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Stochastic Optimal Control of MR Damped Structures with Uncertain Parameters

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ABSTRACT: Magneto-rheological (MR) damper has proved its value in vibration mitigation of engineering structures subjected to dynamic excitations such as seismic ground motion and strong wind. The accurate operation, however, of MR damper still remains a challenge due to incomplete knowledge on the randomness inherent in the dynamical behaviors of the damper. While the classical models of MR damper are most of phenomenal formulation lacking of the in-depth analysis of working mechanism at material scale. The stochastic modeling of MR damper is carried out in this paper of which the variability definition of critical parameters renders to the dynamic yield analysis of Magneto-rheological fluids. A randomly base-excited structure controlled by the MR damper is investigated. Numerical results indicate that the MR damping control can reduce the seismic response significantly, where the distribution range of probability density function becomes narrower comparing with that without control. It is thus remarked that the appropriately designed semi-active controller can achieve almost the same effect as the active controller in probabilistic sense. The randomness, meanwhile, of damper parameters could be neglected safely.

Magneto-rheological (MR) damper is regarded as one of the most promising control devices due to its perfect dynamic damping behaviors. While the accurate operation of the damper still remains a challenge due to the statistical incompleteness on the classical phenomenal models. The authors explored the variability of dynamic yield behavior of MR fluids using a micro-scale method referring to molecular dynamics simulations (Peng and Li, 2011). The previous work provides a path for the stochastic modeling of MR damper through updating the fluctuation of dynamic yield at MR fluid level. This paper firstly addresses the stochastic modeling of MR dampers, and then investigates the optimal semi-active control of structures using the random-parameterized MR dampers in the context of physically based stochastic optimal control.

The physically based stochastic optimal control of structures, hinged on the generalized density evolution equation (GDEE), is proved to be highly efficient for linear and nonlinear structural systems subjected to engineering excitations with non-stationary and non-Gaussian behaviors (Li et al, 2010). In order to reach a good agreement with the dynamic behavior of MR damper, a bounded Hrovat semi-active control strategy is addressed. For illustrative purpose, a randomly base-excited structure controlled by MR damper is investigated, of which viscous damping coefficient is viewed as random variable. Numerical results indicate that the MR damping control can reduce the seismic response significantly, and the appropriately designed semi-active controller can achieve almost the same effect as the active controller in probabilistic sense. The randomness, meanwhile, of damper parameters could be neglected safely.

1. STOCHASTIC MODELING OF MR DAMPERS

The mechanical models representing the dynamics of MR dampers basically could be divided into two modes, i.e. parameterized model and non-parameterized model. The former typically refers to the testing curve of damping force-displacement and of damping force-velocity deriving from the performance test of the MR damper, and then derives the mathematical formulation of damping force through fitting curves with parameter optimization. The parameterized model generally consists of a series of fundamental mechanical units such as the spring units, viscous-damping units, and Coulomb-friction units, ect. These units distribute into a form of series or parallel topology. The classical parameterized models of MR dampers include Bingham model, Gamota-Filisko model, bi-viscous nonlinear model, and simple Bouc-Wen model (Spencer et al, 1997). The non-parameterized model refers to the data of performance test of MR dampers as well, on which the algorithms such as neural networks and fuzzy logics are usually used and the derived models, namely neural-network model, neural-fuzzy model are of interest. While the widely-applied in practices is a parameterized model i.e. Bouc-Wen model.

Since the classical formulation of Bouc-Wen model lacks of ability of revealing the nonlinear behaviors between damping force and velocity in case of low velocity and opposite direction between velocity and acceleration, Spencer and his colleagues proposed an extended Bouc-Wen model; see Fig.1, which has the formulation of MR damping force as follows:

$$F = c_1\dot{y} + k_1(x - x_0) \tag{1}$$

$$\dot{y} = \frac{1}{(c_0 + c_1)}[\alpha z + c_0\dot{x} + k_0(x - y)] \tag{2}$$

$$\dot{z} = -\gamma|\dot{x} - \dot{y}|z|z|^{n-1} - \beta(\dot{x} - \dot{y})|z|^n + A(\dot{x} - \dot{y}) \tag{3}$$

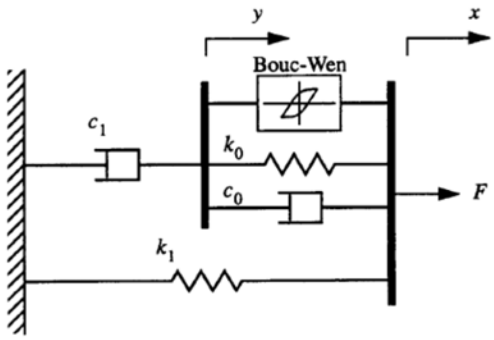
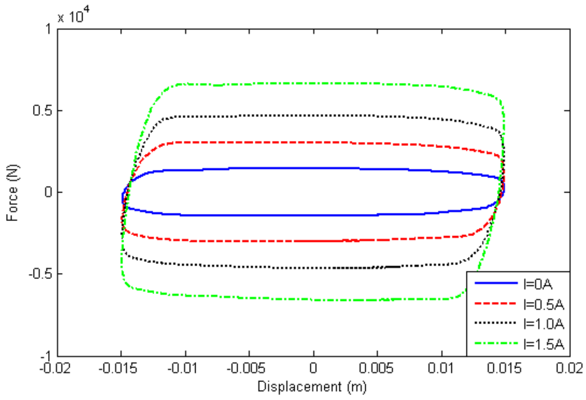


Figure 1: Mechanical model of MR dampers. (Spencer et al, 1997)

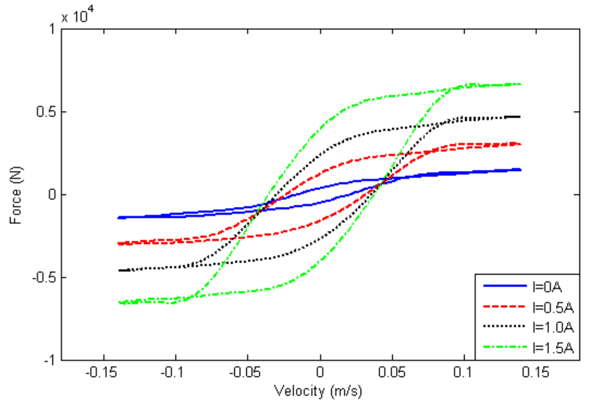
where the coefficient α is defined by the servo system and MR fluids; c_0 denotes the viscous damping coefficient in case of high velocity of damper piston; k_0 denotes the axial spring stiffness in case of high velocity of damper piston; c_1 denotes the viscous damping coefficient in case of low velocity of damper piston; k_1 denotes the equivalent axial spring stiffness of damper; x_0 denotes the initial displacement of the spring k_1.

We made a performance test of MR damper of which the specification is MRD-100-10. Its maximum design capacity is 10kN; outer diameter of cylinder body is 100mm; deploying length is 670mm; piston-motion range is ±55 mm; the maximum input current is 2.0A; the energy consumption is 20W. The piston driven by the servo system moves as rule of harmonic wave and its frequency and amplitude are 1.5Hz and 15mm, respectively. The relationship curves of damping force-displacement and of damping force-velocity are exposed in Fig. 2 where the input current into the MR damper is fixed as four levels: 0A, 0.5A, 1.0A and 1.5A.

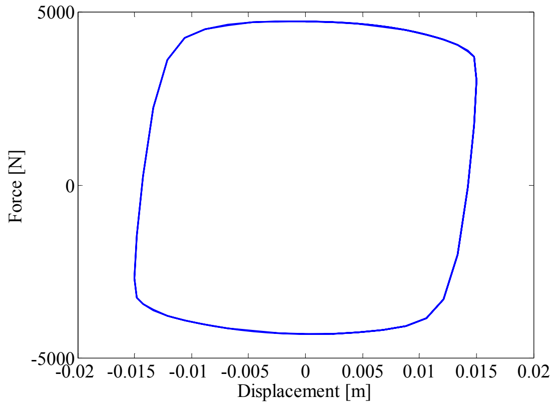
Through the curve fitting of the data in case of current 1.0A, we have the optimized parameters in Eqs. (1) through (3): α=110 N/cm, c_0=52 Ns/cm, k_0=12 N/cm, c_1=910 Ns/cm, k_1=6.2 N/cm, γ=0.1 cm^{-2}, β=0.1 cm^{-2}, n=2, A=210, x_0=20.0 cm. The simulated curves of damping force-displacement and of damping force-velocity are shown in Fig. 3.



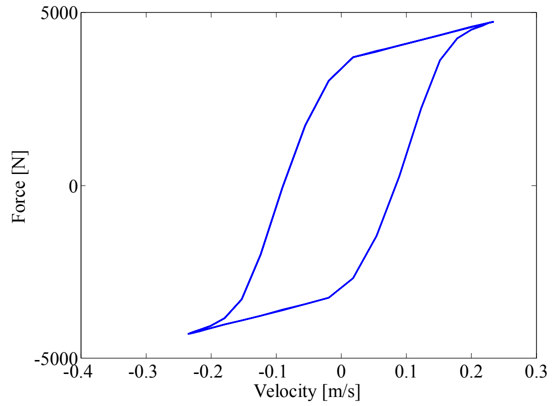
(a)Curves of damping force –displacement.



(b)Curves of damping force –velocity.
Figure 2: Testing curves of MR damper in case of frequency 1.5Hz and amplitude 15mm.



(a)Curve of damping force –displacement.



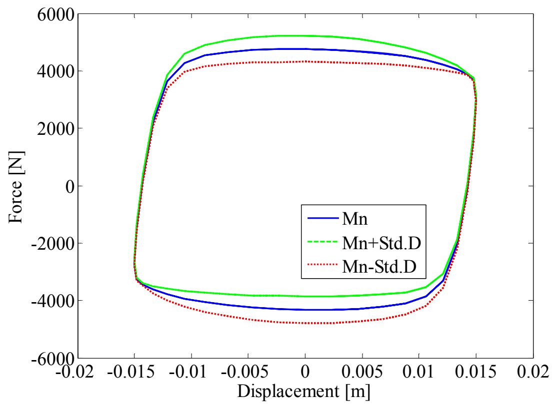
(b)Curve of damping force –velocity.
Figure 3: Simulated curve of MR damper in case of current 1.0A.

As exposed in our previous investigations (Peng and Li, 2011), there exist variability on effective bulk viscosity K^* and strict yield stress τ_0^* of MR fluids. In fact, the viscous damping coefficient c_0 of MR dampers at component scale is relevant to the effective bulk viscosity K^* at material scale, and the initial displacement x_0 of spring k_1 of MR dampers at component scale is relevant to the strict yield stress τ_0^* at material scale. The performance curve of the dampers thus is essentially of randomness. We assume that the viscous damping coefficient c_0 and the initial displacement x_0 both submit to Gaussian distribution, and their relationship to the effective bulk viscosity K^* and strict yield stress τ_0^* is linear. Under this treatment, the variability of parameters of MR damper can be defined by the result of material scale: the mean and variation coefficient of viscous damping coefficient are 52 Ns/cm, 0.5, respectively; while the mean and variation coefficient of the initial displacement are 20 cm, 0.1, respectively. The acting force of MR damper can be written in the function as follows:

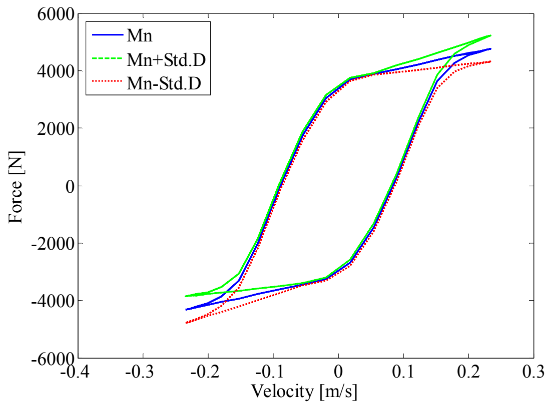
$$F = c_1 \dot{y}(c_0(\xi)) + k_1(x - x_0(\xi)) \qquad (4)$$

where ξ denotes the basic random event representing the randomness inherent in dampers.

Eqs. (2) and (4) show that the damping force has nonlinear relationship with the viscous damping coefficient though the damping force is linear to the initial displacement. Therefore, the analytical solution of statistical moments and probability density function of damping force are difficult to be derived. Using the sampling technique, one could readily obtain their numerical solutions. Fig. 4 shows the mean of MR damping force and its summation with minus and plus standard deviation.



(a)Curves of damping force –displacement.



(b)Curves of damping force –velocity.
Figure 4: Statistical characteristics of model of MR damper.

It is seen that the damping curves arise up a little bit of variability, and the variability reduces in the case of low piston velocity and far-equilibrium-point range; while the variability increases in the case of high piston velocity and near-equilibrium-point range. The maximum value of variability is 0.1085 where the distance of the piston from the equilibrium point is 0.0023m, and the velocity of the piston is -0.2346m/s. There seems that the variability of the damping force is not so significant which even is less than the variability of strength of concrete components (almost 0.15), but the structural response supposes to be sensitive to the damping force especially in case of its tunable state. The following sections investigates the influence of variability of MR damping force upon the controlled structural system in case that the damper often works at the condition of high velocity and moving around the equilibrium point, such as the situation in the seismic response control of MR damped structures.

2. STOCHASTIC OPTIMAL CONTROL OF MR DAMPED STRUCTURES

For a semi-actively controlled structural system attaching MR dampers

$$\mathbf{M}\ddot{\mathbf{Y}}(t)+\mathbf{f}_{\mathrm{s}}(\mathbf{Y},\dot{\mathbf{Y}},t)=\mathbf{B}_{\mathrm{s}}\mathbf{U}_{\mathrm{s}}(\boldsymbol{\Theta},t)+\mathbf{L}_{\mathrm{s}}\boldsymbol{\xi}(\boldsymbol{\Theta},t) \quad (5)$$

where $\mathbf{M}=[M_{ij}]_{n_{\mathrm{d}}\times n_{\mathrm{d}}}$ denotes the mass matrix; $\mathbf{Y}=\{Y_i(\cdot)\}_{i=1}^{n_{\mathrm{d}}}$ denotes the inter-storey drift vector; $\mathbf{f}_{\mathrm{s}}(\cdot)=\{f_{\mathrm{s},i}(\cdot)\}_{i=1}^{n_{\mathrm{d}}}$ denotes the nonlinear internal force vector including the damping and restoring forces; $\mathbf{B}_{\mathrm{s}}=[B_{\mathrm{s},ij}]_{n_{\mathrm{d}}\times m}$ denotes the control influence matrix; $\mathbf{U}_{\mathrm{s}}(\cdot)=\{U_{\mathrm{s},i}(\cdot)\}_{i=1}^{m}$ denotes the control force vector; $\mathbf{L}_{\mathrm{s}}=[L_{\mathrm{s},ij}]_{n_{\mathrm{d}}\times r}$ denotes the excitation influence matrix; $\boldsymbol{\xi}(\cdot)=\{\xi_i(\cdot)\}_{i=1}^{r}$ denotes the stochastic excitation vector. The control force $\mathbf{U}_{\mathrm{s}}(\cdot)$ of MR dampers usually contains two parts, of which the first part is the damping force that cannot be changed by control, the second part is the adjustable force generated by the MR devices' response to the control strategy. $\boldsymbol{\Theta}$ is a stochastic vector denoting the randomness inherent in MR dampers and in external excitations.

A variety of control strategies suitable for MR devices have been studied (Jansen and Dyke, 2000; Casciati et al, 2006). In order to reach a good agreement with the dynamic behavior of MR damper, a bounded Hrovat semi-active control algorithm is proposed (Hrovat, 1983):

$$U_{\mathrm{s}}(\Theta,t)=\begin{cases}C_{\mathrm{d}}\dot{X}(\Theta,t)+U_{\mathrm{dc,max}}\operatorname{sgn}(\dot{X}(\Theta,t)), & \text{Case 1: } U_{\mathrm{a}}\dot{X}<0 \text{ and } |U_{\mathrm{a}}|>U_{\mathrm{d,max}}\\ |U_{\mathrm{a}}|\operatorname{sgn}(\dot{X}(\Theta,t)), & \text{Case 2: } U_{\mathrm{a}}\dot{X}<0 \text{ and } |U_{\mathrm{a}}|<U_{\mathrm{d,max}}\\ C_{\mathrm{d}}\dot{X}(\Theta,t)+U_{\mathrm{dc,min}}\operatorname{sgn}(\dot{X}(\Theta,t)), & \text{Case 3: } U_{\mathrm{a}}\dot{X}>0\end{cases} \tag{6}$$

where $U_{\mathrm{a}}(\Theta,t)$ denotes the control forces needed by the corresponding active optimal control; $U_{\mathrm{d,max}}(\Theta,t)=C_{\mathrm{d}}\left|\dot{X}(\Theta,t)\right|+U_{\mathrm{dc,max}}$ denotes the maximum changeable part force that can be generated by the MR devices; $U_{\mathrm{dc,max}}$, $U_{\mathrm{dc,min}}$ denote the maximum and minimum Coulomb force of MR dampers; C_{d} denotes the damping coefficient; $\dot{X}(\Theta,t)$ denotes the damper velocity, i.e. the motion velocity of piston relative to the cylinder. The control force represented by Eq. (6) can be realized by control the voltage or current applied to the device. Fig. 5 shows the relationship curves between damping force U_{s} and damper velocity $\dot{X}$ in case of a specified sample. It is indicated that the control algorithm of Eq.(6) possesses a good accordance with the curve of force vs velocity of MR damper; see Fig. 3(b).

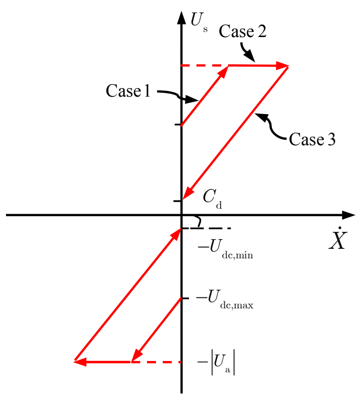


Figure 5: Relationship curves between damping force and damper velocity in case of a specified sample.

Referring to Fig. 4(b), it is seen that the variation of damping force highly relies upon the damping coefficient. While the Coulomb force has not such a significant influence, indicating that the damping coefficient would be a random variable with coefficient of variation 0.1085, accounting for the linear relationship between damping force and inter-storey velocity in Eq.(6).

In order to obtain the similar control effectiveness to the active optimal control, we assume that the MR damping control has the same inter-storey drift with the active controller in case of maximum acting force:

$$U_{\mathrm{s,max}}(\Theta)=C_{\mathrm{d}}\left|\dot{Y}_{\mathrm{s}|U_{\mathrm{s,max}}(\Theta)}\right|+\left|U_{\mathrm{dc,max}}\right|=C_{d}\left|\dot{Y}_{\mathrm{a}|U_{\mathrm{a,max}}(\Theta)}\right|+\left|U_{\mathrm{dc,max}}\right|=U_{\mathrm{a,max}}(\Theta) \tag{7}$$

Assuming the minimum Coulomb force $U_{\mathrm{dc,min}}=0$ and the tunable times of damping force s, then we have

$$U_{\mathrm{s,max}}(\Theta)=sC_{\mathrm{d}}\left|\dot{Y}_{\mathrm{a}|U_{\mathrm{a,max}}(\Theta)}\right|=U_{\mathrm{a,max}}(\Theta) \tag{8}$$

The damping coefficient

$$C_{\mathrm{d}}=\frac{U_{\mathrm{a,max}}(\Theta)}{s\left|\dot{Y}_{\mathrm{a}|U_{\mathrm{a,max}}(\Theta)}\right|} \tag{9}$$

and the maximum Coulomb force

$$U_{\mathrm{dc,max}}=(s-1)C_{\mathrm{d}}\left|\dot{Y}_{\mathrm{a}|U_{\mathrm{a,max}}(\Theta)}\right| \tag{10}$$

Using the control law Eq. (6) in the stochastic dynamical system of Eq. (5), we can get the solutions of the state vector and the control force. Clearly, they are functions of Θ and might be assumed to take the form

$$\mathbf{Y}(t)=\mathbf{H}_{\mathbf{Y}}(\Theta,t) \tag{11}$$

$$\mathbf{U}_{\mathrm{s}}(t)=\mathbf{H}_{\mathbf{U}_{\mathrm{s}}}(\Theta,t) \tag{12}$$

It is seen that all the randomness involved in this system comes from Θ, the augmented systems of components of state and control force vectors $(Y(t),\Theta)$, $(U_{\mathrm{s}}(t),\Theta)$ are thus both probability preserved, and satisfy the GDEEs, respectively, as follows (Li and Chen, 2009)

$$\frac{\partial p_{Y\Theta}(y,\boldsymbol{\theta},t)}{\partial t}+\dot{Y}(\boldsymbol{\theta},t)\frac{\partial p_{Y\Theta}(y,\boldsymbol{\theta},t)}{\partial y}=0 \tag{13}$$

$$\frac{\partial p_{U_{\mathrm{s}}\Theta}(u,\boldsymbol{\theta},t)}{\partial t}+\dot{U}_{\mathrm{s}}(\boldsymbol{\theta},t)\frac{\partial p_{U_{\mathrm{s}}\Theta}(u,\boldsymbol{\theta},t)}{\partial u}=0 \tag{14}$$

The corresponding instantaneous probability density functions (PDFs) of $Y(t)$ and $U_s(t)$ can be obtained by solving the above partial differential equations with given initial conditions

$$p_Y(y,t) = \int_{\Omega_\Theta} p_{Y\Theta}(y,\boldsymbol{\theta},t)\mathrm{d}\boldsymbol{\theta} \qquad (15)$$

$$p_{U_s}(u,t) = \int_{\Omega_\Theta} p_{U_s\Theta}(u,\boldsymbol{\theta},t)\mathrm{d}\boldsymbol{\theta} \qquad (16)$$

where Ω_Θ is the distribution domain of $\boldsymbol{\Theta}$; the joint PDFs $p_{Y\Theta}(y,\boldsymbol{\theta},t)$ and $p_{U_s\Theta}(u,\boldsymbol{\theta},t)$ are the solutions of Eqs. (13) and (14), respectively.

It is noted that due to the randomness inherent in external excitations, the maximum Coulomb force and damping coefficient naturally both rely upon the stochastic vector $\boldsymbol{\Theta}$ even the randomness inherent in MR damper is not considered tentatively. While the parameters of the MR damper are pre-designed deterministic values, which balance the structural state and control force. The primary task, therefore, of stochastic optimal MR damping control on the randomly excited structural system is to optimize the maximum Coulomb force and viscous damping coefficient as rule of tracing the stochastic optimal control force in active modality. The determination of active optimal control force refers to the scheme of physically based stochastic optimal control (Li et al, 2010).

3. CASE STUDIES

A single-degree-of-freedom structure attaching the MR damper subjected to random seismic ground motion is investigated. The properties of the system are as follows: (1) the mass of the first floor is $m = 1\times10^5$ kg; (2) The natural circular frequency of the structural system is $\omega_0 = 11.22$ rad/sec; (3) the damping ratio is 0.05; (4) the tunable times of the damping force is set as s=8. Simulated seismic processes with PGA 0.11g employing the stochastic ground motion model are used as the input excitations (Li and Ai, 2006). The stochastic optimal control force is pre-computed by the dynamic programming method in the context of physically based stochastic optimal control (Li et al, 2010).

Fig. 6 shows the PDF of extreme value of active optimal control force. It is seen that the PDF curve distributes in a wide range with mean 115.44 kN and standard d eviation 34.68 kN. The damper parameters in this case relies on the active optimal control force; see Eqs.(9) and (10), involving the damping coefficient and maximum Coulomb force of the MR damper, which are designed based on the criterion of optimal structural performance.

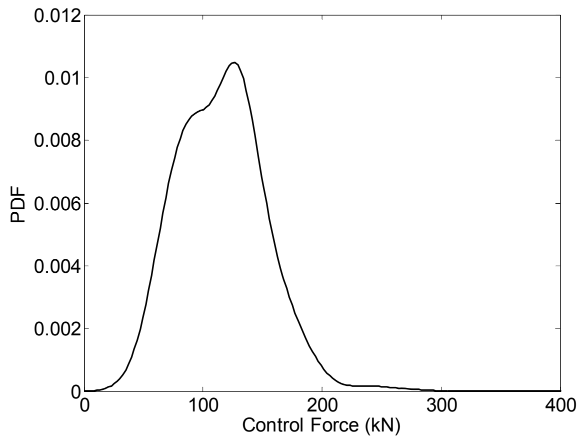
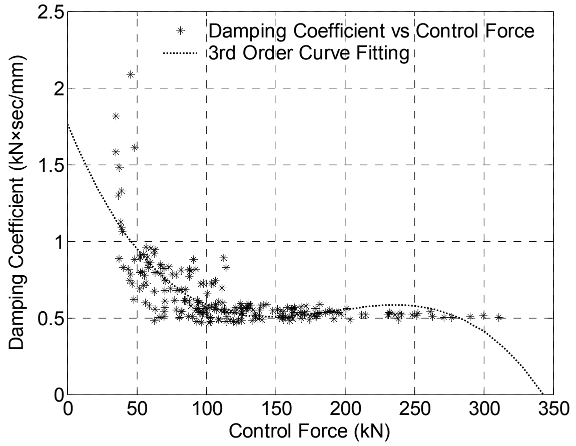


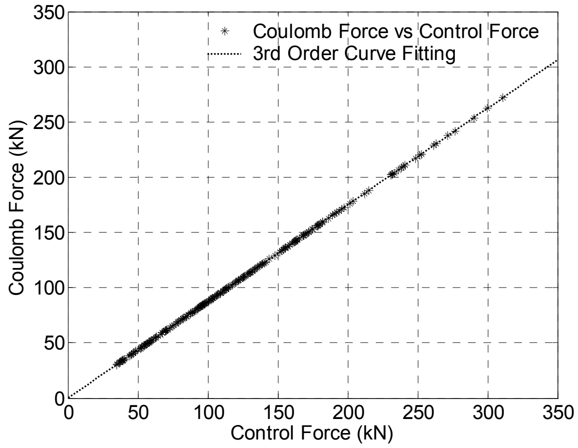
Figure 6: PDF of extreme value of active optimal control force.

Fig. 7 shows the relation between the design damper parameters and the active control force. It is seen that the control force is insensitive to the damping coefficient of MR damper. The definition of the design parameters, meanwhile, should be in accordance with practical capacity of MR damper, e.g. the acting force 200 kN is well-posed; high-viscous rheological liquid is inconvenient for maintenance. Therefore, the damping coefficient and the maximum Coulomb force are designed as 0.6119 kN×sec/mm, 82.28 kN, respectively.

Besides, in order to readily integrate the randomness of damping coefficient of the MR damper into the control effectiveness of structural system, we assume that the damping coefficient submits to Gaussian distribution, of which mean is the designed value and standard deviation is 0.0664 kN×sec/mm.

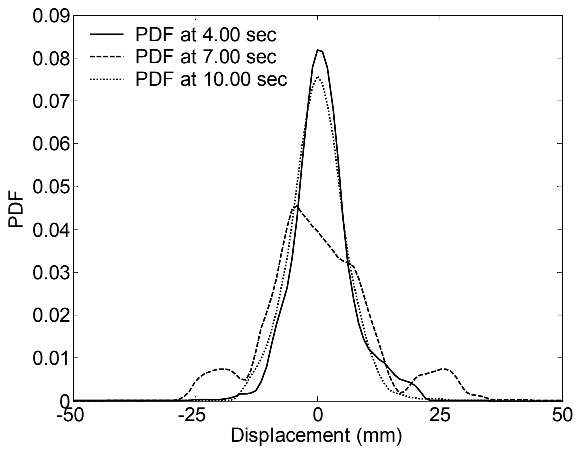


(a)Damping coefficient and control force.

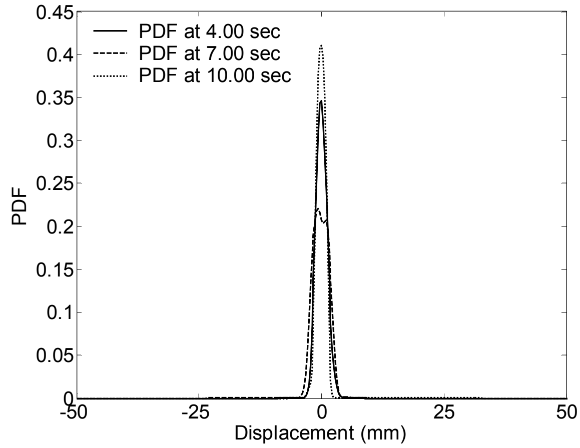


(b)Maximum Coulomb force and control force.

Figure 7: Relation between design parameters and active control force.



(a)Uncontrolled case



(b)MR damping control case

Figure 8: Probability density function of displacement at typical instants with and without MR damping control.

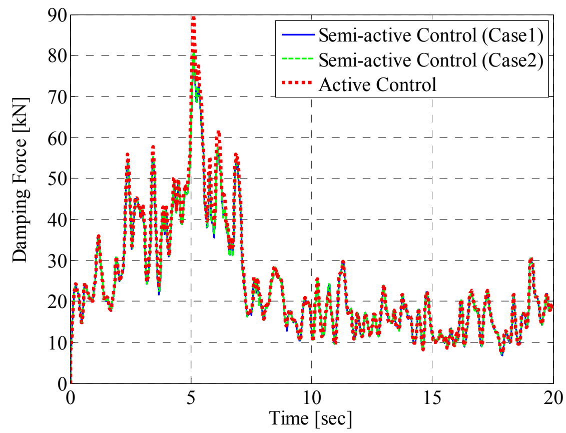


Figure 9: Root-mean-square arguments of MR damping force and of active optimal control force.

Fig. 8 shows the probability density function of displacement at typical instants with and without MR damping control. It is seen that the MR damping control can reduce the structural displacement significantly, where the distribution range of PDF becomes narrower, indicating that the structural system locates at more robust and more safe state.

The root-mean-square arguments of the MR damping force and of the active optimal control force are pictured in Fig. 9. Case 1 of semi-active control involves the situation in this paper where the randomness of damping coefficient is included, while Case 2 treats the damper parameters as deterministic variables. One can see that the MR damping forces of the two cases agree well with each other; the randomness of damping coefficient has no significant influence upon the MR damping force and the structural responses. This remark accords with the knowledge that as exposed in the stochastic modeling of MR damper, the damping coefficient contributes to the randomness of damping force, but it is insensitive to the control force. It is thus indicated that the randomness of damper parameters could be neglected safely.

Besides, the semi-active optimal control force finely traces the active optimal control force in root-mean-square sense, revealing that the appropriately designed MR controller can achieve almost the same effectiveness as the active optimal controller.

4. CONCLUSIONS

This paper addresses the issue of stochastic modeling of MR damper and its application in stochastic optimal control of randomly base-excited structure. Numerical results reveal the applicability and effectiveness of the MR damping control. In order to quantify the randomness inherent in MR damper, the fluctuation of dynamic yield of MR fluid at material scale is directly updated onto the hysteretic model of MR damper at component scale of structure. Numerical investigations show that the variability of MR damping force seems not so sound since the damping coefficient is insensitive to the control force though it contributes to the randomness of damping force. It is remarked that the appropriately designed semi-active controller can achieve almost the same effect as the active controller in probabilistic sense. Parameters design and optimization of MR dampers relevant to the excitation behaviors and risk levels will be addressed in the future work.

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