

Bayesian Updated Time-Dependent Chloride-Induced Corrosion Assessment Using Redundancy Factors

W. Botte

PhD. student, Dept. of Structural Engineering, Ghent University, Ghent, Belgium

R. Caspeele

Professor, Dept. of Structural Engineering, Ghent University, Ghent, Belgium

L. Taerwe

Professor, Dept. of Structural Engineering, Ghent University, Ghent, Belgium

ABSTRACT: In order to assess the structural reliability and redundancy with respect to deterioration, it is required to select appropriate models which describe the deterioration process. The parameters associated with these models have to be estimated through statistical interference, which introduces uncertainties in parameter estimates. As the structural reliability indices which are incorporated in the reliability-based redundancy factor can be considered as random variable, this redundancy factor itself is a random variable as well. In case additional information becomes available, the distribution function can be updated by taking into account this extra information. In this contribution, a framework is developed, which allows for the incorporation of additional information in the uncertain reliability index and the associated redundancy factor through Bayesian updating. It is shown that in case additional information on a main variable is gathered, this has a significant effect on the (mean) value and uncertainty of the reliability index and the associated redundancy factor.

1. INTRODUCTION

In the last decades, deterioration of existing structures has been a growing concern. Deterioration due to corrosion of reinforcement steel has been of particular interest since the reduction of the steel section results in a decrease of the structural safety. Another topic which has been the subject of many publications is the assessment of the structural robustness and redundancy.

In this contribution, both topics are combined: the quantification of the redundancy of reinforced concrete beams subjected to chloride-induced corrosion is discussed. Further, since corrosion is a time-variant process, the associated redundancy factor will be time-dependent. This time-dependency has been explicitly included in redundancy measures by Okasha & Frangopol (2010) and Decò et al. (2011); and in robustness measures by Biondini

(2009) and Biondini & Frangopol (2010).

The assessment of structural reliability and redundancy with respect to deterioration requires the selection of appropriate models which describe the structural behaviour and the deterioration process. The parameters associated with these models have to be estimated through statistical inference, which introduces uncertainties in the parameter estimates. It follows that due to the parameter estimation, the reliability index and the reliability-based redundancy factor can be considered as a random variables.

2. DETERIORATION DUE TO CHLORIDE-INDUCED CORROSION

2.1. General considerations

A common deterioration mechanism in reinforced concrete is corrosion of the reinforcement steel. Corrosion affects the steel as well as the concrete, hence the safety of

deteriorated concrete structures is reduced. According to the Model Code for Service Life Design (*fib* 2006), the process of corrosion of the reinforcement can be divided roughly into two time periods: the initiation period and the propagation period. The first phase is defined as the time until the reinforcement becomes depassivated, either by chloride ingress or by carbonation. During the second phase, the reinforcement itself is affected: the cross-section is reduced. Both phases are governed by different stochastic parameters and can be described by mathematical models. Further in this contribution, the time to corrosion initiation T_i and the associated uncertainties are not considered. For the sake of simplicity, it is assumed that $T_i = 0$. Hence, a possible reduction of the structural safety level during the initiation phase is not considered in this contribution.

2.2. Propagation period

Once corrosion is initiated, the corrosion rate is determined by equation (1), as proposed by Stewart & Suo (2009):

$$i_{corr}(t_p) = i_{corr}(0) \cdot 0.85t_p^{-0.29} \quad (1)$$

where $i_{corr}(t_p)$ [$\mu\text{A}/\text{cm}^2$] is the corrosion rate at time t_p , t_p [years] is the time since corrosion initiation and $i_{corr}(0)$ [$\mu\text{A}/\text{cm}^2$] is the corrosion rate at the start of corrosion propagation. The latter can be calculated from:

$$i_{corr}(0) = (2.70 (1 - w/c)^{-1.64})/a \quad (2)$$

where w/c [-] is the water-cement ratio and a [cm] is the concrete cover. Equation (1) is only valid when no spalling occurs, which is assumed in this paper.

As suggested by Stewart & Rosowsky (1998), the reduction in bar diameter of the reinforcement steel can be derived from the corrosion rate (since $1 \mu\text{A}/\text{cm}^2 = 0.0116 \text{ mm}/\text{year}$):

$$\emptyset(t_p) = \emptyset_0 - 2 \cdot 0.0116 \int_0^{t_p} i_{corr}(t) dt \quad (3)$$

where $\emptyset(t_p)$ [mm] is the reinforcement diameter t_p years after corrosion initiation and \emptyset_0 [mm] is the initial diameter.

3. PREDICTIVE RELIABILITY INDEX

The verification of a structure with respect to a certain limit state requires the definition of a limit state function $g(\cdot)$. The theoretical failure probability P_f for that specific limit state is then defined by:

$$P_f = \int_{g(\mathbf{x}) < 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (4)$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the n -dimensional probability density function (PDF) of the n basic variables X_i ($i=1 \dots n$) and $g(\mathbf{x})$ is the limit state function, defined so that $g(\mathbf{x}) < 0$ corresponds to failure. The n basic variables represent uncertain quantities such as material properties, actions (loads), geometrical properties and model uncertainties. The structural reliability can be quantified through the reliability index β , defined in equation (5).

$$\beta = \Phi^{-1}(1 - P_f) \quad (5)$$

As stated by Der Kiureghian (2008) the aforementioned formulations are a theoretical formulation of the structural reliability problem, since in practice neither the joint PDF $f_{\mathbf{X}}(\mathbf{x})$ nor the limit state function $g(\mathbf{x})$ are precisely known. Hence, the selection of probabilistic or physical models is required and the associated parameters have to be estimated through statistical inference of experimental data and observations. The model for $f_{\mathbf{X}}(\mathbf{x})$ in which the parameters Θ are estimated, is designated $\hat{f}_{\mathbf{X}}(\mathbf{x}|\Theta)$.

Since the model parameters are uncertain, it follows from (4) and (5) that also the failure probability P_f and the corresponding reliability index β are uncertain. Hence, the random variables $P = P_f(\Theta)$ and $B = \beta(\Theta)$ can be introduced. As random variables, P and B have probability density functions, namely $f_P(P_f)$ and $f_B(\beta)$ respectively, and characteristics, such as a mean (μ_P and μ_B) and a variance (σ_P^2 and σ_B^2). The latter expresses the uncertainty of the estimate of the failure probability or reliability index, originating from the parameter uncertainties. Since parameter uncertainties can be reduced by gathering additional information, these variances can also be reduced.

The value of the reliability index that takes into account the influence of parameter

uncertainties is called the predictive reliability index $\tilde{\beta}$. Der Kiureghian (2008) provides different methods for calculating the reliability index and its measures of uncertainty. One of these methods yields a very simple approximation formula for $\tilde{\beta}$:

$$\tilde{\beta} \cong \mu_B / \sqrt{1 + \sigma_B^2} \quad (6)$$

The only approximation used to obtain equation (6) is the assumption that the reliability index follows a normal distribution, which is in many cases an acceptable assumption (Der Kiureghian 2008).

The calculation of the predictive reliability index according to equation (6) requires the mean and variance of the random variable B. These can be calculated by making use of a first-order approximation of the function $B = \beta(\Theta)$. This yields:

$$\mu_B \cong \beta(\mathbf{M}_\Theta) \quad (7)$$

$$\sigma_B^2 \cong (\nabla_{\Theta}\beta)_{\Theta=\mathbf{M}_\Theta}^T \Sigma_{\Theta\Theta} (\nabla_{\Theta}\beta)_{\Theta=\mathbf{M}_\Theta} \quad (8)$$

with \mathbf{M}_Θ the mean vector of Θ , $\Sigma_{\Theta\Theta}$ the covariance matrix of Θ and $(\nabla_{\Theta}\beta)_{\Theta=\mathbf{M}_\Theta}$ the sensitivity vector of the conditional reliability index with respect to the parameters Θ , evaluated at the mean values. The sensitivity vector in equation (8) is given by:

$$\nabla_{\Theta}\beta = \left(\frac{\partial\beta}{\partial\theta_1}, \frac{\partial\beta}{\partial\theta_2}, \dots, \frac{\partial\beta}{\partial\theta_n} \right) \quad (9)$$

$$\frac{\partial\beta}{\partial\theta_i} = \boldsymbol{\alpha}^T \left(\frac{\partial\mathbf{u}}{\partial\theta_i} \right)_{\mathbf{u}=\mathbf{y}} \quad (10)$$

where $\boldsymbol{\alpha}$ is the sensitivity vector, \mathbf{u} the vector of normalized basic variables, \mathbf{y} the design point in standard normal space and $\beta = \boldsymbol{\alpha}^T \mathbf{y}$.

As an example, consider n independent, normal random variables \mathbf{X} with unknown means and standard deviations. Hence the unknown parameter vector $\Theta = (\mathbf{M}, \Sigma) = (M_1, \Sigma_1, M_2, \Sigma_2 \dots M_n, \Sigma_n)$. The unknown mean M_i has a mean value \bar{x}_i and standard deviation σ_{M_i} ; the unknown standard deviation Σ_i has a mean σ_i and a standard deviation σ_{Σ_i} . From equation (10) it follows, in case $\theta_i = M_i$:

$$\frac{\partial\beta}{\partial\theta_i} = \alpha_i \frac{\partial}{\partial M_i} \left(\frac{x_i - M_i}{\Sigma_i} \right)_{M_i=\bar{x}_i; \Sigma_i=\sigma_i} = -\frac{\alpha_i}{\sigma_i} \quad (11)$$

And in case $\theta_i = \Sigma_i$:

$$\frac{\partial\beta}{\partial\theta_i} = \alpha_i \frac{\partial}{\partial \Sigma_i} \left(\frac{x_i - M_i}{\Sigma_i} \right)_{M_i=\bar{x}_i; \Sigma_i=\sigma_i} = +\frac{\alpha_i^2 \beta}{\sigma_i} \quad (12)$$

With (8), (9), (11) and (12) the variance of the reliability index becomes:

$$\sigma_B^2 \cong \sum_{i=1}^n \frac{\alpha_i^2}{\sigma_i^2} \sigma_{M_i}^2 + \sum_{i=1}^n \frac{\alpha_i^4 \beta^2}{\sigma_i^2} \sigma_{\Sigma_i}^2 \quad (13)$$

The first term represents the contribution of the uncertainty on the mean value of the estimated parameters, while the second term represents the contribution of the uncertainty on the standard deviation.

Equations (7) and (8) show that it is sufficient to perform one single calculation of the reliability index along with the parameter sensitivities in order to determine the mean and variance of the reliability index. This calculation is performed using the mean values of the parameters.

4. ROBUSTNESS AND REDUNDANCY

Robustness is the ability of a structure to withstand certain events without being damaged to an extent disproportionate to the original cause. A concept closely related to robustness is ‘redundancy’, which is the ability of a system to redistribute a load which can no longer be sustained by some members. While redundant systems are generally believed to be more robust, there are additional methods of providing robustness that are not related to redundancy (COST TU0601, 2011). Several authors proposed different approaches to quantify structural redundancy and robustness (Sørensen et al. 2012). In general, the different measures can be subdivided in three classes, with increasing complexity:

- Deterministic quantification, based on structural measures;
- Reliability-based quantification, based on the probability of failure of an undamaged and a damaged system;
- Risk-based quantification, based on a complete risk analysis in which consequences are divided into direct and indirect consequences.

The last method is the most general one, but, as stated before, also the most complex one. Therefore, in this contribution, a reliability-based approach is used.

It should be noted that the aforementioned measures focus on two states of a structure: the undamaged and the damaged state. As indicated by Yao (1985) and Frangopol and Curley (1987), ‘damaged’ may refer to any strength deficiency introduced during the design or construction phase of the structure as well as any deterioration of strength caused by external loading (e.g. sudden column loss) and/or environmental conditions (e.g. corrosion) during the life-time of the structure.

One way of quantifying redundancy in a probabilistic way, is through the redundancy index RI introduced by Fu and Frangopol (1990). This index is based on the probability of failure $P_{f,intact}$ of the intact system and the probability of failure $P_{f,damaged}$ of the damaged system and is defined by equation (14).

$$RI = \frac{P_{f,damaged} - P_{f,intact}}{P_{f,intact}} \quad (14)$$

The redundancy index for a very redundant structure is close to zero, while it tends to infinity for structures that are completely damaged.

Alternatively, the redundancy can be quantified by using the reliability index β_i of the intact structure and the reliability index β_d of the damaged structure (Frangopol and Curley 1987). This so-called redundancy factor β_R is defined by equation (15) and takes values between 0, for completely damaged structures, and infinity, for intact structures.

$$\beta_R = \frac{\beta_i}{\beta_i - \beta_d} \quad (15)$$

Note that the reliability-based redundancy factor does not require the description of external consequences, unlike risk-based measures, which simplifies the calculations.

As mentioned in section 3, the reliability index is, in practice, never precisely known because of the estimation of parameters. Hence the reliability indices β_i and β_d are not precisely known and based on formulas (7) and (8), the

mean (μ_{Bi} and μ_{Bd}) and variance (σ_{Bi}^2 and σ_{Bd}^2) of these random variables can be determined.

Since the reliability index of the undamaged and the damaged system are not precisely known, also the redundancy factor, as defined by (15), is not precisely known.

5. EXAMPLE

In order to illustrate the concepts elaborated above, consider a reinforced concrete slab subjected to uniform chloride-induced corrosion.

5.1. Characteristics

The limit state equation with respect to bending, for the most heavily loaded cross-section is defined as in equation (16).

$$g(x) = K_R A_s f_y \left(h - a - 0.5 \Phi_0 - \frac{0.5 A_s f_y}{f_c b} \right) - K_E \cdot (M_G - M_Q) \quad (16)$$

where K_R [-] is the model uncertainty for the resistance effect, A_s [mm²] is the cross-section of the reinforcement, f_y [N/mm²] is the yield strength of the reinforcement, h [mm] is the height of the cross-section, f_c [N/mm²] is the compressive strength of the concrete, b [mm] is the width of the cross-section, K_E [-] is the model uncertainty for the load effect, M_Q [Nmm] is the bending moment induced by the variable load, related here to a 50-year reference period, and M_G [Nmm] is the bending moment induced by the permanent load. The characteristic values of the bending moment induced by the permanent load and the imposed load (i.e. M_{Gk} and M_{Qk}) are defined through equations (17) and (18).

$$\chi = \frac{M_{Qk}}{M_{Gk} + M_{Qk}} \quad (17)$$

$$M_{Rd} = M_{Gk} \cdot \max \left\{ \begin{array}{l} \gamma_G + \psi_0 \gamma_Q \frac{\chi}{1-\chi} \\ \xi \gamma_G + \gamma_Q \frac{\chi}{1-\chi} \end{array} \right\} \quad (18)$$

with χ the load ratio, M_{Gk} the bending moment induced by the characteristic value of the permanent load, M_{Qk} the bending moment induced by the characteristic value of the variable load, γ_G the partial factor for the permanent load (= 1.35), ξ a reduction factor for unfavourable permanent loads, γ_Q the partial

factor for the variable load (= 1.5), and ψ_0 a combination factor (= 0.7). This calculation concept allows to evaluate the reliability of individual structural elements, without requiring additional assumptions with respect to the current and future use of the structure.

The characteristics of the beam are summarized in Table 1 (N: normal distribution; LN: lognormal distribution; GU: Gumbel distribution (max.); DET: deterministic). The characteristics of the variables are based on the Probabilistic Model Code (JCSS 2001) and the Model Code for Service Life Design (*fib* 2006).

Table 1. Characteristics of the RC beam.

		Distribution	μ	σ
h	[mm]	N	200	5
f_c	[MPa]	LN	38.75	4.67
f_y	[MPa]	N	560	30
\emptyset_0	[mm]	N	10	0.2
n^*	[-]	DET	10	-
b	[mm]	DET	1000	-
w/c	[-]	DET	0.4	-
a	[mm]	LN	15	5
M_G	[Nmm]	N	M_{Gk}	$0.1M_{Gk}$
M_Q	[Nmm]	GU	$0.6M_{Qk}$	$0.21M_{Qk}$
K_R	[-]	LN	1.2	0.18
K_E	[-]	LN	1.0	0.10

* The number of reinforcement bars

The diameter of cross-section of the reinforcement decreases in time according to equation 3. Since the cross-section of the reinforcement decreases in time, also the reliability index is a function of time. Moreover, it is assumed that the mean values and standard deviations of h, f_c , f_y , \emptyset_0 and a are estimated, based on prior information. The mean and standard deviation of the estimated parameters are summarized in Table 2. The standard deviation of the mean value \bar{x}'_i is calculated from (19), where n' is the number of samples used to estimate the mean value. The standard deviation of the standard deviation Σ_i is calculated from (20), where v is the number of samples used to estimate the standard deviation ($v = n-1$ in case it is estimated from the same samples).

$$\sigma_{Mi} = s_i / \sqrt{n} \quad (19)$$

$$\sigma_{\Sigma i} = \sqrt{2 / (v - 4)} \quad (20)$$

Table 2. Prior information on the basic variables.

	Mean M_i			Standard deviation Σ_i		
	\bar{x}'_i	σ_{Mi}	n'	s_i	$\sigma_{\Sigma i}$	v'
h	200	0.71	50	5	1.05	49
f_c	38.75	3.81	1.5	4.67	4.67	6
f_y	560	12.25	6	30	7.07	5
\emptyset_0	10	0.03	50	0.20	0.042	49
a	15	2.04	6	5	7.07	5

The prior parameters of the concrete compressive stress are based on a C25 ready mixed concrete (Rackwitz 1983). Further it is assumed that the characteristics of the height h of the cross-section and the initial diameter of the reinforcement \emptyset_0 are based on a large sample size ($n=50$). It is assumed the characteristics of the concrete cover a are based on a small sample size (i.e. $n=6$), hence large uncertainties are associated with these parameters. The Probabilistic Model Code (JCSS 2001) indicates that the prior information on structural steel may be relatively strong and the corresponding sample size is $n \approx 50$. However, no indications about prior information for reinforcement steel were found in literature. Therefore, a low sample size was adopted here.

It should be noted that the cross-section of the reinforcement is the only parameter that changes in time.

Using the characteristics mentioned above, the mean and variance of β are calculated according to equation (7) and (8), by performing one single calculation of the reliability index and the parameter sensitivities at each point in time, using the mean values of the parameters. The result of the FORM calculation with mean values of the parameters, as indicated in Table 2, is shown in Figure 1 as a solid line. The mean value of the reliability index decreases in time, from 3.55 at $t=0$ to 1.34 after 50 years of corrosion.

Further, also the predictive reliability index, according to equation (6), is shown in Figure 1. Note that this predictive reliability index becomes significantly lower than the mean reliability index as time increases.

Figure 2 shows the sensitivity factors for the basic variables as a function of time. The graph shows that the influence of the basic variables

remains more or less constant in time, except for the concrete cover a , which increases significantly in time.

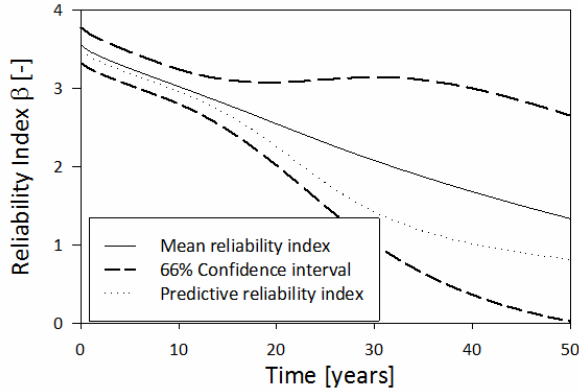


Figure 1. Mean value of the reliability index, 66% CI and predictive reliability index as a function of time.

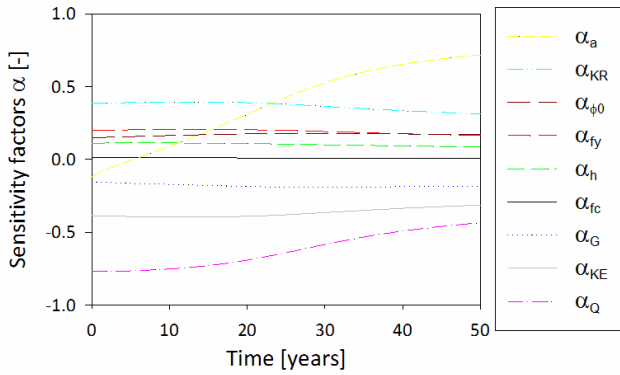


Figure 2. Sensitivity factors as a function of time.

Using these sensitivity factors and the characteristics from Table 2, the standard deviation of the reliability index can be calculated as indicated in section 3. To illustrate the results, the dashed lines in Figure 1 show the interval $[\mu_B - \sigma_B ; \mu_B + \sigma_B]$. This interval contains approximately 66% probability. It is noted that due to the significant increase of the sensitivity factor of the concrete cover over time, also the standard deviation of the reliability index increases significantly from 0.23 at $t = 0$ to 1.32 at $t = 50$ years.

5.2. Updating process

5.2.1. General

The distributions for the reliability index can be updated in case additional information becomes available. The updating of the resistance can be based on a Bayesian approach applied on

individual random variables (e.g. compressive strength) based on a collection of field data.

In case a random variable follows a lognormal-gamma distribution (which is the case for the concrete compressive strength and the concrete cover in this example), the parameters of the prior distribution (i.e. $\bar{x}'_{\ln X}$, n' , $s'_{\ln X}$, ν') can be updated easily when a set of observations ($\bar{x}_{\ln X}$, n , $s_{\ln X}$, ν) are available (Rackwitz 1983):

$$n'' = n' + n \quad (21)$$

$$\nu'' = \nu' + \nu + 1 \quad (22)$$

$$\bar{x}''_{\ln X} = \frac{n' \bar{x}'_{\ln X} + n \bar{x}_{\ln X}}{n''} \quad (23)$$

$$s''_{\ln X} = \frac{1}{\nu''} [(v' s'_{\ln X} + n' \bar{x}'_{\ln X}{}^2) + (v s_{\ln X} + n \bar{x}_{\ln X}{}^2) - n'' \bar{x}''_{\ln X}{}^2] \quad (24)$$

In this section, the updating process of the reliability index and the redundancy factor is illustrated based on the updating of the concrete compressive strength and the concrete cover.

5.2.2. Updating of concrete strength

Consider that 7 compressive strength tests were performed, resulting in a mean value $\bar{x} = 31.5$ MPa and a standard deviation $s = 2.6$ MPa. Consequently, the prior parameters for the concrete compressive strength, mentioned in Table 2, can be updated according to section 5.2.1. The prior and posterior parameters as well as the test data are given in Table 3.

Table 3. Prior and posterior distribution parameters for the concrete compressive strength.

Prior	Test Data	Posterior
$\bar{x}'_{\ln X} = 3.65$	$\bar{x}_{\ln X} = 3.45$	$\bar{x}''_{\ln X} = 3.485$
$n' = 1.5$	$n = 7$	$n'' = 8.5$
$s'_{\ln X} = 0.12$	$s_{\ln X} = 0.082$	$s''_{\ln X} = 0.099$
$\nu' = 6$	$\nu = 6$	$\nu'' = 13$

Subsequently, this updated distribution for the concrete compressive strength is used to update the mean and variance of the reliability index by performing one single calculation of the reliability index and the parameters sensitivities.

Figure 3 shows the updated reliability index $\beta_{\text{updated fc}}$ compared to the reliability index before updating β_{prior} and the confidence interval (CI) $[\mu_B - \sigma_B ; \mu_B + \sigma_B]$ before and after updating.

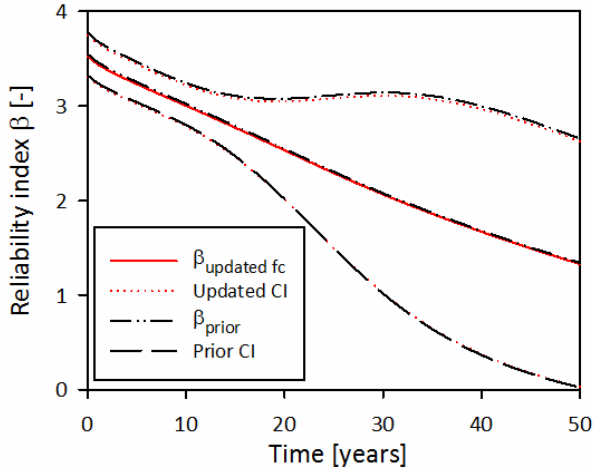


Figure 3. Mean reliability index and 66% CI after updating of the concrete compressive strength

It is noted that updating the concrete compressive strength distribution does not result in significant changes of the reliability index. This is due to the (constant) low sensitivity factor of the compressive strength, as can be seen in Figure 2. Since the updating of the concrete compressive stress does not result in a significant change of the reliability index, this will not result in a significant change in the redundancy factor.

5.2.3. Updating of concrete cover

The reliability index and the redundancy factor can be updated in case additional measurements of the concrete cover are performed. Consider that 10 measurements of the concrete cover were performed, resulting in a mean value of 19.2 mm and a coefficient of variation of 0.25, then the updating of the prior parameters of the concrete cover is shown in Table 4.

Table 4. Prior and posterior distribution parameters for the concrete cover.

Prior	Test Data	Posterior
$\bar{x}'_{lnX} = 2.65$	$\bar{x}_{lnX} = 2.92$	$\bar{x}''_{lnX} = 2.82$
$n' = 6$	$n = 10$	$n'' = 16$
$s'_{lnX} = 0.33$	$s_{lnX} = 0.24$	$s''_{lnX} = 0.27$
$\nu' = 5$	$\nu = 9$	$\nu'' = 15$

Using the updated distribution for the concrete cover, one can update the mean and variance of the reliability index in a similar way

as for the concrete compressive strength. The results are shown in Figure 4.

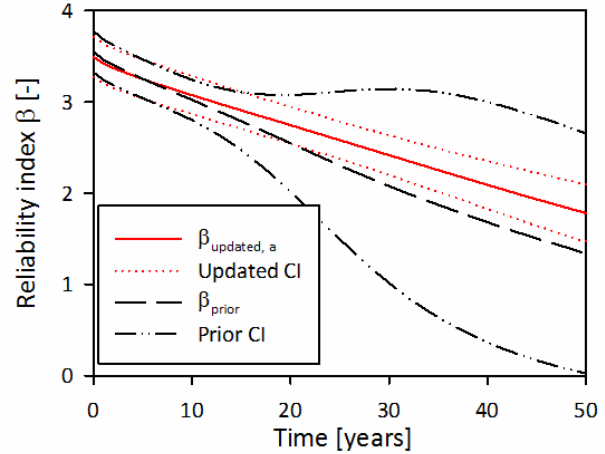


Figure 4. Mean reliability index and 66% CI after updating of the concrete cover.

It can be seen that updating the distribution of the concrete cover results in significant changes in the reliability index. Updating the concrete cover, towards higher values than originally assumed, results in an increase of the mean value of the reliability index compared to the situation before updating, except in the early years after corrosion initiation. The latter is due to the fact that at small corrosion levels, the influence of the concrete cover on the corrosion process is dominated by the influence of the concrete cover on the effective depth of the reinforcement.

Further, it is noted that the standard deviation of the reliability index is significantly reduced due to the updated concrete cover, especially starting from 20 years: at 50 years, the standard deviation is reduced from 1.31 to 0.31.

5.2.4. Redundancy factor

Finally, the probability density function for the redundancy factor before and after updating was determined using Monte Carlo simulations, assuming a normal distribution for the reliability index. The prior and posterior probability density functions of the redundancy factor are shown in Figure 5. Since updating the concrete compressive stress does not result in significant changes of the reliability index, also the influence on the redundancy factor is marginal:

the prior and updated distribution function of the redundancy factor nearly coincide. The influence is however significant in case of updating of the concrete cover: the distribution of the redundancy factor shifts to higher values, indicating a higher redundancy with respect to corrosion than initially anticipated.

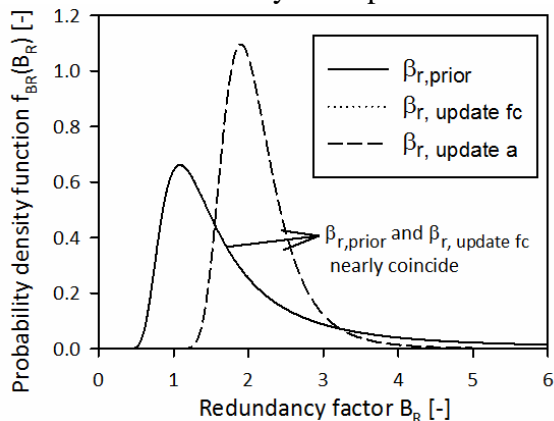


Figure 5. Prior ($\beta_{r,prior}$) and updated ($\beta_{r,update\ fc}$ and $\beta_{r,update\ a}$) redundancy factor.

6. CONCLUSIONS

The quantification of redundancy under parameter uncertainties was investigated. Therefore, the framework, as developed by Der Kiureghian (2008) for the reliability index under parameter uncertainties, was extended towards a reliability-based redundancy factor. Subsequently, this framework was applied on a reinforced concrete beam, subjected to chloride-induced corrosion in order to illustrate the updating process of different individual random variables in case additional information becomes available. It was shown that the effect of the updating is highly dependent on the sensitivity factor of the variable under consideration, obtained through a FORM analysis using the mean values of all random variables. In case additional information on a main variable is obtained, this has a significant effect on the (mean) value and uncertainty of the reliability index and the associated redundancy factor.

7. ACKNOWLEDGEMENTS

Wouter Botte is a Research Assistant of the FWO Research Foundation Flanders. The authors wish to thank the FWO for the financial

support on the research project “Structural reliability and robustness assessment of existing structures considering membrane action effects and Bayesian updating of test information”.

8. REFERENCES

- Biondini, F. (2009) A measure of lifetime structural robustness, *Structures Congress 2009*
- Biondini, F. & Frangopol, D.M. (2010) Time effects on robustness and redundancy of deteriorating concrete structures, *ICOSSAR 2013*
- COST TU0601 –Structural Robustness Design for Practising Engineers – Canisius, T.D.G.
- Decò, A., Frangopol, D.M. and Okasha, N.M. (2011) Time-variant redundancy of ship structures, *SNAME J Ship Res*, 55(3),208-219
- Der Kiureghian, A. (2008) Analysis of structural reliability under parameter uncertainties. *Probabilist Eng Mech*, 23, 351-358.
- fib* (2006) *Model Code for Service Life Design*, FIB-Féd. Int. du Béton.
- Fu, G. & Frangopol, D. M. (1990) Balancing weight, system reliability and redundancy in a multiobjective optimization framework. *Structural Safety*, 7, 165-175.
- Frangopol, D. M. & Curley, J. P. (1987) Effects of damage and redundancy on structural reliability. *J Struct Eng*, 113, 1533-1549.
- JCSS (2001) *Probabilistic Model Code*.
- Okasha, N.M. & Frangopol, D.M. (2010) Time-variant redundancy of structural systems, *Struct Infrastruct E*, 6, 279-301
- Rackwitz, R. (1983) Predictive distribution of strength under control. *Mater & Struct*, 15, 259-267.
- Sørensen, J. D., Rizzuto, E., Narasimhan, H. & Faber, M. H. (2012) Robustness: Theoretical Framework. *SEI*, 22, 66-72.
- Stewart, M. G. & Rosowsky, D. V. (1998) Time-dependent reliability of deteriorating reinforced concrete bridge decks. *Structural Safety*, 20, 91-109.
- Stewart, M. G. & Suo, Q. (2009) Extent of spatially variable corrosion damage as an indicator of strength and time-dependent reliability of RC beams. *Engineering Structures*, 31, 198-207.
- Yao, J. T. P. (1985) *Safety and Reliability of Existing Structures*, Pitman Advanced Publishing Program