Estimation of the Failure Probability of a Floating Wind Turbine Under Environmental Load

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ABSTRACT: This present paper deals with the choice of stochastic expansion models for load processes in structural reliability of floating wind turbines. Two widely used models, the spectral representation method and the Karhunen-Loève (KL) expansion are compared as far as their use in reliability assessment based on real world wind speed data. The use of expansion models makes it possible to compute the outcrossing rate using time invariant reliability methods. We assume stationary conditions and define and compare several outcrossing rate estimators based on a design point computation. We first adapt the PHI2 method to functionals, and specify a closed form of Koo et al. (2005)’s estimator for the KL model. An importance sampling scheme based on the design point with and without an additional Monte Carlo Markov Chain sampler is then suggested and its effectiveness is assessed for the KL model.

Offshore wind farms are garnering considerable interest because the generally superior wind speed makes them more efficient than their onshore equivalents. Due to the influence of wind and wave loads, the mechanical structure is prone to failure which raises the issue of the reliability of floating wind turbine designs. Among structural reliability algorithms, the first and second order reliability methods (FORM/SORM) (Ditlevsen and Madsen (1996)) are popular because they usually require a reasonable amount of response simulations. Nevertheless, the principal underlying hypothesis, namely that the response is approximately linear around a unique design point, suggests the use of other algorithms. An obvious alternative to FORM/SORM is the traditional Monte Carlo Simulation (MCS). Since the sought failure probability is usually small, MCS may not be a suitable option as it requires a significant amount of system response simulations to achieve an acceptable level of accuracy. Importance sampling (IS) can be used in cases where a reasonable approximation of the failure region is available. IS proceeds by sampling from a distribution whose main contribution takes place around the failure domain. However, designing an adequate importance sampling density that yields reasonable estimation variance is not trivial. An additional challenge of this specific time varying reliability analysis is that the response at a given time depends on functional inputs, the wind speed and wave elevation processes. It is therefore necessary to use an expansion of the stochastic processes on a finite functional basis with random coefficients. This study focuses on the wind speed process modelling only, although the approach is more general. Since the performance of virtually all the aforementioned reliability methods is highly dependent on the dimension of the input space, expansion methods that lead to reasonable dimensions are desirable. In this paper, we study the spectral
representation method (Shinozuka and Deodatis (1991)) and the Karhunen-Loève expansion (KL) (Ghanem and Spanos (1991)). The spectral representation method models the time dependent load as a stationary Gaussian process and is widely used in the reliability of offshore structures community (Jensen (2009)), while KL targets more general processes and has some attractive optimality properties. Based on available wind speed measurements, both the KL and spectral representation models are fitted to the data. To this end, the joint distribution of the KL coefficients is estimated via kernel density estimation (KDE) as in Poirion and Zentner (2014) and the spectrum in the spectral representation model is estimated directly from the data assuming input stationarity. This stationarity assumption, motivated by fixed long term conditions, enables the use of time-invariant reliability methods. The failure probability is assessed by estimating the wind turbine response outcrossing rate. Several estimates are provided based on design points computations, which were carried out after careful selection of an optimization algorithm. This work is a first step to the reliability analysis of wind turbines and will serve as point of comparison to other estimators which don’t rely on the design point (subset simulation, importance sampling, etc.). Two FORM approximations are used: the first one is a derivative of the PHI2 method (Andrieu-Renaud et al. (2004)), adapted to the case where the output depends on functional inputs, while the second one is based on Koo et al. (2005)’s work. The outcrossing rate is then evaluated by importance sampling based on the preliminary design point computation. It is expressed as a product of a conditional probability, evaluated by a Monte Carlo Markov Chain (MCMC) sampler, and an unconditional failure probability, which is estimated through IS. These methods are applied to the reliability analysis of a recent wind turbine prototype for both input load models in order to assess the impact on the computing cost and the final result.

1. STOCHASTIC EXPANSION METHODS FOR LOAD PROCESSES

1.1. The spectral representation method

In reliability analysis of offshore structures, it is frequent to model both the wave elevation and wind speed as stationary Gaussian processes with specific spectrum. The resulting expansion is a discrete formulation of the spectral representation theorem, which reads for stationary Gaussian processes:

\[ X(t) = \sum_{i=1}^{n} (u_i \sigma_i \cos(\omega_i t) - \tilde{u}_i \sigma_i \sin(\omega_i t)) \]  

where \( u_i, \tilde{u}_i \) are standard uncorrelated normal variables, \( \omega_i \) are the frequencies with increment \( \omega_{i+1} - \omega_i \) and \( \sigma_i^2 = S(\omega) \omega \) where \( S(\omega) \) is the power spectrum density (p.s.d.) of \( \{X(t), t \geq 0\} \). The previous expansion is sometimes called non-deterministic spectrum amplitude (NSA) and similar to the original spectral representation model which uses a deterministic amplitude with random phases (Shinozuka and Deodatis (1991)).

As pointed out in Grigoriu (1993), there’s usually no clear reason to favour either method. It is to be noted that a spectral representation with equally spaced frequencies produces periodic sample paths with period \( \frac{2\pi}{\omega_1} \), \( \omega_1 \) being the minimum frequency (Shinozuka and Deodatis (1991)). Therefore, simulation of sufficiently long paths requires large \( n \) in (1) which can be prohibitive from a reliability analysis standpoint. To obtain longer trajectories, it is sometimes advocated to use non-equal frequency spacing via iso-energy spectrum discretization for instance: this consists in constructing the frequency grid such that the energy between two successive frequencies \( \int_{\omega_1}^{\omega_{i+1}} S(\omega) d\omega \) is constant. For a fixed \( n \), this usually gives longer periods than the fixed frequency increment method (Jensen (2009)). A similar approach consists in expressing the spectrum in the period domain and using equally spaced discrete periods to discretize this spectrum.

1.2. Karhunen-Loève expansion
1.2.1. Background

Let \( \{X(t), t \in [0, T]\} \) be a mean-square (m.s.) continuous stochastic process. Then there exists a basis of \( L^2([0, T]) \) eigenfunctions \( \{\phi_i, i \geq 1\} \), such
that $\forall t \in [0,T]$,

$$X(t) \overset{m.s.}{=} \mathbb{E}(X(t)) + \sum_{i=1}^{\infty} \sqrt{\lambda_i} \xi_i \phi_i(t)$$

(2)

The zero-mean and uncorrelated random variables $\xi_i, i \geq 1$ are the Karhunen-Loève coefficients, and $\{\lambda_i\}$ is a sequence of decreasing eigenvalues. The latter are solutions to the integral equation

$$\int_0^T R_X(s,t) \phi_i(t) dt = \lambda_i \phi_i(s)$$

where $R_X$ is the process autocovariance function. The KL expansion is clearly more general than the spectral representation model since $X$ need not be stationary nor Gaussian. In addition, it is optimal in the sense that when truncated after a finite number $n$ of terms, it minimizes the mean-square error. A formal criterion to identify the truncation order is the percentage of explained variability defined as

$$\text{varexp}_n = \frac{\int_0^T \mathbb{E} \left[ (X^n(t) - \bar{X}(t))^2 \right] dt}{\int_0^T \mathbb{E} \left[ (X(t) - \bar{X}(t))^2 \right] dt}$$

(3)

where $X^n$ is the expansion truncated after $n$ terms and $\bar{X}(t) = \mathbb{E}(X(t))$. $n$ can be chosen so as to account for a given percentage (e.g. 95%) of the variability of the process.

In order to simulate sample paths according to a truncated KL expansion, the joint distribution of the KL coefficients $\xi_1, \ldots, \xi_n$ needs to be estimated. Note that for Gaussian processes, this distribution is also Gaussian but nothing is known in the general case. A useful tool is kernel density estimation (KDE) where the joint pdf is expressed as a mixture of multivariate kernels centered around the sample KL coefficients $\xi_i(l), l = 1, \ldots, N$.

$$\hat{p}_\xi(x) = \frac{1}{Nh^p} \sum_{i=1}^{N} K \left( \frac{x - \xi(l)}{h} \right)$$

(4)

where $\xi(l) = [\xi_1(l), \ldots, \xi_n(l)]^T$ is the vector of Karhunen-Loève coefficients estimated from the data via

$$\xi_i(l) = \frac{1}{\sqrt{\lambda_i}} \int_0^T [X_l(t) - \bar{X}(t)] \phi_i(t) dt$$

(5)

$K$ is usually not influential on KDE accuracy so we assume a Gaussian kernel from now on. The bandwidth parameter $h$ controls the degree of smoothing and impacts significantly KDE performance. Poirion and Zentner (2014) use KDE to estimate the pdf of $\xi$ and show that, when the kernel is Gaussian, a suitable choice of the bandwidth is $h = 1.06 N^{-\frac{1}{5}}$.

In practice, the KL expansion is constructed from a database of $N$ measurements $\{t \mapsto X_i(t), i = 1, \ldots, N\}$. In this case, under regularity conditions, the KL expansion derived from the empirical autocovariance function is strongly consistent (Poirion and Zentner (2014)) as $N \to \infty$. Another aspect is that the measured realizations $X_i$ are sampled at discrete time steps $t_k, k = 1, \ldots, M$. Performing KL expansion can be done by considering the interpolating processes $\tilde{X}_i(t), s.t. \tilde{X}_i(t_k) = X_i(t_k)$ and performing KL expansion on $\tilde{X}$ (see Ramsay and Silverman (2005)). It can then be shown that KL expansion of $\tilde{X}$ reduces to a matrix eigenvalue problem. In the sequel, the tilde will refer to KL expansion performed on the interpolating process as defined by equations (2), (4) and (5).

2. STRUCTURAL RELIABILITY METHODS

2.1. Outcrossing approach and iso-probabilistic transformation

Let $X(t, \xi)$ be the stochastic load process, where $\xi$ is a random vector of expansion coefficients in a finite functional basis. We will assume that response process at time $t$, $Y(t, \xi)$ only depends on the input load up to time $t$, that is $Y(t, \xi) = F(X_{[0,t]}(\cdot, \xi))$ where other deterministic structural characteristics have been omitted. The limit state function $G(t, \xi) = s - Y(t, \xi)$, where $s$ is a scalar threshold on the response, is strictly positive in the safe domain and negative in the failure domain. In time variant structural reliability, we are concerned with the evaluation of $P(T_h) = \mathbb{P}(\sup_{t \in [0,T_h]} G(t, \xi) \leq 0)$ for a given time horizon $T_h$. A quantity of interest is the outcrossing rate $v^+(t)$ defined as

$$v^+(t) = \lim_{\Delta t \to 0} \mathbb{P} \left( \{G(t, \xi) > 0\} \cap \{G(t+\Delta t, \xi) \leq 0\} \right) \frac{1}{\Delta t}$$

(6)
Indeed, \( v^+ \) is linked to the failure probability through the inequality
\[
P_f(T_h) \leq P(G(0, \xi) \leq 0) + \mathbb{E}(N^+(0, T_h))
\]
where \( \mathbb{E}(N^+(0, T_h)) = \int_0^{T_h} v^+(t)dt \) is the mean number of outcrossings.

In order to compute (6), it is convenient to express the limit state function in the standard Gaussian space \( \mathbb{U} \). This is done thanks to an isoprobabilistic transform \( T \), such as the Nataf transform, that maps the physical space to \( \mathbb{U} \). Letting \( g(t, U) = G(t, T^{-1}(U)) \) the limit state function in the standard space, the outcrossing rate now reads
\[
v^+(t) = \lim_{\Delta t \to 0} \frac{\mathbb{P}(\{g(t, U) > 0\} \cap \{g(t + \Delta t, U) \leq 0\})}{\Delta t}
\]
(8)

We hereafter use the Nataf transform defined as \( T(\xi) = T_2 \circ T_1(\xi) \), \( T_1(\xi) = Z = [\Phi^{-1}(F_1(\xi_1)) \cdots \Phi^{-1}(F_n(\xi_n))]^T \), \( T_2(Z) = L_{0}^{-1}Z \) where \( L_0L_{0}^T = R_0 \) and \( R_0 \) is the correlation matrix of \( Z \). \( \Phi^{-1} \) is the inverse of the standard normal cumulative distribution function (cdf) and \( F_1, \ldots, F_n \) are the marginal cdfs of the input KL coefficient vector. When using the spectral representation model, the Nataf transform is simply the identity function.

If \( \xi \) is the vector of Karhunen-Loève coefficients, since \( \text{Cov}(\xi) = I_n \) (see properties in 1.2.1), the correlation matrix of \( \xi \) is \( C^\xi = I_n \). Besides, if \( C^\xi_{ij} = 0 \), then \( R_{0ij} = 0 \) (Liu and Der Kiureghian (1986)). This leads to \( R_0 = I_n \). Using the kernel density estimate (4) for the distribution of \( \xi \), the corresponding marginal cdfs are
\[
F_i = F_{\xi_i}(x) = \frac{1}{N} \sum_{l=1}^{N} \Phi \left( \frac{x - \xi_i(l)}{h} \right)
\]
From a numerical point of view, the inverse cdf can be computed from the previous expression by pre-computing and storing \( y_j(i) = F_i(x_j(i)) \) where \( x_{\text{min}} = x_1(i) < x_2(i) < \cdots < x_i(i) = x_{\text{max}}(i) \) is a regular grid. \( F_i^{-1}(y) \) can then be computed for all \( y \) by spline interpolation.

Computation of (8) normally requires the evaluation of a joint exceedance probability for all involved time steps. However, in the case of stationary input processes, where the response process at time \( t \) is only a function of \( X \) up until time \( t \), the response is also stationary and \( v^+ \) is independent of \( t \). Its computation thus only requires calculating the probability (8) with a small \( \Delta t \) at a fixed instant \( t \).

2.2. FORM/SORM approximations of the outcrossing rate

FORM/SORM can be used to estimate \( v^+(t) \) via one or two FORM analysis. PHI2 method makes use of two FORM computations which yields
\[
v^+_{PHI2} = \frac{\Phi_2(\beta(t), -\beta(t + \Delta t), \rho_G(t, t + \Delta t))}{\Delta t}
\]
(9)

\( \beta(t) \) is the reliability index at time \( t, \Delta t \) a small time increment and \( \Phi_2 \) the bivariate multinormal cdf. The reliability index is obtained as \( \beta(t) = ||u^*(t)|| \), where \( u^*(t) \) is the design point i.e. the most probable failure point, a solution of constrained optimization problem
\[
u^*(t) = \arg \min_{g(t,u)=0} ||u||^2
\]
(10)
The correlation factor \( \rho_G \) is defined as \( \rho_G(t, t + \Delta t) = -\alpha(t)^T \alpha(t + \Delta t) \), where \( \alpha(t) \) is the vector of FORM

A second more economical method in terms of limit state function evaluations, only resorts to one FORM computation. The main idea, due to Koo et al. (2005), is summed up by the following observation: let \( X_i^\tau \) be the critical wind episode corresponding to design point \( u^\tau = u^*(t) \), and leads to failure at fixed time \( t \). Define \( X_i^\tau(\tau) = X_i^\tau(\tau - \Delta t) \) for \( \Delta t \leq \tau \) and \( X_i^\tau(\tau) = 0 \), for \( 0 \leq \tau \leq \Delta t \). Then, it is clear that \( X_i^\tau \) leads to failure at time \( t + \Delta t \) and may be used to obtain the corresponding design point \( u^\tau = u^*(t + \Delta t) \). The following approximation (Koo et al. (2005)) may then be used to estimate \( v^+ \):
\[
v^+ \approx v^+_{Koo} = \frac{1}{2\pi} \exp \left( -\frac{\beta^2}{2} \right) \left[ \frac{\pi}{2} + \arcsin(\rho_G) \right]
\]
(11)
where \( \beta = ||u^\tau|| \) and \( \rho_G \approx \frac{-u^\tau u^\tau^T}{u^\tau u^\tau} \). When the spectral representation is used, \( u^\tau \) can be computed analytically (see Jensen and Capul (2005)). In the case of KL expansion, we propose to compute a first order approximation of the design point at time \( t + \Delta t \) in...
the physical space $\xi_2^*$, and obtain $u_2^*$ via the Nataf transform $u_2^* = T(\xi_2^*)$. As justified in 1.2.1, $\xi_2^*$ is replaced by $\tilde{\xi}_2^*$, the KL coefficient vector of the interpolating process $\tilde{X}_2^*$. Using (5), $\forall j = 1, \ldots, n$

$$\tilde{\xi}_{2,j}^* = \frac{1}{\sqrt{\lambda_j}} \int_0^T \tilde{X}_{2,c}^*(t) \tilde{\phi}_j(t)dt$$

(12)

where $\tilde{X}_{2,c}^*(t) = \tilde{X}_2^*(t) - \tilde{X}_2^*(t)$ is the centred interpolating process. Let $e_k$, $k = 1, \ldots, M$ be the following linear interpolating functions on $[t_{k-1}, t_{k}]$

$$e_k(t) = \frac{t - t_{k-1}}{\delta \tau} \mathbb{I}_{[t_{k-1}, t_k]}(t) + \frac{t_{k+1} - t}{\delta \tau} \mathbb{I}_{[t_k, t_{k+1}]}(t)$$

where the sampling interval $\delta \tau = t_{k+1} - t_k$ is supposed constant. Then, $X_2^*(t) \approx \tilde{X}_2^*(t) = \sum_{k=1}^M X_2^*(t_{k}) e_k(t)$ and $\phi_j(t) \approx \tilde{\phi}_j(t) = \sum_{k=1}^M c_{j,k} e_k(t)$, $c_{j,k} = \phi_j(t_k)$, $j = 1, \ldots, n$, $l = 1, \ldots, M$. Then, (12) becomes

$$\tilde{\xi}_{2,j}^* = \frac{1}{\sqrt{\lambda_j}} \int_0^T \tilde{X}_{2,c}^*(t - \Delta t) \tilde{\phi}_j(t)dt$$

$$= \frac{1}{\sqrt{\lambda_j}} \sum_{1 \leq k \leq M} X_{1,c}^*(t_{k}) c_{j,l} \int_{\Delta t} e_k(t - \Delta t) e_l(t)dt$$

(13)

(14)

After some basic integration, we have for $\Delta t \to 0$,

$$\tilde{\xi}_{2,j}^* = \xi_{1,j}^* + \frac{1}{\sqrt{\lambda_j}} X_{1,c}^*(t_{j,2} - c_{j,1}) \Delta t$$

$$+ \frac{\Delta t}{\sqrt{\lambda_j}} \sum_{k=2}^{M-1} \left( c_{j,k} - \frac{c_{j,k-1}}{2} - \frac{c_{j,k+1}}{6} \right) X_{1,c}^*(t_{k})$$

$$- \frac{1}{\sqrt{\lambda_j}} X_{1,c}^*(t_M)(c_{j,M-1} + c_{j,M}) \Delta t/2 + o(\Delta t)$$

(15)

Note that approximations (9) and (11) assume there is a unique design point whose distance to the origin is significantly less than other local minimas; this may not always be the case.

2.3. Estimating the outcrossing rate via importance sampling

Importance sampling (IS) is a practical standard Monte Carlo variance reduction method. It is useful to compute the expectation of a function of random parameters, when the underlying density is either difficult to sample from or yields high variance estimate. Here, we are interested in approximating (8) via a finite element type approximation defined by the following expectation

$$v^+(t) \approx \frac{\mathbb{E}(\mathbb{G}(g(t,U) > 0, g(t + \Delta t, U) \leq 0))}{\Delta t}$$

(16)

Since we are in the standard space, $U \sim \varphi_n$ where $\varphi_n$ is the standard $n$-dimensional multinormal pdf. The MC estimate of (16) is $v_{MC}^+(t) = \frac{1}{N_{MC} \Delta t} \sum_{i=1}^{N_{MC}} \mathbb{G}(g(t,u_i) > 0, g(t + \Delta t, u_i) \leq 0)$ where $u_i \sim \varphi_n$, $i = 1, \ldots, N_{MC}$. Importance sampling proceeds by drawing samples $u_i$, $i = 1, \ldots, N_{IS}$ from a proposal density $\varphi$ and reweighting them appropriately providing the IS estimate of (16)

$$v_{IS}^+(t) \approx \frac{1}{N_{IS} \Delta t} \sum_{i=1}^{N_{IS}} \varphi(u_i) \mathbb{G}(g(t,u_i) > 0, g(t + \Delta t, u_i) \leq 0)$$

(17)

A suitable choice of $\varphi$ usually leads to a lower variance of the estimator. In this case, a frequent choice is to choose a proposal density, for instance a Gaussian pdf, centred on the design point. The IS estimation variance reads

$$\hat{\sigma}_{IS}^2 = \frac{1}{N_{IS}} \sum_{i=1}^{N_{IS}} \left( \mathbb{G}(g(t,u_i) > 0, g(t + \Delta t, u_i) \leq 0) \varphi^2(u_i) - \frac{v_{IS}^2}{\varphi_n^2(u_i)} \right)$$

(18)

Since the numerator in (8) might be a few magnitude orders smaller than the failure probability $\mathbb{P}(g(t + \Delta U, U) \leq 0)$, we decomposed it into $\mathbb{P}(g(t + \Delta U, U) \leq 0) = \mathbb{P}(g(t + \Delta U, U) \leq 0) \mathbb{P}(g(t, t + \Delta U) \leq 0)$. $p_1 = \mathbb{P}(g(t + \Delta U, U) \leq 0)$ can then be computed by an importance sampling procedure with a proposal centred on the design point at time $t + \Delta t$ as outlined previously. As for the conditional probability $p_2 = \mathbb{P}(g(t + \Delta U, U) > 0 | g(t + \Delta U, U) \leq 0)$, an MCMC sampler targeting $\mathbb{P}(g(t + \Delta U, U) \leq 0)$ may be used to estimate the estimator. Indeed, when estimating $\mathbb{P}(g(t, U) > 0 | g(t + \Delta U, U) \leq 0)$ by IS, the $n_i$ samples that fall in the failure domain $\{g(t + \Delta U, U) \leq 0\}$ during the IS procedure can be used as seeds of independent Markov chains. If a population of size $N$ is needed, then each chain can be run with
\( c[N_{\text{ns}}] + b \) iterations where \( b \) is a burnin parameter and \( c \) a thinning parameter used (meaning one out of \( c \) MCMC samples is kept). In this study, the modified Metropolis-Hastings (MH) described in (Au and Beck (2001)) was used. Letting \( \hat{p}_1 \) be the estimator of \( P(g(t + \Delta t, U) \leq 0) \) and \( \hat{p}_2 \) that of \( P(g(t, U) > 0 | g(t + \Delta t, U) \leq 0) \), the outcrossing rate estimate is \( v_{IS+MH}^{+} = \frac{\hat{p}_1 \hat{p}_2}{\Delta t} \) and the corresponding variance \( \sigma_{IS+MH}^2 \) can be estimated by noticing that the estimate \( \hat{p}_1 \) is independent of \( \hat{p}_2 \) since the dependency of the samples used to estimate \( \hat{p}_2 \) w.r.t. to those used for \( \hat{p}_1 \), vanishes after sufficient burn-in in the Markov chain.

\[
\sigma_{IS+MH}^2 = \frac{\text{Var}(\hat{p}_1) \text{Var}(\hat{p}_2)}{\Delta t^2} + \frac{\text{Var}(\hat{p}_1) \text{E}(\hat{p}_2)^2}{\Delta t^2} + \frac{\text{Var}(\hat{p}_2) \text{E}(\hat{p}_1)^2}{\Delta t^2}
\]

\( \text{Var}(\hat{p}_2) \) can be estimated as in Au and Beck (2001) while \( \text{Var}(\hat{p}_1) \) is a standard IS variance estimate. Moreover, \( \text{E}(\hat{p}_1) = p_1 \) since IS provides unbiased estimates. We can therefore estimate it by \( \hat{p}_1 \). As for \( \text{E}(\hat{p}_2) \), since \( \hat{p}_2 \) is an MCMC estimate, it is biased for a finite sample size. Nevertheless, we assume asymptotic conditions in which case \( \text{E}(\hat{p}_2) \approx p_2 \).

3. Assessment of a wind turbine short-term failure probability subject to wind load

3.1. Test case

We consider the problem of estimating the short-term failure probability of a wind turbine, that is the failure probability assuming stationary conditions. In reliability analysis of offshore structures, the loads are classically described in terms of stochastic processes that are stationary within a given time interval. This window duration is 10 minutes for the wind speed process and 3 hours for the wave elevation. A bound on the short-term failure probability may be obtained by computing the outcrossing rate \( v^+(t) \) for fixed long-term parameters. We consider the 5 MW OC4 wind turbine, with a 90 m hub height and 126 m rotor diameter whose response is modelled by the NREL FAST code. To model short-term stationary conditions based on real wind speed data, we have extracted 10 minute recordings of the wind speed at the Hornsrev site (Denmark), available at winddata.com. Those selected 65 measurements have a mean wind speed equal to the rated speed, i.e. \( U_{10} = 11.5 \text{ m/s} \) and turbulence intensity \( I_u = 6\% \), i.e. \( \sigma_u = 0.7 \). Assuming stationarity during a 10 minute interval, the time-invariant outcrossing rate may be computed at any time \( t \) in the interval. In practice, it is necessary to discard the initial transient part of the response which means that the stationarity is actually valid after the system memory effects have disappeared, say one minute after the beginning of the simulation if wind loads are only present and up to 3 or 4 minutes if wave loading is considered. \( v^+ \) is computed at a fixed time \( t \) after the transient part. We therefore only kept the first \( T = 5 \) minutes of each wind speed measurement. Without accounting for hydrodynamic effects, it is sufficient to simulate the first 60 seconds of the response. The limit state function we consider is therefore \( G(t_0, \xi) = 0.5 - Y(t_0, \xi) \) where \( Y \) is the wind turbine tower-top displacement in meters, where \( t_0 = 1 \) minute.

3.2. Stochastic model fitting

We have then fitted the spectral representation model as well as the Karhunen-Loeve model to the data. For the spectral representation model, it is necessary to specify a spectral density. This has been done by computing averaged smoothed periodograms of each wind speed measurement (see fig. 1). This p.s.d. estimate was then discretized by selecting \( n_{\text{harm}} \) frequencies \( \omega_1, \ldots, \omega_{n_{\text{harm}}} \) via an iso-energy discretization scheme, where \( \omega_1 = \frac{2\pi}{T} \) and \( \omega_{\text{harm}} = \frac{\pi}{\delta \tau} \), \( \delta \tau = 0.1 \) s being the input sampling step. The comparison of both models was carried out by setting \( n = 60 \) harmonics for the KL model and \( n_{\text{harm}} = 50 \): this corresponds to a relative \( L^2 \) error of the autocovariance function less than 0.1% for KL expansion and of 6% for the spectral representation model. Be advised that the dimension of the expansion is \( d = 2n_{\text{harm}} = 100 \) in the spectral representation expansion.

3.3. Simulation experiments

3.3.1. Design point computation

Given both input stochastic models, several constrained optimizations solvers were compared in order to identify the most probable failure point \( u^* \).
defined by (10). The sequential quadratic programming (SQP) algorithm was the most efficient in terms of computing cost and the gradient-free sequential quadratic approximation (SQA) stood out in terms of number of calls to the limit state function. For practical reasons SQP was used with an initialization at the origin of the U space. The optimization is thus performed in dimension \( d = 2n_{harm} = 100 \) for the spectral representation model and \( d = 60 \) for the KL expansion.

### 3.3.2. Outcrossing rate estimation via a FORM approximation

As stated in 2.2, we compute two approximations for the outcrossing rate: the extended PHI2 approximation \( \nu_{PHI2}^+ \) and \( \nu_{Koo}^+ \). The first one involves two FORM computations at time \( t_0 \) and \( t_0 + \Delta t \) while \( \nu_{Koo}^+ \) only requires one FORM analysis at time \( t_0 \), the design point \( u^*_2 \) at time \( t_0 + \Delta t \) being a function of \( u^*_1 \) and \( \Delta t \). For both approximations, \( \Delta t = 0.01 \) s. The results are presented in table 1, where the number of calls to the limit state function is in brackets. Both estimators \( \nu_{PHI2}^+ \) and \( \nu_{Koo}^+ \) seem to agree regardless of the input model. However, the type of stochastic expansion plays a significant role even when the model parameters for each expansion are estimated directly from the data. The outcrossing rate with a spectral representation of the input load yields is roughly 100 times less than that obtained with a KL expansion. This indicates that the assumption of a Gaussian process is a rather strong one for the wind velocity. As expected the number of calls to the limit state function for the spectral representation is more important, most notably because of the use of finite differences in the gradients computation. For validation purposes only, the multiple design point algorithm by Der Kiureghian and Dakeessian (1998) was applied and suggested no other significant failure mode.

### Table 1: Outcrossing rate estimation (FORM approximation)

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<tr>
<td>( \beta(t) )</td>
<td>8.57 (4051)</td>
<td>7.96 (2295)</td>
</tr>
<tr>
<td>( \nu_{PHI2}^+ )</td>
<td>( 2.68 \times 10^{-17} ) (10039)</td>
<td>( 2.39 \times 10^{-15} ) (7267)</td>
</tr>
<tr>
<td>( \nu_{Koo}^+ )</td>
<td>( 2.90 \times 10^{-17} ) (4051)</td>
<td>( 2.57 \times 10^{-15} ) (2295)</td>
</tr>
</tbody>
</table>

### 3.4. Outcrossing rate estimation via Importance sampling

Based on the design point, we have used \( N = 2000 \) samples to estimate \( \nu^+ \) via \( \nu_{IS}^+ \) or \( \nu_{IS+MH}^+ \) as defined in 2.3 for the KL model. The estimates are reported in table 2 along with the coefficient of variation and the number of g calls including the design point computation. The importance density used for both estimators is the normal density centred at the design point and with identity covariance matrix. Using "straight-up" importance sampling seems to yield high estimation variance as evidenced by the c.o.v. The estimator \( \nu_{IS+MH}^+ \) which splits the probability into two terms, one being estimated by IS and the other through MCMC by Metropolis-Hastings seems to lessen the estimation variance even if it remains non-negligible.

### Table 2: Importance sampling estimators of the outcrossing rate

<table>
<thead>
<tr>
<th></th>
<th>IS</th>
<th>IS+MH</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu^+ )</td>
<td>( 2.88 \times 10^{-15} )</td>
<td>( 3.44 \times 10^{-16} )</td>
</tr>
<tr>
<td>c.o.v.</td>
<td>0.99</td>
<td>0.82</td>
</tr>
<tr>
<td># g calls</td>
<td>4295</td>
<td>10831</td>
</tr>
</tbody>
</table>

### 4. Conclusion

Finite expansion models of stationary or non-stationary input loads are of practical importance in
reliability analysis of mechanical structures. Practitioners seek to account for maximum signal variability while keeping a sufficiently compact representation in terms of the dimension of the random parameters. This dimension impacts directly the computational cost of reliability algorithms since the number of calls to the simulator necessary to estimate failure probabilities grows rapidly with the dimension. We have compared two popular expansion schemes, the spectral representation method and the Karhunen-Loève expansion by fitting both models on real wind speed measurements in an approximate stationary setting. Prior to any reliability analysis, it is confirmed that the Karhunen-Loève expansion is significantly more economical in terms of number of random parameters. The estimation of the short-term failure probability is taken from the point of view of an outcrossing rate analysis. The identification of the design point is used as a starting block for the outcrossing rate computations via FORM/SORM type approximations or Importance sampling estimators with or without an MCMC sampler. The use of a KL model makes it possible to extend the PHI2 method to functional inputs, by rewriting the limit state function in terms of the expansion coefficients. This leads to an estimator of the outcrossing rate that requires two FORM analyses. In the case of KL expansion we derived an explicit approximation of the correlation factor in Koo’s outcrossing rate formula (Koo et al. (2005)) which only requires one design point computation. Additionally a Monte Carlo type estimator is studied. The estimation of outcrossing rate is split into an instantaneous failure probability computation, estimated via importance sampling based on a previously determined design point, and a conditional probability calculation achieved through MCMC sampling. In future developments, design point(s) computation will be based on the SQA optimization algorithm as it is more economical in terms of calls to the limit state function which will mean a lesser computing cost compared to other solvers, one the wave loads have been taken into account. We also plan to adress the estimation of the outcrossing rate without a preliminary design point computation.

5. REFERENCES