stage, and b) Network reliability stage. At the bridge fragility stage, uncertainties associated with structural modeling parameters, aging parameters, as well as the computational error from the application of a predictive metamodel for bridge fragilities are considered. These uncertainties are combined to predict the 95% confidence interval around the mean bridge fragility estimate. At the network reliability stage, the uncertainty originates from the application of a surrogate model, which is aggregated with the propagated errors from bridge fragilities stage to estimate the uncertainty bounds around the network

connectivity reliability, which is the final output in this study.

Surrogate models are employed at both bridge fragility and network reliability computation stages to efficiently approximate the failure probabilities and avert the computational burden associated with traditional techniques. Development of these models require an experimental design matrix which is the input for the model selection process (Hastie et al. 2009). The reliability estimates associated with rows in the input matrix are evaluated using traditional methods [e.g., finite elements analysis for bridges and Monte Carlo or Recursive Decomposition (Liu and Li 2009) for the network] to provide the response vector. The surrogate model is trained using the provided input matrix and response vector, and the application error (test error) is estimated using samples not used in the training process. The described procedure provides an unbiased estimate of the error introduced by the surrogate model, which can be aggregated with uncertainty from other sources, as discussed next.

In multiple-input and single output systems, several methods exist which quantify the propagation of uncertainty, such as Monte Carlo analysis, sensitivity analysis, statistical linearization, and first order analysis using Taylor series expansion (Lei and Schilling 1994). While each method has its advantages and shortcomings, this study adopts the first order analysis method because of superior computational efficiency and direct applicability to the case at hand. Highway bridge reliability problems are typically characterized by low curvature of the failure domain (Ghosh et al. 2013). For such moderately nonlinear problems, the first order analysis method for error propagation assessment provides an excellent approximation to the output uncertainty in the response while allowing partitioning of the error into its input sources (Lei and Schilling 1994; Stein et al. 2008). Moreover, this method is simpler statistical linearization in terms of computational complexity. The first order analysis method uses the Taylor series linear approximation of the response function around

the mean of the predictor variables with the nonlinear components truncated. For example, if the output response y is a function of input variable vector $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$ as:

$$y = f(\mathbf{x}) \quad (1)$$

then, the Taylor series expansion of the response can be approximated as:

$$y \cong f(E[\mathbf{x}]) + \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}}(\mathbf{x} - E[\mathbf{x}]) \quad (2)$$

The expected value and the variance of y can thus be evaluated as:

$$E[y] \cong f(E[\mathbf{x}]) \quad (3)$$

$$Var[y] \cong \sum_{i=1}^n \left(\frac{\partial f(\mathbf{x})}{\partial x_i} \right)^2 Var[x_i] \quad (4)$$

Equation 4 depicts the overall output model uncertainty as a function of the variances of the individual input parameters and thus helps to capture the propagation of uncertainty through the process. In many cases, the output response is not an exact function of the input variable, but involves model fitting errors. As is the case for this study, highway bridge fragilities are often obtained analytically after fitting a metamodel to the input predictor variables, and thereby introducing a metamodel error. In such cases, the variance of y can be expressed as:

$$Var[y] \cong \sum_{i=1}^n \left(\frac{\partial f(\mathbf{x})}{\partial x_i} \right)^2 Var[x_i] + \sigma_m^2 \quad (5)$$

where, σ_m is the predictive metamodel error. Using a case study example in South Carolina as described next, Section 4 will demonstrate the error propagation method to quantify the contribution of individual error sources to overall output uncertainty through a case study.

3. CASE STUDY EXAMPLE

The highway bridge network around the Charleston metropolitan area in South Carolina, USA, has been previously studied by the authors to evaluate its network connectivity reliability (Rokneddin, et al. 2014a). A subset of that network, comprised of 22 bridges (Figure 1), is

studied in this paper to quantify the uncertainties in bridge-to-network reliability computations. The development of surrogate models and the associated uncertainties for the bridges and network are discussed next.

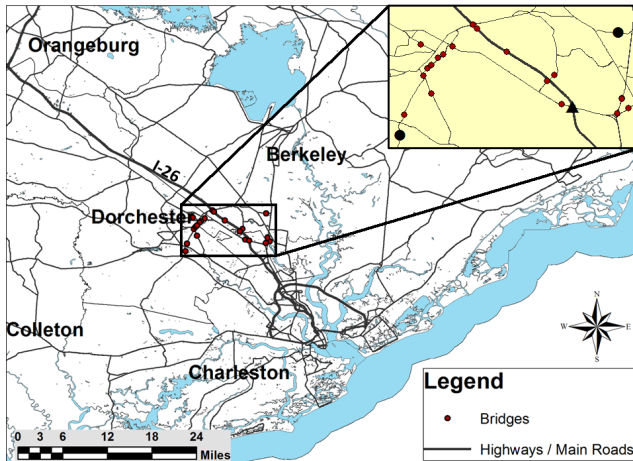


Figure 1. Location and extent of the 22-bridge network case study. Dark circles in the zoomed-in figure indicate the location of origin nodes while the dark triangle shows the destination node.

3.1. Surrogate models to develop parameterized bridge fragility functions.

The bridges in the depicted network comprise two bridge types: Multi-span Simply Supported Slab (MSSS Slab) and Multi-span Simply Supported Concrete girder (MSSS Concrete) bridges. Both of these bridge types have been previously identified by Nielson (2005) as being seismically vulnerable due to inadequate seismic detailing of bridge components such as bridge columns and elastomeric pad bearings. The seismic performance of these bridges can be further exacerbated due to potential aging and deterioration of critical bridge components. As identified by Ghosh and Padgett (2012) and Rokneddin et al. (2014a) the reinforcing steel in bridge columns and bearing dowel bars are prone to corrosion deterioration from the chloride laden sea water in the atmosphere, while the elastomeric bearing pads are susceptible to time-dependent stiffening due to thermal oxidation. The following subsection discusses the development of time dependent parameterized seismic fragility models for the bridge classes which will subsequently be

applied to individual bridges within the network by defining appropriate structure specific parameters. These fragility models are used to investigate the contribution of different sources of input parameter uncertainty to the overall error around the mean fragility estimate.

3.1.1. Experimental design and finite element simulations

Development of multidimensional bridge fragility models consists of the following steps: a) Formulate the experimental design matrix and conduct finite element simulation of bridge models, b) Fit metamodels to predict the seismic response of bridge components and subsequently develop parameterized seismic fragility curves.

The experimental design matrix for the two bridge types is developed through a systematic combination of the predictor variables described in Fang et al. (2006). The considered variables include the peak ground acceleration (*PGA*) of the earthquake record, critical bridge modeling parameters, deterioration affected structural parameters and geometric parameters. While *PGA* is an uncontrolled parameter in the experimental design, the other parameters constitute the vector \mathbf{x} (Equations 1 to 5). A complete list and descriptions of these parameters are presented in Table 2.

Table 2: Input parameter predictor variables for case study bridge types

Predictor Variable	Variable Description	
	MSSS Slab	MSSS Concrete
<i>PGA</i>	Peak Ground Acceleration	Peak Ground Acceleration
x_1	Concrete Strength	Steel Strength
x_2	Bearing pad friction	Bearing pad friction
x_3	Abutment gap	Dowel gap
x_4	Shear modulus	Shear modulus
x_5	Dowel Strength	Dowel Strength
x_6	Column rebar area	Column rebar area
x_7	Cover Depth	Cover Depth
x_8	Column height	Column height
x_9	Span Length	Span Length
x_{10}	Deck Width	Deck Width

This study adopts the Latin Hypercube experimental design with optimal spacing for computer experiments. Construction of the design sequence is followed by nonlinear dynamic time history analysis of three dimensional finite element bridge models, with bridge characteristics for each nonlinear time history run informed from the generated experimental design matrix.

3.1.2. Parameterized bridge fragility development

This study uses metamodels in two steps to develop generalized bridge class specific multidimensional vulnerability functions which are subsequently used for error propagation analysis. In the first step, the ensemble learning meta-algorithm known as Adaptive Boosting or AdaBoost (Hastie et al. 2009) is used to predict the seismic response of bridge components, such as columns, bearings and abutments, as a function of the predictor variables (Table 2). This algorithm uses a single composite strong learner by iteratively adding weak learners. For both bridge classes within the network, AdaBoost is found to perform well while predicting the bridge component responses resulting in high R^2 values and low mean square errors after repeated random sub-sampling cross validation. In the second step, the logistic regression metamodel is used to develop parameterized bridge fragility models after comparing the component seismic demands from the AdaBoost models with the component capacity estimates from Nielson and DesRoches (2007). While the details of this procedure can be found elsewhere (Ghosh et al. 2013), bridge system failure probability (P_f) conditioned on PGA and parameter vector $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$ can be represented as:

$$[P_f | PGA, \mathbf{x}] = \frac{e^{\tau_0 + \tau_{pga} \cdot PGA + \sum_{i=1}^n \tau_i x_i}}{1 + e^{\tau_0 + \tau_{pga} \cdot PGA + \sum_{i=1}^n \tau_i x_i}} \quad (6)$$

where, τ_0 , τ_{pga} and τ_1, \dots, τ_n are the logistic regression coefficients. The conditioned bridge

failure probability corresponds to the variable y in Equations 1 to 5.

3.2. The network surrogate model

The development of network surrogate model is elaborated in Rokneddin et al. (2014b). Each row in the network-level experimental design matrix represents the bridge failure probabilities, as evaluated using the metamodeling procedure in Section 3.1, against a synthetic seismic scenario which is generated following the logic tree methodology recommended by the USGS (Peterson et al. 2014). In total, 398 seismic scenarios are sampled for this study to train the model, and the peak ground accelerations at the location of the 22 bridges are evaluated for each scenario using the recommended GMPEs (only the mean predicted PGA value is used per scenario; therefore, each scenario produces one intensity map). Subsequently, Equation 6 is used to estimate the set of bridge failure probabilities per scenario, resulting in 398 records for the input matrix to develop the network surrogate model. Once the network reliability associated with each record of bridge failure probabilities is evaluated by Monte Carlo simulations, 200 records out of 398 are assigned to model training and validation, and the rest are reserved for testing in order to evaluate the order of error induced to the network reliability assessment by surrogate model application.

Random Forests (Breiman 2001) are selected as the statistical learning method for network surrogate model development as it has been shown to provide superior performance in network reliability applications (Rokneddin et al. 2014b). Random Forests, similar to AdaBoost, are an ensemble learning method which build a predictive model by averaging the outcome of a set of B regression trees as follows:

$$\hat{f}_{rf}^B(\mathbf{y}) = \frac{1}{B} \sum_{b=1}^B T_b(\mathbf{y}) \quad (7)$$

where, $\hat{f}_{rf}^B(\mathbf{y})$ denotes the outcome of random forest prediction from a total of B regression trees and $T_b(\mathbf{y})$ is a regression tree formed out of a

randomly selected subset of input variables. For the network surrogate model, the input variables in \mathbf{y} are the conditional bridge failure probabilities y_i , $i = 1, \dots, 22$. The generic form of the regression tree can be described as

$$T_b(\mathbf{y}) = \sum_{m=1}^M c_m \mathbf{I}(\mathbf{y} \in R_m) \quad (8)$$

where, the 22-dimensional input space described by \mathbf{y} is divided into an optimal number of sub-regions (M) each associated with a weight c_m (Hastie et al. 2009).

Figure 2 demonstrates the evolution of the root of mean squared error for predicted network reliability estimates as the number of the regression trees (B) grows. The model with an acceptable number of trees to provide the least error (around 400) is selected which results in an error in the order of 0.02 for network connectivity reliability estimates.

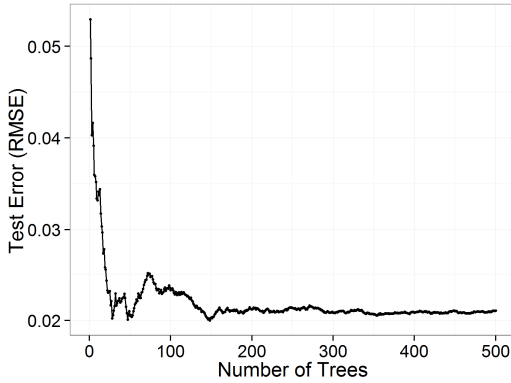


Figure 2. The test error of the developed network surrogate model with 200 training records

4. RESULTS AND DISCUSSION

This section presents the evaluation of bridge failure probabilities and network reliability along with their associated uncertainties against a $M_w = 6.7$ synthetic seismic event which was selected for this case study. This event was excluded from development of the surrogate models to provide an unbiased estimate of the associated errors in bridge and network failure probabilities.

To demonstrate the uncertainty propagation at the bridge level, a MSSS Concrete girder bridge within the network is selected as an example. Among input parameters x_1 to x_{10} , the sources of

uncertainty for this bridge are assumed to only stem from parameters listed in Table 3 along with their probability distributions. The distribution parameters for bridge modeling parameters x_1 and x_2 are taken from Nielson (2005), while the aging and deterioration parameters x_4 to x_6 are adopted from Rokneddin et al. (2014a). Given a chosen ground motion scenario, deterministic parameters for this bridge are the bridge dowel gap ($x_3=25.4\text{mm}$) and cover depth ($x_7=38.1\text{mm}$) (Nielson 2005). Additionally, the bridge geometric parameters (x_8 to x_{10}) are obtained from the National Bridge Inventory database (FHWA 2010).

Table 3: Probability distribution of the random variables affecting bridge fragility. C.O.V refers to the associated coefficient of variation.

Variable	Unit	Distribution	Mean	C.O.V.
x_1^*	MPa	Lognormal	463	0.08
x_2^*	-	Lognormal	1	0.10
x_4^\dagger	MPa	Uniform	2.50	0.30
x_5^\dagger	kN	Lognormal	56.04	0.10
x_6^\dagger	cm ²	Lognormal	5.67	0.10

*Nielson (2005) † Rokneddin et al. (2014a)

The contribution of different parameter uncertainties and the logistic regression metamodel error to the uncertainty around the predicted fragility estimate is calculated using the first order analysis method (Section 2) for extensive damage state. At this limit state, the damage to bridge components is visible, requires repair and results in closure of the bridge for at least a week following a seismic event. Figure 3 shows the mean predicted bridge fragility estimate (calculated using Equation 6) and the 95% confidence bounds for each individual source of uncertainty represented by the outer extremities of the different colored band. The figure also shows the 95% confidence interval of the total error obtained by combing the different error sources in the logit space. Although not presented here, the total lumped uncertainty around the mean fragility is also verified using Monte Carlo simulations, showing close agreement with the first order analysis.

For the chosen MSSS Concrete bridge (and in general observed for other MSSS Concrete bridges within the network), Figure 3 reveals that column longitudinal rebar (x_6) area has the highest impact on the output uncertainty followed by bearing pad shear modulus (x_4) and steel strength (x_1). The error contribution stemming from the parameters bearing pad friction and dowel strength (x_2 and x_5) are found to be negligible and not shown in the figure. The contribution from the metamodel error to the overall uncertainty is found to be the highest. Similarly, for the MSSS Slab bridges within the network, the metamodel error is predominantly higher than that of all other input variables, followed by the uncertainties stemming from column rebar area (x_6), bearing pad friction (x_2) and bearing dowel strength (x_5).

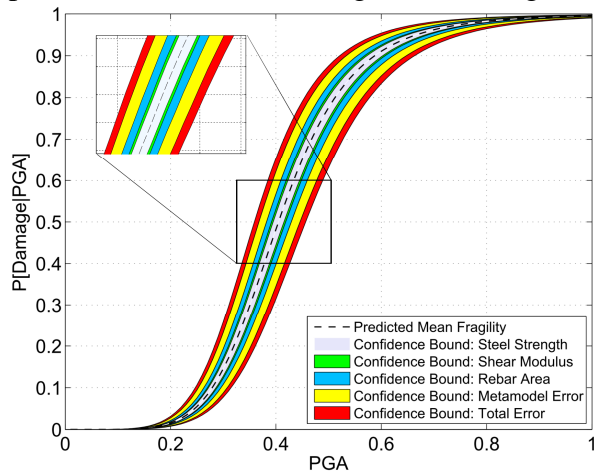


Figure 3: Predicted mean fragility estimate for the MSSS Concrete bridge type with 95% confidence bounds for each source of uncertainty represented by the different colored bands. Also shown is the 95% confidence interval from the total combined error.

Using the mean predicted bridge failure probabilities, the network failure probability estimate is 0.264. Applying the 95% confidence bounds in bridge fragilities results in a range of 0.145 to 0.455 for the network failure probability estimate. Finally, applying the 0.02 expected error from the network surrogate model extends the range to [0.143-0.457], as shown in Table 4. The observed range of uncertainty is significant, emphasizing the need to incorporate the known sources of uncertainty into risk assessment studies. For this example, the uncertainties

associated with the network surrogate model are small compared to the variability from the bridge fragility metamodels and bridge input parameters. However, the error from the network surrogate model may become more significant for different network performance metrics (e.g., travel time) and topologies.

Table 4. The range of uncertainties in network reliability estimates for the case study example

Lower Bounds		Mean	Upper Bounds	
Network	Bridge		Bridge	Network
0.143	0.145	0.264	0.455	0.457

5. CONCLUSIONS

Although uncertainty quantification is considered an integral part of many tracks across different sciences and engineering, their application to seismic loss evaluations (travel time or loss of access) in highway transportation networks is in its infancy. This study identifies major sources of uncertainties, and quantifies the impact of structural and computational uncertainties on network connectivity reliability estimates for a case study network in South Carolina, USA. The developed multi-dimensional bridge fragility functions in this study are conditioned on the ground motion intensity, deterioration parameters, bridge modeling parameters, and geometric parameters. The contribution of each uncertainty source to the aggregate uncertainty around the mean estimated fragility is computed using a first order analysis method. The metamodel uncertainty contributes the most to the overall error followed by column rebar area, bearing shear modulus and steel strength for the concrete girder bridges, and column rebar area, bearing pad friction and bearing dowel strength for the slab bridges. Finally, the uncertainty from network surrogate model is aggregated with the uncertainties propagated from bridge fragilities to evaluate the overall uncertainties around the network reliability estimate. The uncertainties around mean losses from each event influence the risk curve, especially at tail losses. In future, the authors will explore this impact in a framework which includes the hazard, structural, and

computational uncertainties to improve the confidence in risk estimates.

6. ACKNOWLEDGEMENTS

This study was supported in part by the National Science Foundation under Grant No. CMMI-1234690. Any opinions, findings, conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

7. REFERENCES

- Adhikari, S., Friswell, M. I., Lonkar, K., and Sarkar, A. (2009). "Experimental case studies for uncertainty quantification in structural dynamics." *Probabilistic Engineering Mechanics*, 24(4), 473–492.
- Alpak, F. O., Vink, J. C., Gao, G., and Mo, W. (2013). "Techniques for effective simulation, optimization, and uncertainty quantification of the in-situ upgrading process." *Journal of Unconventional Oil and Gas Resources*, 3–4, 1–14.
- Aslani, H., Cabrera, C., and Rahnema, M. (2012). "Analysis of the sources of uncertainty for portfolio-level earthquake loss estimation." *Earthquake Engineering & Structural Dynamics*, 41(11), 1549–1568.
- Bazzurro, P., Park, J., Tothong, P., and Jayaram, N. (2008). *Effects of spatial correlation of ground motion parameters for multi-site seismic risk assessment: Collaborative Research with Stanford University*.
- Breiman, L. (2001). "Random Forests." *Mach. Learn.*, 45(1), 5–32.
- Choe, D.-E., Gardoni, P., Rosowsky, D., and Haukaas, T. (2009). "Seismic fragility estimates for reinforced concrete bridges subject to corrosion." *Structural Safety*, 31(4), 275–283.
- Dror, M., L'Ecuyer, P., and Szidarovszky, F. (2006). *Modeling Uncertainty: An Examination of Stochastic Theory, Methods, and Applications*. Springer US.
- Fang, K., Li, R., and Sudjianto, A. (2006). *Design and modeling for computer experiments*. CRC Press.
- FEMA. (2009). "FEMA Library - HAZUS®MH MR4 Earthquake Model User Manual." <<http://www.fema.gov/library/viewRecord.do?id=3732>> (Mar. 11, 2011).
- FHWA. (2010). "NBI ASCII Files - NBI - Programs - Integrated - Bridge - FHWA." <<http://www.fhwa.dot.gov/bridge/nbi/ascii.cfm?year=2010>> (Mar. 11, 2011).
- Ghosh, J., and Padgett, J. (2010). "Aging Considerations in the Development of Time-Dependent Seismic Fragility Curves." *Journal of Structural Engineering*, 136(12), 1497–1511.
- Ghosh, J., and Padgett, J. (2012). "Impact of Multiple Component Deterioration and Exposure Conditions on Seismic Vulnerability of Concrete Bridges." *Earthquakes and Structures*, 3(5), 649–673.
- Ghosh, J., Padgett, J. E., and Dueñas-Osorio, L. (2013). "Surrogate modeling and failure surface visualization for efficient seismic vulnerability assessment of highway bridges." *Probabilistic Engineering Mechanics*, 34, 189–199.
- Hastie, T., Tibshirani, R., and Friedman, J. (2009). *The Elements of Statistical Learning: Data Mining, Inference, and Prediction, Second Edition*. Springer Science & Business Media.
- Lei, J., and Schilling, W. (1994). "Parameter uncertainty propagation analysis for urban rainfall runoff modelling." *Water Science & Technology*, 29(1-2), 145–154.
- Liu, W., and Li, J. (2009). "An improved recursive decomposition algorithm for reliability evaluation of lifeline networks." *Earthquake Engineering and Engineering Vibration*, 8(3), 409–419.
- Nielson, B. G. (2005). "Analytical fragility curves for highway bridges in moderate seismic zones." PhD Thesis, Georgia Institute of Technology, Atlanta, Georgia.
- Nielson, B. G., and DesRoches, R. (2007). "Analytical Seismic Fragility Curves for Typical Bridges in the Central and Southeastern United States." *Earthquake Spectra*, 23(3), 615–633.
- Peterson, M. D., Moschetti, M. P., Powers, P. M., Mueller, C. S., Haller, K. M., Frankel, A. D., Zeng, Y., Rezaeian, S., Harmsen, S. C., Boyd, O. S., Field, N., Chen, R., Rukstales, K. S., Luco, N., Wheeler, R. L., Williams, R. A., and Olsen, A. H. (2014). *Documentation for the 2014 Update of the United States National Seismic Hazard Maps*. Open-File Report 2014–1091, 243.
- Rokneddin, K., Ghosh, J., Dueñas-Osorio, L., and Padgett, J. E. (2014a). "Seismic Reliability Assessment of Aging Highway Bridge Networks with Field Instrumentation Data and Correlated Failures, II: Application." *Earthquake Spectra*, 30(2), 819–843.
- Rokneddin, K., J Ghosh, L Dueñas-Osorio, and JE Padgett. (2014b). "Seismic reliability assessment of bridge networks by statistical learning." *Safety, Reliability, Risk and Life-Cycle Performance of Structures and Infrastructures*, CRC Press, 613–620.
- Stein, A., Shi, W., and Bijker, W. (2008). *Quality Aspects in Spatial Data Mining*. CRC Press.