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The SL-AVV Approach to System Level Reliability-Based Design Optimization of Large Uncertain and Stochastic Dynamic Systems

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ABSTRACT: Recently a number of efficient reliability-based design optimization methodologies have been proposed for optimizing uncertain dynamic systems subject to stochastic excitation. While these methods are capable of handling a large number of uncertain parameters, they are generally applicable to problems characterized by small design variable vectors. This paper focuses on the development of a new reliability-based design optimization methodology for uncertain dynamic systems subject to stationary stochastic wind excitation that is capable of handling large design variable vectors, a characteristic of many practical design problems, while considering system level performance constraints.

1. INTRODUCTION

Reliability based design optimization (RBDO) is a powerful tool for obtaining structural systems that satisfy a number of probabilistic constraints posed with the aim of ensuring a satisfactory performance of the system. The advantages of such an approach over more classic deterministic optimization strategies are well known. However, this approach is far more computationally involved compared to its deterministic counterpart. This has hindered the widespread adoption of RBDO, especially for large and dynamic structural systems. This computational burden led to many of the early approaches to RBDO being based on classic analytical approximations of the reliability integrals of the probabilistic constraints. Recent advances in computational capabilities as well as achievements in the field of simulation-based reliability assessment have spawned a new generation of simulation-based RBDO strategies that are capable of handling problems characterized by large numbers of random variables, a typical property of uncertain structural systems subject to stochastic excitation (Schuëller and Jensen, 2008; Valdebenito and Schuëller, 2010). This paper focuses on the development of a novel simulation-based RBDO strategy aimed at solving problems that are not only characterized by a high number of random variables, but also by a high-dimensional design variable vector which, significantly complicates the situation (Valdebenito and Schuëller, 2010). This class of problems is often encountered in structural design optimization where a large number of members are to be designed (Spence and Kareem, 2014), or in applications of topology optimization where a high number of design variables are required in order to provide an adequate discretization of the design domain (Bobby et al., 2014). In particular, the method proposed in this work is based on the generalization of a recently proposed component-level simulation-based RBDO strategy (Spence and Kareem, 2014; Spence and Gioffrè, 2012) to wind excited systems characterized by system level constraints.

2. PROBLEM DEFINITION

The RBDO problems of interest to this work my be cast in the following form:

$$\text{Find} \qquad \mathbf{x} = \{x_1, ..., x_m\}^T \tag{1}$$

$$\text{to minimize} \qquad W = f(\mathbf{x}) \tag{2}$$

$$\text{s. t.} \quad P_f(\mathbf{x}) \leq P_0 \tag{3}$$

$$x_k \in \mathbf{X}_k \quad k = 1, ..., m \tag{4}$$

where $\mathbf{x}$ is a high-dimensional design variable vector containing the parameters that fully define the state of the system, e.g. section sizes, W is the material weight of the structural system, P_f is the system level failure probability, P_0 is the target system level failure probability while $\mathbf{X}_i$ is the discrete set to which the kth design variable must belong.

The RBDO problem outlined above is characterized, for the problems that are of interest to this study, by a high-dimensional design variable vector $\mathbf{x}$, a high-dimensional uncertain vector $\mathbf{U}$ describing the model uncertainty as well as a multi-variate stationary stochastic process, $\mathbf{F}$, describing the wind excitation. In order to describe damage, the following system level demand to capacity ratio will be considered:

$$d(\mathbf{u},\hat{\mathbf{r}},\mathbf{x}) = \max_{i=1,\ldots,N_r} \left\{ \max_{t\in[0,T]} \frac{|r_i(t;\mathbf{u},\mathbf{x})|}{c_i} \right\} \tag{5}$$

where N_r is the total number of components defining the system response, T is the event duration, $r_i(t)$ are the component response processes, $\hat{\mathbf{r}}$ is the vector collecting the largest values of r_i to occur during a given event of duration T while c_i is the capacity of the system. Obviously c_i is characterized by a significant amount of uncertainty and will therefore be modeled as a random variable in this work. Under these circumstances, a predefined damage state will occur if d is larger than 1. Therefore the limit state function of interest is the following:

$$g(\mathbf{u},\hat{\mathbf{r}},\mathbf{x}) = 1 - d(\mathbf{u},\hat{\mathbf{r}},\mathbf{x}) \tag{6}$$

while the failure probability of interest is:

$$\begin{aligned} P_f(\mathbf{x}) =& P(g(\mathbf{u},\hat{\mathbf{r}},\mathbf{x}) \leq 0) \\ =& \iint\limits_{g(\mathbf{u},\hat{\mathbf{r}},\mathbf{x})\leqslant 0} p(\hat{\mathbf{r}}|\mathbf{u})p(\mathbf{u})\mathrm{d}\hat{\mathbf{r}}\mathrm{d}\mathbf{u} \end{aligned} \tag{7}$$

where p indicates the conditional and non-conditional joint probability density functions of the uncertain vectors $\mathbf{U}$ and $\hat{\mathbf{R}}$. As indicated in Eq. (7), the random N_r-dimensional vector $\hat{\mathbf{R}}$ will in general depend on $\mathbf{U}$.

3. MECHANICAL MODELING

3.1. Excitation

It is common to describe the intensity of wind storms through the maximum wind speed, v_H to occur during the event at a height of interest H (e.g. building height) averaged over a fixed interval T (e.g. an hour) while considering a site specific roughness length z_0. Generally, wind speed data, v, is only available averaged over a period τ (often 3 s) and collected at a meteorological height H_{met} at regional airports characterized by a roughness length z_{01}. A probabilistic model for transforming this information into site specific data is the following:

$$\begin{aligned} v_H(T,z_0) = e_7 e_3(\tau,T) \left(\frac{e_5 z_0}{e_6 z_{01}}\right)^{e_4\delta} \\ \frac{\ln[H/(e_5 z_0)]}{\ln[H_{met}/(e_6 z_{01})]} e_2 e_1 v(\tau, H_{met}, z_{01}) \end{aligned} \tag{8}$$

where e_1 and e_2 are random variables modeling observational and sampling errors in v; e_4, e_5, and e_6 are random variables modeling the uncertainties with respect to the actual values of the empirical constant $\delta = 0.0706$ and of the roughness lengths z_0 and z_{01}, respectively; $e_3(\tau,T)$ is the conversion factor that accounts for the uncertainty in converting between wind speed averaging times; while e_7 is a model uncertainty to be used in the case of hurricanes and tornadoes.

The stochastic wind loads can be estimated directly from wind tunnel tests carried out on rigid scale models. In particular, each realization of the multi-variate stationary stochastic process defining the wind loads may be related to a scaled (length and time scales) realization of its wind tunnel counterpart, $\mathbf{f}_w(t)$, through the expression:

$$\mathbf{f}(t;\mathbf{u}) = w_1 w_2 w_3 \left(\frac{v_H}{v_{H_m}}\right)^2 \mathbf{f}_w(t) \tag{9}$$

where v_{H_m} is the simulated hazard intensity used in the wind tunnel tests while w_1, w_2 and w_3 are components of $\mathbf{U}$ and model the uncertainties associated with the estimation of building aerodynamics through the use of wind tunnels.

3.2. Component Response

In this work it is assumed that the response process, $r_i(t)$, associated with the ith failure mode of the system may be written in the following form:

$$r(t;\mathbf{u}) = s_1 \Gamma_r^T \left[\mathbf{K}\Phi_n \mathbf{q}_{r_n}(t) + \mathbf{f}(t)\right] \tag{10}$$

where Γ_r is a vector of influence functions giving the response in r due to a unit load acting at each degree of freedom of the system, $\Phi_n = [\phi_1, ..., \phi_n]$ is the mass normalized mode shape matrix of order n, $\mathbf{q}_{r_n}(t) = \{q_{r_1}(t), ..., q_{r_n}(t)\}^T$ is the vector of resonant modal displacement responses, $\mathbf{K}$ is the nominal (mean) stiffness matrix while s_1 is a random variable modeling the epistemic uncertainty in the load effect model of Eq. (10).

In Eq. (10) each component of $\mathbf{q}_{r_n}(t)$ is given by the solution of the following uncertain modal equation:

$$\ddot{q}_j(t) + 2s_{3_j}\zeta_j s_{2_j}\omega_j\dot{q}_j(t) + \left(s_{2_j}\omega_j\right)^2 q_j(t) = \phi_j^T\mathbf{f}(t) \tag{11}$$

where q_j, $\dot{q}_j$ and $\ddot{q}_j$ are the jth generalized displacement, velocity and acceleration response, ω_j is the nominal value of the jth circular frequency, s_{2_j} is an uncertain parameter modeling the variability in the estimate of ω_j, while ζ_j is the nominal value of the generalized damping ratio with s_{3_j} an uncertain parameter modeling the uncertainty that exists in the nominal value of ζ_j.

The jth component of $\mathbf{q}_{r_n}(t)$ is simply given by $q_{r_j}(t) = q_j(t) - q_{b_j}(t)$, where the background modal displacement q_{b_j} is given by:

$$q_{b_j}(t) = \frac{1}{\left(s_{2_j}\omega_j\right)^2}\phi_j^T\mathbf{f}(t) \tag{12}$$

4. Reliability Problem

As outlined in Eq. (7), in order to calculate the failure probability of the system, the conditional distributions of the largest values of the response functions $r_i(t)$ are needed. In particular, to this end, it is convenient to consider the following random reduced variate:

$$\hat{\Psi}_i(\mathbf{u}) = \frac{\hat{R}_i(\mathbf{u}) - \mu_{r_i}(\mathbf{u})}{\sigma_{r_i}(\mathbf{u})} \tag{13}$$

where μ_{r_i} and σ_{r_i} are the mean and standard deviation of the stationary response process $r_i(t)$ conditioned on $\mathbf{u}$. For Gaussian systems (a typical characteristic of the stochastic response of multistory buildings), the conditional distribution of $\hat{\psi}_i$ can be estimated from classic results of time-variant reliability:

$$P(\hat{\Psi}_i \leq \hat{\psi}_i|\mathbf{u}) = [1 - G_0(\hat{\psi}_i)]\exp\left[\frac{-\upsilon^+(\hat{\psi}_i,\mathbf{u})T}{1 - G_0(\hat{\psi}_i)}\right] \tag{14}$$

where $\upsilon^+(\hat{\psi}_i,\mathbf{u})$ is the up-crossing rate of the normalized threshold $\hat{\psi}_i$ conditional on $\mathbf{u}$ while $G_0(\hat{\psi}_i,\mathbf{u})$ is the probability that the system response is above the threshold $\hat{\psi}_i$ at time equal to zero and is given by:

$$G_0(\hat{\psi}_i) = \exp\left[-\frac{\hat{\psi}_i}{2}\right] \tag{15}$$

The conditional crossing rate may be estimated as:

$$\upsilon^+(\hat{\psi}_i,\mathbf{u}) = \kappa(\hat{\psi}_i,\mathbf{u})r^+(\hat{\psi}_i,\mathbf{u}) \tag{16}$$

where r^+ is given by the conditioned Rice formula as:

$$r^+(\hat{\psi}_i,\mathbf{u}) = \frac{\sigma_{\dot{r}_i}(\mathbf{u})}{2\pi\sigma_{r_i}(\mathbf{u})}G_0(\hat{\psi}_i) \tag{17}$$

with $\sigma_{\dot{r}_i}$ the standard deviation of the derivative of the response process $r_i(t)$ while κ is the correction factor accounting for any dependency between successive crossings of the normalized threshold $\hat{\psi}_i$ and can be modeled as:

$$\kappa(\hat{\psi}_i,\mathbf{u}) = 1 - \exp\left[-\frac{(k(\mathbf{u}))^{1.2}}{(2\pi)^{0.1}}\hat{\psi}_i\right] \tag{18}$$

where k is the spectral shape factor given by:

$$k(\mathbf{u}) = \sqrt{2\pi\left(1 - \frac{\gamma_1^2(\mathbf{u})}{\gamma_0(\mathbf{u})\gamma_2(\mathbf{u})}\right)} \tag{19}$$

where γ_p for $p = 0, 1, 2$ are the spectral moments of r_i given by:

$$\gamma_p(\mathbf{u}) = \int_0^\infty \omega^p S_{r_i}(\omega;\mathbf{u})\mathrm{d}\omega \tag{20}$$

where S_{r_i} is the one-sided spectrum of r_i while ω is the circular frequency.

4.1. Conditional Response Statistics

In defining the distributions of the reduced variates $\hat{\psi}_i$, the conditional (on $\mathbf{u}$) response statistics are needed. To this end, the mean conditioned response, μ_{r_i}, is simply given by the expected value of Eq. (10) given $\mathbf{u}$ and therefore by:

$$\mu_{r_i}(\mathbf{u}) = s_1 \Gamma_{r_i}^T \bar{\mathbf{f}} \tag{21}$$

where $\bar{\mathbf{f}}$ is the expected value of $\mathbf{f}$. The second order conditioned response statistic, $\sigma_{r_i}^2$, may be estimated in the frequency domain as:

$$\sigma_{r_i}^2(\mathbf{u}) = \sigma_{r_{i_b}}^2(\mathbf{u}) + \int_0^\infty S_{r_{r_n}}(\omega;\mathbf{u})\mathrm{d}\omega \tag{22}$$

where $\sigma_{r_{i_b}}^2$ is the background response variance while $S_{r_{r_n}}$ is the one-sided resonant response spectrum estimated considering the participation of n vibration modes. The estimate of the background contribution to σ_r^2 is given by:

$$\sigma_{r_{i_b}}^2(\mathbf{u}) = s_1^2 \Gamma_{r_i}^T \mathbf{C_f}(\mathbf{u}) \Gamma_{r_i} \tag{23}$$

where $\mathbf{C_f}(\mathbf{u})$ is the covariance matrix of the excitation $\mathbf{f}$ conditioned on $\mathbf{u}$. The resonant response contribution to $\sigma_{r_i}^2$ can be efficiently estimated through a double modal spectral proper orthogonal decomposition (POD) of $\mathbf{f}(t)$ (Carassale et al., 2001; Spence and Kareem, 2013). Following this framework, $S_{r_{r_n}}$ is estimated as:

$$S_{r_{r_n}}(\omega;\mathbf{u}) = \sum_{i=1}^{l} \tilde{F}_i(\omega;\mathbf{u})\tilde{F}_i^*(\omega;\mathbf{u}) \tag{24}$$

where the symbol * indicates the transposed complex conjugate, l is the number of spectral loading modes used in the estimation of $S_{r_{r_n}}$ while $\tilde{F}_i$ is given by:

$$\tilde{F}_i(\omega;\mathbf{u}) = s_1 \Gamma_{r_i}^T \mathbf{K} \Phi_n \mathbf{H}_{r_n}(\omega;\mathbf{u}) \Phi_n^T \chi_i(\omega)\sqrt{\Lambda_i(\omega)} \tag{25}$$

where χ_i is the ith frequency dependent loading eigenvector, Λ_i is the corresponding ith frequency dependent loading eigenvalue while $\mathbf{H}_{r_n}$ is the diagonal resonant mechanical transfer function (Spence and Kareem, 2013).

The other spectral moments, i.e. γ_1 and $\gamma_2 = \sigma_{\dot{r}_i}^2$, may be efficiently estimated through Eq. (20) where S_{r_i} is assessed in a strictly modal setting by simply substituting into $\tilde{F}_i$ (Eq. 25) the classic mechanical transfer function instead of $\mathbf{H}_{r_n}$.

4.2. Solution Strategy

In defining an efficient solution strategy for the reliability integral Eq. (7), it is convenient to write the integral in terms of the vector of reduced variates, $\hat{\Psi} = \{\hat{\psi}_1, ..., \hat{\psi}_{N_r}\}^T$:

$$P_f(\mathbf{x}) = \iint_{g(\mathbf{u},\hat{\psi},\mathbf{x})\leqslant 0} p(\hat{\psi}|\mathbf{u})p(\mathbf{u})\mathrm{d}\hat{\psi}\mathrm{d}\mathbf{u} \tag{26}$$

For the problems of interest to this work, the above integral will be of high dimensions therefore ruling out the use of classical reliability methods (Schuëller et al., 2003). In particular, in this work Monte Carlo simulation is adopted. In generating conditional samples of the random vector $\hat{\Psi}$, the components are considered independent and therefore fully described by their marginal distributions given in Eq. (14). The grounds for making this assumption lies in the fact that the dependency between the components of $\hat{\Psi}$ models the dependency between the peaks of the normalized response processes, which are only weakly dependent on $\mathbf{u}$ through the crossing rate. Therefore $\hat{\Psi}$ will be only weakly dependent on parameters such as the mean wind speed v_H defining the intensity of the wind event, which significantly contributes to the dependency of the peaks of the various response functions $r_i(t)$.

5. THE SYSTEM LEVEL RBDO APPROACH

This section presents a new system level RBDO algorithm, SL-AVV, that leverages the recently introduced concept of Auxiliary Variable Vector (Spence and Kareem, 2014).

5.1. Problem Definition

The failure probability of Eq. (7) is a system failure probability defined as the union of the following component level failure events:

$$F_i = \left\{ \frac{|\hat{r}_i|}{c_i} > 1 \right\} \qquad i = 1, ..., N_r \tag{27}$$

Here the failure probability of Eq. (7) is first written in terms of the component failure modes as:

$$P_f = P\left\{\bigcup_{i=1}^{N_r} F_i\right\} = \sum_{i=1}^{N_r} P\{F_i\} - \alpha\left(\sum_{i=1}^{N_r} P\{F_i\}\right) \tag{28}$$

where α is simply given by:

$$\alpha = \left(\sum_{i=1}^{N_r-1}\sum_{j>i}^{N_r} P\{F_i \cap F_j\} - \sum_{i=1}^{N_r-2}\sum_{j>i}^{N_r-1}\sum_{k>j}^{N_r} P\{F_i \cap F_j \cap F_k\} + \ldots - (-1)^{N_r-1} P\{F_i \cap \ldots \cap F_{N_r}\}\right) \Bigg/ \left(\sum_{i=1}^{N_r} P\{F_i\}\right) \tag{29}$$

It is then assumed that the component failure probabilities can be approximated as exponential distributions. This allows the failure probability to be written as (where the dependency on the design variables is now indicated):

$$P_f(\mathbf{x}) = (1-\alpha(\mathbf{x}))\left(\sum_{i=1}^{N_r} \exp\left[-\frac{1}{\mu_{D_i}(\mathbf{x})}\right]\right) + \Delta(\mathbf{x}) \tag{30}$$

where μ_{D_i} is the expected value of the random component demand to capacity ratio given by $\hat{R}_i/C_i$ while Δ is the error term introduced to account for the assumption made on the component failure distributions. If it is assumed that α and Δ of Eq. (30) are independent of the design variable vector $\mathbf{x}$, then the dependency of the system level failure probability on $\mathbf{x}$ is exclusively in terms of the mean values of the random component demand to capacity ratios D_i. The problem therefore becomes the description of the dependency of μ_{D_i} on $\mathbf{x}$. To this end the concept of AVV (Spence and Kareem, 2014) can be leveraged.

Before continuing, it should be observed that the assumption of independence of α from $\mathbf{x}$ is equivalent to assuming that the change in the probability of the joint occurrence of the various failure modes due to a change in $\mathbf{x}$, will in general follow the trend of the sum of the probabilities of the individual failure events.

5.2. *The AVVs*

In order to derive the aforementioned AVVs, it is first convenient to consider the following variables defined for each realization of $\mathbf{U}$ and $\hat{\Psi}$ and for the current design variable vector $\mathbf{x}_0$:

$$\Upsilon_i(\mathbf{u}, \hat{\psi}_i, \mathbf{x}_0) = \frac{\hat{\psi}_i(\mathbf{u},\mathbf{x}_0)\mathbf{C_L}(\mathbf{u},\mathbf{x}_0)\Gamma_{r_i}(\mathbf{x}_0)}{\sigma_{r_i}(\mathbf{u},\mathbf{x}_0)} \tag{31}$$

where $\mathbf{C_L}$ is the covariance matrix of the following vector:

$$\mathbf{L}(t;\mathbf{u},\mathbf{x}_0) = s_1\left[\mathbf{K}\Phi_n\mathbf{q}_{r_n}(t) + \mathbf{f}(t)\right] \tag{32}$$

Dividing by the value of the capacity contained in $\mathbf{u}$, the following static relationship is determined for the damage ratio d_i associated with the realizations of $\mathbf{u}$ and $\hat{\psi}$:

$$d_i(\mathbf{u},\mathbf{x}_0) = \frac{1}{c_i}\Gamma_{r_i}^T(\mathbf{x}_0)\Upsilon_i(\mathbf{u},\mathbf{x}_0) = \Gamma_{r_i}^T(\mathbf{x}_0)\Upsilon_{d_i}(\mathbf{u},\mathbf{x}_0) \tag{33}$$

where Υ_{d_i} is simply given by the ratio between Υ_i and c_i. The realizations of the vector Υ_{d_j}, generated during the simulation process used to estimate the reliability of the system, may be used to define the following AVV:

$$\tilde{\Upsilon}_i(\mathbf{x}_0) = \bar{\Upsilon}_{d_i}(\mathbf{x}_0) \tag{34}$$

where $\bar{\Upsilon}_{d_i}$ is the expected value of Υ_{d_i}.

The significance of the AVV, and so of $\tilde{\Upsilon}_i$, is that if it is statically applied to the nominal structure it will cause a response in r_i that is equal to the expected value of the damage ratio D_i. In other words the following relationship holds:

$$\mu_{D_i}(\mathbf{x}_0) = \Gamma_{r_i}^T(\mathbf{x}_0)\tilde{\Upsilon}_i(\mathbf{x}_0) \tag{35}$$

This relationship is particularly useful as it allows the failure probability of Eq. (30) to be written in terms of what may be considered, for all intents and purposes, a series of static load distributions.

5.3. *The Approximate Subproblem*

The definition $\tilde{\Upsilon}_i$ together with the formulation of the failure probability given in Eq. (30) can be used to define a approximate optimization subproblem, the sequential definition and solution of which will lead to a final optimal solution. To this end, if the assumption of independence of α and Δ from the design variable vector is extended to the AVV $\tilde{\Upsilon}_i$, then the following optimization problem can be cast that is completely defined from the results of a single reliability analysis carried out in the current design point $\mathbf{x}_0$:

$$\text{Find} \qquad \mathbf{x} = \{x_1, ..., x_m\}^T \tag{36}$$

$$\text{to minimize} \qquad W = f(\mathbf{x}) \tag{37}$$

s. t.

$$(1-\alpha)\left(\sum_{i=1}^{N_r} \exp\left[-\frac{1}{\Gamma_{r_i}^T(\mathbf{x})\tilde{\Upsilon}_i}\right]\right) + \Delta \leqslant P_0 \tag{38}$$

$$x_k \in \mathbf{X}_k \quad k = 1, ..., m \tag{39}$$

Not only does the subproblem outlined above decouple the reliability problem from the optimization loop, but it also takes on an extremely convenient form as it can be easily made explicit in terms of $\mathbf{x}$ by simply defining an explicit expression of Γ_{r_i} in terms of $\mathbf{x}$. In defining this explicit relationship, a number of classic results can be used that were developed for the optimization of deterministic structures under static loads. In particular, here the solution proposed by Chan et al. (1995) was adopted. Once the subproblem has been made explicit in $\mathbf{x}$, any gradient-based optimization strategy can be used to find solutions. Here the pseudo-discrete optimality criteria (Chan et al., 1995) is used, therefore allowing the discrete nature of the design space to be fully considered.

Once a solution is found to a subproblem defined in $\mathbf{x}_0$, a new sub-problem must be formulated in the design point found from the solution of the previous sub-problem. This updating procedure defines a design cycle and will continue until the design point of two successive design cycles are identical.

6. Case Study

6.1. *Description*

The case study consists of a 45-story rectangular building with an offset core (Fig. 1a). The columns consist of steel box sections and are grouped in plan as indicated in Figure 1b (C1 to C18). In particular, it is required that the mid-line diameter of the box sections, D_i, belong to the discrete set $\{0.2\text{ m}, 0.25\text{ m}, 0.3\text{ m}, 0.35\text{ m}, ..., 2\text{ m}\}$ while the flange thickness is fixed at $D_i/20$. The columns are grouped over three consecutive floors. The beams, B1 to B6, are grouped in plan as indicated in Figure 1b and are required to belong to the family of AISC W24 steel profiles. The beams are also grouped three floors at a time (for a total of 6×15 groups). The diagonal bracings are also required to belong to the AISC W24 steel profiles and are grouped as pairs over the height of the building. The aforementioned grouping results in 375 independent design variables. Initially the structure is designed with all columns having a mid-line diameter equal to 0.6 m while all beams and diagonals are set to W24×176 profiles. The first three structural modes, with mean circular frequencies $\omega_1 = 1.023$ rad/s, $\omega_2 = 1.118$ rad/s and $\omega_3 = 1.847$ rad/s, are considered sufficient for describing the resonant response. The mean modal damping ratios were taken as 1.5 %. The uncertain parameters S_1, S_{2_j} and S_{3_j} with $j = 1, ..., 3$ were modeled as independent lognormal random variables with coefficients of variation 0.025, 0.015, and 0.3 respectively.

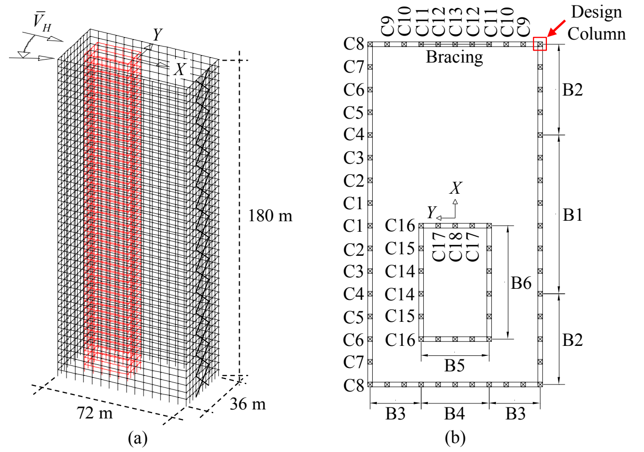

Figure 1: (a) 3D view of the 45-story building; (b) structural layout of the building showing beam and column assignments and the critical column.

Non-structural damage is to be controlled for wind blowing down the X direction (Fig. 1). In particular, it is assumed that damage is associated with the X and Y interstory drift response of the critical column line illustrated in Fig. 1b. The capacities, C_i, are taken as independent lognormal random variables with mean 1/400 of the story height and coefficient of variation 0.25.

The building is considered to be located in the Miami area of Florida, USA. The extreme wind hazard is given by the risk of hurricanes. In particular, the site of the building is characterized by a roughness length $z_0 = 2$ m while an averaging time of $T = 3600$ s was considered. The set of wind speeds used to defined the distribution of v (Eq. (8)) was obtained from the simulated hurricane database of the National Institute of Standards and Technology while considering milepost 1450 where a roughness length of $z_0 = 0.05$ m was assumed together with a meteorological height of $H_{met} = 10$ m. The averaging time for this data was $\tau = 60$ s. The distributions and parameter values assumed for E_1 to E_7 can be found in Table 2 of Spence and Kareem (2014) while considering a mean for E_3 of 0.8065. The stochastic loads acting on the building were derived from the Tokyo Polytechnic University Wind Pressure Database. In particular, the mean wind speed during the tests was $v_{H_m} = 11$ m/s. Through integration and appropriate scaling, realizations of $\mathbf{f}_w$ were obtained. The distributions and parameter values assumed for W_1 to W_3 can be found in Table 2 of Spence and Kareem (2014) while considering a coefficient of variation of 0.05 for W_1. For the case study under investigation the reliability integral needs to be calculated over the space of $\mathbf{U} = \{C_1, ..., C_{90}, S_1, \mathbf{S}^T_{2_n}, \mathbf{S}^T_{3_n}, W_1, ..., W_3, E_1, ..., E_7, V\}^T$, with $n = 3$, and $\hat{\Psi}$, resulting in a total dimension of 198 which constitutes a large scale reliability problem. Considering the size of $\mathbf{x}$, $m = 375$, the problem truly represents a large scale RBDO optimization problem.

6.2. Results

Two cases are presented in this section with the first considering the components of random vector $\hat{\Psi}$ independent and the second considering the components of $\hat{\Psi}$ fully correlated with the aim of quantifying the assumption of Sec. 4.2. P_0 was set at 2×10^{-2}. Figure 2 shows the convergence history of the objective function which here coincides with the weight of the material composing the structural system while Fig. 3 reports the history of the system level failure probability. From Figs. 2 and 3 it is immediately evident the strong convergence properties of the proposed SL-AVV algorithm. Indeed, the problem practically converges after only 6 design cycles which means that the simulation-based reliability analysis only had to be invoked 6 times before solutions are found. It is also evident that the assumption that the random vector $\hat{\Psi}$ has independent components would, at least for this example, seem to be perfectly reasonable with little difference between the two extreme cases. Figure 4 reports the variation of α (Eq. (29)) during the optimization process. While α is seen to change, its variation quickly drops off after design cycle 4 therefore allowing for the quick overall convergence seen in Figs 2 and 3. Fig. 5 shows the initial and final failure distributions for the two cases. Once again the effectiveness of the proposed

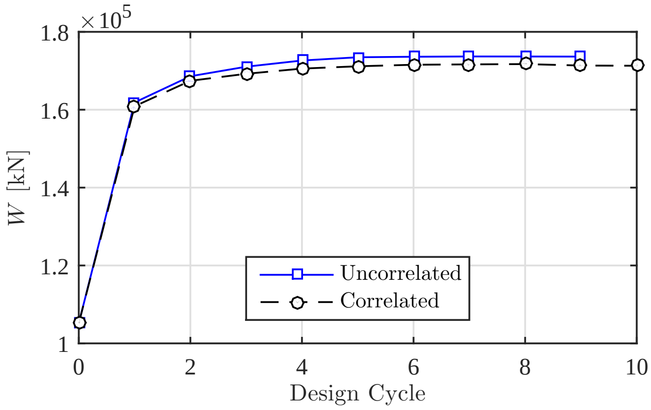

Figure 2: Design history of the objective function.

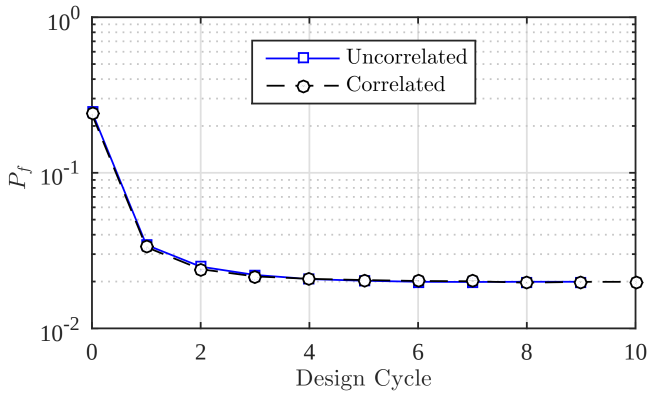

Figure 3: Failure probability history.

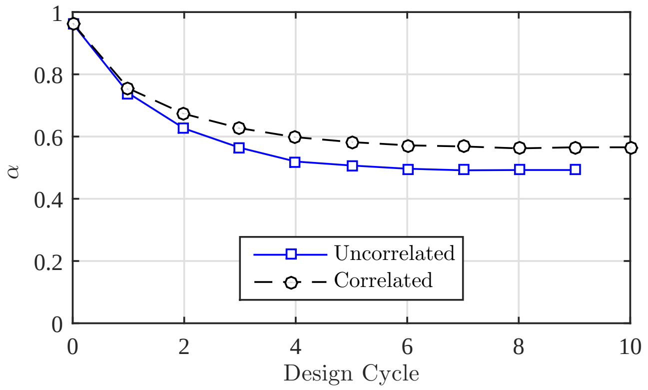

Figure 4: History of the parameter α (Eq. (29)).

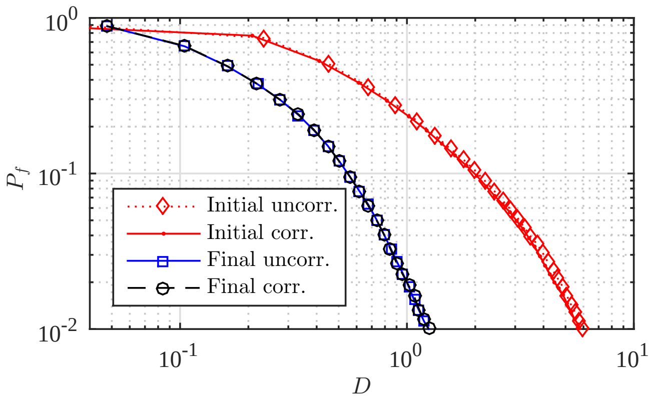

Figure 5: Initial and final failure probability distributions.

method is evident. Finally, it should be observed that the failure probability of the converged design is exact as for any design cycle in which **x** does not change (e.g. at convergence), all approximations used in the proposed approach are exact.

7. Conclusions

This paper presented a new simulation-based system level reliability based design optimization algorithm, SL-AVV, for large scale uncertain and dynamic systems excited by stationary stochastic wind excitation. The method was based on the concept of decoupling the reliability analysis from the optimization loop through the definition of a sequence of high quality subproblems defined in terms of a number of Auxiliary Variable Vectors (AVVs) that allow the system failure probability to be written in terms of a sum of exponential functions modeling the failure modes of the system. The structure of the AVVs allows the subproblem to be easily made explicit in the design variable vector, and therefore any gradient-based optimization algorithm may be used for its resolution. The effectiveness of the proposed approach lies in how each subproblem can be defined exclusively in terms of a single simulation-based reliability analysis carried out in the current design point. The practicality and strong convergence properties of the proposed approach were illustrated on a full scale building example characterized by a high-dimensional design variable vector as well as a high-dimensional system-level reliability integral.

8. References

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