Probabilistic Damage Identification of the Dowling Hall Footbridge through Hierarchical Bayesian Model updating

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ABSTRACT: In this paper, a Hierarchical Bayesian finite element model updating framework is applied for probabilistic identification of simulated damage on the Dowling Hall Footbridge. The footbridge is located at Tufts campus and is equipped with a continuous monitoring system, including 12 accelerometers. Structural damage is simulated by the addition of mass on a small segment of the footbridge, and the Hierarchical framework is used to identify the location and extent of the damage (added mass), and to quantify the prediction uncertainties. This framework is well suited for applications to civil structures, where the structural properties (stiffness, mass) can be considered time-variant due to changing environmental conditions such as temperature, wind speed, or traffic.

1. INTRODUCTION

Measured dynamic or static response of structures can provide useful information for the assessment of structural health and performance. This information should include the location and severity of a potential structural damage (Inman et al., 2005). Finite Element (FE) model updating techniques have been introduced as a useful tool to achieve this goal (Friswell and Mottershead, 1995). However, the accuracy of structural identification results can be significantly affected by different sources of uncertainties including modeling errors, changing environmental/ ambient conditions, and measurement noise (Doebling et al., 1996; Friswell, 2007). Therefore, probabilistic identification frameworks have become an integral part of damage diagnosis applications. Beck and Katafygiotis (1998) proposed a Bayesian model updating framework for structural identification. In this framework, the optimum model parameters and their estimation uncertainties can be estimated based on the measured structural responses. Beck et al. (2001) formulated the Bayesian FE model updating technique for damage identification of civil structures and based on identified natural frequencies and mode shapes. In this framework the posterior probability distribution of the updating structural parameters $\boldsymbol{\theta}$ can be estimated using the Bayes theorem:

$$p(\boldsymbol{\theta}|\mathbf{D}) \propto p(\mathbf{D}|\boldsymbol{\theta})\, p(\boldsymbol{\theta}) \tag{1}$$

where $\mathbf{D}$ refers to the identified modal parameters extracted from a vibration test data, $p(\mathbf{D}|\boldsymbol{\theta})$ is the likelihood function and $p(\boldsymbol{\theta})$ is the prior probability distribution. If the posterior has a unique global maximum (globally identifiable), Laplace asymptotic approximation can be used to estimate the posterior distribution (Beck et al., 2001; Papadimitriou et al., 1997); otherwise, the posterior needs to be sampled numerically (Beck and Au, 2002; Ching and Chen, 2007).

In this framework and in the case of having N_t number of independent data sets, the joint posterior probability distribution of the updating model parameters can be calculated as:

$$p\left(\boldsymbol{\theta}|\mathbf{D}_{1,\ldots,t,\ldots,N_t}\right) \propto \prod_{t=1}^{N_t} p(\mathbf{D}_t|\boldsymbol{\theta})\, p(\boldsymbol{\theta}) \tag{2}$$

More information about structural identification framework can be found in Yuen (2010) and Beck (2010).

The effects of data accumulation in Eq. (2) was also discussed in Beck et al. (2001) and it was observed that the posterior standard deviation of the updating structural parameters were reduced by increasing the number of data sets. However, in the presence of changing ambient and environmental conditions, the updating structural parameters such as mass or stiffness are expected to have a level of inherent variability which is unreducible in the absence of an underlying model to explicitly consider these ambient and environmental effects. Changes in ambient temperature, temperature gradient, wind speed, traffic loads, and rain/snow result in varying structural stiffness and mass (Cornwell et al., 1999; Cross et al., 2013; Moser and Moaveni, 2011). Beck (2010) and Jaynes (2003) also confirm that model uncertainties can be expected due to imperfect understanding of the system behavior and/or incomplete information.

A Hierarchical Bayesian model updating framework is proposed to address this issue by considering an underlying variability in the updating structural parameters (Behmanesh et al., 2015a). Hyper-parameters are introduced for updating structural parameters to represent the structural variability. More information on the theoretical aspects of the Hierarchical Bayesian modeling can be found in Gilks et al. (1998) and Gamerman and Lopes (2006). The main advantage of this framework is its ability to explicitly include different sources of uncertainties in the identification process. For example in (Behmanesh et al., 2015b), it is shown that the effects of ambient temperature on the Elastic Young's Modulus of concrete can be included in the updating process when the temperature measurements are available during each data collection $\mathbf{D}_t$. In this paper, the Hierarchical approach is briefly reviewed and applied for damage identification of the Dowling Hall footbridge.

2. DOWLING HALL FOOTBRIDGE

Dowling Hall footbridge is located at Tufts University, Medford campus. More information about the footbridge and its monitoring system can be found in Moser and Moaveni (2013). Structural damage is simulated by the addition of mass on different segments of the footbridge. Three damage scenarios with different extent and location of added mass are considered; however, only one damage scenario will be presented in this paper. This damage scenario were previously studied by Behmanesh and Moaveni (2014) using the Bayesian FE model updating framework without the hyper-parameters.

2.1. Simulating the Effects of Structural Damage

Figure 1 shows the addition of 2.24 tons of concrete blocks on a small segment of the footbridge deck. The effects of the added mass on the identified natural frequencies of the first bending mode and first torsional mode of the structure are shown in Figure 2. The first six modes, including 4 vertical bending modes and 2 torsional modes, will be used for damage identification. Overall 1355 and 72 sets of data are available for the structure in the undamaged and damaged states, respectively. All data are recorded in warm seasons. The minimum, average, and maximum air temperature is recorded as 14, 26, and 37 degrees Celsius, respectively.

2.2. Finite Element Model

A linear FE model of the footbridge is created using a MATLAB based program, FEDEASLAB (Filippou and Constantinides, 2004; MathWorks, 2014). This model consists of 461 nodes, 406 frame elements, and 324 shell elements. The initial FE model is carefully tuned to a reference model based on the average modal parameters at the undamaged state of the footbridge. Table 1 presents the natural frequencies of the reference model, the average of identified natural frequencies in the undamaged state, and the MAC values between the model-calculated and the average of identified mode shapes.

For damage identification process, the footbridge deck is divided into 7 segments and the addition of mass on each segment is defined as one updating structural parameter. The segments are shown in Figure 3 and are defined based on the location of the accelerometers.

Figure 1: Added mass on the footbridge deck.

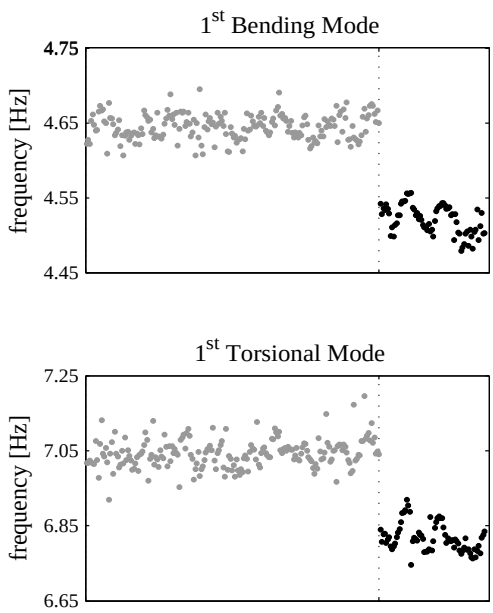

Figure 2: Natural Frequencies in undamaged state (gray dots) and in damaged state (black dots).

Table 1: Modal parameters of the initial FE model and identified modal parameters.

Mode	*1*	*2*	*3*	*4*	*5*	*6*
FE model	*4.60*	*6.13*	*7.02*	*8.75*	*12.97*	*13.66*
Identified	*4.65*	*6.10*	*7.05*	*8.88*	*13.07*	*13.53*
MAC (%)	*99.4*	*99.9*	*99.2*	*99.7*	*99.3*	*99.6*

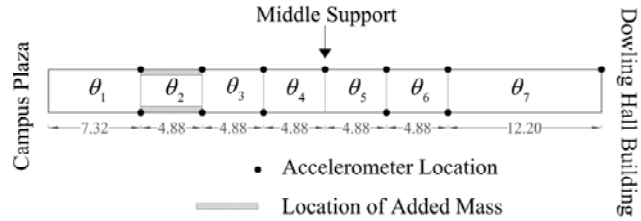

Figure 3: Seven considered segments of footbridge deck correspond to seven updating structural parameters.

3. HIERARCHICAL BAYESIAN FE MODEL UPDATING

This section briefly reviews the general framework of Hierarchical Bayesian model updating for structural identification. Subsections 3.1 to 3.4, explain the specific assumptions and formulations that are used in this paper for damage identification of the Dowling Hall footbridge.

In linear damage identification applications, damage is usually defined as a loss of stiffness and therefore, the structural updating parameters are usually stiffness parameters such as Elastic Young's Modulus. The Hierarchical approach begins with assuming a probability distribution for the updating structural parameters with unknown hyper-parameters, i.e., $\boldsymbol{\theta} \sim p\left(\boldsymbol{\mu}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}\right)$. This distribution represents the variability of the structural stiffness due to changing environmental/ambient conditions. The two hyper-parameters $\boldsymbol{\mu}_{\boldsymbol{\theta}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ are modeled by hyper-prior probability distributions. Therefore for a given data set *t*:

$$p\left(\boldsymbol{\theta}_t, \boldsymbol{\mu}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}, \sigma_e^2 \mid \mathbf{D}_t\right) \propto p\left(\mathbf{D}_t \mid \boldsymbol{\theta}_t, \sigma_e^2\right) p\left(\boldsymbol{\theta}_t \mid \boldsymbol{\mu}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}\right) p\left(\boldsymbol{\mu}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}, \sigma_e^2\right) \tag{3}$$

where σ_e^2 is the model error parameter and defines the variance of the error functions, which are usually defined as the discrepancy between the identified and model-calculated modal parameters. The sub-index *t* refers to the values of the updating structural parameters during test *t*. Therefore, the parameter $\boldsymbol{\theta}_t$ is a realization of

$\boldsymbol{\theta}$ given the structural conditions during test t. If N_t sets of independent data sets are available, the joint posterior probability distribution of updating parameters can be written as:

$$p\left(\boldsymbol{\Theta},\boldsymbol{\mu}_{\boldsymbol{\theta}},\boldsymbol{\Sigma}_{\boldsymbol{\theta}},\sigma_e^2 \mid \mathbf{D}_{1,\ldots,t,\ldots,N_t}\right) \propto \prod_{t=1}^{N_t} p\left(\mathbf{D}_t \mid \boldsymbol{\theta}_t,\sigma_e^2\right) p\left(\boldsymbol{\theta}_t \mid \boldsymbol{\mu}_{\boldsymbol{\theta}},\boldsymbol{\Sigma}_{\boldsymbol{\theta}}\right) p\left(\boldsymbol{\mu}_{\boldsymbol{\theta}},\boldsymbol{\Sigma}_{\boldsymbol{\theta}},\sigma_e^2\right) \tag{4}$$

where $\boldsymbol{\Theta}=\left\{\boldsymbol{\theta}_1,\ldots,\boldsymbol{\theta}_t,\ldots,\boldsymbol{\theta}_{N_t}\right\}$. The Likelihood can be defined by assuming uncorrelated, zero-mean error functions (Beck et al., 2001) and can be approximated as a Normal distribution through Laplace asymptotic approximation (Papadimitriou et al., 1997). This approximation can significantly reduce the computational time in the Hierarchical updating approach, although stochastic simulation and parallel computing can also be used for more accurate estimations (Angelikopoulos et al., 2012). By assuming a Normal distribution for the prior probability of $\boldsymbol{\theta}$, Inverse Gamma distributions for the variance components of $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$, no correlations between the updating structural parameters, and uniform priors for $\boldsymbol{\mu}_{\boldsymbol{\theta}}$ and σ_e^2, the full conditional probability distributions of each parameter can be represented by either Inverse Gamma or Normal distribution. Thus, Gibbs Sampler can be comfortably used to sample the joint probability distribution of Eq. (4). However in this study, damage is simulated by addition of mass and our updating structural parameters are defined also as the addition of mass that are considered as positive values. Therefore, we cannot use the asymptotic approximation to estimate the likelihood function. The specific assumptions and computational procedure that are used in this study are presented in the following subsections.

3.1. Likelihood Function

The likelihood function is defined as the difference (between model and data) of changes in modal parameters from a reference state to the current state. By defining the eigen-value and mode shape error functions of mode m as Eq. (5-6), the likelihood function can be written as Eq. (7-8).

$$\lambda_m^c\left(\boldsymbol{\theta}_t\right)-\tilde{\lambda}_{tm}^c\frac{\lambda_m^r}{\bar{\tilde{\lambda}}_m^r}=e_{\lambda_{tm}}\left(\boldsymbol{\theta}_t\right)\sim N\left(\mathbf{0},\bar{\tilde{\lambda}}_m^c\sigma_e^2\right) \tag{5}$$

$$\left(\boldsymbol{\Phi}_m^c\left(\boldsymbol{\theta}_t\right)-\boldsymbol{\Phi}_m^r\right)-\left(\tilde{\boldsymbol{\Phi}}_{tm}^c-\bar{\tilde{\boldsymbol{\Phi}}}_m^r\right)=\mathbf{e}_{\boldsymbol{\Phi}_{tm}}\left(\boldsymbol{\theta}_t\right)\sim N\left(\mathbf{0},w\mathbf{I}\sigma_e^2\right) \tag{6}$$

$$p\left(\mathbf{D}_t \mid \boldsymbol{\theta}_t,\sigma_e^2\right)\propto\frac{1}{\sigma_e^{N_m^t\left(N_s+1\right)}}\exp\left(\frac{-J\left(\boldsymbol{\theta}_t,\mathbf{D}_t\right)}{2\sigma_e^2}\right) \tag{7}$$

$$J\left(\boldsymbol{\theta}_t,\mathbf{D}_t\right)=\sum_{m=1}^{N_m^t}\left\{e_{\lambda_{tm}}^2\left(\boldsymbol{\theta}_t\right)+\mathbf{e}_{\boldsymbol{\Phi}_{tm}}^T\left(\boldsymbol{\theta}_t\right)\mathbf{e}_{\boldsymbol{\Phi}_{tm}}\left(\boldsymbol{\theta}_t\right)\right\} \tag{8}$$

where the superscript c refers to the current state and superscript r refers to the reference state. $\tilde{\lambda}_{tm}^c$ is the identified eigenvalue, $\tilde{\lambda}_{tm}^c=\left(2\pi\tilde{f}_{tm}^c\right)^2$ and $\tilde{\boldsymbol{\Phi}}_{tm}^c$ is the identified mode shape of mode m from test t. λ_m^r and $\boldsymbol{\Phi}_m^r$ are the eigenvalue and mode shape of the reference FE model at mode m, reported in Table 1. Also, N_m^t is the number of available identified modes at test t, N_s is the number of identified mode shape components, $\mathbf{I}$ is the identity matrix of size N_s, and w is a weight factor between eigenvalue and mode shape error functions.

In this likelihood, the optimum $\boldsymbol{\theta}_t$ provides the closest changes of model-calculated modal parameters to the observed changes in the identified modal parameters. The reference data is the average of 1355 sets of modal parameters in the undamaged state and are shown by $\bar{\tilde{\lambda}}_m^r$ and $\bar{\tilde{\boldsymbol{\Phi}}}_m^r$ for mode m.

3.2. Prior Probabilities

We use Log-Normal distribution for the prior probability distribution of $\boldsymbol{\theta}$:

$$p\left(Log\left(\boldsymbol{\theta}_t\right) \mid \boldsymbol{\mu}_{\boldsymbol{\theta}},\boldsymbol{\Sigma}_{\boldsymbol{\theta}}\right)\approx N\left(\boldsymbol{\mu}_{\boldsymbol{\theta}},\boldsymbol{\Sigma}_{\boldsymbol{\theta}}\right) \tag{9}$$

Uniform prior distributions are assumed for $\boldsymbol{\mu}_{\boldsymbol{\theta}}$ and σ_e^2. The covariance matrix of the updating

structural parameter is assumed as a diagonal matrix with Inverse Gamma prior probability distributions for the variance of each updating structural parameter, i.e. $\sigma_{\theta_q}^{-2} \sim G(\alpha, \beta)$. Please note that q refers to q^{th} updating structural parameter, and the same prior is assumed for all the N_q number of updating structural parameters.

3.3. Conditional Distributions for Gibbs Sampler

In the Gibbs sampling technique, samples are generated from the full conditional probability distribution of each parameter until convergence is reached. In the following, the full conditional probability distribution of each updating parameter is presented:

$$p\left(Log\left(\boldsymbol{\theta}_t\right)|.\right) \propto \exp\left(-\sigma_e^{-2}\sum_{m=1}^{N_m^t} J\left(\boldsymbol{\theta}_t, \mathbf{D}_t\right) - \sum_{p=1}^{N_q}\sigma_{\theta_q}^{-2}\left(Log\left(\theta_{tq}\right) - \mu_{\theta_q}\right)^2\right) \tag{10}$$

$$p\left(\mu_{\theta_q}|.\right) = N\left(\frac{1}{N_t}\sum_{t=1}^{N_t} Log\left(\theta_{tq}\right), \sigma_{\theta_q}^2 / N_t\right) \tag{11}$$

$$p\left(\sigma_{\theta_q}^{-2}|.\right) = G\left(\frac{N_t}{2} + \alpha, \left(\beta^{-1} + \frac{1}{2}\sum_{t=1}^{N_t}\left(Log\left(\theta_{tq}\right) - \mu_{\theta_q}\right)^2\right)^{-1}\right) \tag{12}$$

$$p\left(\sigma_e^{-2}|.\right) = G\left(\frac{\left(N_s+1\right)}{2}\prod_{t=1}^{N_t} N_m^t + 1,\ 2\left(\sum_{t=1}^{N_t} J\left(\boldsymbol{\theta}_t, \mathbf{D}_t\right)\right)^{-1}\right) \tag{13}$$

where the sign "|." refers to the conditional distributions given the available data and all the updating parameters ($\boldsymbol{\Theta}, \boldsymbol{\mu}_{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\boldsymbol{\theta}}, \sigma_e^2$) except the one that is written on the left hand side of "|.".

As it can be seen all the conditional distributions except Eq. (10) are either Normal or Gamma. To simplify the sampling process the first term of the right hand side of Eq. (10), the likelihood, is approximated as a Log-Normal distribution.

3.4. Simplified Sampling Process

Replacing the first term of the right hand side of Eq. (10) with a Log-Normal distribution yields:

$$\exp\left(-\sigma_e^{-2}\sum_{m=1}^{N_m^t} J\left(\boldsymbol{\theta}_t, \mathbf{D}_t\right)\right) \propto \prod_{q=1}^{N_q} N\left(Log\left(\theta_{tq}\right) | \mu_{\hat{\theta}_{qt}}, \sigma_{\hat{\theta}_{tq}}^2\right) \tag{14}$$

A two-step procedure is used to estimate the two parameters of the approximated Log-Normal distribution: (1) the most probable $\boldsymbol{\theta}_t$ is estimated by minimizing the objective function of Eq. (8) for each data set, separately. Therefore, given the fact that the peak (mode) of the Log-Normal distribution of parameter q is at $\exp\left(\mu_{\hat{\theta}_{tq}} - \sigma_{\hat{\theta}_{tq}}^2\right)$, the estimated mean can be obtained as:

$$\mu_{\hat{\theta}_{tq}} \approx Log\left(\hat{\theta}_{tq}\right) + \sigma_{\hat{\theta}_{tq}}^2 \tag{15}$$

where:

$$\hat{\boldsymbol{\theta}}_t = Arg\min_{\boldsymbol{\theta}_t}\left(J\left(\boldsymbol{\theta}_t, \mathbf{D}_t\right)\right) \tag{16}$$

(2) The variance of $\boldsymbol{\theta}_t$, which represents the parameter estimation uncertainties due to modeling errors and incomplete modal information, is estimated through Adaptive Metropolis Hastings algorithm of Andrieu and Thoms (2008). In this study, this variance is assumed to be constant for all data sets. For more accurate estimations of the updating parameters a Metropolis-within-Gibbs sampling techniques should be used. The average of the modal parameters in the damaged state is used for this estimation and 25,000 samples are generated with 44% acceptance ratio. Figure 4 shows the histogram of the structural parameters and the fitted Log-Normal distributions.

The conditional probability distribution of Eq. (10) can now be approximated as:

$$p\left(Log\left(\theta_{tq}\right)|.\right) \approx N\left(\frac{\mu_{\hat{\theta}_{tq}}\sigma_{\theta_q}^2 + \mu_{\theta_q}\sigma_{\hat{\theta}_{tq}}^2}{\sigma_{\theta_q}^2 + \sigma_{\hat{\theta}_{tq}}^2}, \frac{\sigma_{\theta_q}^2\sigma_{\hat{\theta}_{tq}}^2}{\sigma_{\theta_q}^2 + \sigma_{\hat{\theta}_{tq}}^2}\right) \tag{17}$$

The Gibbs Sampler can now be applied easily since all the conditional distributions are either

Normal or Gamma. Two sets of 10,000 Gibbs samples are generated at both damaged and undamaged states.

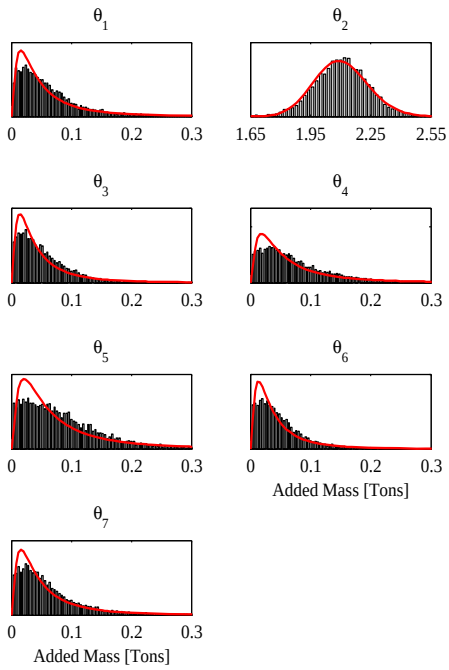

Figure 4: Histogram of updating parameters using average of modal parameters in the damaged state.

4. DAMAGE IDENTIFICATION RESULTS

The Hierarchical updating process is performed twice based on the data in the undamaged and damaged states. Figure 5 shows the histogram of the generated Gibbs samples from 72 sets of data at the damaged state. The first row shows the histogram of θ_{21} and θ_{41}, the added mass on segment 2 and 4 at test 1, respectively. The second row shows the distribution of the mean and standard deviation of θ_2, while the third row plots the mean and standard deviation of θ_4. Please note the estimation uncertainties of the mean and standard deviation of updating structural parameters. Figure 6 shows the "most probable" posterior distributions of θ_2 and θ_4 before and after damage, which correspond to the maximum a-posteriori estimates of the mean and standard deviation values.

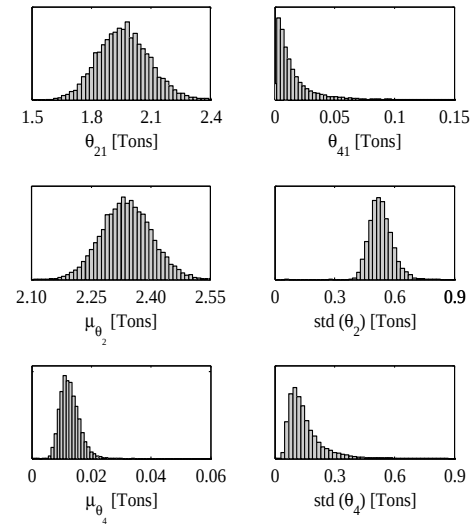

Figure 5: Histogram of updating parameters from Gibbs Samples.

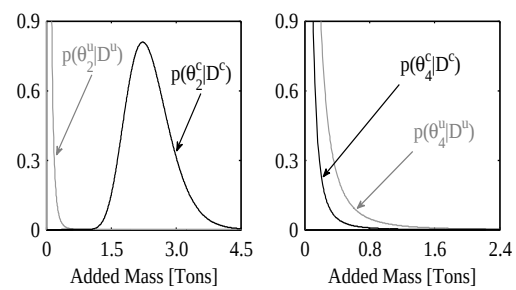

Figure 6: Most probable Posterior probability distributions before and after damage (loading).

The probability of added load (damage) at segment q exceeding d_q (tons) can be expressed as:

$$P\left[d_q\right] = P\left[\theta_q^c - \theta_q^u > d_q \mid \mathbf{D}^c, \mathbf{D}^u\right] \tag{18}$$

where u refers to the undamaged state and c refers to the current (damaged) state. The probability of damage exceeding a given d_q in Eq. (18) cannot be analytically calculated; however, it has a closed-form solution if $\mu_{\theta_q}^c, \sigma_{\theta_q}^{2^c}$, and θ_q^u are given (see Eq. (9)):

$$P\left[\theta_q^c > \theta_q^u + d_q \mid \theta_q^u, \mu_{\theta_q}^c, \sigma_{\theta_q}^{2^c}\right] = \frac{1}{2}\left(1 - erf\left(\frac{Log\left(\theta_q^u + d_q\right) - \mu_{\theta_q}^c}{\sqrt{2\sigma_{\theta_q}^{2^c}}}\right)\right) \quad (19)$$

where *erf* is the Gauss Error Function. Therefore, Monte Carlo simulation can be used to estimate $P[d_q]$ given different possible values of $\mu_{\theta_q}^c, \sigma_{\theta_q}^{2^c}$, and θ_q^u. From the results of Section 3.4, 10,000 Gibbs samples are available for $\mu_{\theta_q}^c, \sigma_{\theta_q}^{2^c}, \mu_{\theta_q}^u, \sigma_{\theta_q}^{2^u}$. To generate θ_q^u samples, one random value is generated from $N\left(\mu_{\theta_q}^u, \sigma_{\theta_q}^{2^u}\right)$ given each pair of the 10,000 previously generated Gibbs samples of $\mu_{\theta_q}^u, \sigma_{\theta_q}^{2^u}$. The PDF of the $P[d_q]$ can be estimated based on these 10,000 samples using kernel density estimation.

The probability of damage at segment 2 exceeding d_2 values are plotted in Figure 7. The color map represents the probability distribution at each damage value or each confidence level. The posterior uncertainties in the estimated probabilities stems from insufficient information of the data and modeling errors. Adding more data sets will narrow the width of the colored band, but the overall trend of the band centerline will remain the same as it represents the underlying variability of estimated mass. From this figure, the probability distribution of damage can be estimated at a specific confidence level as it is shown at 50% which corresponds to the most probable damage distribution. The line P=50% corresponds to the most probable damage. Similarly, the probability distribution of damage (added mass) to exceed a certain value can be obtained as it is shown in the figure for 2.24 tons.

5. CONCLUSION

In this paper, we applied a newly-developed Hierarchical Bayesian model updating method for damage identification of the Dowling Hall footbridge based on the identified modal parameters of the structure in the damaged and undamaged states. The main objective of this study is to represent the effects of changing environmental and ambient conditions in the posterior identification results. These effects are modeled as by introducing hyper-parameters for the updating structural parameters.

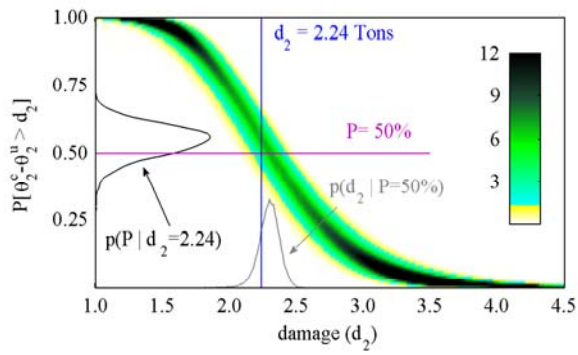

Figure 7: Posterior probability of damage exceedance.

A Gibbs Sampler was used to estimate the updating parameters and the corresponding damage distributions. Unlike our previous study in (Behmanesh and Moaveni, 2014) where the uncertainties of the updating structural parameters were reduced continuously by adding more data sets, the variability of the updating structural parameters are converged to their underlying variability using the implemented Hierarchical approach. The convergence is reached once the additional data do not correspond to new environmental and/or ambient conditions. It is worth noting that in this framework, addition of data sets reduces parameter estimation uncertainties (e.g., for the mean and standard deviations).

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