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Variability of Allowable Bearing Capacity of Soft Soil Stabilized by End-bearing Deep Mixed Columns

Jie Zhang

Associate Professor, Department of Geotechnical Engineering, Tongji University, Siping Road 1239, Shanghai 200092, China.

Hongzhi Chen

Research Assistant, Department of Geotechnical Engineering, Tongji University, Siping Road 1239, Shanghai 200092, China.

Hongwei Huang

Professor, Department of Geotechnical Engineering, Tongji University, Siping Road 1239, Shanghai 200092, China.

ABSTRACT: Deep mixed (DM) columns are commonly used to reduce the settlement of foundations constructed on the soft soils. This paper investigate the variability of the allowable bearing capacity of soft soil stabilized by end-bearing DM columns. A continuum model is used to calculate the allowable bearing capacity corresponding to a settlement threshold. To enhance the computational efficiency, the kriging-based response surface method is used to model the relationships between the random variables and the allowable bearing capacity. The coefficient of variation (COV) of the bearing capacity of the reinforced ground is assessed with Monte Carlo simulation. For the example investigated in this paper, the COV of the bearing capacity and the resistance factor are most sensitive to Young's modulus of the DM columns, the shear strength parameters of the DM columns, but are insensitive to soil properties. The COV and the resistance factor are not sensitive to the thickness of the soft soil layer and the allowable settlement threshold when it varies between 5 - 15 cm. The results obtained in this study may be particular to soft ground reinforced with end-bearing DM columns. If the DM columns are floating, the uncertainties in soil properties may be more important.

1. INTRODUCTION

Deep mixed (DM) columns are commonly used to reduce the settlement of foundations constructed on the soils (Huang et al., 2009). In general, the behavior of the stabilized soil depends on the properties of the natural soil, the reinforced columns, and the interaction between the soil and the reinforced columns. The properties of the reinforced columns are related to the construction methods and properties of in situ soil, and existing data have shown that properties of the reinforced column are highly variable (e.g., Kazemain and Barghchi, 2012). In view of the uncertainties involved in the design of DM ground, it is highly desirable to quantify how such uncertainties can affect the bearing capacity of the DM ground.

Several researches have been conducted to study the variability of the ultimate bearing capacity of foundation stabilized by the DM columns. For example, Al-Naqshabandy et al. (2012) studied the effect of spatial variability of the strength properties in lime-cement columns on the embankment stability, in which the CPT data is used to evaluate the variability of the undrained shear strength of the DM columns. Kasama et al. (2012) studied the effect of spatial variability on the ultimate bearing capacity of cement-treated ground using numerical limit analyses combined with random field theory.

Existing probabilistic studies on ground reinforced with DM columns mainly focus the reliability of the DM ground against ultimate limit state. In many cases, the design of the DM ground is governed by the serviceability limit state, which is seldom addressed in the literature. The objective of this paper is thus to study the variation of allowable bearing capacity of DM ground where DM columns are end-bearing, and to investigate its implication on the design of DM columns. This paper is organized as follows. First, the numerical model for predicting the allowable bearing capacity of the DM ground is introduced. Then, the method for analyzing the variability of the allowable bearing capacity is described. Finally, factors affecting the variability of the allowable bearing capacity are discussed, and their impact on the design of DM columns is analyzed.

2. NUMERICAL MODEL FOR GROUND STABILIZED BY DM COLUMNs

Figure 1 shows the soft ground stabilized by DM columns studied in this paper. The thickness of the soft soil layer is 12.1 m. It is assumed that below the soft soil layer is a hard incompressible stratum, i.e., the DM columns are end-bearing. The diameter and length of the DM columns are 0.8 m and 12.1 m, respectively. The width of the surface loading is 7.7 m. Suppose the allowable settlement of the foundation is 10 cm (MOT, 1996). Let q_{10} denote the load corresponding to a maximum settlement of 10 cm. Let q denote the actual applied load. The performance function in this example can be written as

$$g(\boldsymbol{\theta}) = q_{10} - q \qquad (1)$$

where $\boldsymbol{\theta}$ = uncertain variables. Let E_s, c_s, and φ_s denote the cohesion, the friction angle and the elastic modulus of the soft soil, respectively. Let E_p, c_p, and φ_p denote the cohesion, the friction angle and the elastic modulus of the DM columns, respectively. In this example, E_s, c_s, φ_s, E_p, c_p, and φ_p are modeled as random variables, and hence $\boldsymbol{\theta} = \{ E_s, c_s, \varphi_s, E_p, c_p, \varphi_p \}$. The statistics of random variables are shown in the Table 1. For simplicity, all the random variables are assumed to be lognormally distributed and statistically independent. The Poison's ratio and unit weight of the clay and DM columns are modeled as deterministic values: Poison's ratio for the soil and DM columns are 0.33 and 0.20, respectively. The unit weights of the soil and DM columns are 18 kN/m^3 and 20 kN/m^3, respectively.

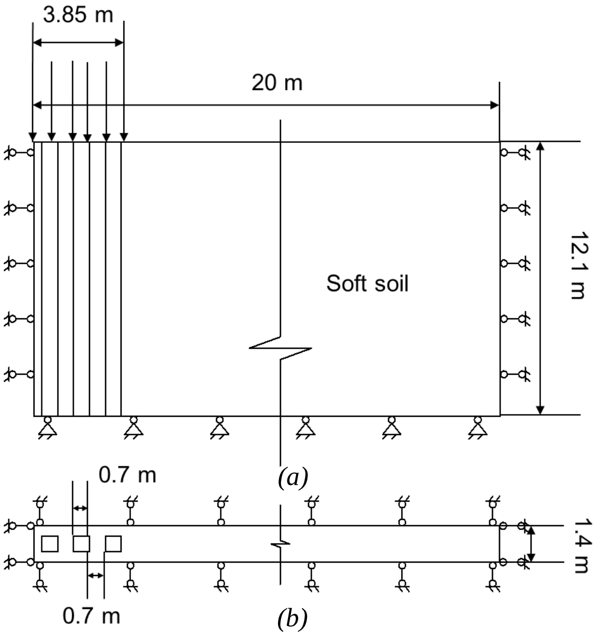
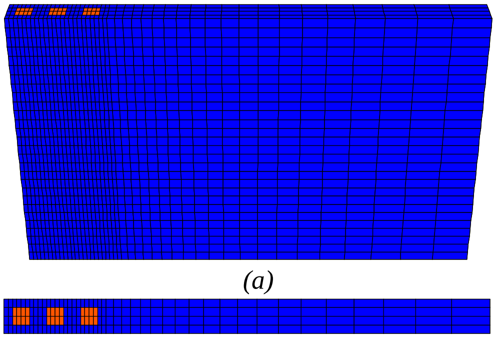


Figure 1: Layout of the reinforced columns: (a) Cross-section; (b) Plan view.

Tab.1 The distribution and statistics of random variables

	E_s(MPa)	c_s(kPa)	φ_s(°)	E_p(MPa)	c_p(kPa)	φ_p(°)
Mean	4.5	16	8	120	16	35
COV	0.3	0.15	0.25	0.4	0.3	0.2

As the allowable bearing capacity corresponding to the settlement limit does not have an analytical solution, the finite difference program FLAC3D is used to calculate the allowable bearing capacity. The mesh of the finite difference model is shown in the Figure 2. For simplicity, the circular isolated columns are modeled by square columns of equivalent area (Huang et al., 2009). The widths of the square columns are 0.7 m. Due to symmetry and the repetition of the span along the longitudinal direction, only half of the cross-section and half span along the longitudinal direction is modeled. The DM columns and the soil are both modeled as linearly elastic-perfectly plastic materials with Mohr-Coulomb failure criteria. The boundary conditions are explicitly shown in Figure 1.


(a)

(b)

Figure 2: FLAC3D mesh. (a)Elevation view; (b) plan view

The finite difference model is first brought to equilibrium by turning on the gravity. In principle, the maximum settlement corresponding to a certain applied load can be calculated with the FLAC3D model by applying the uniform surcharge. However, it is quite time-consuming to analyze such models. In this study, the load-displacement curve of the foundation is obtained by applying equal velocities to the nodes at the ground surface which are subjected to surface loading. With the current velocity controlled method, the ground settlement profile is uniform. Based on the load-displacement curve, the load corresponding to the 10 cm displacement is then regarded as the allowable bearing capacity. As an example, Figure 3 shows the load-settlement curve of the foundation when the mean values of the random variables in Table 1 are used as input for the FLAC3D model. Based on this curve, the allowable bearing capacity corresponding to a settlement of 10 cm is 284 kPa. If a uniform surcharge of 284 kPa is applied to the model, the maximum settlement of the ground is 9.75 cm, which shows that the maximum settlement calculated by directly applying the surcharge and applying the velocity are very close. Thus, to achieve computational efficiency, the velocity control method is employed in this study.

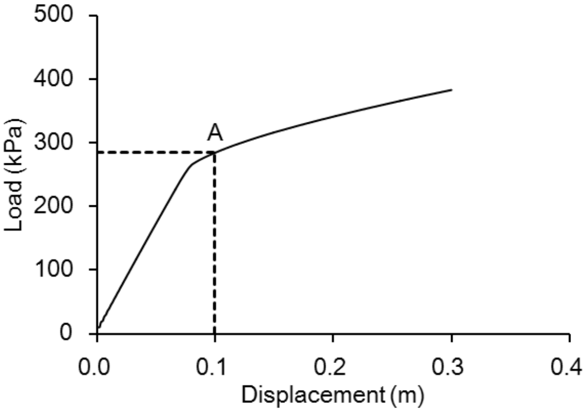


Figure 3: Load-displacement curve obtained from displacement control method.

3. METHOD OF UNCERTAINTY ANALYSIS

In principle, one can use methods like Monte Carlo simulation (MCS) to evaluate the variability of the allowable bearing capacity of the DM ground directly based on the FLAC3D. However, it is computationally prohibitive to do so. In this study, a kriging-based response surface is used to approximate the relationship between the allowable bearing capacity and the random variables, and the variability of the allowable bearing capacity is assessed based on the kriging model. For ease of illustration, let $y = g(\boldsymbol{\theta})$ denote the relationship between the allowable bearing capacity y and the random variables $\boldsymbol{\theta}$ as implied in the FLAC3D model. In the kriging model used in this study, it is assumed that $y = g(\boldsymbol{\theta})$ can be decomposed into a deterministic trend $t(\boldsymbol{\theta}) = b$ and a zero mean, Gaussian stationary random error function $\varepsilon(\boldsymbol{\theta})$ with the following correlation function

$$R\left(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j\right) = \exp\left[-\sum_{m=1}^{k}\left(\frac{\theta_{mi} - \theta_{mj}}{\delta_m}\right)^2\right] \qquad (2)$$

where δ_m = a correlation parameter that reflects the correlation between points $\boldsymbol{\theta}_i$ and $\boldsymbol{\theta}_j$ along the m^{th} axis; θ_{mi} = m^{th} element of $\boldsymbol{\theta}_i$; and θ_{mj} = m^{th} element of $\boldsymbol{\theta}_j$. The kriging model with the above structure is quite flexible and is applicable to a wide range of geotechnical problems (e.g., Zhang et al. 2011).

Let $y_1, y_2, \ldots$, and y_n denote the values of the bearing capacity at points $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_2$, … , and $\boldsymbol{\theta}_n$, respectively. Let $\boldsymbol{\theta}_{n+1}$ denote a new point and y_{n+1} denote the bearing capacity value at this point. Let $\mathbf{Y}_1 = \{y_1, y_2, \ldots , y_n\}$. Based on the above model, when $\mathbf{Y}_1$ is known, the mean of y_{n+1}, i.e., $\mathrm{E}(y_{n+1})$, can be estimated as

$$\mathrm{E}\left(y_{n+1}\right) = t\left(\boldsymbol{\theta}_{n+1}\right) + \mathbf{r}^T \mathbf{R}_{\mathbf{Y1}}^{\mathbf{-1}}\left(\mathbf{Y}_1 - \boldsymbol{\mu}_{\mathbf{Y1}}\right) \qquad (3)$$

where $\boldsymbol{\mu}_{\mathbf{Y}1} = \{t(\boldsymbol{\theta}_1), t(\boldsymbol{\theta}_2), \ldots, t(\boldsymbol{\theta}_{\mathrm{n}})\}$; $\mathbf{r}$ = a column vector containing the correlation coefficients between y_{n+1} and the elements in $\mathbf{Y}_1$; and $\mathbf{C}_{\mathbf{Y}1}$ is the covariance matrix of $\mathbf{Y}_1$. Taking $\mathrm{E}(y_{n+1})$ as the estimate for y_{n+1}, Eq. (3) can thus be used to predict y_{n+1}.

To calibrate the kriging model, 250 calibration samples are first uniformly drawn from the hypercube of $\theta_{i,\min} < \theta_i < \theta_{i,\max}$ (i = 1, 2, …, 6) using the Latin hypercube sampling method, where $\theta_{i,\min}$ and $\theta_{i,\max}$ are the lower and upper bound values of θ_i corresponding to the 99.865% confidence level, respectively. The optimal values of the unknown parameters in the kriging model (i.e., b, σ_ε, δ_1, δ_2, .. , and δ_k) are then determined with the maximum likelihood method. Figure 4 compares the allowable bearing capacity values predicted from FLAC3D and those from the kriging model for another 50 randomly generated verification points. We can see that the kriging model can predict the allowable bearing capacity with reasonable accuracy. To verify the accuracy of the kriging model for evaluating the COV of the allowable bearing capacity, another 300 samples are drawn for the random variables, and the allowable bearing capacity corresponding to these 300 samples are calculated based on FLAC3D model. Based on these 300 bearing capacity values calculated with FLAC3D, the COV of the bearing capacity value is 0.24. Based on the same 300 samples of the random variables, 300 bearing capacity values can also be calculated with the kriging model, and the COV of these bearing capacity values is also 0.24. It seems that the current kriging model can evaluate the COV of the bearing capacity values quite accurately. In the above verification, only 300 samples are used for evaluating the COV of the allowable bearing capacity value as it is very time consuming to calculate the bearing capacity based on FLAC3D. To minimize the statistical error, in the following study 5000 samples will be used to evaluate the COV of the bearing capacity based on the kriging model.

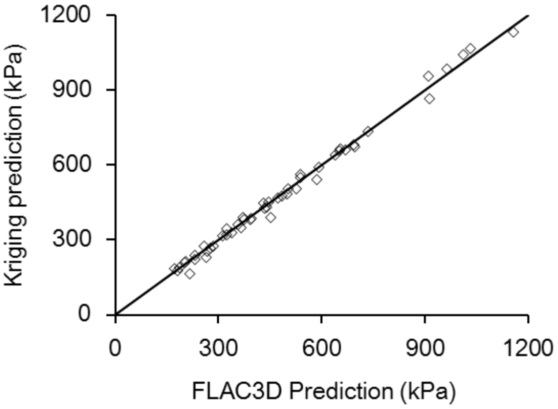
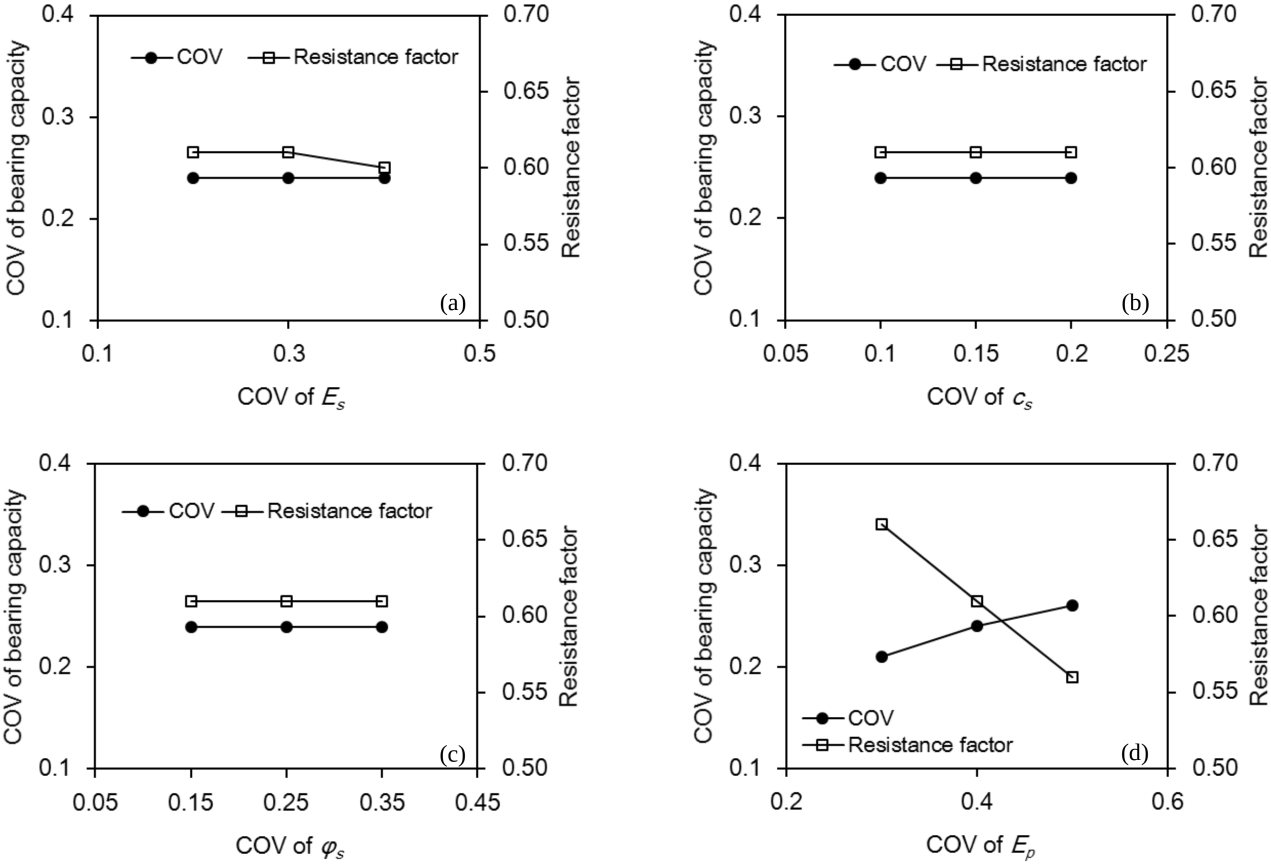


Figure 4: Comparison of allowable bearing capacity predicted from FLAC3D model and a kriging model.

4. VARIABILITY OF ALLOWABLE BEARING CAPACITY

4.1 Effect of uncertainties in basic random variables

To investigate how the uncertainties in the basic random variables affect the variability of the allowable bearing capacity, the COV of the allowable bearing capacity is calculated by changing the COV of each basic random variable in Table 1 in turn while keeping the statistics of other random variables unchanged. The obtained relationship between the COV of the allowable bearing capacity and the COV of each of the random variables in Table 1 are shown in Figure 5(a) - 5(f), respectively. For instance, Figure 5(a) is obtained by changing the COV of E_s while the statistics of other random variables are the same as those specified in Table 1. As we can see from these figures, as the COV of the basic random variables increases, the COV of the allowable bearing capacity also increases. However, the sensitivity of the COV of the allowable bearing capacity to different random variables is not the same. Among the six random variables investigated, the COV of the allowable bearing capacity is most sensitive to the COV of E_p. The COV values of c_p and φ_p also affect the COV of the allowable bearing capacity. The COV of the allowable bearing capacity is insensitive to the COV values of c_s φ_s and E_s, probably because the load are mainly carried by the DM columns and the soil is still in the elastic state. The above phenomenon may be related to the boundary condition assumed in this study, i.e., both the bottom of the DM columns and soft soil are constrained, indicating the DM columns and soft soil are underlain by a incompressible soil layer. If the DM columns are floating, it is expected that the soil-DM columns interaction will be more obvious. In such a case, the soil may carry more load, and the uncertainties in the soft soil may be more important.



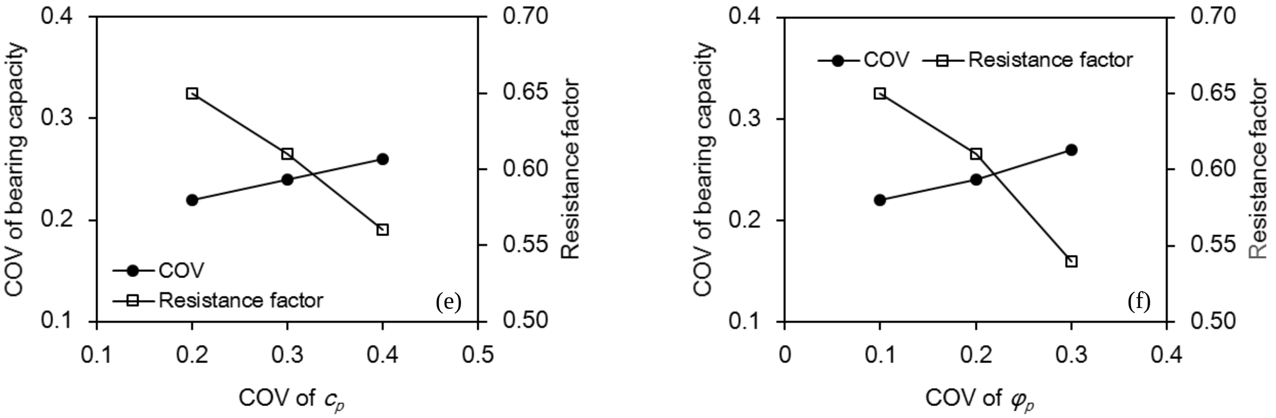


Figure 5: Effect of COV of different variables on the COV of the allowable bearing capacity and resistance factor: (a)E_s; (b) c_s; (c)φ_s; (d)E_p; (e)c_p; (f)φ_p

4.2 Effect of thickness of the soft soil layer

In the above analysis, the thickness of the soft soil layer is 12.1 m. To study the effect of thickness of the soft soil layer, the COV of the allowable bearing capacity is calculated assuming the thickness of the soil layer is 6.1 m and 18.1m, respectively, and the relationship between COV of the allowable bearing capacity and the thickness of the soft soil layer is shown in Figure 6. The COV of the bearing capacity is not very sensitive to the thickness of the soil layer. This is also probably because in this study the DM columns are assumed to be end-bearing which limits the soil-structure interaction.

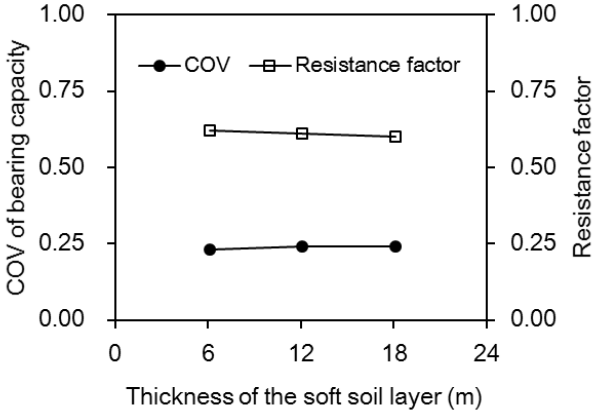


Figure 6: Effect of thickness of the soft soil layer

4.3. Effect of allowable settlement

To investigate the effect of the allowable settlement, the COV of the bearing capacity is assessed assuming the allowable settlement is 5 cm and 15 cm, respectively. The relationship between the allowable settlement and COV of the bearing capacity is shown in Figure 7. The COV of the bearing capacity is not sensitive to the allowable settlement adopted. This is probably because that even when the allowable settlement is 15 cm, the reinforced ground is still largely within the elastic state around the limit state function. As such, the relationship between the allowable bearing capacity and the basic random variables are not highly non-linear and is not very much affected by the allowable settlement.

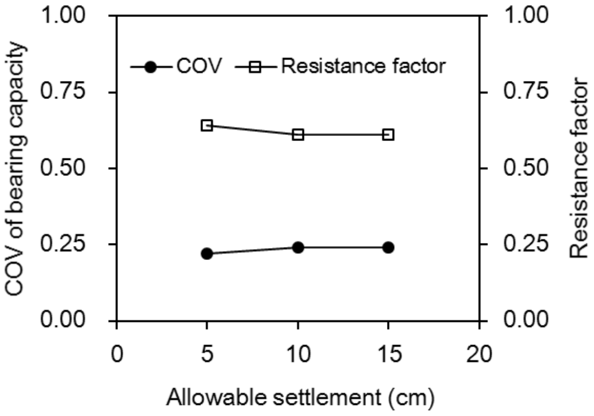


Figure 7: Effect of allowable settlement on the resistance factors

5. IMPLICTION ON LOAD RESISTANCE FACTOR DESIGN

5.1. *Load resistance factor design*

The performance function for the ground stabilized by DM columns can be written in terms of allowable bearing capacity R as follows

$$R - Q_D - Q_L = 0 \qquad (4)$$

where Q_D = dead load and Q_L = live load. The design equation for the above performance function can be written as

$$\gamma_R R_n = \gamma_{QD} Q_{nD} + \gamma_{QL} Q_{nL} \qquad (5)$$

where γ_R = resistance factor for R; γ_{QD} and γ_{QL} = load factors for Q_D and Q_L, respectively; and R_n, Q_{nD}, and Q_{nL} = nominal values for R, Q_L, and Q_D, respectively.

Given a target reliability index β_T, γ_R can be calculated approximately as follows (e.g.,Barker et al., 1991):

$$\gamma_R \approx \frac{\lambda_R\left(\gamma_{QD}\dfrac{Q_{nD}}{Q_{nL}}+\gamma_{QL}\right)\sqrt{\dfrac{1+COV_{QD}^2+COV_{QL}^2}{1+COV_R^2}}}{\left(\lambda_{QD}\dfrac{Q_{nD}}{Q_{nL}}+\lambda_{QL}\right)\exp\left[\beta_T\sqrt{\ln\left(1+COV_R^2\right)\left(1+COV_{QD}^2+COV_{QL}^2\right)}\right]} \qquad (6)$$

where COV_{QD} and COV_{QL} = COV of Q_D and Q_L, respectively; COV_R = COV of the allowable bearing capacity R; λ_{QD} and λ_{QL} = bias factors for Q_D and Q_L, respectively; and λ_R = bias factor for the allowable bearing capacity R.

It is reasonable to assume that the FLAC3D model used is unbiased, $\lambda_R = 1$. The COV of R can be assessed based on the kriging-based response surface as described in this study. To determine the resistance factor based on Eq. (6), the statistics of loads shall be assessed. As an illustration, the load statistics from the AASHTO LRFD specifications (AASHTO 2007) are adopted: $\lambda_{QL} = 1.15$, $\lambda_{QD} = 1.08$, $COV_{QL} = 0.18$, $COV_{QL} = 0.13$, $\gamma_{QD} = 1.25$, and $\gamma_{QL} = 1.75$. Previous studies showed that reliability index is relatively insensitive to the ratio of Q_{nD} to Q_{nL}, i.e., Q_{nD}/Q_{nL} (Barker et al., 1991; Zhang, 2004). Thus, this ratio is fixed to be a typical value of 3 in the present study.

5.2 *Effect of target reliability index*

As mentioned previously, when the statistics of random variables as shown in Table 1 are adopted and when the allowable settlement is 10 cm, the COV of the bearing capacity is 0.24, i.e., $COV_R = 0.24$. Substituting this value into Eq. (6), the required resistance factor to achieve a certain target reliability index can be found. Figure 8 shows the relationship between the target reliability index and the required resistance factor. We can see as the target reliability index varies from 2.5 to 3.5, the resistance factor varies from 0.68 to 0.53, i.e., a more conservative resistance factor should be used if the desired reliability is higher. While such an observation is largely within expectation, the reliability analysis provides a quantitative way for assessing the resistance factor considering the uncertainties involved. In the following the discussion will be based on a target reliability index of 3.0.

5.3 *Effect of COV of basic random variables*

To study the effect of the COV of the basic random variables, the resistance factor is calculated based on Eq. (6) using the COV values of the allowable bearing capacity as shown in Figure 5, and the results are also shown in Figure 5. Consistent with the relationship between COV of the bearing capacity and the COV of the basic random variables, the resistance factor is most sensitive to the COV of E_p, and is insensitive to the COV of soil properties. In general, the resistance factor decreases as the COV of the basic random variables increases. For the range of COV values of the basic random variables investigated in this paper, the required resistance factor is in the range of 0.54-0.65.

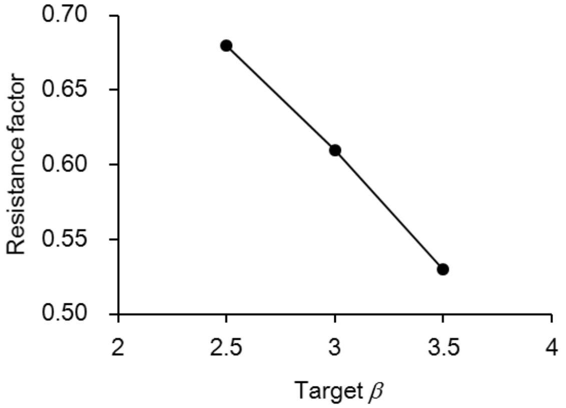


Figure 8: Resistance factors corresponding the target reliability indexes varies from 2.5 to 3.5.

5.4 Effect of thickness of the soft soil layer and allowable settlement

To investigate the effect of thickness of soft soil layer and allowable settlement, the resistance factor is calculated based on Eq. (6) using COV values of the allowable bearing capacity as shown in Figure 6 and Figure 7, respectively, and the results are also shown in these two figures. As expected, because the COV of the allowable bearing capacity is not sensitive to the thickness of the soft soil layer and the allowable settlement, the resistance factor is also not sensitive to these two parameters.

6. SUMMARY AND CONCLUSIONS

This paper investigate the variability of the allowable bearing capacity of a soft ground reinforced by end-bearing DM columns considering the serviceability requirement. For the example investigated in this paper, the COV of the bearing capacity and the resistance factor are most sensitive to Young’s modulus and shear strength parameters of the DM columns, but are insensitive to soil parameters. The COV and the resistance factor are also not sensitive to the thickness of the soft soil layer and the allowable settlement threshold when it varies between 5 - 15 cm. These results may be particular to end-bearing DM columns. If the DM columns are floating, the uncertainties in soil may be more important for design of the DM columns.

7. ACKNOWLEDGEMENT

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