A New Probabilistic Model of Fully Non-Stationary Ground Motion and its Application

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ABSTRACT: A numerical simulation scheme is presented that combines the advantages of the spectral representation method (SRM) and the idea of random function to generate fully non-stationary ground motion. In the paper, firstly, a new family of spectral representation method is proposed. Secondly, these standard orthogonal random variables in SRM formula can be defined as the orthogonal function form of a basic random variable, and the original stochastic process can be represented as a function of the basic random variable. Its advantage is proved that fully non-stationary ground motion can be completely represented by a dimension-reduced spectral model with just few elementary random variables through defining the high-dimensional standard orthogonal random variables of classical spectral representation into the low-dimensional orthogonal random function. Finally, it provides an opportunity to incorporate the probability density evolution method (PDEM), and study the stochastic dynamics in engineering.

It is well understood in earthquake engineering community that the rational description and model of random earthquake ground motions underlies the seismic analysis and reliability assessment of engineering structures (Wang and Li, 2011). The systematic development on the study of random seismic ground motion began with the pioneered work contributed by Housner in 1947 (Douglas and Aochi, 2008), who modeled the seismic acceleration as a pulse-structured random process. While the nonlinear random analysis, in practices, of seismic structures involves the transform of earthquake ground motion models from frequency-domain representation to temporal-domain counterpart. The practical demand highly prompts the enthusiasm of researchers on the stochastic simulation of random processes. There has been tens of simulation techniques so far among which, nevertheless, the spectral representation is widely used due to its rigorous mathematical formulation and easier-to-be-implemented algorithm (Liu et al, 2014, 2015).

In the paper, a numerical simulation scheme is presented that combines the advantages of the spectral representation method (SRM) and the idea of random function to generate fully non-stationary ground motion. Its advantage is proved that fully non-stationary ground motion can be completely represented by a dimension-reduced spectral model with just few elementary random variables through defining the high-dimensional standard orthogonal random variables of classical spectral representation into the low-dimensional orthogonal random functions. The proposed technique provides an opportunity to employ the probability density evolution method (PDEM) which has been developed by Li and Chen in the past few years to study the stochastic nonlinear responses of general structural systems (Li and Chen, 2009). An example, which deals with a MDOF aqueduct structure subjected to ground motions, is investigated to illustrate the proposed procedure.

1. EVOLUTIONARY POWER SPECTRUM OF FULLY NON-STATIONARY GROUND MOTION

In this paper, based on the Kanai-Tajimi power spectrum of stationary ground motion acceleration process, the evolutionary power spectrum (two-side spectrum) of fully non-stationary ground motion acceleration process is given by

$$S_{X_g}(t,\omega) = A^2(t)\times \frac{\omega_g^4(t)+4\xi_g^2(t)\omega_g^2(t)\omega^2}{\left[\omega^2-\omega_g^2(t)\right]^2+4\xi_g^2(t)\omega_g^2(t)\omega^2}\cdot S_0(t) \quad (1)$$

where $\omega_g(t)$ and $\xi_g(t)$ are the frequency and the damping ratio of site soil, respectively; $S_0(t)$ is the spectral intensity factor; $A(t)$ is deterministic intensity modulation function.

In Eq. (1), $S_0(t)$ can be expressed as

$$S_0(t) = \frac{A_P^2}{\gamma^2\left\{\pi\omega_g(t)\left[2\xi_g(t)+\frac{1}{2\xi_g(t)}\right]\right\}} \quad (2)$$

where A_P denotes the ground motion peak acceleration, γ denotes the peak factor.

In Eq. (1), the parameters $\omega_g(t)$ and $\xi_g(t)$ can be expressed as (Cacciolaand Deodatis, 2011)

$$\omega_g(t) = \omega_0 - a\frac{t}{T},\ \xi_g(t) = \xi_0 + b\frac{t}{T} \quad (3)$$

where T is the duration of ground motion, ω_0, ξ_0 and a, b are site soil parameters.

For the intensity modulation function $A(t)$ is given as

$$A(t) = \left[\frac{t}{c}\exp\left(1-\frac{t}{c}\right)\right]^d \quad (4)$$

where c denotes the time of ground motion peak acceleration, d is the parameter controlling the shape of $A(t)$.

2. THE SPECTRAL REPRESENTATION AND RANDOM FUNCTIONS METHOD OF NON-STATIONARY PROCESSES

In general, we may assume that $X_g(t)$ is a non-stationary ground motion process with zero mean, the spectral representation can be expressed (Liu, 2015)

$$X_g(t) \approx \sum_{k=1}^{N}\sqrt{2S_{X_g}(t,\omega_k)\Delta\omega}\times \left[\cos(\omega_k t)X_k + \sin(\omega_k t)Y_k\right] \quad (5)$$

which $\omega_k = k\Delta\omega$ and $\Delta\omega$ denotes the discrete frequency step; N denotes the truncated number; $S_{X_g}(t,\omega)$ is the two-side evolutionary power spectrum density function.

In Eq. (5), these standard orthogonal random variables $\{X_k, Y_k\}$ $(k=1,2,\cdots,N)$ must satisfy the basic conditions as follow

$$E\left[X_k\right] = E\left[Y_k\right] = 0,\ E\left[X_k Y_m\right] = 0 \quad (6a)$$

$$E\left[X_k X_m\right] = E\left[Y_k Y_m\right] = \delta_{km} \quad (6b)$$

where $E[\cdot]$ denotes the mathematical expectation, δ_{km} is the Kronecker symbol.

For the standard orthogonal random variables $\{X_k, Y_k\}$ $(k=1,2,\cdots,N)$ in Eq. (5), the random function is suggested to represent the standard orthogonal random variables by Liu (2014). Firstly, suppose two orthonormal random variables $\bar{X}_n$ and $\bar{Y}_n$ are functions of the basic random variable Θ, respectively, namely

$$\bar{X}_n = \sqrt{2}\cos(n\Theta + \pi/4), n = 1, 2, \cdots, N \quad (7b)$$

$$\bar{Y}_n = \sqrt{2}\sin(n\Theta + \pi/4), n = 1, 2, \cdots, N \quad (7b)$$

where the basic random variable Θ satisfy uniform distribution in the interval $[0, 2\pi)$. It is easy to prove that the orthonormal random variables $\{\bar{X}_n, \bar{Y}_n\}$ $(n = 1, 2, \cdots, N)$ defined by Eq. (7) can satisfy the basic conditions of Eq. (6).

Secondly, the orthonormal random variables $\{\bar{X}_n, \bar{Y}_n\}$ $(n = 1, 2, \cdots, N)$ need be converted to the standard orthogonal random variables $\{X_k, Y_k\}$ $(k = 1, 2, \cdots, N)$ in Eq. (5) by deterministic one-to-one mapping way.

3. SIMULATION OF FULLY NON-STATIONARY GROUND MOTION

In the above evolutionary power spectrum, the parameters' values of the evolutionary power spectrum are shown in Table 1. Moreover, the ground motion peak acceleration A_p = 196 cm/s^2.

Table 1: Parameter values.

$\omega_0 / \mathrm{s}^{-1}$	ξ_0	a / s^{-1}	b
24.94	0.67	8	0.1
c / s	d	γ	T / s
5.5	1.8	3.0	20

In the spectral representation and random function method of fully non-stationary ground motion, the basic random variables Θ is uniformly discretized in the interval $[0, 2\pi)$, and the discrete points' number n_{sel} = 610. Taking each discrete point value Θ_i $(i = 1, 2, \cdots, n_{se1})$ into Eq. (7) to generate a set of discrete values of orthonormal random variables $\{\bar{X}_n, \bar{Y}_n\}$ $(n = 1, 2, \cdots, N)$, and convert $\{\bar{X}_n, \bar{Y}_n\}$ into the standard orthogonal random variables $\{X_k, Y_k\}$ $(k = 1, 2, \cdots, N)$ in Eq. (5) by deterministic one-to-one mapping way. So, the representative history time for fully non-stationary ground motion acceleration process can be generated. The representative time histories are shown in Figure 1.

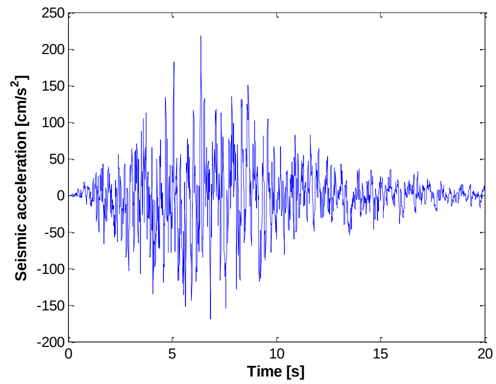

Figure 1: Generated representative function of non-stationary ground motion acceleration

Comparison between the mean and standard deviation by the 610 representative time histories and the targets are shown in Figure 2. It is shown that the samples' ensemble values are consistent with the target values.

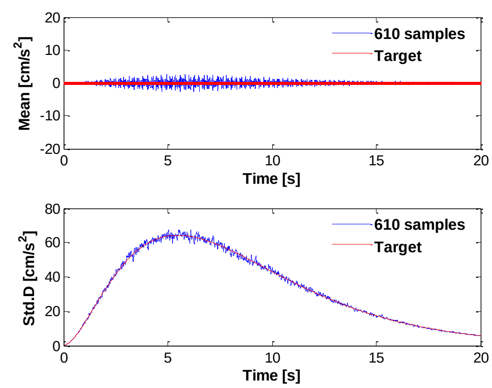

Figure 2: Comparison between mean and standard deviation from samples ensemble and from the target

4. APPLICATION EXAMPLE

Recently, the probability density evolution method (PDEM) has made significant progress in stochastic dynamics of structures (Li and Chen, 2009). In this paper, combining the proposed probabilistic model of fully non-stationary ground motion and PDEM, stochastic seismic response analysis and dynamic reliability evaluation on engineering structures can be implemented.

In order to illuminate the practicability of proposed probabilistic model of fully non-stationary ground motion, an aqueduct structure as shown in Figure 3 subjected to stochastic ground motion is investigated. The total length of aqueduct structure is 440m, and the length of single span is 40.0m. The cross section of aqueduct body is a single rectangular sink with clear width of the bottom being 8.4m and the height of the aqueduct body being 6.4m.

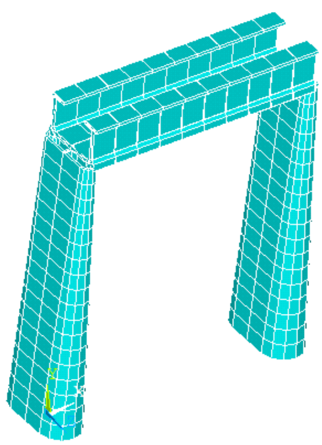

Figure 3: The finite element model of aqueduct structure

In employing the PDEM, stochastic seismic response analysis and dynamic reliability evaluation for the aqueduct displacement of the mid-span node is carried out to capture the probability information, as shown in Figure 4-7. Pictured in Figure 4 are the mean and the standard deviation. Figure 5 shows the typical PDF at certain instants of time. As shown in Figure 6 and 7, where Figure 6 is the surface constructed by the PDF at different instants of time, while Figure 7 is the contours of the surface, respectively. From the Figure 4-7, the probability density function (PDF) has the typical evolutionary characteristics.

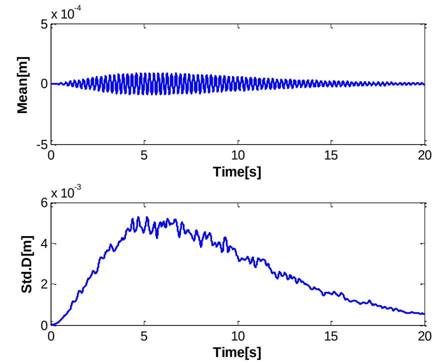

Figure 4: Mean values and standard deviations

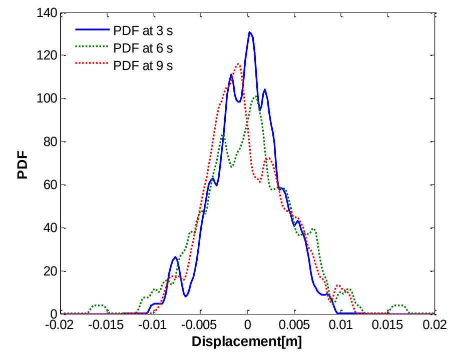

Figure 5: Typical probability density function at different time instants

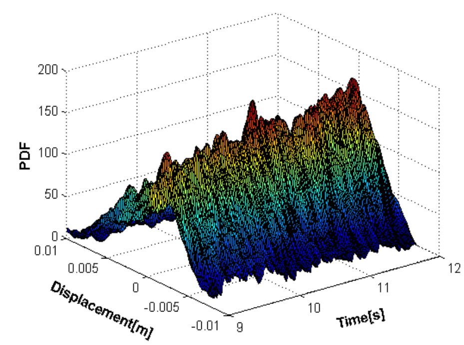

Figure 6: Probability density function evolution surface

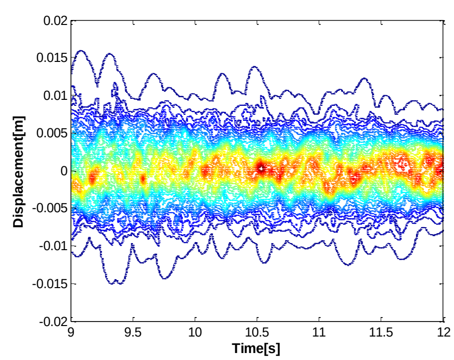

Figure 7: Contours of the PDF surface

Based on the equivalent extreme-value event (Li and Chen, 2007), under the control criteria on displacement of the mid-span node, the cumulative distribution function (CDF) can be captured, as shown in the Figure 8. Actually, the CDF is the dynamic reliability of aqueduct structure.

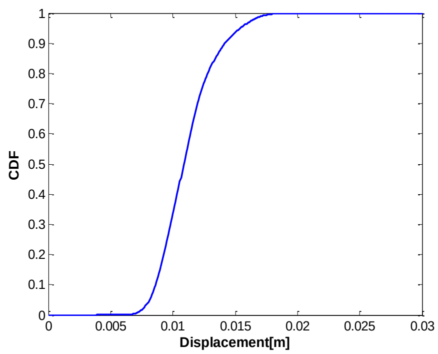

Figure 8: The dynamic reliability of aqueduct structure

5. CONCLUSIONS

Rational representation and models of random engineering excitations underlie the stochastic dynamics of structures. A random function and spectral representation scheme are proposed in this paper. The updated scheme is used for the simulation of seismic acceleration processes. Employing the above representation of ground motion, the PDEM can be implemented to capture the instantaneous PDF of responses for general MDOF structure. An aqueduct structure subjected to stochastic seismic excitation is studied to illustrate its applications, showing the effectiveness of the proposed method.

6. ACKNOWLEDGMENTS

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