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Numerical Description Of Size And Load Configuration Effects In Glulam Structures

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ABSTRACT: The limit states design requires realistic strength values for the verification of reliable glulam structures. Here, the problem arises that the probabilistic system effect strongly affects load-carrying capacities. Considering this effect, modification factors for the glulam bending and tensile strength values are determined, based on strength simulations on various member sizes and structural systems. Using the proposed factors in the practical design of the treated problems leads to a structural reliability, which is more consistent, since the proposed factors cover the inherent system effects correspondingly.

1. INTRODUCTION

The design rules for timber structures are based on the limit states method rather than on a direct application of the reliability analysis. With respect to safety, the 5th percentiles of the resistance and the strength (f_k), respectively, are among others employed to describe the limit state. In a practical design situation, the 5th percentile, originally related to a member with reference size, is adjusted with modification and calibrated safety factors to meet the target reliability, see EN 1990. Thus, the limit states design is a compromise for a reliability analysis. The magnitude of the 5th percentile of the strength has, therefore, a crucial influence on the reliability of timber members or structures made thereof.

Related to this compromise, the paper aims at showing variations in the 5th percentile of glulam strength values which are obviously caused by changes in member sizes, in the structural system or in the load configuration but actually related to the probabilistic system effect as concisely described e.g. by Thelandersson (2003).

Since the examined changes basically represent variations of the stressed volume, the problem treated here concerns the theory of Weibull applied to timber by e.g. Colling (1986a/b) and Isaksson (2003).

The direct and practical benefit of the work is the calculation of more balanced strength values for members or structures, which partly deviate from standardised reference sizes. Thus, the presented strengths have the meaning of effective strength values ($f_{k,ef}$) coupled with a special member size or system. According to the concept in EN 1990 for calculating design values, the paper proposes a row of modification factors (k_i, i = 1-9) to adapt the 5th percentile of the strength to changed conditions often occurring in practice. The corresponding format of this adaption is given in Equation (1).

$$f_{k,ef} = k_i \cdot f_k \tag{1}$$

The size- and system-dependent strength variation is treated by means of the 5th percentile based on the counting method. Thus, restrictions occur regarding the interpretation of the true structural reliability.

The method used for the strength variation employs Monte Carlo Simulations. The members and structures were modelled and computed with an individually configured stochastic finite element program for glulam. Performing a parameter variation, the load-carrying capacity and strength, respectively, of bending and tension members as well as continuous beams and frames were determined. Similar works, which gave impulses for these examinations, were performed by Foschi et al. (1996) as well as Hansson and Thelandersson (2003).

A further benefit of the work concerns the application of finite element modelling on structures which are more complex like continuous beams or frames. Thus, the experiences gained in this numerical examination should also help to identify deficiencies in modelling and purposeful amendments.

The paper is organised as follows: Section 2 describes the approach including the paper's general concept, the members and systems with reference sizes, the computer model and the performed variations. In section 3, the simulated strength values are stated in normalised form and described in three separate subsections. Based on these results, the modification factors are finally proposed.

2. METHODS OF THE EXAMINATION

2.1. Concept

The first and main focus of the parameter variations concerns the member size and the structural system. These variations are the basis for simulated strength values applying to conditions which deviate from the reference systems. The second focus (partly superposed with the first one) is on the variation of the material quality (MQ). The purpose is to examine, whether different material qualities and material strengths, respectively, have an influence on the resulting effective member or system strength modification. Thus, several common and theoretical strength grading processes were empirically represented with the computer model. They result in different material qualities (A to I). The material quality A is the result of a simple strength grading process (leading to high yield and low strength) while quality I is the result of a demanding process (leading to low yield and high strength).

Table 1 gives the outline of the numerical examination. The 1st column contains the nine modification factors to be determined. The 2nd one contains the corresponding member (bending or tension) and system type (continuous beam or frame), respectively for which the *k*-factors are valid. The 3rd one shows which parameters are subject to variation. Except for the frames and tension members, size or structural and material aspects are examined together. The size variation of the bending members is based on a constant h/ℓ-ratio (=1/18) while that of the tension members is performed with h- and ℓ-values which are independent of each other. The last column contains the total number of performed single simulations.

Table 1: k-factors and parameter variation.

	Member type/ system	*Variation*	*Simulations*
k_1	*Bending member*	*Size and material quality*	*30000 (+3000)*
k_2	*Tension member*	*Length and depth*	*70000*
k_3	*Continuous beams*	*2 spans and material quality*	*9000*
k_4		*3 spans and material quality*	*9000*
k_5		*4 spans and material quality*	*9000*
-	*2-hinged frames*	$h = \infty$	*3000*
k_6		$h = 3\ell/2$	*3000*
k_7		$h = \ell/2$	*3000*
k_8		$h = \ell/4$	*3000*
k_9		$h = \ell/8$	*3000*

Once a load-carrying capacity (F or M = function of F) is simulated with the computer model, the effective strength is calculated with this capacity and the corresponding cross-sectional values. In case of the continuous beams and the frames, the effective strength is always calculated with the global maximum bending moment present in the structure in the state of ultimate failure.

2.2. Reference systems

The initial configuration of the member size and of the system is shown by the reference systems with a beam depth h of 600 mm and a dependent length ℓ (Figure 5). In case of the bending and tension members, the parameters α, β and γ are set to 1. For the continuous beams the bending member with $\alpha = 1$ and $n = 1$, respectively, states the reference system. $\delta = \infty$ (infinite) describes the reference system for the frames; statically seen, a simple beam with uniformly distributed load (q).

Further details are: The position of the concentrated load F is in the third points of the bending member. The action line of F coincides with the centre line in the tension member. For simplification reasons, frames are modelled as 3-span-beams with equal lengths of the end spans (Figure 1). Thus, normal forces are not modelled and the bending resistance of the frame corner is assumed to be the same as in the straight section.

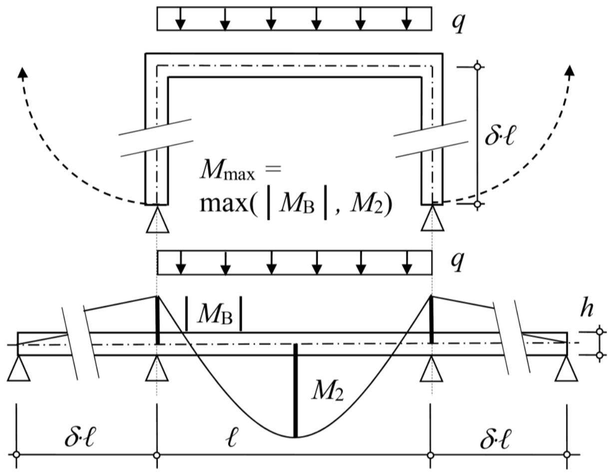

Figure 1: 3-span beams as a substitute for frames

2.3. Computer model

The computational strength examinations were conducted with a validated finite element based computer model. It is based on the structural analysis system ANSYS (release 14) and on its corresponding processors. The integrated design language was employed to control the general programme flow and to build the model with realistic mechanical properties of glulam.

Figure 2 shows the basic model used to simulate the bending strength. It exemplifies the bending test later shown in Figure 5 and applies to arbitrary members and structures analogously. The member size of the modelled body depends on the numbers of elements (150 mm in length and 30 mm in depth) along the total length and depth. The main features of the computer model are described in the following. For further model characteristics, see Frese (2010).

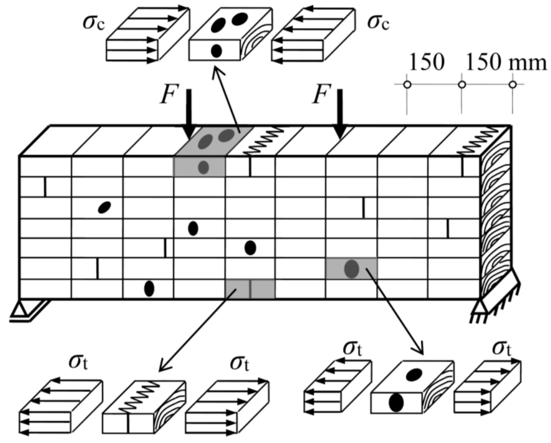

Figure 2: Discretised glulam model with selected elements under tensile and compressive stresses.

To obtain one of the material qualities (A-I), a corresponding grading process is activated prior to the Monte Carlo analysis. That results in statistically distributed values of features (knots and density) which influence the mechanical properties (Figure 3, 1st step). After being empirically represented by regression equations (Fig. 3, 2nd step), stochastically distributed and auto-correlated mechanical properties of glulam are assigned discretely and systematically to the elements of the model (Fig. 3, 3rd step). In doing so, the natural occurrence of knots and glulam specific characteristics as finger joints are taken into account with regard to the local mechanical properties. That enables a computation of realistic resulting tensile and compressive stresses (σ_t and σ_c) for the loaded member. An orthotropic material model is used. In the compression zone of simple beams under bending ideal elastoplasticity and in the tension zone, in principle, linear elasticity until tensile failure is assumed. A uniaxial failure criterion is used for both compression and tension. With the continuous beams and frames, the compression and tension zone alters its position. For simplification reasons, linear elasticity in the spatially limited compression zones of continuous beams and frames is modelled instead of ideal elastoplasticity.

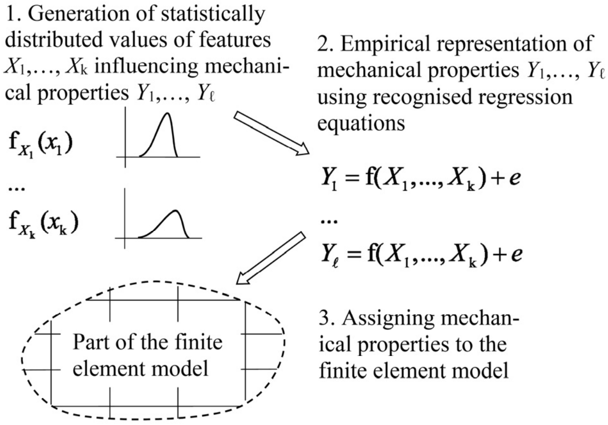

Figure 3: Computation of mechanical properties

The load (F or q) is applied through a stepwise displacement. That results in a particular number of load steps for a simulated test. During these load steps, element failure in the tension zone outside the outermost laminations is allowed (s. Figure 4, a); corresponding elements are identified after each load step and are deactivated by multiplying their stiffness by a severe reduction factor. That leads to an apparently non-linear behaviour in the tension zone.

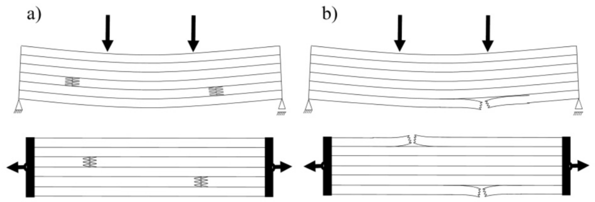

Figure 4: Locally limited failure accepted during simulated loading (a) and ultimate member failure terminating loading (b).

Figure 4, b exemplifies the ultimate failure criterion used for the simulated tests. This criterion is based on the assumption that no further loading is mechanically possible if the locally calculated tensile stress (confer σ_t, Figure 2) of any element in the outermost laminations equals the individual element tensile strength. In case of tension members, failure in either of the two outermost laminations constitutes ultimate failure.

2.4. Variations of the reference systems and simulation of the load-carrying capacities

Figure 5 shows the parameter variations for the two basic members and the two systems. The variation range of α, β, γ, δ, n and the material quality is stated by the corresponding lower and upper value. Using the computer model, each defined single parameter combination is examined with 500 up to 3000 single simulations resulting in the total number of simulations given in Table 1. Each single simulation finally leads to an individual load-carrying capacity used to calculate a corresponding effective strength value.

3. RESULTS

3.1. Effective and normalised strengths

The effective bending (f_m) and tensile strength (f_t), respectively, is computed according to Eq. (2) with the simulated load-carrying capacity F_{max} or q_{max}.

$$f_m = \frac{|\omega \cdot F_{max} \cdot \ell|}{W} = \frac{M_{max}}{W}$$
$$f_m = \frac{|\omega \cdot q_{max} \cdot \ell^2|}{W} = \frac{M_{max}}{W} \qquad (2)$$
$$f_t = F_{max} / A$$

Here, ω is a common tabular value to compute the global maximum bending moment according to elementary beam theory. With the continuous beams, ω depends on the number of spans (n) and the load configuration (e.g. n = 2 → ω = -1/3). For the frames, the ω-values are independent of the stiffness ratio between beam and column, which amounts to 1; in the present cases, ω-values are calibrated to the frame length ℓ (e.g. δ = 3/2 → ω = 1/12 or δ = 1/8 → ω = -1/13). W is the section modulus and A the cross-sectional area.

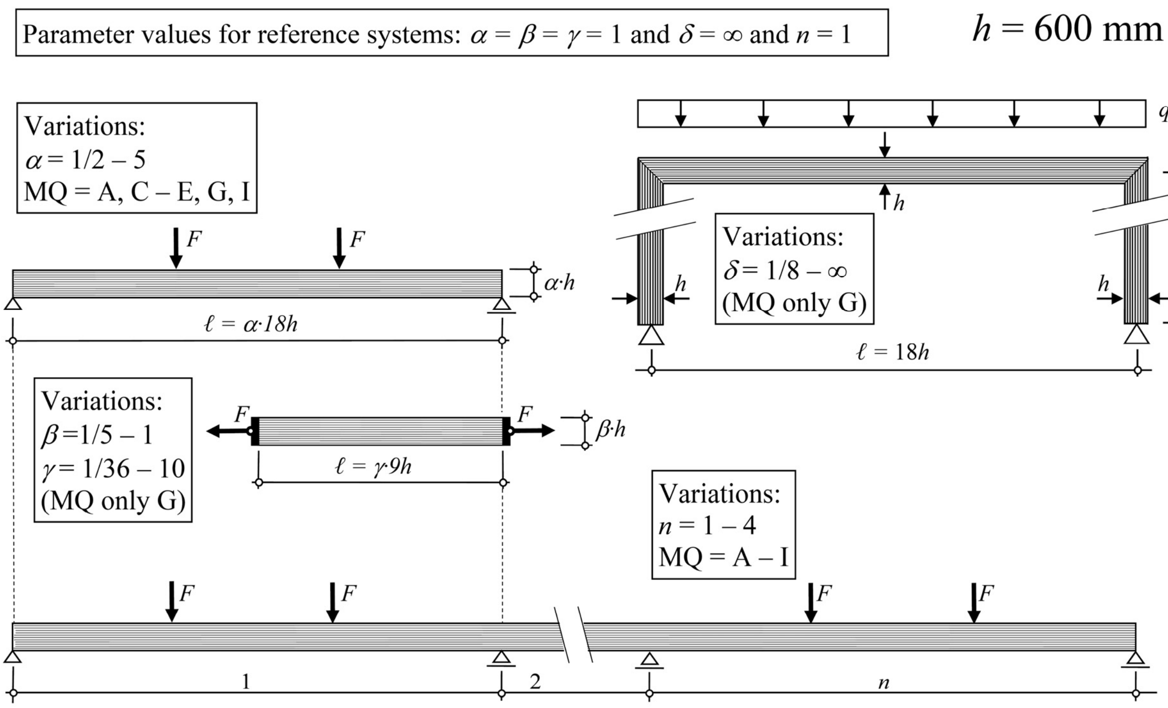

Figure 5: Reference configurations as well as size and material variations with the basic members and systems

Table 2: Reference strengths.

1	k	*Material*	*β*	$f_{k,ref}$ N/mm²
2	k_1	*A*	-	*24.2*
3		*C*	-	*27.5*
4		*D*	-	*28.6*
5		*E*	-	*31.4*
6		*G*	-	*32.3 (32.8*)*
7		*I*	-	*36.7*
8	k_2	*G*	*1/5-1*	*28.5*
9	k_3-k_5	*A*	-	*22.9*
10		*B-H*	-	*24.8-35.2*
11		*I*	-	*38.6*
12	k_6-k_9	*G*	-	*33.8*

**Assuming linear elasticity in the compression zone for comparable purpose, based on 3000 simulations*

For the graphic representation, the strength values f (either 5th percentiles of simulated populations or individual simulated strength values) are normalised according to Eq. (3) where $f_{k,ref}$ is the reference 5th percentile stated in Table 2.

$$f_{norm} = f \,/\, f_{k,ref} \tag{3}$$

With the normalisation, two cases are distinguished: first, normalising of 5th percentiles of simulated populations and second, of single simulated values. With the first case, which applies to the bending and tension members (later on treated in Figure 6), f_{norm} amounts to approximately 1 for the corresponding reference member ($\alpha = \gamma = 1$). With the second case, which applies to the continuous beams and frames (treated in Figure 7 and 8), the normalised individual strength amounts to 1 for the cumulative frequency of 0.05 and the reference system ($n = 1$ or $\delta = \infty$).

3.1.1. Variations of the member size

Figure 6 shows the relation between the normalised 5th percentile of the bending and tensile strength, respectively, and the corresponding size parameter. The diagram for the bending strength contains individual courses for each employed material quality (MQ) and the one for the tensile strength for each β value. In the variation for α and γ, there is a clear dependence between the stressed member size and the normalised 5th percentile. The courses change from a steep negative gradient for small α- and γ-values to a slight negative inclination for higher values. Both the material quality and the depth of tension members show a low influence on the effective strength; however, these effects have to be judged carefully. As in reality, an increasing material quality usually reduces the strength variation within the material, the simulation results state a contradiction. The reason for that could be a model simplification concerning the generation of the errors e in the regression equations; this generation is independent of the material quality. The apparent independence of the tensile strength with regard to the member width could partly be explained by the ultimate failure criterion. The probability of weak spots in the outermost laminations is independent of the member width, causing most likely the same strength level for narrow and wide members.

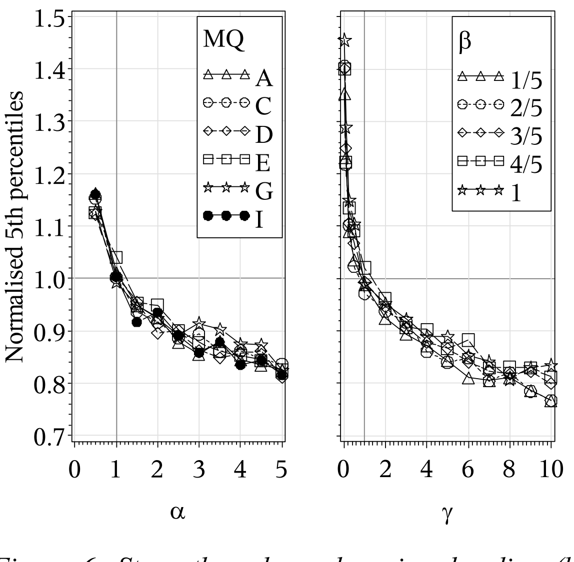

Figure 6: Strength and member size; bending (left) and tensile strength (right)

3.1.2. Continuous beams

The diagrams in Figure 7 show parts of the cumulative frequency distributions between 0 and 20 % of the normalised individual values of the bending strength. The upper diagram exemplifies the simulation results for material quality A and the lower one for I. The horizontal line at 0.05 and its points of intersection with the distributions help to read the normalised 5th percentiles of the bending strength for the continuous beams with 2, 3 and 4 spans. According to the agreement on the strength of the reference system, the normalised 5th percentile amounts to 1 for the simple beams (n = 1). Thus, the intersection points with the distributions for n = 2, 3 and 4 indicate the change in effective strength for the corresponding continuous beam. Due to the probabilistic system effect, the 2-span beam, for instance, possesses an effective bending strength which is at least 1.2 times as high as the reference strength. The resulting strength difference makes in particular 2-span beams more efficient compared to simple beams since both of them have the same design moment (under comparable loading and span).

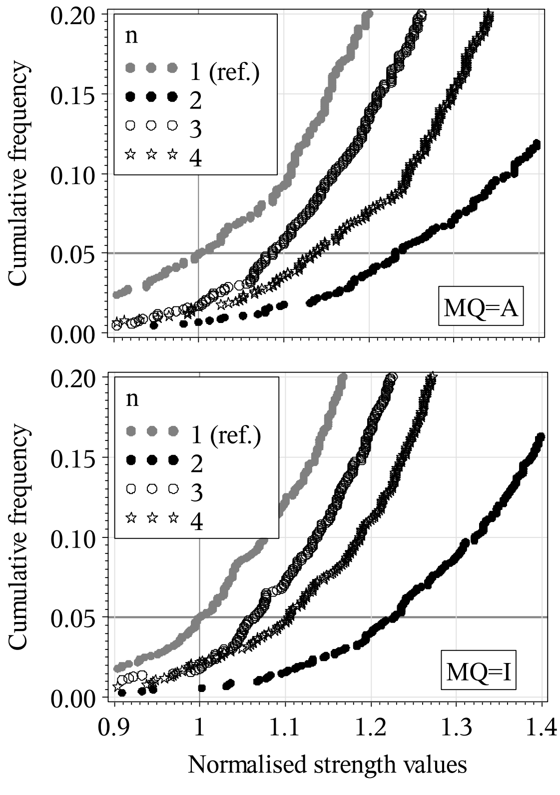

Figure 7: Cumulative frequency distributions of the bending strength in continuous beams: material A (top) and I (bottom).

3.1.3. Frames

In agreement with the graphic representation of the results for the continuous beams, Figure 8 shows the same proportion of the cumulative frequency distribution. The vertical reference lines (through the intersection points between the reference line at 0.05 and the distributions) exemplify the order of the strength modification, which is to be expected for the examined frame variations.

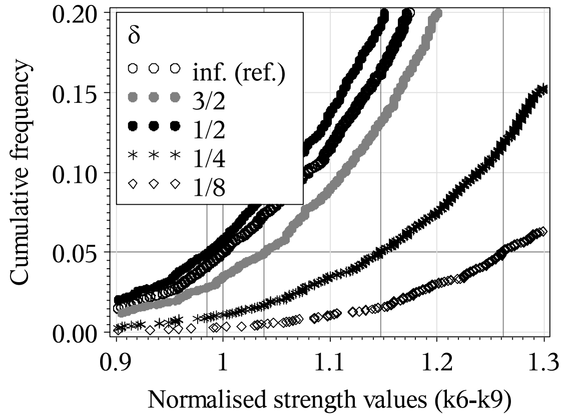

Figure 8: Cumulative frequency distributions of the bending strength in frames; vertical reference lines show target values for the factors k_6 to k_9.

3.1.4. Comparison between two different load configurations

The reference strength of 32.8 N/mm² (Table 2, line 6) reflects the load configuration with two single concentrated loads (2 x *F*) while that of 33.8 N/mm² (line 12) reflects a configuration with equally distributed load (*q*). In both cases, the systems are simple beams, 600 mm in depth and 10,800 mm in length; the material corresponds quality G. The ratio of both values is 33.8/32.8 = 1.03. Hence, there is a slight increase in bending strength for the configuration with equally distributed load. This numerically determined ratio does not contradict an analytical calculation. Based on formulas, imparted by Isaksson (2003) for the same comparison, and under the assumption of the Weibull exponent of 1/10.8 from Eq. (5), Isaksson's formulas result in a ratio of 1.034. Consequently, the comparison shows that the numerical approach is also sensitive to minor changes in the load configuration.

3.2. Modification factors

The diagram in Figure 9 combines the representations in Figure 6. In place of the single courses for the different material qualities, a nonlinear regression curve represents the dots, stating the normalised 5th percentiles for the bending strength. The same procedure was applied to the tensile strength. A nonlinear regression curve was fitted to all the depth-dependent normalised 5th percentiles (circles). Eq. (4) and (5) define the course of both curves and the calculation of k_1 and k_2, respectively.

$$k_1 = \left(\frac{1}{\alpha}\right)^{1/8.29} \quad \text{with} \quad \frac{1}{\alpha} = \frac{600}{h} \quad (h \text{ in mm}) \quad (4)$$

$$k_2 = \left(\frac{1}{\gamma}\right)^{1/10.8} \quad \text{with} \quad \frac{1}{\gamma} = \frac{5400}{\ell} \quad (\ell \text{ in mm}) \quad (5)$$

Figure 10 shows the relation between the normalised 5th percentiles of the bending strength for the 2-, 3- and 4-span-beams and the absolute reference strength for the different material qualities.

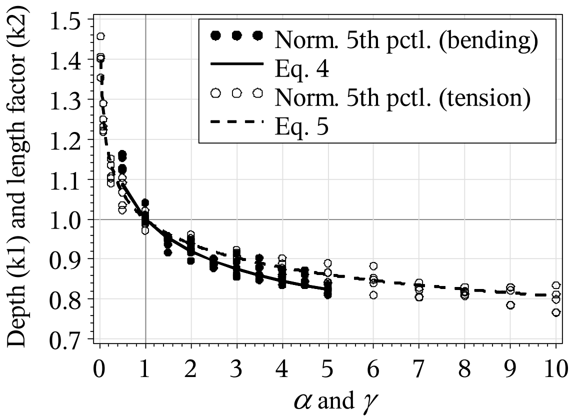

Figure 9: Bending and tensile strength modification

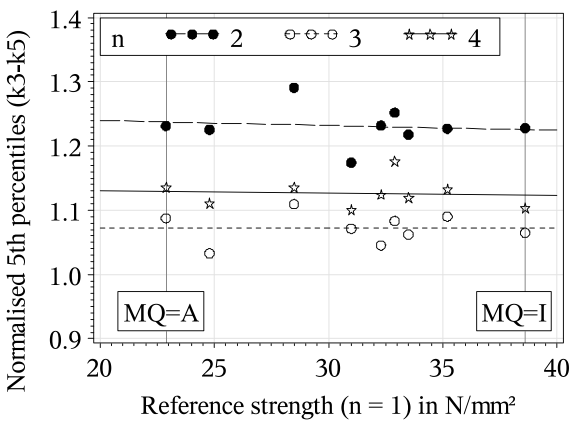

Figure 10: Determination of the factors k_3 to k_5.

As an example, vertical lines mark the qualities A and I. The regression lines were fitted to each of the three relations. They prove more or less that the strength modification is independent on the simulated material quality and strength, respectively; all straight lines run almost horizontally. Based on Figure 10, the three modification factors can be read as constant values. Eq. (6) specifies their order.

$$\begin{aligned} n = 2 &\rightarrow k_3 = 1.23 \\ n = 3 &\rightarrow k_4 = 1.07 \\ n = 4 &\rightarrow k_5 = 1.13 \end{aligned} \quad (6)$$

The modification factors for the examined frame variations are specified in Eq. (7). As no material variation is performed in the frame examination, the factors were directly taken from Figure 8. The values below agree with those of the vertical reference lines running through the intersection points between the distributions and the 5th percentile line. Since each of the frames is examined with 3000 single simulations, stable *k*-factors for the frames can be assumed, although no material variation was superposed.

$$\begin{aligned} \delta = 3/2 &\rightarrow k_6 = 1.04 \\ \delta = 1/2 &\rightarrow k_7 \approx 0.99 \\ \delta = 1/4 &\rightarrow k_8 = 1.15 \\ \delta = 1/8 &\rightarrow k_9 = 1.26 \end{aligned} \quad (7)$$

4. CONCLUSIONS

The numerical examination of bending and tension members and various structural systems made of glulam reveals pronounced differences in strength values caused by a change in the stressed member size and the applied structural system, respectively.

The application of the modification factors, proposed in the paper, lead to a more balanced reliability in particular between small and large as well as short and long glulam members under bending and tension, respectively. The examination of various structural systems shows that 2-span-beams and 2-hinged-frames with short and stiff columns provide a favourable probabilistic system effect so far not fully reflected if a standard verification of the bending strength is performed.

The extension of the strength simulation to more complex structural systems still requires suitable input data concerning stress interactions between normal and shear stresses, its implementation in the computer model and appropriate models including the effect of normal forces in frames. The interactions concern particularly areas with high discontinuity as supports or frame corners.

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