Nonlinear Stochastic Dynamic Analysis For Performance Based Multi-objective Optimum Design Considering Life Cycle Seismic Loss Estimation

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ABSTRACT: A performance-based multi-objective design optimization framework for nonlinear/hysteretic multi-degree-of-freedom (MDOF) structural systems subject to evolutionary stochastic excitation is formulated. The core of the developed framework is an efficient approximate dimension reduction technique based on the concepts of statistical linearization and of stochastic averaging for determining the non-stationary system response amplitude probability density functions (PDFs); thus, computationally intensive Monte Carlo simulations are circumvented. Note that the approach can handle readily stochastic excitations of arbitrary non-separable evolutionary power spectrum (EPS) forms that exhibit strong variability in both the intensity and the frequency content. Further, approximate closed-form expressions are derived for the non-stationary inter-story drift ratio amplitude PDFs corresponding to each and every DOF. In this regard, considering appropriately defined damage measures structural system related fragility curves are determined at a low computational cost as well. Finally, the structural system design optimization problem is formulated as a multi-objective one to be solved by a Genetic Algorithm based approach. A building structure comprising the versatile Bouc-Wen (hysteretic) model serves as an example for demonstrating the efficiency of the methodology.

The performance-based engineering (PBE) framework aims at providing information for facilitating risk-based decision-making via performance assessment and design methods that properly account for the presence of uncertainties. As far as the decision variable (DV) is concerned, the seismic life-cycle cost (LCC) accounting for the structure lifetime expected damage costs is commonly adopted; see (Wen and Kang 2001; Ellingwood and Wen 2005; Taflanidis and Beck 2009) for some indicative references.

Further, several approaches have been developed for relating the seismic hazard to the system fragility and for producing corresponding fragility curves. These range from the ones that employ a limited number of nonlinear time-history analyses with prescribed IM level compatible scaled real earthquake records (Vamvatsikos and Cornell 2002), to the ones that employ standard or efficient Monte Carlo simulation (MCS) based methodologies such as importance/line sampling, and subset simulation (Schueller et al. 2004). Nevertheless, note that there are cases where the computational cost of the MCS based techniques can be significantly high; thus, rendering their use computationally cumbersome, or even prohibitive. In this regard, an interesting contribution relating to the development of an efficient approximate technique is the work by Der Kiureghian and Fujimura (Der Kiureghian and Fujimura 2009), where an efficient tail-equivalent linearization based approach was applied for fragility analysis of a nonlinear building structure.

In this paper, a PBE multi-objective design optimization framework for nonlinear/hysteretic multi-degree-of-freedom (MDOF) structural systems subject to evolutionary stochastic earthquake excitation is formulated. The core of the developed framework is an efficient approximate analytical dimension reduction approach for determining the system response evolutionary power spectrum (EPS) matrix based on the concepts of statistical linearization and stochastic averaging; thus, computationally intensive Monte Carlo simulations are circumvented (Kougioumtzoglou and Spanos 2013). Further, approximate closed-form expressions are derived for the non-stationary response amplitude PDFs of the inter-story drift ratios (IDRs) corresponding to each and every DOF. In this regard, considering appropriately defined damage measures (DMs) structural system related fragility curves are determined at a low computational cost as well. Furthermore, note that a multi-objective optimization treatment (Jensen 2009) allows for objectives that exhibit potentially conflicting requirements to be treated simultaneously. In the present formulation, solving the multi-objective optimization problem yields the determination of a set of Pareto optimal solutions (Pareto front). Each solution of the Pareto front constitutes an acceptable design configuration compromising the potentially conflicting sub-objectives of the problem.

1. NONLINEAR STOCHASTIC RESPONSE DETERMINATION

1.1. Statistical linearization treatment

Consider an n-degree-of-freedom nonlinear structural system governed by the equation

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} + \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{f}(t) \qquad (1)$$

where $\ddot{\mathbf{q}}$, $\dot{\mathbf{q}}$ and $\mathbf{q}$ denote the response acceleration, velocity and displacement vectors, respectively, defined in relative coordinates; $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ denote the $(n \times n)$ mass, damping and stiffness matrices, respectively; $\mathbf{g}(\mathbf{q}, \dot{\mathbf{q}})$ is assumed to be an arbitrary nonlinear $(n \times 1)$ vector function of the variables $\mathbf{q}$ and $\dot{\mathbf{q}}$; and $\mathbf{f}(t)^T = \left(f_1(t), f_2(t), \dots, f_n(t)\right)$ is a $(n \times 1)$ zero mean, non-stationary stochastic vector process defined as $\mathbf{f}(t) = -\mathbf{M}\{\gamma\}\ddot{\alpha}_g(t)$, where $\{\gamma\}$ is the unit column vector, and $\ddot{\alpha}_g(t)$ is a stochastic non-stationary excitation process (e.g. earthquake excitation) characterized by an evolutionary power spectrum (EPS) $S_{\ddot{\alpha}_g}(\omega, t)$. Further, $\mathbf{f}(t)$ possesses an EPS matrix $\mathbf{S_f}(\boldsymbol{\omega}, \mathbf{t})$ of the form

$$\mathbf{S_f}(\boldsymbol{\omega}, \mathbf{t}) = \begin{bmatrix} m_1{}^2 S_{\ddot{\alpha}_g}(\omega, t) & 0 & \cdots & 0 \\ 0 & m_2{}^2 S_{\ddot{\alpha}_g}(\omega, t) & \cdots & 0 \\ \vdots & \ddots & \cdots & \vdots \\ 0 & 0 & \cdots & m_n{}^2 S_{\ddot{\alpha}_g}(\omega, t) \end{bmatrix} \qquad (2)$$

while the non-stationary stochastic process $\mathbf{f}(t)$ is regarded to be a filtered stationary stochastic process. In this regard, the excitation EPS matrix Eq.(2) can be written as

$$\mathbf{S_f}(\omega, t) = \mathbf{A}(\omega, t)\mathbf{S}_{\tilde{\mathbf{f}}}(\omega)\mathbf{A}(\omega, t)^{\mathbf{T}*} \qquad (3)$$

where the superscripts (T) and (*) denote matrix transposition and complex conjugation, respectively; $\mathbf{A}(\omega, t)$ is the modulating matrix which serves as a time-variant filter; and $\mathbf{S}_{\tilde{\mathbf{f}}}(\omega)$ is the power spectrum matrix corresponding to the stationary stochastic vector process $\tilde{\mathbf{f}}(t)$.

In the following, a statistical linearization approach (Roberts and Spanos 2003) is employed for determining the response EPS matrix $\mathbf{S_q}(\omega, t)$. In this regard, a linearized version of Eq.(1) is given in the form

$$\mathbf{M}\ddot{\mathbf{q}} + \left(\mathbf{C} + \mathbf{C_{eq}}\right)\dot{\mathbf{q}} + \left(\mathbf{K} + \mathbf{K_{eq}}\right)\mathbf{q} = \mathbf{f}(t) \qquad (4)$$

Relying next on the standard assumption that the response processes are Gaussian, the time-dependent elements of the equivalent linear matrices $\mathbf{C_{eq}}$ and $\mathbf{K_{eq}}$ are given by the expressions

$$c_{i,j}^{eq} = E\left\{\frac{\partial g_i}{\partial \dot{q}_j}\right\}, \quad k_{i,j}^{eq} = E\left\{\frac{\partial g_i}{\partial q_j}\right\} \qquad (5)$$

In general, for a linear MDOF system subject to evolutionary stochastic excitation a matrix input-output spectral relationship of the form

$$\mathbf{S_q}(\omega,t) = \mathbf{H_{gen}}(\omega,t)\mathbf{S_{\tilde{f}}}(\omega)\mathbf{H_{gen}^{T^*}}(\omega,t), \qquad (6)$$

can be derived, where

$$\mathbf{H_{gen}}(\omega,t) = \int_0^t \mathbf{h}(t-\tau)\,\mathbf{A}(\omega,\tau)e^{-i\omega(t-\tau)}d\tau \quad (7)$$

In Eq.(7) $\mathbf{h}(t)$ denotes the impulse response function matrix. Further, the time dependent cross–variance of the response displacement can be evaluated by the expression

$$\sigma_{q_i q_j}^2(t) = \int_{-\infty}^{\infty} S_{q_i q_j}(\omega,t)\,d\omega. \qquad (8)$$

It can be readily seen that Eqs.(5-8) constitute a coupled nonlinear system of algebraic equations to be solved numerically for the system response covariance matrix. Next, omitting the convolution of the impulse response function matrix with the modulating matrix can lead to substantial reduction of computational effort, especially for the case of MDOF systems. In this manner, Eq.(7) takes the form

$$\mathbf{H_{gen}}(\omega,t) = \mathbf{H}(\omega)\mathbf{A}(\omega,t) \qquad (9)$$

where $\mathbf{H}(\omega)$ is the frequency response function (FRF) matrix defined as

$$\mathbf{H}(\omega) = \left(-\omega^2\mathbf{M} + i\omega\left(\mathbf{C} + \mathbf{C_{eq}}\right) + (\mathbf{K} + \mathbf{K_{eq}})\right)^{-1} \quad (10)$$

Consequently, taking into account Eq.(3), Eq. (6) becomes

$$\mathbf{S_q}(\omega,t) = \mathbf{H}(\omega)\mathbf{S_f}(\omega,t)\mathbf{H^{T*}}(\omega) \qquad (11)$$

Note that Eq.(11) can be regarded as a quasi-stationary approximate relationship which, in general, yields satisfactory accuracy in cases of relatively stiff systems. Considering next Eqs.(2), (8) and (11) yields for the i-th degree of freedom

$$\sigma_{q_i}^2(t) = \int_{-\infty}^{\infty} (|H_{i1}(\omega)|^2 m_1{}^2 + \cdots + |H_{in}(\omega)|^2 m_n{}^2) S_{\ddot{\alpha}_g}(\omega,t)d\omega \qquad (12)$$

and

$$\sigma_{\dot{q}_i}^2(t) = \int_{-\infty}^{\infty} \omega^2 (|H_{i1}(\omega)|^2 m_1{}^2 + \cdots + |H_{in}(\omega)|^2 m_n{}^2) S_{\ddot{\alpha}_g}(\omega,t)d\omega \qquad (13)$$

Eqs.(12) and (13) hold true in the approximate quasi-stationary sense delineated earlier. Clearly, Eq.(11) can be used in conjunction with Eq.(5) and (7-8) to form a nonlinear system of algebraic equations to be solved for determining the MDOF system response covariance matrix at a low computational cost (Kougioumtzoglou and Spanos 2013); thus, circumventing computationally intensive Monte Carlo simulations

1.2. Dimension reduction approach

Following next the dimension reduction/ decoupling approach developed in (Kougioumtzoglou and Spanos 2013) an auxiliary effective linear time-variant (LTV) oscillator corresponding to the i-th DOF can be defined as

$$\ddot{q}_i + \beta_{aux,i}(t)\dot{q}_i + \omega_{aux,i}^2(t)q_i = f_i(t), \qquad (14)$$

where the time-varying equivalent stiffness $\omega_{aux,i}^2(t)$ and damping $\beta_{aux,i}(t)$ elements can be determined by equating the variances of the response displacement and velocity expressed utilizing the quasi-stationary FRF of Eq.(14) with the corresponding ones determined via Eqs.(12-13); this yields

$$\sigma_{q_i}^2(t) = \int_{-\infty}^{\infty}\left(\frac{1}{(\omega_{aux,i}^2(t)-\omega^2)^2 + (\beta_{aux,i}(t)\omega)^2}\right) m_i{}^2 S_{\ddot{\alpha}_g}(\omega,t)d\omega, \qquad (15)$$

and

$$\sigma_{\dot{q}_i}^2(t) = \int_{-\infty}^{\infty}\omega^2\left(\frac{1}{(\omega_{aux,i}^2(t)-\omega^2)^2 + (\beta_{aux,i}(t)\omega)^2}\right) m_i{}^2 S_{\ddot{\alpha}_g}(\omega,t)d\omega, \qquad (16)$$

Clearly, Eqs.(15) and (16) constitute a nonlinear system of two algebraic equations to be solved for the unknowns $\omega_{aux,i}^2(t)$ and $\beta_{aux,i}(t)$. Further, relying primarily on the assumption of light damping, a stochastic averaging technique (Kougioumtzoglou and Spanos 2013) is applied. Next, the system non-stationary response amplitude α_i is assumed to follow a time-dependent Rayleigh distribution of the form

$$p(\alpha_i, t) = \frac{\alpha_i}{c_i(t)} \exp\left(-\frac{{\alpha_i}^2}{2c_i(t)}\right) \tag{17}$$

yielding a first-order ODE of the form

$$\dot{c}_i(t) = -\beta_{aux,i}(t)c_i(t) + \frac{\pi S_f(\omega_{aux,i}(t),t)}{\omega^2_{aux,i}(t)} \tag{18}$$

which can be solved via standard numerical integration schemes such as the Runge-Kutta; see also (Kougioumtzoglou and Spanos 2013) for a more detailed presentation of the topic.

Further, the herein considered damage states (DS) are expressed in terms of the inter-story drift ratio (IDR) that is defined as the difference of the horizontal displacements between two successive stories, normalized by the inter-story height h. Considering in the ensuing analysis the IDR amplitude $A_i(t)$, a direct transformation of the response amplitude PDF $p(\alpha_i, t)$ yields the non-stationary IDR amplitude PDF in the form

$$p(A_i, t) = h^2 \frac{A_i}{c_i(t)} \exp\left(-\frac{h^2 {A_i}^2}{2c_i(t)}\right) \tag{19}$$

Furthermore, of particular interest from a reliability assessment perspective is the time instant where the IDR amplitude reaches its most critical value, i.e. $p_{cr}(A_i) = p(A_i, t = t_{cr})$. In the following, this is assumed to be the time where $c_i(t)$ reaches its peak value, and thus, the PDF of Eq.(19) takes its most broad-band form yielding higher failure probabilities. Specifically, the failure probability P_i defined as the probability of exceeding various levels of damage δ_{ds} conditioned upon the peak ground acceleration (PGA), is expressed as

$$P_i[A_i(t) \geq \delta_{ds} = \delta \middle| PGA = \alpha_{pga}] = \\ = 1 - \int_0^{\delta} p_{cr}(A_i \middle| PGA = \alpha_{pga})\, dA. \tag{20}$$

2. LIFE-CYCLE COST PBE FRAMEWORK

The PBE methodology serves as a potent stochastic framework for assessing the performance of engineering structural systems subject to various hazards via an appropriately defined decision variable (DV). The uncertainty in seismic ground motions is normally described in terms of the probability distribution of a seismic intensity measure, such as the PGA. In this regard, the seismic hazard is presented as a mean seismic hazard curve $H(\alpha_{pga})$, which provides the annual probability of exceeding specified levels of PGA (Cornell et al. 2002); that is,

$$H(\alpha_{pga}) = P[PGA \geq \alpha_{pga}] \tag{21}$$

In various PBEE studies as well as in the ensuing analysis, discrete DS are considered. The non-stationary IDR amplitudes $A_i(t)$ serve as global EDPs while the employed relationship between the EDP and the DS is based on the work by Ghobarah (Ghobarah 2004) related to ductile reinforced concrete (RC) moment resisting frames (see Table 1).

Table 1: Damage states, IDR limits and costs

Damage State	IDR (%)	Cost (% C_{in})
(I)-None	$0.0 \leq \delta_{ds} < 0.1$	0
(II)-Slight	$0.1 \leq \delta_{ds} < 0.2$	0.5
(III)-Light	$0.2 \leq \delta_{ds} < 0.4$	5
(IV)-Moderate	$0.4 \leq \delta_{ds} < 1.0$	20
(V)-Heavy	$1.0 \leq \delta_{ds} < 1.8$	45
(VI)-Major	$1.8 \leq \delta_{ds} < 3.0$	80
(VII)-Destroyed	$3.0 \leq \delta_{ds}$	100

Further, the seismic fragility curves serving as a quantitative tool of the structure vulnerability are evaluated for various damage levels. Specifically, based on the approximate nonlinear stochastic dynamics technique outlined in section 1, the seismic fragility curves are efficiently determined by simply integrating the critical non-stationary response IDR amplitude PDF $p_{cr}(A_i)$ for the time instant t_{cr}; see Eqs.(19-20). Next, considering the i-th DOF of the MDOF system, the annual probability of exceeding a given state of damage can be defined as

$$P_{i,a} = \int P_i[A_i(t) \geq \delta_{ds} = \delta \middle| PGA = \alpha_{pga}] \\ \left|\frac{dH(\alpha_{pga})}{d\alpha_{pga}}\right| d\alpha_{pga}, \tag{22}$$

In the herein study, the earthquake occurrence is assumed to follow a Poisson process (Ellingwood and Wen 2005). Further, the expected value of the life-cycle cost (LCC) due to seismic hazard can be expressed in the form

$$\mathrm{E}\left[\mathrm{LCC}\left(\mathrm{A_i}(\mathbf{x},\mathbf{t})\right)\right] = \frac{1}{\lambda T_d}\left(1-\exp\left(-\lambda T_d\right)\right) \times \ldots$$

$$\sum_{i=1}^{n_{dof}}\sum_{m=1}^{n_{ds}}\left(-C_m\left[\ln\left(1-P_{i,T_d}(A_i(t)>\delta_m)\right)-\cdots\right.\right.$$

$$\left.\left.\ldots-\ln\left(1-P_{i,T_d}(A_i(t)>\delta_{m+1})\right)\right]\right) \tag{23}$$

where n_{ds} is the total number of damage states considered; n_{dof} is the number of degrees of freedom of the MDOF system, λ is a constant discount rate/year, T_d is the design life of the structure, C_m is the cost associated with the m-th damage state, given in Table.1 as a percentage of the initial cost; P_{i,T_d} refers to the i-th DOF and represents the T_d-year probability of exceeding the m-th damage state given by the expression

$$P_{i,T_d} = 1-\exp\left(-P_{i,a}T_d\right) \tag{24}$$

Note that in the herein proposed LCC model the contribution of each and every DOF is considered resulting in a better account of the system overall performance; this is not the case with commonly used LCC models in PBEE studies where the system performance is associated with the most critical component only.

3. NUMERICAL APPLICATION

3.1. Three-story Bouc-Wen hysteretic building

In this section, the proposed methodology is applied to a 3-story reinforced concrete building which is modeled as a nonlinear/hysteretic 3-DOF structural system subject to evolutionary stochastic earthquake excitation, with h=3 m and $m_{plate} = 3.5\times10^4\mathrm{kg}$. A Young's modulus of $E = 30\times10^9\mathrm{Pa}$ and mass density of $\rho = 2{,}5\times 10^3\mathrm{kg/m^3}$ are considered herein. The nonlinearity is assumed to be in the form of the Bouc-Wen hysteretic model (Ikhouane and Rodellar 2007). Columns' cross-section dimensions for a given floor are assumed to be equal, and thus, the vector of design variables $\mathbf{x}$ has one component for every story, i.e. the width of the cross-section. The 3-DOF nonlinear structural system is governed by Eq.(1) where

$$\mathbf{q}^T = (q_1\ q_2\ q_3\ z_1\ z_2\ z_3) \tag{25}$$

and

$$\mathbf{g}(\mathbf{q},\dot{\mathbf{q}})^{\mathbf{T}} = \left(0\ 0\ 0 - g_1(\dot{q}_1,z_1)\ - g_2(\dot{q}_2,z_2)\ - g_3(\dot{q}_3,z_3)\right) \tag{26}$$

In the Bouc-Wen model the additional state z_i is associated with the relative displacement q_i via the nonlinear differential equation

$$g_i(\dot{q}_i,z_i) = -\gamma|\dot{q}_i|z_i|z_i|^{n-1} - \ldots$$
$$-\beta\dot{q}_i|z_i|^n + A\dot{q}_i \tag{27}$$

In Eq.(27) the parameters γ,β,A and n are capable of representing a wide range of hysteresis loops. The values $a = 0.15$, $\beta=\gamma=0.5$, $n=1$ and $A=1$ are considered herein. Further, the elements c_{eq_i} and k_{eq_i} are given by the expressions (Roberts and Spanos 2003)

$$c_{eq_i} = \sqrt{\frac{2}{\pi}}\left[\gamma\frac{E(\dot{q}_i z_i)}{\sqrt{E(\dot{q}_i^{\,2})}} + \beta\sqrt{E(z_i^{\,2})}\right] - A, \tag{28}$$

and

$$k_{eq_i} = \sqrt{\frac{2}{\pi}}\left[\gamma\sqrt{E(\dot{q}_i^{\,2})} + \beta\frac{E(\dot{q}_i z_i)}{\sqrt{E(z_i^{\,2})}}\right], \tag{29}$$

respectively. The damping matrix of the structural system $\boldsymbol{C}$ takes the form

$$\mathbf{C} = \begin{bmatrix}\mathbf{C_{11}} & \mathbf{C_{12}}\\ \mathbf{C_{21}} & \mathbf{C_{22}}\end{bmatrix} \tag{30}$$

where $\mathbf{C_{11}}$ is assumed to be proportional to the stiffness matrix of the structure according to the expression

$$\mathbf{C_{11}} = c_0\cdot\mathbf{K_{11}} \tag{31}$$

where c_0 is considered to be equal to 2×10^{-2}. Regarding the excitation EPS $S_{\ddot{\alpha}_g}(\omega,t)$, it is assumed to have the form

$$S_{\ddot{\alpha}_g}(\omega,t) = |g(t)|^2 S_{CP}(\omega) \tag{32}$$

where $S_{CP}(\omega)$ represents a stationary process power spectrum and $g(t)$ denotes a time-modulating function described by

$$g(t) = k\left(e^{-b_1 t} - e^{-b_2 t}\right) \qquad (33)$$

where $b_1 = 0.1$ and $b_2 = 0.3$; k is a normalization constant so that $g(t)_{max} = 1$. The widely used Kanai-Tajimi spectrum appropriately modified by Clough and Penzien (Clough and Penzien 1993) is considered for $S_{CP}(\omega)$; that is

$$S_{CP}(\omega) = S_0 \frac{(\omega/\omega_f)^4}{(1-(\omega/\omega_f)^2)^2 + 4\xi_f^2(\omega/\omega_f)^2} \times \frac{\omega_g^4 + 4(\xi_g)^2\omega_g^2\omega^2}{(\omega_g^2-\omega^2)^2 + 4\xi_g^2\omega_g^2\omega^2}, \qquad (34)$$

where S_0 is the amplitude of the bedrock excitation spectrum, modeled as a white noise process; ξ_g and ω_g are the damping factor of the soil and the fundamental natural frequency, respectively; and ξ_f and ω_f are parameters describing the Clough-Penzien filter. The parameters values chosen are $\xi_g = 0.7, \omega_g = 2\ \mathrm{rad/s}$, $\xi_f = 0.6$, $\omega_f = 12.5\ \mathrm{rad/s}$. The duration of the earthquake excitation t_o is taken equal to 20 seconds. Note that in the ensuing analysis the following definition for α_{pga} is adopted; i.e.,

$$\alpha_{pga} = E\left[\max\left(\left|\ddot{\alpha}_g(t)\right|\right)\right], 0 \le t \le t_o \qquad (35)$$

Thus, to provide with a mapping between the α_{pga} and the modulated C-P excitation spectrum intensity factor S_0, several MCS are conducted for various S_0 values via the spectral representation approach (Shinozuka and Deodatis 1991). In this manner, the relationship $S_0(\alpha_{pga})$ depicted in Fig.(1) is obtained.

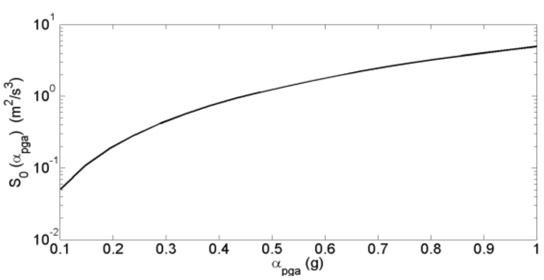

Figure 1: Mapping between the amplitude $S_0(\alpha_{pga})$ of the excitation spectrum and α_{pga}.

Next, the seismic hazard curve of Eq.(21) is expressed in the approximate form used in (Cornell et al. 2002), i.e.,

$$H(\alpha_{pga}) = k_0 \times \alpha_{pga}{}^{-k_1} \qquad (35)$$

where $k_0 = 6.734 \times 10^{-5}$ and $k_1 = 2.857$ were considered. Note that when dealing with the evaluation of the expected value of LCC (see Eq.(23)), and for the purpose of taking into account all possible earthquake scenarios a structure is anticipated to encounter during its lifetime, all seismic events with acceleration input α_{pga} values between 0.1 and 1g are considered. In this setting, a wide range of imposed seismic inputs α_{pga} is regarded while neglecting those with ground acceleration less than 0.1g that are not expected to cause significant damage to the structure.

3.2. Pareto Optimal Set

The objective function is defined as a weighted linear combination of the initial cost function and of the expected value of the LCC. Further, the response of the structural system is constrained in terms of the modes (i.e. most probable values) of the non-stationary response IDR amplitude PDFs of every DOF of the hysteretic MDOF system. The design variables are the dimensions of the square cross-section of the column elements. Columns' cross-section dimensions for a given floor are assumed to be equal, and thus the vector of design variables $\mathbf{x}$ has three components, one for every story. Next, assuming an initial design $\mathbf{x}^{in} = [0.30m, 0.25m, 0.20m]^T$ and boundary constraints $x_i{}^{in} \le x_i \le x_i{}^{ub}$, $i = 1, \dots, n_{dof}$, where $\mathbf{x}^{ub} = [0.55m, 0.55m, 0.55m]^T$ the optimization problem takes the form

$$\min_{x \in D}(C_{in}, E[LCC(A_i(\mathbf{x}, \mathbf{t}))]) \qquad (36)$$

under the stochastic constraints

$$\boldsymbol{\mu}_{\mathbf{o,i}}(S_o^*, \mathbf{x}, \mathbf{t}) = \frac{\sqrt{\mathbf{c}_i(t)}}{h} \le \boldsymbol{\delta}_{ds}^{Limit} \qquad (37)$$

and

$$\boldsymbol{\omega}_{aux,i}(S_o^*, \mathbf{x}, \mathbf{t})_{,\max} \le \boldsymbol{\omega}_{cr,L}$$

$$\text{or} \quad \boldsymbol{\omega}_{aux,i}(S_o^*, \mathbf{x}, \mathbf{t})_{,\min} \geq \boldsymbol{\omega}_{cr,R} \tag{38}$$

and the deterministic constraint

$$x_i \geq x_{i+1}, \quad i = 1, \ldots, n_{dof}. \tag{39}$$

In Eq.(36) $C_{in}(\mathbf{x})$ stands for the initial cost; $E[LCC(A_i(\mathbf{x}, \mathbf{t}))]$ is the expected value of the LCC, evaluated at the design variables vector $\mathbf{x}$. In Eq.(37) $\boldsymbol{\mu}_{o,i}(S_o^*, \mathbf{x}, \mathbf{t})$ is a vector of the modes (i.e. most probable values) of the non-stationary response IDR amplitude PDFs of every DOF of the hysteretic MDOF system. The structure design service life T_d is considered to be equal to fifty years while the discount ratio, λ, is taken to be equal to 3%. Regarding the stochastic constraints of Eqs.(37) and (38) the critical excitation was selected to be the one with intensity factor S_o^* yielding an earthquake input α_{pga} equal to 0.34g, whereas $\boldsymbol{\delta}_{ds}^{\text{Limit}}$ =0.2%.

The technique not only provides with the system response amplitude PDF for each and every DOF, but also decouples the original n-DOF system of Eq.(1) into n SDOF LTV oscillators of the form of Eq.(14) yielding time-varying effective stiffness $\omega_{aux,i}^2(t)$ and damping $\beta_{aux,i}(t)$ elements. This important additional output of the technique is exploited in the constraint of Eq.(38) for avoiding "moving resonance" phenomena. In this regard, it facilitates the optimization process to avoid unnecessary optimal design searching in areas where surely optimal designs do not exist. Specifically, considering the quasi-stationary treatment of the LTV oscillator in Eq.(12), it can be reasonably argued that the maximum response variance of the original MDOF system occurs when the excitation EPS $S_{\ddot{\alpha}_g}(\omega, t)$ resonates with the LTV oscillator equivalent natural frequency $\omega_{aux,i}(t)$. To avoid resonance phenomena, the constraint of Eq.(38) is formulated so that $\omega_{aux,i}(t)$ is kept outside a critical range in the frequency domain $[\omega_{cr,L}, \omega_{cr,R}]$ where the excitation EPS $S_{\ddot{\alpha}_g}(\omega, t)$ takes its largest values. In this regard, the expression

$$S_{\ddot{\alpha}_{g,L}}(\boldsymbol{\omega}, t) \leq \varepsilon \times S_{\ddot{\alpha}_{g,P}}^*(\boldsymbol{\omega}, t) \tag{40}$$

is adopted, where $S_{\ddot{\alpha}_{g,L}}(\boldsymbol{\omega}, t)$ is a selected EPS value given as a percentage ε of the peak EPS value $S_{\ddot{\alpha}_{g,P}}^*(\boldsymbol{\omega}, t)$ corresponding to the time instant where $|g(t)|^2$ takes its peak value. In the herein considered application, ε was taken equal to 75%. Further, according to (Wen and Kang 2001)

$$E\big[C_{Total}\big(A_i(\mathbf{x}, \mathbf{t})\big)\big] = C_{in}(\mathbf{x}) + \ldots$$
$$\ldots + E\big[LCC\big(A_i(\mathbf{x}, \mathbf{t})\big)\big] \times C_{in}(\mathbf{x}) \tag{41}$$

The Pareto front curves for both the expected value of the LCC and the expected value of the total cost with respect to the initial cost are presented in Fig.(2)

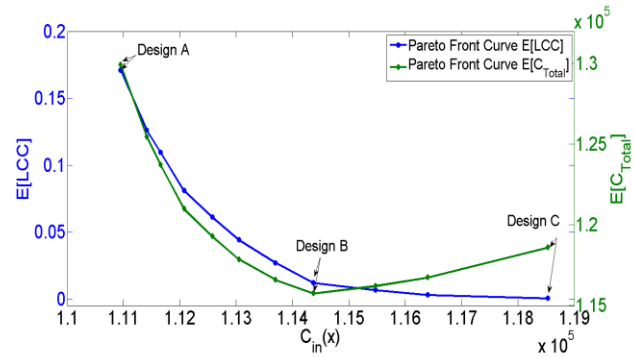

Figure 2: Pareto front curves for the expected values of LCC and total cost against the initial cost.

Next, to highlight the flexibility of the proposed methodology, the compromise design solution from the Pareto front curve exhibiting the lowest expected value of the total cost, as well as the ones corresponding to the two tails (see Fig.(2)) are presented in Table 2. In this setting, the designer/analyst possesses a considerable amount of information for every compromise solution configuration regarding the initial cost and the expected values of both the LCC and the total cost.

Table 2: Designs from Pareto front curves

Designs		$\mathbf{x}$(m)	$C_{in}(\mathbf{x})$ $(\times 10^5)$	E[LCC]	$E[C_{Total}]$ $(\times 10^5)$
A	1st 2nd 3rd	0.3892 0.3701 0.3294	1.1095	17.1103 $\times 10^{-2}$	1.2993
B	1st 2nd	0.4750 0.4749	1.1415		1.1544

	3rd	0.3981		1.1302×10^{-2}	
C	1st 2nd 3rd	0.5492 0.5489 0.5471	1.1853	5.5901×10^{-4}	1.1860

4. CONCLUSIONS

In this paper, a performance-based multi-objective design optimization framework considering LCC has been developed for nonlinear/hysteretic MDOF structural systems subject to evolutionary stochastic excitations. Although the developments herein have been tailored specifically for earthquake engineering related applications, they can be readily modified to account for other hazard kinds as well.

The core of the developed framework is an efficient approximate dimension reduction technique for determining the non-stationary system response amplitude probability density functions (PDFs) based on the concepts of statistical linearization and of stochastic averaging; thus, computationally intensive Monte Carlo simulations are circumvented. In this regard, considering appropriately defined damage measures structural system related fragility curves for each story are determined at a low computational cost as well. Finally, the structural system design optimization problem is formulated as a multi-objective one to be solved by a Genetic Algorithm based approach; thus, various compromise solutions are obtained providing the designer with enhanced flexibility regarding decision-making analysis. A building structure comprising the versatile Bouc-Wen (hysteretic) model has served as a numerical example for demonstrating the efficiency of the proposed methodology.

5. REFERENCES

Roberts J. B., Spanos P.D. (2003). Random vibration and statistical linearization, New York: Dover Publications.

Jensen H. A.(2009). Tradeoff analysis of non-linear dynamical systems under stochastic excitation, *Probab. Eng. Mech; 24: 585-599.*

Cornell C. A., Krawinkler H., Progress and challenges in seismic performance assessment, *PEER Center News; Spring 2000. http://peer.berkeley.edu/news/2000spring/index.html*

Wen Y. K., Kang Y. J., (*2001*) Minimum building life-cycle cost design criteria. I: methodology, *J Struct Eng*; *127(3): 330-7.*

Taflanidis A. A., Beck J. L. (*2009*), Life-cycle cost optimal design of passive dissipative devices, *Struct Safety; 31: 508-22.*

Schueller, G. I., Pradlwarter, H. J. (*2004*), and Koutsourelakis, P. S., A Critical Appraisal of Reliability Estimation Procedures for High Dimensions, *Prob. Eng. Mech., 19, pp. 463–474.*

Vamvatsikos D, Cornell C. A (*2002).* Incremental dynamic analysis,*Earthquake Engineering and Structural Dynamics; 31:491–514,*

Kougioumtzoglou I. A., Spanos P. D. (*2013).* Nonlinear MDOF system stochastic response determination via a dimension reduction approach, *Comp. and Struct.; 126: 135-148.*

Der Kiureghian A., Fujimura K. *(2009).* Nonlinear stochastic dynamic analysis for performance-based earthquake engineering, *Earthquake EngngStruct. Dyn.; 38:719–738*.

Ellingwood B, Wen Y. K., (2005). Risk-benefit-based design decisions for low probability/high consequence earthquake events in Mid-America. Prog Struct. Eng. Mater (7):56–70.

Ikhouane F., Rodellar J., (2007) Systems with hysteresis: analysis, identification and control using the Bouc-Wen model, John Wiley and Sons.

Clough R. W., Penzien J., (1993). *Dynamics of structures*, McGraw-Hill.

Shinozuka M., Deodatis G., (*1991).* Simulation of stochastic processes by spectral representation, *Applied Mechanics Reviews, vol. 44, no. 4, pp. 191-204.*

Cornell C. A., Jalayer F., Hamburger R., Foutch D., (2002). Probabilistic basis for 2000 SAC federal emergency management agency steel moment frame guidelines, *ASCE J StructEng; 128: 526–32.*

Ghobarah A. (2004). On drift limits associated with different damage levels. Fajfar, Krawinkler (eds) *International workshop on performance-based seismic design. Bled, Slovenia*.