Defining a Braking Probability to Estimate Extreme Braking Forces on Road Bridges

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ABSTRACT: A probabilistic approach is necessary to estimate the magnitude of the braking force that compares in terms of return period with the vertical traffic loads present in bridge design codes. A data set from a Danish Field Operational Test served as support to identify hard braking episodes and to compute travelled distances by road hierarchy. The rate of braking events per vehicle travelled distance is used as estimate of the braking probability. Concurrently a structural model was developed for stochastic analysis of the dynamic response of bridges to braking events from realistic traffic configurations. This paper presents results for extreme braking forces on bridges with site-specific traffic data and within a probabilistic framework. Such evaluation is economically advantageous when compared to the application of structural codes, while still meeting the reliability targets they impose.

1. INTRODUCTION
In many European countries the design value of the braking force on road bridges significantly increased when the Eurocodes came into force replacing each country’s national standards. Hence, safety assessments of existing bridges may show a lack of compliance with the new safety requirements of the Eurocodes. In such cases, uneconomical structural retrofitting operations may be avoided if the safety verification to the braking force follows a probabilistic site-specific approach instead of applying the load model enforced by EN 1991-2 (Eurocode 1 - Part 2) [European Committee for Standardization (CEN 2003)]. In fact, the braking force model from (CEN 2003) was derived from deterministically defined simplistic traffic scenarios with characteristics that do not correspond with actual traffic measurements, as described in the background studies of the Eurocodes’ traffic load models by (Merzenich and Sedlacek 1995).

As a consequence of following a deterministic approach, the return period of the braking force defined in (CEN 2003) is unclear and might deviate significantly from the 1000-year target used to calibrate the vertical traffic...
load models. Consequently, further investigation into the braking force model is relevant not only for assessment of existing bridges, but also to derive a load model for the braking forces that compares in terms of return period with the vertical load models. In this context, it is the purpose of this study to develop a probabilistic framework to derive site-specific braking force models considering real-world traffic data and aiming at a magnitude of the braking force that verifies a return period of 1000 years.

There are two main types of sources of real-world traffic data that will be used to realistically model traffic loads on road bridges. The first type is measurement stations installed on roads. In this group Weigh-in-Motion (WIM) stations are the most used for estimating traffic loads on bridges because they measure axle weights of moving vehicles. The second type is on-road studies. The data collected with these studies usually comes from instrumented vehicles and focuses on aspects of driving behaviour, as discussed by (Carsten et al. 2013).

The model presented in this paper resorts to Monte Carlo simulations to combine both types of traffic data and reproduce the randomness of braking events. The result is a probability distribution of the braking force that is specific for a given bridge on a given point of the road network where traffic characteristics are known.

In order to identify the quantile of the probability distribution of the braking force that corresponds to a 1000-year return period, information about the rate of braking events is also necessary. In the case of vertical loads, the time frame associated to the history of loading events is simply equal to the period of traffic flow considered. However, a time-history of crossing vehicles can be decomposed in much more convoy configurations than those that are likely to brake on a bridge during this period of time. In order to be able to estimate the expected number of braking events as a function of the number of vehicles and travelled distance, the concept of braking probability is introduced in Section 2 of this paper.

In Section 3 of this paper, the analysis of data from an on-road study held in Denmark illustrates the proposed procedure to estimate the braking probability. This is applied in Section 4 to compute extreme values of the braking force from realistic traffic situations, which are taken from traffic microsimulation. Finally, Section 5 concludes the paper with some final remarks and an outlook on future developments.

2. BRAKING PROBABILITY
In this paper the braking probability $\beta$ is defined as the likelihood that a vehicle will engage in a hard braking event per metre travelled for a reason unrelated to road infrastructure constraints, such as traffic lights or pedestrian crossings. This definition aims at isolating the situations that are most likely to occur on road bridges. In this definition, a hard braking event consists of a continuous period of time during which brakes are applied and the maximum deceleration reached is higher than $4 \text{ m/s}^2$. This threshold should have little influence in the computed value of the braking force for a 1000-year return period. However, if this threshold is too high, the number of braking events identified in on-road studies is too small to statistically characterize them, and the severity of the design situation may be exceeded. Also, if the threshold is too low, there is a waste of computational time with simulations of braking events that are not relevant for safety assessment because the associated braking forces are too small.

Following the definition above, an estimate of the braking probability is given by the rate of braking events per vehicle travelled distance in meters. Hence, the braking probability is dimensionless and associated to a Bernoulli trial of initiating or not initiating braking for every meter travelled. As a consequence, the rate of braking events has to be computed per vehicle meter travelled $[(\text{veh} \cdot \text{m})^{-1}]$ in order to match the proposed definition of the braking probability. In the definition of the distance of reference, which in this study measures one meter, it is important that it is short enough to only have two possible outcomes: one hard braking event or none at all.
The initiation of more than one hard braking episode is unrealistic in one meter travelled.

It is assumed that the probability of being involved in a braking event is the same for all vehicles and constant along a road section of length $l$, in meters, since further detailing is impossible with the available data. Hence, the expected number of vehicles braking, $n_{b,v}$, for a given traffic flow $q$, during a period of time $\Delta t$ in this road section, is given by

$$n_{b,v} = \beta \cdot q \cdot \Delta t \cdot l \quad (1)$$

Furthermore, the braking force that corresponds to a return period of 1000 years depends directly on the number of braking events, not vehicles, in 1000 years, $n_{b, ev1000}$. This variable is computed as the ratio between the total number of braking vehicles in 1000 years, from equation (1), and the average number of vehicles per braking event with a deceleration higher than 4 m/s$^2$. Hence, the quantile of the probability distribution of the braking force that corresponds to a 1000-year return period has a probability of non-exceedance $P$ given by

$$P = 1 - \frac{1}{n_{b, ev1000}} \quad (2)$$

Virginia Tech Transportation Institute has supported several on-road traffic studies. The first large scale one was the 100-Car Naturalistic Driving Study described by (Dingus et al. 2006), which collected data from 100 light vehicles covering a total of 3.3 MVKT (million vehicle kilometre travelled). A total of 233 incidents were identified for a deceleration threshold of 4 m/s$^2$, yielding a rate of braking events of $7.1 \times 10^{-8}$ (veh·m)$^{-1}$. In this study, incidents were defined as conflict situations that required an evasive manoeuvre, such as braking or steering, to avoid crashing.

Moreover, (Olson et al. 2009) combined two data sets from on-road studies with commercial vehicles to investigate driver distraction. In a total of approximately 5 MVKT, the number of conflicts analysed was 3,237, comprising crash, near-crash and other crash-relevant conflicts. One of the triggers used to identify these conflicts was a deceleration higher than 2 m/s$^2$, but swerving or activating the critical incident button also signalled a conflict. A rate of events of $6.5 \times 10^{-7}$ (veh·m)$^{-1}$ is computed using this total number of conflicts. This value is about ten times higher than the rate of braking events computed with data from the 100-Car study, but it is less coherent with the proposed definition of braking probability, since it classifies as relevant events other episodes than just hard braking events.

3. ANALYSIS OF ON-ROAD DATA

The aforementioned studies indicate an order of magnitude of the braking probability between $10^{-8}$ and $10^{-6}$. However, these studies focus on safety issues and, therefore, hard braking events in situations that represented no risk to the driver might be neglected, whereas conflict situations with evasive manoeuvres other than hard braking might be counted.

In order to estimate the braking probability in a way that unequivocally fits in this study, a series of procedures was specifically developed. Data from an on-road study conducted in North Denmark is used to illustrate these procedures. This data includes speed and GPS coordinates, measured with 1 Hz frequency, of five vehicles that travelled a total of approximately $3.2 \times 10^3$ km in October 2013.

Section 3.1 shows the methodology proposed to identify braking events in agreement with the definition of braking probability and to categorize them by road hierarchy. In section 3.2, an algorithm is introduced to estimate the distance travelled distinguishing roads of different hierarchies. Finally, section 3.3 presents the estimates of the braking probability computed using the rate of braking events per vehicle meter travelled.

3.1. Identifying braking events

The travelled distance, speed and longitudinal acceleration computed from GPS coordinates are similar to those computed having speed records as starting point, but not perfectly coincident. The acceleration computed from speed records was chosen to identify braking events, since it
only requires one derivative. As a result, 31 braking events were identified, leading to a rate of braking events of $9.7 \times 10^{-6} \text{ (veh \cdot m)}^{-1}$.

Using a free online convertor from UTM coordinates to latitude and longitude (http://users.tpg.com.au/adslly6v/UtmGoogleStreetView.html accessed in October 2014) and analysing the location of the braking events with Google Maps (http://maps.google.com accessed in October 2014), braking events were categorized by road hierarchy in local, regional (an intermediate level of the road hierarchy that comprises A-class routes) and motorway. It was also possible to identify braking events that were most likely initiated by road constraints, such as traffic lights, pedestrian crossings or road junctions. The other braking events occurred in road sections that are straight or slight curves, and have no road constraints in sight. As Table 1 shows, the majority of braking events, 22 out of 31, was likely caused by road constraints.

<table>
<thead>
<tr>
<th>Road constraint</th>
<th>No road constraint</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>Regional</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>Motorway</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sum</td>
<td>22</td>
<td>9</td>
</tr>
</tbody>
</table>

Updating the rate of braking events to consider only the events that most likely were unrelated to road constraints, the value computed is reduced to $2.8 \times 10^{-6} \text{ (veh \cdot m)}^{-1}$. The statistical significance of this result is weak, given the small size of the sample, but the procedure is simple enough to be efficiently reproduced in a larger set of data.

3.2. Travelled distance by road type

Unlike the case of braking events, which represent only a reduced number of points in the dataset, a user-based categorization of all road sections in the dataset is very cumbersome. However, in order to estimate the travelled distance by road type, each instant of the dataset must be associated to local road, regional road or motorway. To that end, a supervised learning classification algorithm was developed to expedite the classification of all the locations recorded in the data set. Following a metaheuristic procedure the algorithm was run several times varying the values of several parameters and aiming at an optimization of the algorithm’s capacity of correctly predicting the road hierarchy using only the speed profile.

In order to assess the algorithm’s capacity of predicting the road hierarchy, 500 instants from the data set were randomly chosen. This training dataset includes 134 instances of motorway, 225 regional roads and 141 local roads, classified using the same interface and criteria as for braking events. This information was used as baseline to compute a confusion matrix for each run of the algorithm, quantifying all possible combinations between predicted and real road category. The matrices were computed in percentage terms relative to the user identifications because the data set is unbalanced due to the preponderance of regional roads.

The developed algorithm, designed for speed measurements recorded with a frequency of 1 Hz, consists of the following steps, where the variable parameters are identified in italic:

1. Every instant the speed is larger than $v_{\min M}$ the road section is classified as motorway.
2. For every situation identified as entering motorway, the following $n_{\text{clust1}}$ instants are analysed. If the percentage of instants identified as motorway is lower than $\%_{\min}$, the first consecutive set of instants identified as motorway goes back to being unclassified. Isolated excessive speed situations are thus obliterated.
3. For every situation identified as entering motorway, the previous $n_{\text{clust nearM}}$ instants are analysed. If the maximum speed is larger than $v_{\max \text{ nearM}}$ or the mean speed is larger than $v_{\text{mean nearM}}$, they are classified as being motorway. They might be temporary episodes of travelling at lower speed on a motorway.
4. For every situation identified as exiting motorway, if in less than $n_{\text{clust2}}$ instants the classification goes back to motorway, this gap is considered still motorway. Otherwise, the verification of step 3 is repeated in the following $n_{\text{clust nearM}}$ instants, thus discarding temporary lower speed episodes.

5. Every remaining instant that registered a speed larger than $v_{\text{minR}}$ is classified as regional road (similar to step 1).

6. Repeat steps 2 to 4, for regional roads instead of motorway, using $n_{\text{clust nearR}}$, $v_{\text{max nearR}}$ and $v_{\text{mean nearR}}$, instead of $n_{\text{clust nearM}}$, $v_{\text{max nearM}}$ and $v_{\text{mean nearM}}$.

7. Remaining road sections are classified as local roads.

Table 2 presents the mean and standard deviation, for random number generation using the Gaussian distribution, of the parameters that the algorithm requires. The presented values are already the result of an iterative procedure that starts with more disperse distributions. At each step of the procedure, the algorithm is run 500 times, and the mean and standard deviation for each random variable of the 100 runs that produce the best confusion matrices are stored for the next iteration, until an acceptable degree of convergence is achieved. The performance of each run of the algorithm is measured by computing the average of the diagonal terms of the confusion matrix. To promote accurateness in all road types, terms of the diagonal below 70% are penalized by 3%, and terms between 70% and 80% (exclusive) are penalized by 1%.

The algorithm was run 2000 times, which took less than 20 minutes, with the top 500 runs showing a performance criterion higher than 81.5%. Table 3 shows the 90% two-sided confidence intervals of each entry of the confusion matrices of the final top 500 runs of the algorithm, as well as the associated travelled distance in each type of road. L, R and M stand for local road, regional road and motorway, and travelled distances were computed from the GPS data, instead of integration of the speed profile.

| Table 2: Mean and standard deviation for random number generation of the algorithm’s parameters. |
|-----------------------------------------------------|------------------|------------------|
| $v_{\text{minM}}$ (km/h) | 93 | 3 |
| $v_{\text{minR}}$ (km/h) | 49 | 2 |
| $n_{\text{clust1}}$ | 367 | 10 |
| $n_{\text{clust2}}$ | 245 | 20 |
| $\%_{\text{min}}$ | 70% | 5% |
| $n_{\text{clust nearM}}$ | 127 | 15 |
| $v_{\text{max nearM}}$ (km/h) | 127 | 6 |
| $v_{\text{mean nearM}}$ (km/h) | 81 | 2 |
| $n_{\text{clust nearR}}$ | 40 | 20 |
| $v_{\text{max nearR}}$ (km/h) | 100 | 12 |
| $v_{\text{mean nearR}}$ (km/h) | 42 | 5 |

| Table 3: 90% confidence interval of confusion matrices and travelled distances. |
|-----------------------------------------------------|-----|-----|-----|
| Predicted | L | R | M |
| Real | 80%-89% | 8%-17% | 2%-4% |
| 13%-20% | 75%-82% | 3%-7% |
| 7%-10% | 4%-10% | 81%-87% |

The high relative values in Table 3, larger than 80%, show a good performance of the algorithm in correctly identifying local roads and motorways, and reasonably good, higher than 75%, for regional roads. Table 3 also reveals a bias of the predictions since the components in the lower triangle are significantly larger than their “mirror” entries in the upper triangle. This means that the algorithm tends to downgrade the road hierarchy. A possible explanation is that there are more situations leading to a speed profile more indicative of a road of lower hierarchy, for instance lower speeds in traffic jams, than situations of excessive speed. As a consequence, the travelled distance in local roads is probably overestimated, while being underestimated for regional roads and motorways.

This biased behaviour of the algorithm might actually lead to a better estimation of the braking probability. In fact, since braking events
near road constraints are discarded, the travelled distance around these points of the road should also be discarded, thus keeping focus on circumstances that might occur on bridges. Table 4 shows that with this procedure up to 17% less road sections than the original 225 are identified as regional, and, for motorways, up to 10% less than the original 134. On the other hand, the upper bounds of the confidence intervals do not exceed the original number of identifications.

Table 4: 90% confidence interval of confusion matrices (absolute values).

<table>
<thead>
<tr>
<th></th>
<th>Predicted</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
<td>R</td>
<td>M</td>
</tr>
<tr>
<td>Real</td>
<td>113-126</td>
<td>11-24</td>
<td>3-6</td>
</tr>
<tr>
<td></td>
<td>30-46</td>
<td>168-185</td>
<td>7-15</td>
</tr>
<tr>
<td></td>
<td>10-14</td>
<td>6-13</td>
<td>109-116</td>
</tr>
<tr>
<td>Total</td>
<td>155-185</td>
<td>187-219</td>
<td>121-134</td>
</tr>
</tbody>
</table>

The algorithm was validated with a validation dataset of 100 additional road sections randomly picked. As in the training dataset, the top 500 of 2000 runs of the algorithm showed a performance criterion based only on these 100 locations higher than 81.5%. Since this threshold is equal to the one computed during the calibration, the algorithm is deemed valid to analyse the whole dataset.

3.3. Braking probability by road type

The ratios between the number of braking events identified (Table 1) and the estimated travelled distances (Table 3) yield a set of intervals for the rate of braking events per vehicle meter travelled. These values are presented in Table 5 as intervals for the braking probability that are fit to use in the determination of extreme braking forces on road bridges given the definition proposed in this paper. Table 5 also includes the values of the braking probability that consider all events. If the data set had not included GPS positioning, road constraints would not have been identified and these would have been the only values computed.

<table>
<thead>
<tr>
<th></th>
<th>No road constraint</th>
<th>All events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regional</td>
<td>(5.2-5.9)×10^6</td>
<td>(1.4-1.6)×10^7</td>
</tr>
<tr>
<td>Motorway</td>
<td>(6.8-7.2)×10^-7</td>
<td>(1.4-1.4)×10^-6</td>
</tr>
<tr>
<td>All roads</td>
<td>2.8×10^-6</td>
<td>9.7×10^-6</td>
</tr>
</tbody>
</table>

Table 5 shows an order of magnitude of difference between the braking probability on motorways and regional roads, while the braking probability for all road types has an intermediate value. Moreover, only the estimate of the braking probability for motorway bridges lies in the range between 10^-8 and 10^-6 mentioned before. This range might be inadequate for regional roads because it was deduced from a combination of all road types or due to fundamental differences in driver behaviour and traffic conditions between both studies. On the other hand, the estimates in Table 5 have less statistical significance, since they are based on an on-road study with approximately 1000 times less travelled distance than the ones in Section 2. In fact, the estimation of the braking probability on motorways is based on only one braking event in about 1400 km. Therefore, the presented values are provisional and depend on validation from estimations with larger datasets.

This study produces no conclusions regarding braking on local roads because a probabilistic approach does not make sense in a location where it is certain that most vehicles will brake. A realistic worst-case scenario should be considered instead.

4. EXTREME BRAKING FORCES

The values of the braking force presented in this paper result from Monte Carlo simulations of the dynamic response of bridges to braking vehicles. The traffic configurations used to model the braking events are captured from a time-history of bridge crossing vehicles generated with traffic microsimulation tools. A previous version of this model is presented by (Martins et al. 2014). The simulations are able to take into account the dynamic response of the bridge, while also
reproducing the randomness associated to drivers’ reactions in case of braking events and the statistical characteristics of traffic.

Given the high axial stiffness of bridge decks, a linear elastic viscously damped single degree of freedom (SDOF) system is used to compute the response of the bridge. The SDOF system is characterized by a natural vibration period and a damping ratio that simulate the dynamic behaviour of the bridge in the longitudinal direction when subject to the action of braking vehicles. The braking force is computed via closed-form expressions so that it matches the static force that causes a longitudinal displacement of the bridge equal to the maximum displacement computed for the SDOF system. Thus, it can be directly compared with the braking force model from (CEN 2003).

The excitation force is built as a succession of possibly overlapping braking reactions of vehicles that travel in a convoy. The maximum deceleration of each vehicle and the response time of following drivers are randomly generated with parameters taken from studies of driver behaviour and braking systems. These studies are beyond the scope of this paper.

The set of braking forces that results from the Monte Carlo simulations serves as support to define an empirical probability distribution of the braking force, which is specific of the bridge and traffic conditions that are given as input. The characteristic value of the braking force $F_k$, i.e., the value of the braking force that has a return period of 1000 years and should be used in safety verifications, is the quantile of the braking force distribution that has the probability of non-exceedance given by equation (2).

Figure 1 shows the variation of $F_k$ as a function of the bridge length. It results from Monte Carlo simulations performed for bridges with a length between 25 and 200 m, a damping ratio of 0.07 and a natural frequency in the longitudinal direction of 1 Hz. The traffic input was a time-history of vehicles covering a period of one week with the statistical characteristics of traffic on a Swiss motorway. To that end, traffic simulation software Aimsun developed by [Transportation Simulation System (TSS 2011)] was used to combine data from a WIM station on Switzerland’s A2 motorway near Monte Ceneri (2 lanes and traffic flow $q \approx 20\times10^3$ veh/day in each direction) with records from a close by Swiss automatic road traffic count station. The values of $F_k$ were computed for a braking probability $\beta$ between $10^{-8}$ and $10^{-6}$. Figure 1 also shows the load model for the braking force in Eurocode 1 - Part 2 (EC1-2) (CEN 2003).

Figure 1: Characteristic braking forces from EC1-2 (CEN 2003) model and probabilistic approach.

Figure 1 shows that a probabilistic approach to compute the characteristic value of the braking force with site-specific data may yield a force with lower magnitude than what the load model in EC1-2 (CEN 2003) provides. This is particularly clear for lower values of the braking probability and bridges longer than 100 m. The highest estimate of the braking probability on motorways, $10^{-6}$, yields values of the braking force that concur with the code model up to a bridge length of 100 m but flattens after that, unlike the load model. This indicates that the deterministic scenarios from (Merzenich and Sedlacek 1995) having four or more 25-tonne vehicles braking simultaneously is too conservative, even for a 1000-year return period.

Due to the sparseness of braking events, one week of traffic configurations is enough to model more braking events than it is expected to occur in 1000 years. Therefore, there was no need to resort to extreme value theory to compute the quantile of the empirical probability distribution.
that corresponds to the characteristic value of the braking force.

5. CONCLUSIONS
This paper presents a site-specific probabilistic model to compute the characteristic value of the braking force on road bridges. The model simulates realistic traffic conditions from a Swiss motorway and takes into account the length and dynamic properties of the bridge. Results show that such evaluation leads to lower values than those included in structural codes, while more accurately meeting the safety criterion target of a 1000-year return period. Therefore, for existing bridges, this type of models might safely discard the need to retrofit or, at least, endorse retrofit solutions that are less expensive than those required when load models from the codes are applied. This holds in particular for long bridges, where the assumptions behind the load model in the Eurocode seem to be extremely conservative.

To enable a probabilistic approach to the estimation of extreme values of the braking force on road bridges, the concept of braking probability is introduced and the braking probability is defined as the likelihood that a vehicle will engage in a braking event per metre travelled for a reason unrelated to road infrastructure constraints. Results from previous studies indicate a range for the braking probability between $10^{-8}$ and $10^{-6}$, but do not distinguish between different road hierarchies.

This paper presents a supervised learning algorithm to compute the distance travelled per road type. With this information and a matching categorization of the braking events, it was possible to estimate a braking probability on motorways of circa $7 \times 10^{-7}$ and on regional road of circa $5.5 \times 10^{-6}$.

The goal of this research is to update the braking force model from (CEN 2003) using the probabilistic framework here outlined. That will require a more thorough evaluation, considering traffic from different locations and, ideally, larger data sets of on-road studies to improve the statistical characterization of braking events and the estimates of the braking probability. The dynamic model and the selection of traffic configurations can still be improved too, but it is beyond the scope of this paper to give further details concerning these aspects.

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This research work was financed through a grant by the Swiss Federal Roads Office (Project AGB 2011-003). Associate Professor Agerholm from the Traffic Research Group of Aalborg University kindly supplied part of the GNSS based Floating Car Data project ITS Platform in North Denmark. Both contributions are gratefully acknowledged.

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