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The Role of Fiber Volume Fraction in Tensile Strength of Fibrous Composites

Rostislav Rypl
Research assistant, Dept. of Mechanics, Czech Tech. Univ. in Prague, Czech Republic

Miroslav Vořechovský
Professor, Dept. of Struct. Mechanics, Brno Univ. of Technology, Brno, Czech Republic

Rostislav Chudoba
Senior researcher, Inst. of Structural Concrete, RWTH Aachen Univ., Aachen, Germany

ABSTRACT: It has been proved experimentally that a finer crack pattern in brittle matrix composites with heterogeneous fibrous reinforcement increases the reinforcement efficiency in terms of fiber strength by up to 100 %. In the present paper, we simulate this phenomenon by a semi-analytical model of a composite crack bridge based on probabilistic fiber bundle models. The model is able to quantify the reinforcement efficiency increase with finer crack spacing given the information on the reinforcement heterogeneity. Possible sources of heterogeneity are variabilities in fiber diameter, modulus of elasticity or bond quality. With finer crack spacing, the heterogeneous stress state of the reinforcement is homogenized which leads to a more efficient load bearing behavior. Since the crack spacing is (within the practical range of values) a monotonic function of the fiber volume fraction and fiber diameter, these quantities should be taken into account in structural analysis and design of composites with heterogeneous reinforcement.

1. INTRODUCTION

The combination of brittle matrix (ceramic, cementitious) with fibrous reinforcement provides the possibility to design composites with tuned properties, in particular with a favorable quasi-ductile behavior and high load bearing capacity, see Phoenix (1993); Curtin (1993). The high tensile stiffness and strength of micro-fibers that create the load bearing component in brittle matrix composites can only be utilized if cracks form in the matrix. Fibers then stretch and transmit tensile load between the crack planes providing the composite with high ductility and stress redistribution capacity (Evans and Zok (1994)). The debonded lengths of fibers, and thus the compliance of a crack bridge, grow with the applied load and fiber radius, and decreases with the bond strength Cox (1952); Marshall and Evans (1985); Aveston et al. (1971).

When loaded in tension, well designed brittle matrix composites exhibit multiple cracks developing in the matrix perpendicularly to the loading direction over a range of applied stresses up to a state of crack saturation and ultimate failure Aveston et al. (1971); Hui et al. (1995); Curtin (1991). During this process, which is accompanied by damage evolution and significant stress redistribution, fibers debond in all crack bridges. Starting from the crack planes, the debonding process advances until the debonded zones meet between two adjacent matrix cracks. From that point on fibers behave like fixed between the cracks and the compliance of crack bridges upon further crack opening remains constant (if not affected by growing fiber damage). The qualitative and quantitative characteristics of the whole stress-strain response of composites depend on the mechanical, geometrical (size effect) and statistical properties of the constituents and their interface Ibnabdeljalil and Curtin (1997); Phoenix and Raj (1992).

The present modeling framework is based on a special class of mechanical models – probabilistic models. These models use probabilistic methods for the evaluation of representative mechanical responses of composite materials Curtin (1993); Hui et al. (1995); Thouless and Evans (1988); Smith (1982) and provide a fully probabilistic output in terms of statistical distributions of the analyzed measures (e.g. strength, stiffness, toughness). There are good reasons for the use of probabilistic methods to model the mechanical behavior of composites: a) the random nature of fiber failure and fiber properties in general; b) the large number of fibers (of the order of 10^4-10^8) in the composite. If one incorporated these features in deterministic models, computational limits would be exceeded very fast Chudoba et al. (2006).

Even though the models referenced so far have contributed important insights and are, in general, methodologically sound, the set of assumptions they use represents the material structure with a high level of idealization. All material and interface properties are deemed to be perfectly homogeneous with fiber strength as the only considered source of randomness and the fibers do not interact in any way. Probabilistic models with these idealizations have a great ability to predict qualitative tendencies, such as tough-to-brittle transitions Curtin (1993) and size effects Phoenix and Raj (1992). However, when reinforcement is far from being perfectly homogeneous, e.g. due to variability in interface quality, fiber diameter or fiber stiffness, the composite's response changes dramatically, rendering the predictions of common probabilistic models inaccurate Rypl et al. (2013). Experimental observations regarding textile reinforced concrete (TRC) yield in some aspects reversed tendencies than predicted by existing models for composites with homogeneous reinforcement Weichold and Hojczyk (2009); Rypl et al. (2013).

2. COMPOSITE CRACK BRIDGE MODEL

2.1. *Assumptions and notation*

A unidirectional composite with constant cross-sectional area containing fibers of volume fraction V_f is considered. The fibers exhibit linear elastic behavior with the modulus of elasticity E_f and brittle failure upon reaching their breaking strain ξ. The fiber cross-section is assumed circular with radius r. Elastic deformations of the matrix are neglected so that it is assumed to be rigid. This is justified for cross-sections with much higher matrix stiffness compared to the stiffness of the reinforcement $E_\mathrm{m}(1-V_\mathrm{f}) \gg E_\mathrm{f}V_\mathrm{f}$, where E_m denotes the matrix modulus of elasticity. Matrix cracks in a composite subjected to tensile load are assumed to be planar and perpendicular to the loading direction. Any residual force carried by the matrix crack planes is neglected so that the force is transmitted solely by the fibers. When the tensile load is increased, fibers debond at the bond strength τ and slide against a constant frictional stress τ at the fiber-matrix interface along the debonded length a (Fig. 2).

Although detailed analyses of stress profiles within a fiber cross-section have been performed in the past Nairn (1997); Xia et al. (2002), the stress concentrations at the fiber perimeter close to the matrix crack plane are assumed to have a minor effect (see also Curtin (1993)). Therefore, the fundamental assumption of shear-lag models Cox (1952); Nairn (1997) of constant fiber stress over the cross-section can be anticipated. Nevertheless, the stress is variable for individual fibers due to the parameters which affect the fiber-matrix bond and which are assumed to be of random nature. The mechanical idealization of the composite can thus be described as a parallel set of independent 1D springs representing the fibers coupled to a rigid body representing the matrix through a (possibly random) frictional bond.

2.2. *Homogenized composite response*

The (quasi-static) matrix crack width w is chosen as control variable of the composite loaded in tension because it enables a model formulation with random properties of fibers and fiber-matrix interface. Also, this way the composite response can be tracked along the complete descending branch. Note that the far field composite stress taken as control variable would result in an unstable (dynamic) damage process if monotonically increased beyond the peak stress. The task of the present model is to evaluate the far field composite stress σ_c given a value of w. The composite stress $\sigma_{\mathrm{c},\mathbf{X}}$ is defined as the sum of (random) fiber forces $f_{\mathrm{f},i}(w, X_i)$, $i \in$

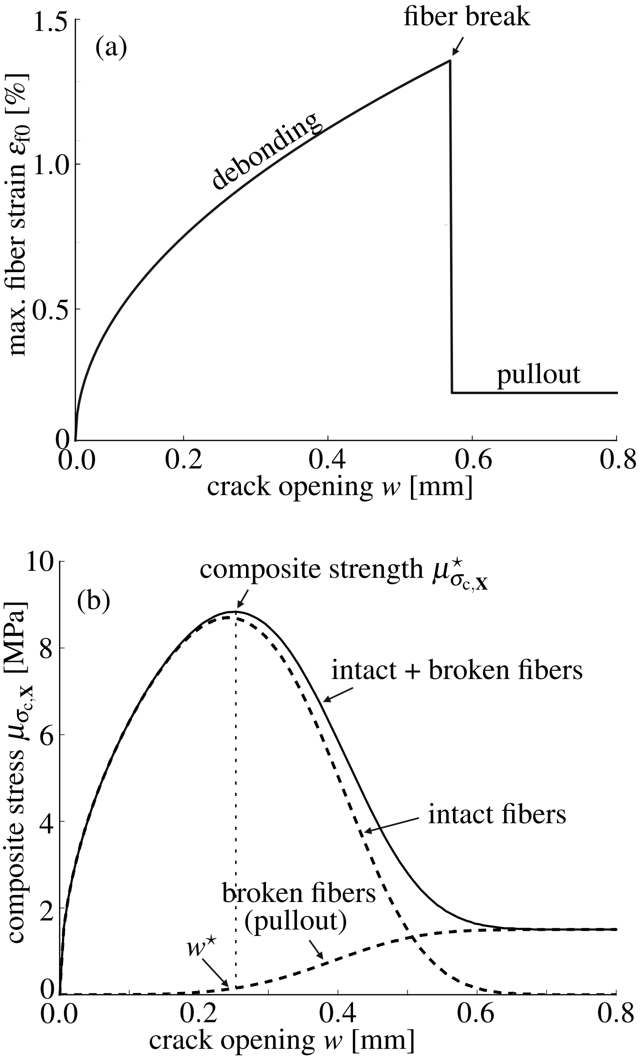

Figure 1: Typical fiber crack bridge function (a) and mean composite crack bridge function (b).

$1,2\ldots n_f$ transmitted by the n_f fibers within a crack plane at a given nonnegative crack opening w yielding the total transmitted force, which is divided by the composite cross-sectional area A_c

$$\sigma_{c,\mathbf{X}}(w,\mathbf{X}) = \frac{1}{A_c}\sum_{i=1}^{n_f} f_{f,i}(w,X_i), \quad w \geq 0. \tag{1}$$

Here, X_i is a sampling point from the $\mathbf{X} \in \mathbb{R}^n$ sampling space of the n considered random variables with the joint distribution function $G_{\mathbf{X}}$. Hence, the sampling points X_i are random n-dimensional vectors containing the fiber and bond properties. The force of a single fiber, $f_{f,i}(w,X_i)$, maps the vector X_i on a nonnegative scalar – the fiber force – as a function of the crack opening w. The $\sigma_{c,\mathbf{X}}(w,\mathbf{X})$ function then sums the random fiber contributions and is therefore itself a random variable sharing the same sampling domain as the fibers $\mathbf{X} \in \mathbb{R}^n$. One realization of the random variable $\sigma_{c,\mathbf{X}}(w,\mathbf{X})$ is thus the sum of randomly chosen samples (fiber forces) divided by A_c. These realizations have unique global maxima $\sigma^{\star}_{c,\mathbf{X}}(\mathbf{X})$ in the w dimension at some nonnegative crack opening $w^{\star}$. Such a maximum is called 'composite strength'.

Assuming a large number of fibers, the term $\sum_{i=1}^{n_f} f_{f,i}(w,X_i)$ in Eq. (1) can be approximated by expected value stating that

$$\sum_{i=1}^{n_f} f_{f,i}(w,X_i) \approx n_f \mathrm{E}[f_{f,\mathbf{X}}(w,\mathbf{X})] \tag{2}$$

where $f_{f,\mathbf{X}}(w,\mathbf{X})$ is the fiber force as a continuous function spanning the $\mathbb{R}^{n+1}$ space (n random variables + the crack opening w). The formula can be interpreted as stating that the sum of random fiber forces is asymptotically equal to the mean fiber force multiplied by the total number of fibers. Similarly, A_c can be for large n_f substituted by

$$A_c \approx n_f \frac{\mathrm{E}[A_f]}{V_f}, \tag{3}$$

where $A_f = \pi r^2$ is the single fiber cross-sectional area. It is assumed that for a nonnegative w the fibers exhibit linear elastic behavior, i.e.

$$f_{f,\mathbf{X}}(w,\mathbf{X}) = A_f E_f \,\varepsilon_{f0,\mathbf{X}}(w,\mathbf{X}) \tag{4}$$

with $\varepsilon_{f0,\mathbf{X}}(w,\mathbf{X}) \in \mathbb{R}^{n+1}$ standing for the fiber strain at the matrix crack derived below. Then, the substitution of Eqns. (2) and (3) into Eq. (1) is the expected value of $\sigma_{c,\mathbf{X}}$ denoted as $\mu_{\sigma_{c,\mathbf{X}}}$ and referred to as the 'mean composite crack bridge function'. With the dependencies on w and $\mathbf{X}$ omitted, it is derived as

$$\begin{aligned}\sigma_{c,\mathbf{X}} \approx \mu_{\sigma_c,\mathbf{X}} &= V_f \frac{\mathrm{E}[f_{f,\mathbf{X}}]}{\mathrm{E}[A_f]} = V_f \frac{\mathrm{E}[A_f E_f \,\varepsilon_{f0,\mathbf{X}}]}{\mathrm{E}[A_f]} \\ &= E_f V_f \mathrm{E}\left[\frac{A_f}{\mathrm{E}[A_f]}\varepsilon_{f0,\mathbf{X}}\right].\end{aligned} \tag{5}$$

The fraction in the square brackets in Eq. (5) is defined as the dimensionless fiber cross-section

$$v_f(r) = \frac{A_f}{\mathrm{E}[A_f]} = \frac{r^2}{\mathrm{E}[r^2]}, \tag{6}$$

so that the general form of the mean composite crack bridge function reads

$$\mathrm{E}\left[\sigma_{\mathrm{c},\mathbf{X}}(w,\mathbf{X})\right] = E_{\mathrm{f}}V_{\mathrm{f}}\,\mathrm{E}\left[v_{\mathrm{f}}(r)\,\varepsilon_{\mathrm{f0},\mathbf{X}}(w,\mathbf{X})\right],\ \ w \geq 0 \tag{7}$$

which we will use the notion $\mu_{\sigma_\mathrm{c},\mathbf{X}}(w)$ for in further text. The expectation in the formula (referred to as the 'mean composite crack bridge function') is evaluated as

$$\mu_{\sigma_\mathrm{c},\mathbf{X}}(w) = E_{\mathrm{f}}V_{\mathrm{f}} \int_{\mathbf{X}} v_{\mathrm{f}}(r)\,\varepsilon_{\mathrm{f0},\mathbf{X}}(w,\mathbf{X})g_{\mathbf{X}}(\mathbf{X})\mathrm{d}\mathbf{X} \tag{8}$$

with $g_{\mathbf{X}}$ being the joint probability density function of the random variables. Since the expectation operator $\mathrm{E}[\cdot]$ maps the $\mathbb{R}^n$ sampling space of the random variables onto a scalar (the mean value), the result, $\mu_{\sigma_\mathrm{c},\mathbf{X}}(w)$ is defined in $\mathbb{R}$ – the dimension of the control variable w. The maximum of the mean composite crack bridge function will be referred to as the 'mean composite strength' and is defined as

$$\mu^{\star}_{\sigma_\mathrm{c},\mathbf{X}} = \sup\{\mu_{\sigma_\mathrm{c},\mathbf{X}}(w); w \geq 0\}. \tag{9}$$

In order to evaluate Eqns. (7) and (9), the unknown fiber strain $\varepsilon_{\mathrm{f0},\mathbf{X}}(w,\mathbf{X})$, which shall be referred to as 'fiber crack bridge function', has to be resolved. The formulation of $\varepsilon_{\mathrm{f0},\mathbf{X}}(w,\mathbf{X})$ is considered in the next section.

2.3. *Fiber crack bridge function*

Individual fibers in a composite with rigid matrix are mechanically independent so that their strain can be defined regardless of the strain state of neighboring fibers. When a matrix crack opens, the bridging fibers debond and transmit an amount of force that is linearly proportional to their debonded lengths a and the bond strength τ at the fiber-matrix interface. The debonded length is a function of the random variables from the $\mathbf{X}$ sampling space and of the crack opening w, i.e. $a = f(w,\mathbf{X})$. Following differential equilibrium condition for the debonded fibers at the longitudinal distance z from the matrix crack is stated

$$E_{\mathrm{f}}\varepsilon'_{\mathrm{f},\mathbf{X}}(z) + T_z(z,\mathbf{X}) = 0, \tag{10}$$

where $\varepsilon_{\mathrm{f},\mathbf{X}}(z)$ is introduced as the longitudinal fiber strain, $\varepsilon'_{\mathrm{f},\mathbf{X}}(z)$ as its derivative with respect to z and

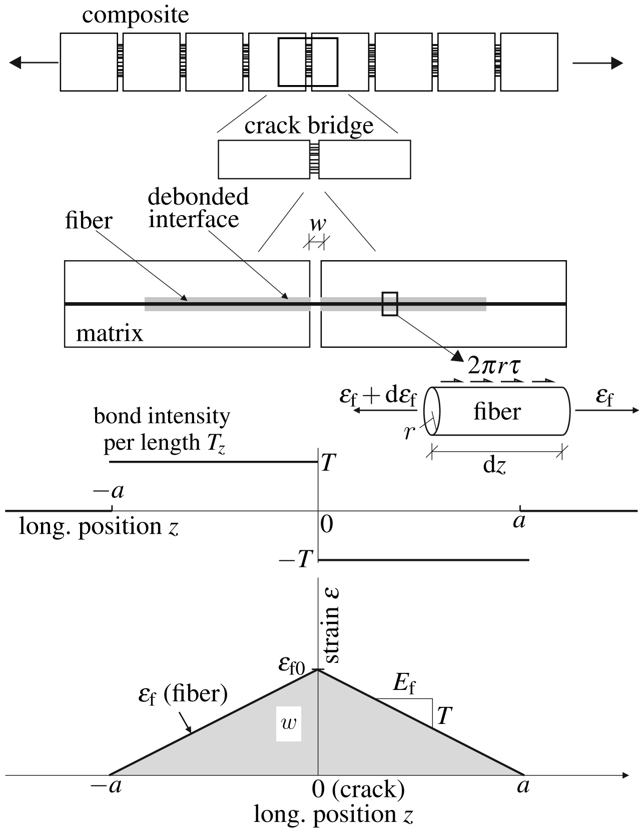

Figure 2: Multi-scale modeling approach diagram.

$T_z(\mathbf{X},z)$ as the bond intensity. The function $T_z(\mathbf{X},z)$ is defined as the interface shear flow $2\tau\pi r$ acting on the fiber cross-section πr^2 within the debonded length a (Fig. 2) with the corresponding sign depending on the longitudinal distance z from the matrix crack

$$T_z(z,\mathbf{X}) = T(\mathbf{X}) \cdot \mathrm{sign}(z) \tag{11}$$

with

$$T = \frac{2\pi r\tau}{\pi r^2} = \frac{2\tau}{r}. \tag{12}$$

Since the sampling domain $\mathbf{X}$ and the distance variable z are included in T_z, Eq. (10) has the dimension $\mathbb{R}^{n+1}$. The fiber strain derivative for the debonded range of a fiber with respect to the longitudinal position is derived from Eq. (10) as

$$\varepsilon'_{\mathrm{f},\mathbf{X}}(z) = -\frac{T_z(z,\mathbf{X})}{E_{\mathrm{f}}}. \tag{13}$$

Analyzing the fiber strain $\varepsilon_{\mathrm{f},\mathbf{X}}$ along z, the maximum is found at the crack plane $z = 0$ and with growing distance from the crack the function decays linearly with the slope $-T/E_\mathrm{f}$ until it reaches zero at $z = \pm a$ (Fig. 2a). An explicit expression for the fiber strain can be obtained by integrating Eq. (13). For the complete z domain, the fiber strain then yields

$$\varepsilon_{\mathrm{f},\mathbf{X}}(w,z,\mathbf{X}) = \begin{cases} \dfrac{Ta - T_z(z,\mathbf{X})z}{E_\mathrm{f}} & : |z| < a \\ 0 & : |z| > a. \end{cases} \quad (14)$$

Note that these formulas involve the debonded length a which is a function of w and $\mathbf{X}$. The dimension of $\varepsilon_{\mathrm{f},\mathbf{X}}$ is thus $\mathbb{R}^{n+2}$ corresponding to $\mathbf{X}$, z and w. The maximum fiber strain $\varepsilon_{\mathrm{f0},\mathbf{X}}(w,\mathbf{X}) = \varepsilon_{\mathrm{f},\mathbf{X}}(w,z=0,\mathbf{X})$ then reduces the z dimension. At his point, we refer to Rypl et al. (2013) for detailed derivation and give the fiber crack bridge function in the final form as

$$\varepsilon_{\mathrm{f0},\mathbf{X}}(w,\mathbf{X}) = \varepsilon_{\mathrm{f0},\mathbf{X}}^{\mathrm{intact}}(w,\mathbf{X}) + \varepsilon_{\mathrm{f0},\mathbf{X}}^{\mathrm{broken}}(w,\mathbf{X}). \quad (15)$$

The two parts of the fiber crack bridge function are the contributions of the intact and broken fibers, respectively (see Fig. 1). The first term has the form

$$\varepsilon_{\mathrm{f0},\mathbf{X}}^{\mathrm{intact}}(w,\mathbf{X}) = \varepsilon_{\mathrm{f0},r\tau}(w,\mathbf{X}) \cdot H(\xi - \varepsilon_{\mathrm{f0},r\tau}(w,\mathbf{X})) \quad (16)$$

where

$$\varepsilon_{\mathrm{f0},r\tau}(w,\mathbf{X}) = \sqrt{\frac{Tw}{E_\mathrm{f}}} \quad (17)$$

and $H(\cdot)$ denotes the Heaviside step function defined as

$$H(x) = \begin{cases} 0 & : x < 0 \\ 1 & : x \geq 0. \end{cases} \quad (18)$$

Broken fibers contribute with the strain

$$\varepsilon_{\mathrm{f0},\mathbf{X}}^{\mathrm{broken}} = \frac{\xi}{m+1} \cdot H(\varepsilon_{\mathrm{f0},r\tau} - \xi), \quad (19)$$

with m being the Weibull modulus of the fiber strength distribution, see Rypl et al. (2013).

2.4. *Strength of multiply cracked composites*

With an *a priori* known (or approximately predicted) periodic crack spacing ℓ_CS, which is a monotonic function of the fiber volume fraction V_f, the composite crack bridge model can be adapted to reflect the periodic stress field of a multiply cracked composite. Once the debonded lengths reach the value $\ell_\mathrm{CS}/2$ (see Fig. 3b), fibers can be assumed as fixed to the matrix at the distance $\ell_\mathrm{CS}/2$ from the matrix crack. For these fibers, further debonding is not possible so that they only stretch elastically with the composite stiffness $E_\mathrm{f}V_\mathrm{f}/\ell_\mathrm{CS}$ resulting in a linear response upon crack opening (see Fig. 3b). To include this constraint, the fiber crack bridge function for intact fibers with infinite strength has to be modified to take on the linear form

$$\varepsilon_{\mathrm{f0},r\tau}^{\mathrm{MC}}(w) = \frac{w}{\ell_\mathrm{CS}} + \frac{T\ell_\mathrm{CS}}{4E_\mathrm{f}}, \text{ for } (a > \ell_\mathrm{CS}/2) \quad (20)$$

where the superscript $^{\mathrm{MC}}$ denotes the multiple cracking state. The derivation of Eq. (20) is straightforward and details can be found in Rypl et al. (2013).

For the mean pullout length of broken fibers in a multiply cracked composite, the approximation $\mu_\ell \approx a_\xi/2$, which is derived and justified in Phoenix (1993), can be applied. The variable a_ξ denotes the debonded length of the fiber at the instant of its rupture. This assumption becomes accurate as the composite approaches its ultimate state where the matrix crack spacing can be assumed narrow and the fiber strains high. The contribution of broken fibers to the fiber crack bridge function can be written as

$$\varepsilon_{\mathrm{f0},\mathbf{X}}^{\mathrm{MC,broken}}(w,\mathbf{X}) = \frac{\xi}{2} \cdot H(\varepsilon_{\mathrm{f0},r\tau}^{\mathrm{MC}}(w) - \xi). \quad (21)$$

To remain consistent with the structure of the fiber crack bridge function (Eq. 15), the fiber crack bridge function for a multiply cracked composite is written as

$$\varepsilon_{\mathrm{f0},\mathbf{X}}^{\mathrm{MC}}(w,\mathbf{X}) = \varepsilon_{\mathrm{f0},\mathbf{X}}^{\mathrm{MC,intact}}(w,\mathbf{X}) + \varepsilon_{\mathrm{f0},\mathbf{X}}^{\mathrm{MC,broken}}(w,\mathbf{X}), \quad (22)$$

where $\varepsilon_{\mathrm{f0},\mathbf{X}}^{\mathrm{MC,intact}}$ is obtained as

$$\varepsilon_{\mathrm{f0},\mathbf{X}}^{\mathrm{MC,intact}}(w,\mathbf{X}) = \varepsilon_{\mathrm{f0},r\tau}^{\mathrm{MC}}(w) \cdot H(\xi - \varepsilon_{\mathrm{f0},r\tau}^{\mathrm{MC}}(w)). \quad (23)$$

2.4.1. *Conclusions and discussion*

Existing models predict a strength reduction of multiply cracked composites when compared to a composite with a single crack Phoenix and Raj

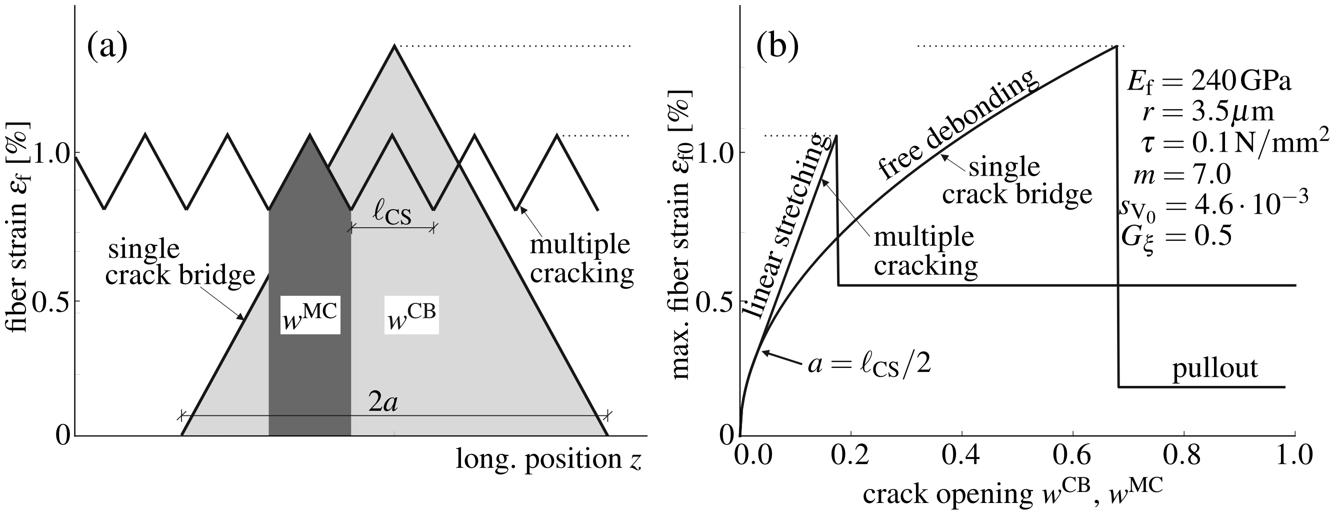

Figure 3: Comparison of single fiber crack bridge with free debonding (single crack bridge) and boundary conditions (multiple cracking): (a) fiber strain profiles along z; (b) fiber crack bridge functions.

(1992); Phoenix (1993); Curtin (1993). This can be explained by the higher average fiber strain within the length $\pm a$ from a matrix crack as compared to the case of a single matrix crack (Fig. 3a). This source of strength reduction can be implemented in the fiber breaking strain distribution G_ξ, see Phoenix and Raj (1992); Rypl et al. (2013), and for composites with homogeneous reinforcement, it is the only source of interaction of strength with crack density.

However, experimental investigations of textile reinforced concrete involving a single crack bridge and multiple cracks show the opposite effect, see Rypl et al. (2013). The strengths of multiply cracked specimens were up to 1.7 times higher than the strengths of specimens with a single matrix crack. Textile reinforced concrete is known for its pronounced heterogeneity of bond quality. Therefore, the strength increase for the multiple cracking state observed with textile reinforced concrete specimens has to be connected with the reinforcement heterogeneity. In the following paragraphs, the effect of boundary conditions (crack spacing) on the strength of composites with heterogeneous reinforcement is explained mathematically and phenomenologically.

Using the presented model, a stress-homogenizing effect of the periodic boundary conditions on fibers due to multiple cracking can be observed. The more uniform is the stress in the reinforcement, the higher load it can transmit – this is a general principle of materials mechanics. The variance of fiber strain in a crack bridge can thus be considered as a measure of the crack bridge's performance in the sense that a high variance denotes low strength. For the two respective cases – single and multiple cracking – the variance operator, $\mathrm{D}[\cdot]$, is applied on Eqs. (17) and (20) (assuming randomness in T and omitting the effect of fiber rupture) as follows

$$\mathrm{D}[\varepsilon_{\mathrm{f}0,r\tau}] = \mathrm{D}\left[\sqrt{\frac{Tw}{E_\mathrm{f}}}\right] = \frac{w}{E_\mathrm{f}}\mathrm{D}\left[\sqrt{T}\right] \qquad (24)$$

and

$$\mathrm{D}[\varepsilon^{\mathrm{MC}}_{\mathrm{f}0,r\tau}] = \mathrm{D}\left[\frac{w}{\ell_{\mathrm{CS}}} + \frac{T\ell_{\mathrm{CS}}}{4E_\mathrm{f}}\right] = \frac{\ell^2_{\mathrm{CS}}}{16E^2_\mathrm{f}}\mathrm{D}\left[T\right]. \qquad (25)$$

Analyzing these formulas, it is apparent that the variability of strains in the single crack bridge case grows linearly with w while the variability of fibers bridging cracks in a multiply cracked composite is independent of w and decreases quadratically with decreasing crack spacing ℓ_{CS}. When $\ell_{\mathrm{CS}} \to 0$, the variance completely vanishes and the strain in all fibers is uniform. Therefore, the growing crack density can be said to cause strain homogenization in the fibers despite the scatter in the bond intensity T and therefore increases the overall composite strength.

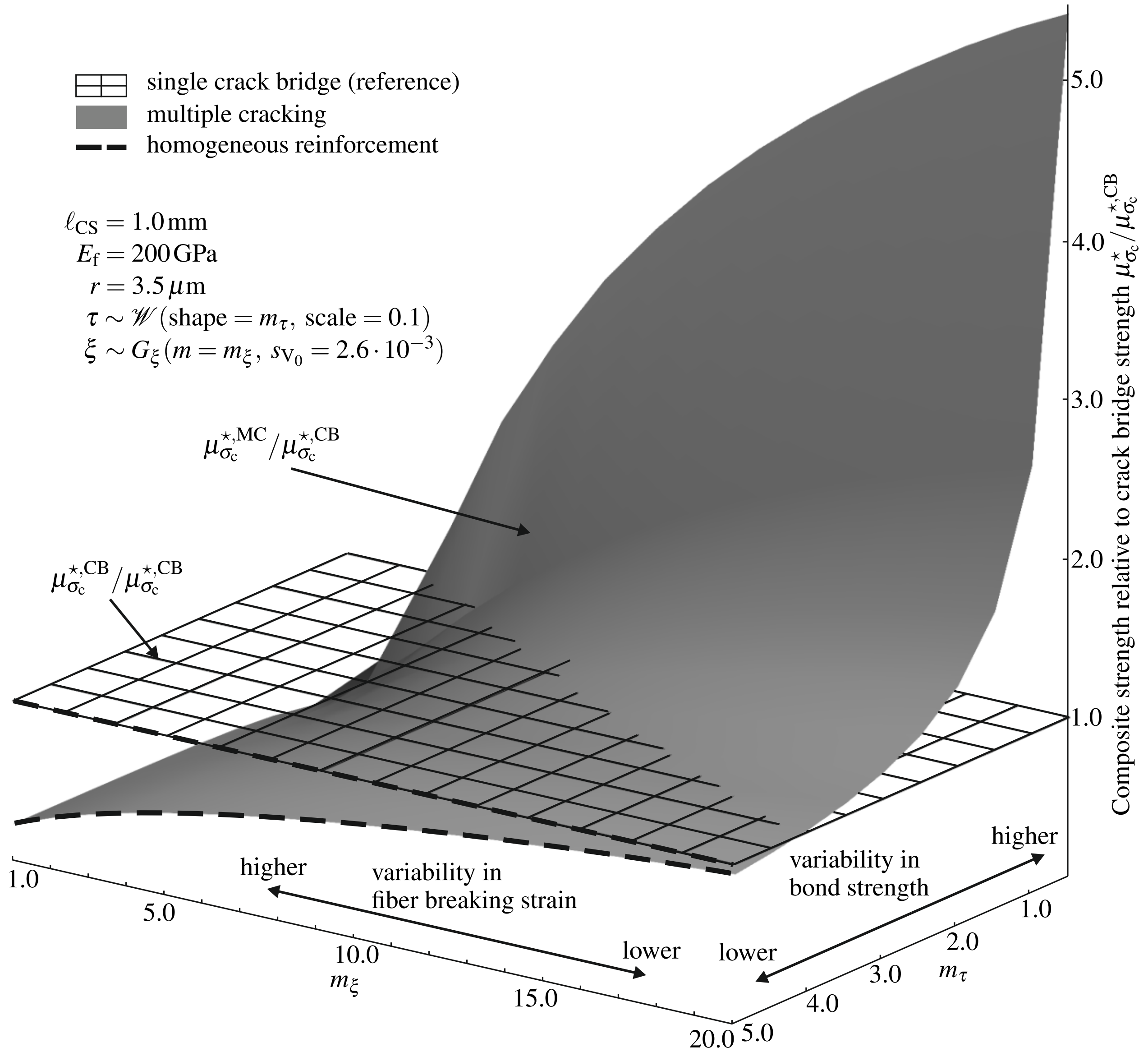

Figure 4: Effect of boundary conditions on the mean composite strength.

Comparing the strength of composite specimens with a single crack (in the sense of a few isolated cracks corresponding to a low V_f) and multiple interacting cracks, an unambiguous conclusion cannot be drawn. This is because the crack spacing influences the composite strength in two opposite ways. It reduces the strength because the average fiber strain along the specimen grows but at the same time, variability in fiber strains is reduced which increases the composite strength. Generally, it depends on the ratio of variability in T to the variability in ξ which of the two effects of the crack spacing will take the upper hand. Will it be the homogenizing effect, the multiply cracked specimen will have higher strength than a single crack bridge. If the more severe stress state effect is stronger, the multiply cracked specimen will be weaker.

The interaction of these effects is depicted in Fig. 4, which shows the ratio between the single and multiple cracking strength (assuming 1 mm crack spacing). The bond strength and the fiber strength in the study are assumed to follow the two parameter Weibull distribution with shape parameters m_τ and m_ξ, respectively. The variable $\mu^{\mathrm{CB}}_{\sigma_c,\mathbf{X}}$ is the mean composite crack bridge function of a single crack and $\mu^{\mathrm{MC}}_{\sigma_c,\mathbf{X}}$ the mean composite crack bridge function in a multiply cracked composite. It is worth noting that for homogeneous reinforcement, the multiple cracking strength approaches the single crack bridge strength when the fiber breaking strength is a deterministic value ($m_\xi \to \infty$, bold dashed curves in Fig. 4). For the studied material, textile reinforced concrete, the scatter of bond strength is very high and therefore the homogenization due to increasing fiber volume fraction which increases the crack density is likely do dominate.

3. ACKNOWLEDGMENT

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4. REFERENCES

Aveston, J., Cooper, G., and Kelly, A. (1971). “Single and multiple fracture, the properties of fibre composites.” *Proc. Conf. National Physical Laboratories*, IPC Science and Technology Press Ltd. London, 15–24.

Chudoba, R., Vořechovský, M., and Konrad, M. (2006). “Stochastic modeling of multifilament yarns. I. random properties within the cross-section and size effect.” *International Journal of Solids and Structures*, 43(3-4), 413–434.

Cox, H. (1952). “The elasticity and strength of paper and other fibrous materials.” *British Journal of Applied Physics*, 3, 72–79.

Curtin, W. (1991). “Exact theory of fiber fragmentation in a single-filament composite.” *Journal of Materials Science*, 26, 5239–5253.

Curtin, W. (1993). “The tough to brittle transition in brittle matrix composites.” *Journal of the Mechanics and Physics of Solids*, 41(2), 217–245.

Evans, A. and Zok, F. (1994). “The physics and mechanics of fiber-reinforced brittle-matrix composites.” *Journal of Materials Science*, 29(15), 3857–3896.

Hui, C.-Y., Phoenix, S., Ibnabdeljalil, M., and Smith, R. (1995). “An exact closed form solution for fragmentation of Weibull fibers in a single filament composite with applications to fiber-reinforced ceramics.” *Journal of the Mechanics and Physics of Solids*, 43(10), 1551–1585.

Ibnabdeljalil, M. and Curtin, W. (1997). “Strength and reliability of fiber-reinforced composites: Localized load-sharing and associated size effects.” *International Journal of Solids and Structures*, 34(21), 2649–2668.

Marshall, D. and Evans, A. (1985). “Failure mechanisms in ceramic-fiber ceramic-matrix composites.” *J. Am. Ceram. Soc.*, 68(5).

Nairn, J. (1997). “On the use of shear-lag methods for analysis of stress transfer in unidirectional composites.” *Mechanics of Materials*, 26(2), 63–80.

Phoenix, S. (1993). “Statistical issues in the fracture of brittle-matrix fibrous composites.” *Composites Science and Technology*, 48(1-4), 65–80.

Phoenix, S. and Raj, R. (1992). “Scalings in fracture probabilities for a brittle matrix fiber composite.” *Acta Metallurgica et Materialia*, 40(11), 2813–2828.

Rypl, R., Chudoba, R., Scholzen, A., and Vořechovský, M. (2013). “Brittle matrix composites with heterogeneous reinforcement: Multi-scale model of a crack bridge with rigid matrix.” *Composites Science and Technology*, 89, 98–109.

Smith, R. (1982). “The asymptotic distribution of the strength of a series-parallel system with equal load-sharing.” *The Annals of Probability*, 10(1), 137–171.

Thouless, M. and Evans, A. (1988). “Effects of pull-out on the mechanical-properties of ceramic-matrix composites.” *Acta Metallurgica*, 36(3), 517–522.

Weichold, O. and Hojczyk, M. (2009). “Size effects in multifilament glass-rovings: the influence of geometrical factors on their performance in textile-reinforced concrete.” *Textile Research Journal*, 79(16), 1438–1445.

Xia, Z., Okabe, T., and Curtin, W. (2002). “Shear-lag versus finite element models for stress transfer in fiber-reinforced composites.” *Composites Science and Technology*, 62(9), 1141–1149.