Sensitivity of Ground Motion Simulation Validation Criteria to Filtering

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ABSTRACT: The validation of ground motion synthetics has received increased attention over the last few years due to advances in physics-based deterministic and hybrid simulation methods. Validation of synthetics is necessary in order to determine whether the available simulation methods are capable of faithfully reproducing the characteristics of ground motions from past earthquakes. Some validation methods use filters to evaluate the quality of the fit between synthetics and data within different frequency bands. This is done, primarily, to weight the contribution of different wavelengths so that low frequencies are given more weight than high frequencies. One particular method of interest is the goodness-of-fit (GOF) criterion introduced by Anderson (2004). In this method, the degree of similarity between two signals is quantified by means of a set of ten (error) metrics which are projected on a 0–10 scoring scale. These metrics are based on ground motion characteristics commonly used in seismology and earthquake engineering. The scores are used to evaluate each given pair of signals in the three components of motion and within different frequency ranges. The scores of each frequency band, component, and metric are then combined to obtain a final GOF score. In this paper we study the sensitivity of the GOF scores, and thus the sensitivity of the validation itself, to the filtering process when different filter parameters are considered. Our initial analysis of results show that GOF methods are susceptible to the design of filters. The filter's order, for instance, seems to significantly affect the interpretation of the validation—especially for metrics that are time-dependent (e.g., peak ground response). We evaluate the implications of the variability in GOF scores on the case study of the 2008 M_w 5.4 Chino Hills earthquake, and investigate the sensitivity of GOF criteria to the type of filters used to decompose the signals. We analyze the consistency and correlation of the results obtained using various metrics by means of a filtering fitting function. Our work indicates that elliptic infinite impulse response filters lead to more reliable results, over other more commonly used filters; and we provide guidance on the selection of elliptic filter parameters.

1. INTRODUCTION AND BACKGROUND

With recent advances in earthquake ground motion simulation using physics-based methods and hybrid approaches (e.g., Bielak et al., 2010; Graves and Pitarka, 2010), verification and validation (V&V) of synthetics have become increasingly relevant. V&V imply processing signals in preparation to perform qualitative and quantitative comparisons. We are particularly interested in investigating the effect that different filtering procedures and parameters can have on quantitative validations done using the goodness-of-fit (GOF) method introduced by Anderson (2004). In this method, the degree of similarity between two signals is quantified by means of a set of ten (error) metrics, labeled C1 through C10. These metrics include ground mo-

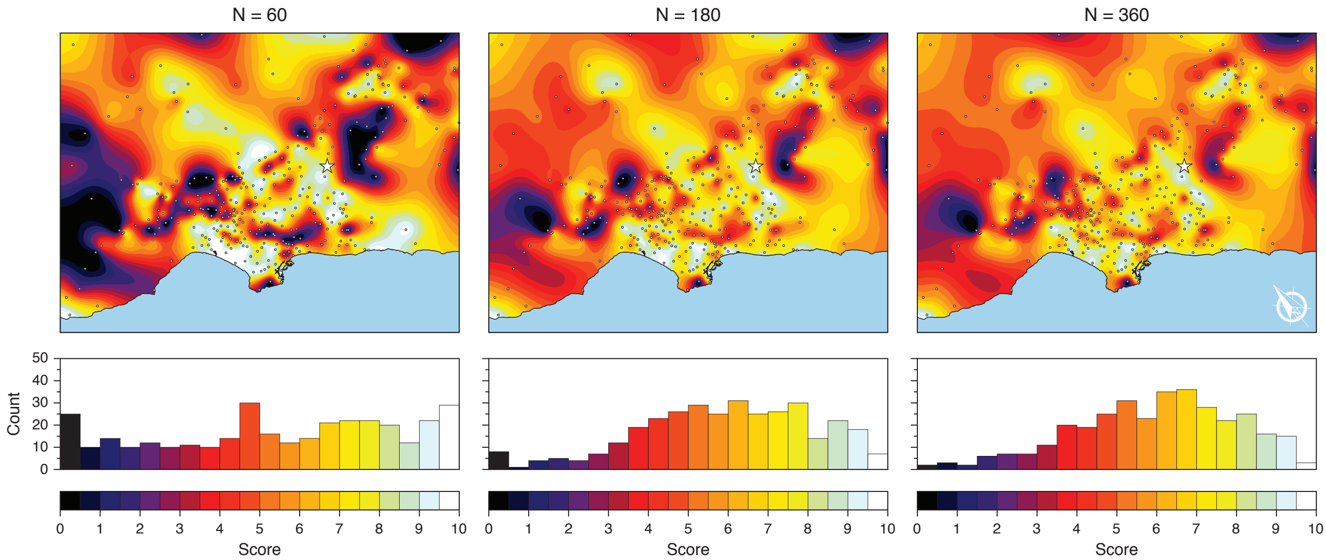

Figure 1: Variation of PGA GOF scores for the case study of the 2008 M_w 5.4 Chino Hills, California, earthquake using FIR Chebyshev filters with order $N = 60$ (left), 180 (center) and 360 (right). Top frames show contours drawn based on GOF values derived from 336 stations with records used for the comparisons against synthetics from Taborda and Bielak (2013). Bottom frames show histogram distributions in the GOF scale introduced by Anderson (2004).

tion characterization parameters commonly used in seismology and earthquake engineering. The quantitative comparisons are projected on a 0–10 scoring scale, where a score of 10 corresponds to a perfect match. When using Anderson's method, signal pairs are compared by component of motion and at different frequency ranges to weight the contribution of different wavelengths so that low frequencies are given more weight than high frequencies. The scores of each frequency band, component, and metric are then combined to obtain a final GOF value.

In two initial studies (Khoshnevis and Taborda, 2014a,b) we found that filtering, in particular, can play a significant role in the output of the GOF scores. In Khoshnevis and Taborda (2014a) we showed that the variation of the order of a finite impulse response (FIR) filter has a substantial influence on the GOF scores, especially on those metrics that are strongly time dependent. Therefore, among the metrics considered in the GOF method, the Arias intensity score, and the peak acceleration, velocity and displacement scores are most sensitive; followed by the scores derived from the energy integral, energy duration, Arias duration, cross correlation, response spectra and Fourier spectra; with the latter being the least sensitive of all.

The overall effect of different filter orders on the validation of simulations is illustrated in Figure 1. This figure shows the contour maps of the GOF scores obtained for a simulation of the 2008 M_w 5.4 Chino Hills, California, earthquake (Taborda and Bielak, 2013). These maps are composed using the GOF values obtained from comparisons between 336 signal pairs (synthetics and data). In particular, the maps correspond to the GOF scores for peak ground acceleration (PGA) metric C5 in Anderson (2004), where Chebysehv FIR filters with order $N = 60$, 180 and 360, were used in the band-pass frequency analysis. We recognize these levels of a filter's order may sound very high in the context of seismology and earthquake engineering, where other types of filters have been traditionally preferred, yet as we will see in the following section, they are perfectly normal. In all, Figure 1 shows that GOF maps can be influenced by the filter characteristics.

More important, even though FIR filters approach a perfect filter with increasing order, we have found that GOF scores do not necessarily con-

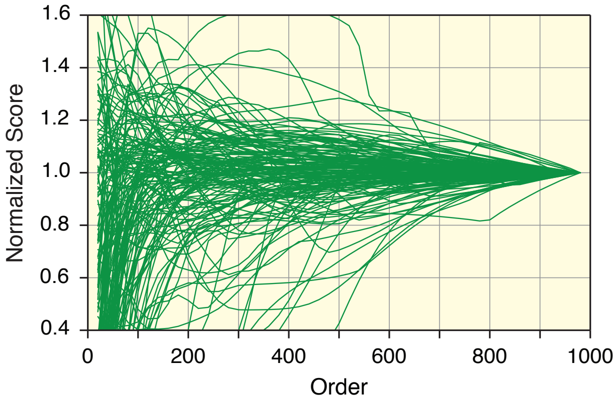

Figure 2: Variation of PGA GOF scores with increasing order of FIR Chebyshev filter for the same 336 stations used for the validation of the Chino Hills earthquake simulation.

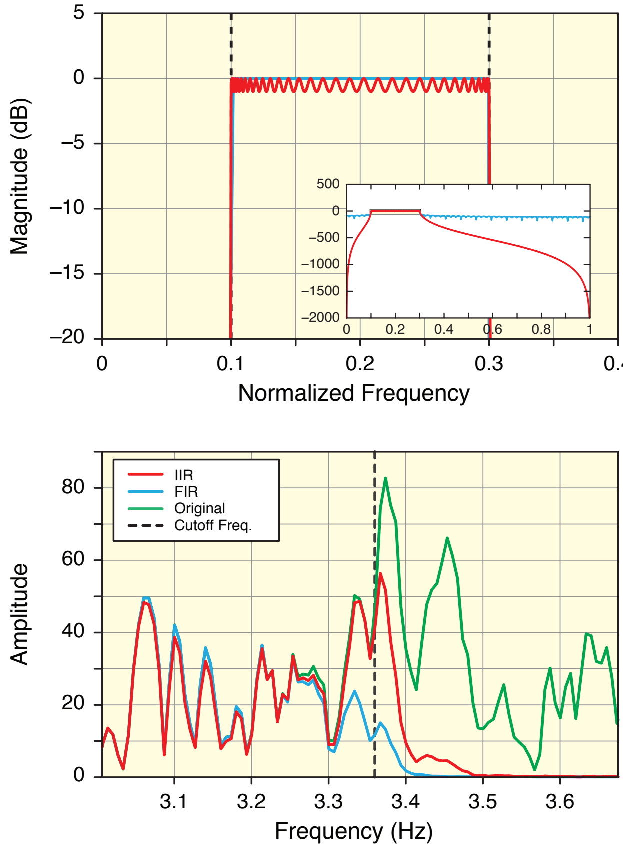

Figure 3: Comparison between FIR and IIR filters. Top: amplitude response (dB) of the bandpass filters' design in normalized frequency for corner frequencies equal to 0.1 and 0.3 of the maximum frequency. Bottom: detail of a Fourier amplitude spectra around the upper cutoff frequency of a signal filtered with both IIR and FIR filters (the original signal is also included).

verge with increasing filter order. Figure 2 shows the variation of PGA GOF scores with filter order for the same 336 signal pairs from the Chino Hills simulation. Each score trend is normalized with respect to the value obtained with the highest order. Despite the chosen normalization, the substantial variability over the range of order values suggests that, even for higher order filters, there is not one stable score.

Using different types of filters can also be influential on the results. In Khoshnevis and Taborda (2014b) we implemented a Chebyhsev I infinite impulse response (IIR) filter, instead. As we will explain in greater detail below, IIR filters have sharper cutoffs at the corner frequencies, yet they present ripples in the passband. Figure 3 shows the frequency response of a FIR and an IIR bandpass filter and a comparison of a portion of the Fourier amplitude spectra around the cutoff frequency of a signal filtered using these two type of filters, along with the unfiltered signal. As it can be seen in this figure, the amplitudes of the FIR filtered signal and the original signal are nearly identical within a portion of the passband, but they differ from each other over a wider transition zone than that of the IIR filter. In turn, the IIR filter does not have a perfect match within the passband, but it exhibits a sharper decay past the cutoff frequency.

Arguably, for most applications, these differences are minor and inconsequential. Their effect on GOF scores used for validation may, however, become relevant, especially in the absence of a well-accepted standard. Moreover, since simulations involve numerous unknowns and assumptions, it is imperative to minimize the additional uncertainties that post-processing of the data may have on the analysis of results. In this paper we seek to contribute to reducing such additional uncertainties by investigating the characteristics of filters used in ground motion signal processing for validation purposes.

While it would be difficult to decide on a final ideal filter, it is desirable to define a filter that can be used consistently in validation studies with minor effects on the analysis. Here, we contribute to such

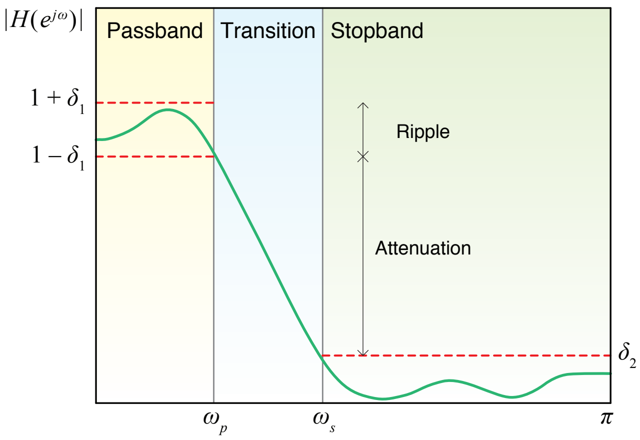

Figure 4: Specification for effective frequency response for the case of low-pass filter. (the discrete-time system).

an objective. We review the characteristics of the most commonly used FIR and IIR filters, and make initial recommendations on the selection of suitable filter for ground motion simulation validation.

2. FINITE AND INFINITE IMPULSE RESPONSE FILTERS

Filters are a particularly important class of linear time-invariant systems. Strictly speaking, the term frequency-selective filter suggests the existence of a system that passes certain frequency components and rejects all others. In practice, however, filters are far from ideal, and there are different kinds of filters, each of them with advantages and disadvantages. To aid the discussion, in Figure 4, we show the main characteristics of a low-pass filter in the frequency domain. Here, δ_1 is the maximum ripple's amplitude in the passband, δ_2 is maximum ripple's amplitude in the stopband, ω_p is the normalized cut-off frequency in the passband, and ω_s is the normalized cut-off frequency in the stopband.

Digital filters can be broadly classified into two groups: infinite impulse response (IIR) and finite impulse response (FIR). FIR filters are very attractive because they are inherently stable (do not have poles) and can be designed to have linear phase. In FIR filters, the higher the order, the closer to an ideal filter. FIR filters, however, tend to have wider transient zones (see Figure 4) and become computationally expensive with increasing orders, due to the underlying convolution computation. FIR filters are almost entirely restricted to discrete-time implementations. Consequently, their design is based on directly approximating the desired frequency response of the discrete-time system. The simplest method of FIR filter design is called the window method. This method generally begins with an ideal desired frequency response that can be represented as

$$H_d\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} h_d[n]e^{-j\omega n} \,, \qquad (1)$$

where $h_d[n]$ is the corresponding impulse response sequence. Many idealized systems are defined by piecewise-constant or piecewise-functional frequency responses with discontinuities at the boundaries between bands. As a result, these systems have impulse responses that are non casual and infinitely long. The most straightforward approach to obtain a causal FIR approximation to such systems is to truncate the ideal response. This, however, introduces undesired wiggles. This is known as the Gibbs phenomenon (Oppenheim et al., 1989). Different windowing functions have been defined to reduce this effect. For our study, we examined four commonly used windowing methods: Chebyshev, Hamming, Hanning, and Gaussian.

IIR filters, on the other hand, are attractive because they are continuous in time, and thus can be used in real-time applications. They have also enjoyed preference because they transitioned naturally from analog to digital technologies, thus they continued to be used in practice as the technology evolved over the years. Typical frequency-selective continuous-time approximations are: Butterworth, Chebyshev, and elliptic filters. The former two being traditionally preferred in earthquake engineering for no evident reason other than, perhaps, the fact that closed-form design formulas of these continuous-time approximations facilitates their design. A Butterworth continues-time filter, for instance, is monotonic in both the pass-band and the stopband. A Type I Chebyshev filter has an equiripple characteristic in the passband and varies monotonically in the stopband. An elliptic filter is equiripple in both the passband and the stopband. All these approximation methods yield digital fil-

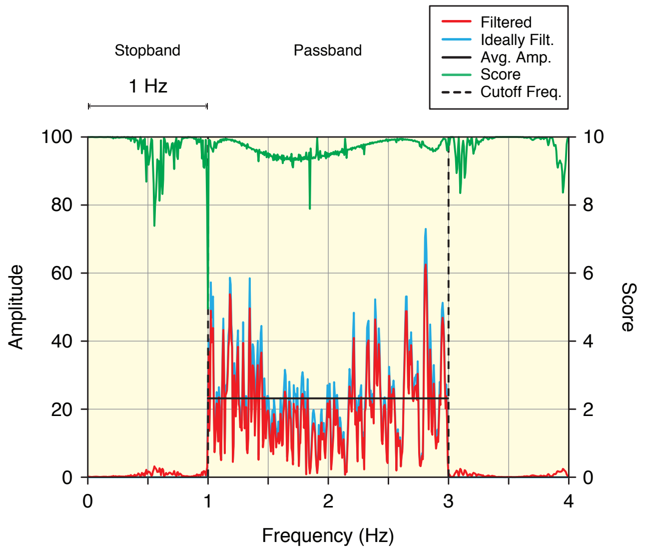

Figure 5: Evaluation of coherency between the ideal frequency response and filtered signal's response. Left vertical axes is the amplitude and right vertical axes is the score. The mean of amplitude in the inside section is shown as dashed line.

ters with non-constant group delay or, equivalently, non-linear phase. The greatest deviation from the constant group delay occurs, in all, cases at the edge of the passband or in the transition band. If phase linearity is not an issue, then elliptic approximation yields the lowest order system function, and therefore elliptic filters will generally require the least computation to implement a given filter specification (Oppenheim et al., 1989).

Altogether, the primary advantage of IIR filters over FIR filters is that the former typically meet a given set of specifications with a much lower filter order than its corresponding FIR filter. Although IIR filters have nonlinear phase, data processing using software such as MATLAB® is commonly performed at a post-processing stage (off-line), and therefore the entire data sequence is available for filtering. This allows one to use non-causal, zero-phase filtering approach by forward and backward filtering (two-pass) the signals (e.g., via the `filtfilt` function). This process eliminates the nonlinear phase distortion of IIR filters. Here, we adopt this strategy and all of our filters are zero phase. Consequently, we do not address phase variations and concentrate only on amplitude effects.

3. EVALUATION METHOD

It follows from the previous section that designing a filter entails a trade-off between the accuracy of the response of the passband, the transition zone, and the stopband. Different applications will therefore require different methods to assess the efficacy of a filter. The most common alternative for this is to compare the Fourier amplitude spectra. This evaluation is typically done visually. We, however, seek to define standards that provide a solid reference framework for simulation validation. That is, we are interested in filters that are good for narrow bands at low frequencies and wider bands at higher frequencies, with sharp transition zones, minimum ripples in the passband amplitudes, and with sufficient attenuation in the stopbands (see Figure 4).

In the context of the GOF analysis, Anderson (2004) defined the expression to evaluate the similarity between two Fourier amplitude spectra as:

$$S(p_1,p_2) = 10\exp\left\{-\left[\frac{p_1-p_2}{min(p_1,p_2)}\right]^2\right\}, \quad (2)$$

where p_1 and p_2 are the frequency amplitude of two waveforms evaluated at every frequency. Since the Fourier amplitude score in Anderson's GOF method is the least sensitive parameter to filtering, we are interested in using a similar function to evaluate the filters' selection.

Eq. (2) decreases monotonically as the difference between the parameters p_1 and p_2 increases. While this offers a good measure for almost all scales, it cannot characterize the differences when one of the parameters vanishes (becomes zero). This is important because in order to quantify the efficacy of the filtering process, we also need to consider the values in the stop band, which should be compared to zero. Therefore, inspired in (2), we define a new scoring function specifically to evaluate filters based on the Fourier amplitude in the stopband. The proposed expression is:

$$S(p_1,p_2) = 10\exp\left\{-\left[\frac{p_1-p_2}{F(a)}\right]^2\right\}, \quad (3)$$

where $F(a)$ is a function of the amplitude inside the passband. The scoring scale defined by Eq. (3)

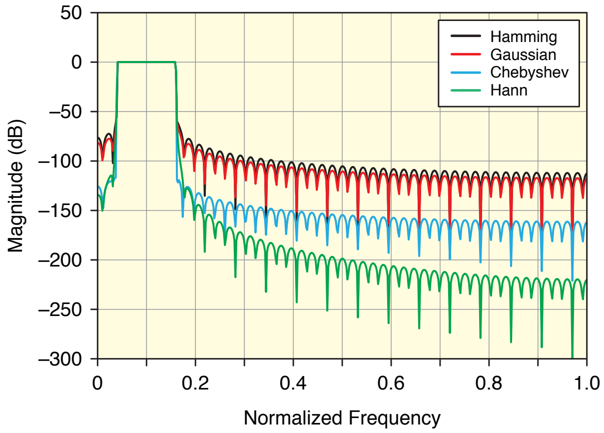

Figure 6: Comparison of the frequency response of four FIR filters.

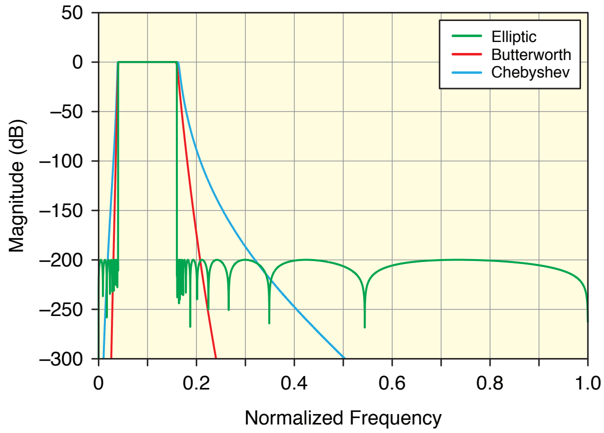

Figure 7: Comparison of the frequency response of three IIR filters.

avoids the division by zero and yet offers a signal-specific measure of the accuracy of the filter with respect to an ideal filter with amplitude zero in the stopbands.

We use Eq. (2) and Eq. (3) to evaluate the efficacy of filters by comparing the Fourier amplitudes of filtered signals with respect to the result that would be expected from an ideal filter, where the amplitude in the passband is the same amplitude of the original signal within the cutoff frequencies, and zero elsewhere. Figure 5 shows an example of this comparison and the resulting scores, using Eq. (2) for scoring the amplitude similarity in the passband and Eq. (3) for scoring the filtered signal remnant amplitude in the stopbands. The whole domain for comparing is max ±1 Hz above and below the cutoff frequencies. For the the reference amplitude in Eq. 3, we assume

$$F(a) = 0.25\,\mathrm{mean}\left([f_1, f_2]\right)\ . \tag{4}$$

The values are computed for each discrete frequency and the final score is the mean of the score at all frequencies within the comparison range.

4. RESULTS

As we mentioned before, we are interested in defining a standard set of filter parameters that can be used for accurate and consistent GOF analysis in validation of ground motion simulation. To facilitate this, and for practical reasons, all the filters we considered here are designed using functions in the Signal Processing Toolbox of MATLAB®.

We first consider four different FIR filters: Hamming, Gaussian, Chebyshev, and Hanning. Their magnitude in frequency is shown in Figure 6. All these filters were designed using windowing methods with order equal 4000. Note again that FIR filters allow very high orders. Such orders are uncommon in signal processing in earthquake engineering or seismology, where low-order (yet unstable) IIR filters such as Butterworth have been traditionally preferred. In Khoshnevis and Taborda (2014a) we showed that, for FIR filters, GOF values converge as the filter's order increases. Figure 6 also shows that in the case of FIR filters, all four options have very similar steepness at the cutoff frequencies, yet different levels of attenuation. From these filters, we choose Chebyshev as representative of the FIR family.

Figure 7, on the other hand, shows the magnitude of three IIR filters: Elliptic, Butterworth, and Chebyshev. These filters were designed with order values of 57, 57 and 17, respectively. We chose a different order for latter because higher orders for Chebyshev rapidly develop strong ripples. In other words, order 17 for Chebyshev is as high an order as 57 is for Elliptic and Butterworth. In this case, it is easily seen that while both the Butterworth and Chebyshev filters have a strong attenuation in the stopband, their decay is not as sharp as that of the elliptic filter. We therefore choose the elliptic filter from the IIR family, as this is a desired characteristic. In addition, elliptic filters offer more con-

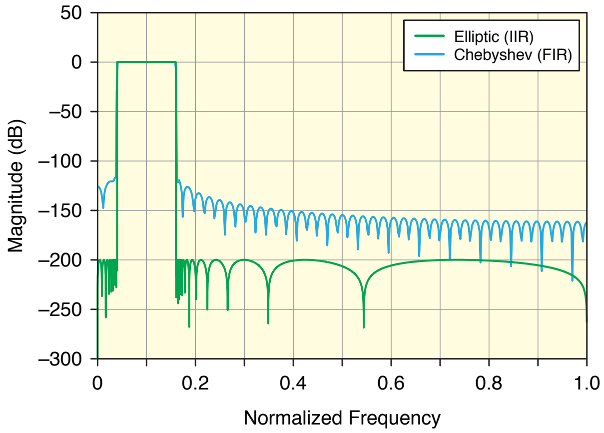

Figure 8: Comparison of frequency response of FIR and IIR filters

trol over the magnitudes of the passband ripples, the stopband attenuation, and the transition width.

Figure 8 shows a comparison between the selected FIR and IIR filters, and illustrates the advantages offered by the IIR elliptic filters over the general shape of FIR filters, strong attenuation with a sharp transition zone. Choosing the best filter strategy and parameters is, however, non trivial. For validation using the GOF method described before, we focus our attention on bandpass filters. Bandpass filtered signals, however, can also be obtained by low- and high-pass filtering. Thus one needs to these alternatives as well.

To evaluate our selection of elliptic IIR filters, we processed the same set of synthetics and data for the 2008 Chino Hills simulation used before. We followed different procedures to obtain bandpass filtered signals and evaluated the filtered sets using Eqs. (2) and (3); and found that low- and then high-pass filtering the signals leads to a less sensitive outcome for GOF validation purposes. This is due to the fact that by separating the bandpass filtering process, one has more control over the filtering parameters for the two (low and high) cutoff frequencies. There are, however, some tradeoffs. Increasing the filter's order and attenuation factor of the stopband, and decreasing the maximum amplitude of the ripples in the pass band altogether at once—assuming that each, separately, should bring the filter closer to an ideal condition—does not work. Therefore, a balance must be found.

To find this point of balance we did a sensitivity analysis based on the different options of the elliptic filter. In this analysis we assumed that the parameters of low- and high-pass filters are the same, and focused simply on finding the best parameters for the process. Figure 9 shows the sensitivity of the GOF scores with respect to the filtering order and attenuation factor (R_s) for the case of an elliptic IIR zero phase filter. We can see here that for low attenuation factors ($R_s = 40$), GOF converge with increasing orders at about 10–15. Larger attenuation factors, however, may have negative effects and may even become unstable (e.g., $R_s = 150$) at higher order values. Similarly, Figure 10 shows the variation of the GOF scores goodness with varying attenuation factor. Again, increasing R_s gener-

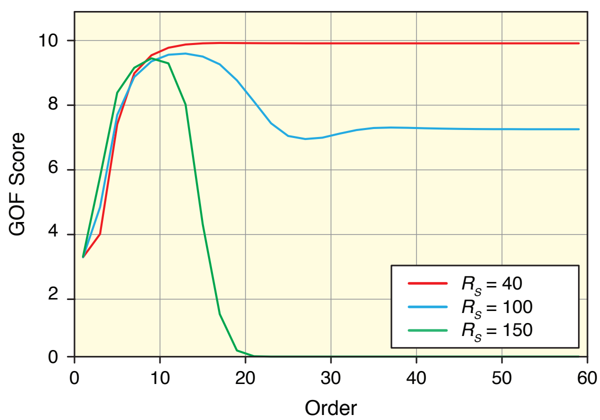

Figure 9: Variation of GOF scores with increasing the filter order, and variable attenuation factor (R_s in dB).

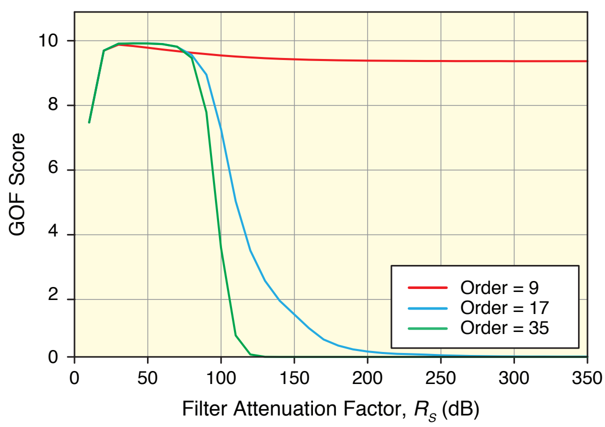

Figure 10: Variation of GOF scores with increasing the attenuation factor (R_s), and variable filter order.

ally affects the stability of the scores. These results were obtained for a subset of 40 signal pairs from the Chino Hills test case, considering five different bands in the ranges 0.1–0.25, 0.25–0.5, 0.5–1, 1–2, 2–4 Hz. In all cases, the amplitude of ripples in the passband (R_p) was set to 0.001 dB. We found that all bands exhibited the same behavior, although with different initial and mean values.

5. FINAL REMARKS AND RECOMMENDATIONS

We present a study about the effects that different filtering approaches have on validation of ground motion simulations. According to our evaluation of different filters, elliptic IIR filters offer the most consistent results for validation purposes, where near-ideal filters are desirable. We reached this conclusion based on a quantitative evaluation of the similarity of a filter's result with that expected from an ideal filter. The quantitative measure introduced here is based on similar ideas first put forward by Anderson (2004), but modified to facilitate the evaluation of the stopband alongside the passband. The proposed scoring method helps find optimal parameters and serves as a starting point to find the best possible filter.

Even though we can increase the attenuation of stopband and decrease the allowable ripple magnitude in the passband, we found that there is a trade off between these factors and modifying them simultaneously may not always yield better results. After conducting a sensitivity analysis, we found that in the case of elliptic filters, an attenuation factor between 35–100, allowable ripple magnitudes in passband less 0.01, and filter orders bigger than 17, are good initial values for filters' design. While the present study focuses on the application of these filters for ground motion validation purposes, it is possible to think that our evaluation may be used in other contexts, especially in seismology and earthquake engineering, where other filters have been traditionally used without much attention.

We find this to be particularly relevant for ground motion verification and validation of simulations because this seems to be a field where there is a significant lack of consensus on how to evaluate the accuracy of results. Therefore, establishing post-processing standards on how to handle both recorded and simulated signals is an effort worth pursuing. This, in turn, will help constrain the uncertainties introduced when processing results and limit the analysis to the simulation parameters, models, and assumptions.

6. ACKNOWLEDGEMENTS

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