Return Period Determination for Several Extreme Rainfall-Induced Events Using the IDF Relationship Obtained via Copulas

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ABSTRACT: Flash floods, debris flows and shallow landslides are all induced by intensive rainfall events, where antecedent precipitation conditions can play an important role in triggering these kinds of natural catastrophic events. This study presents the methodology used for the construction of the intensity-duration-frequency (IDF) relationship using the Frank copula function and the evaluation of the different empirical rainfall threshold curves for several extreme events, which happened in Slovenia in the last 25 years and caused 17 casualties and about 500 million Euros of economic losses. The results show that different empirical rainfall threshold curves should be included in the early warning system where measuring rainfall network should have an adequate spatial resolution. The differences in estimated return periods with the use of classical and copula approaches can be significant, however differences can also be the result of relatively short series. The results demonstrate that in case of shorter data series the classical univariate methodology should be preferred.

1. INTRODUCTION

Floods and other similar natural catastrophic events (e.g., landslides, debris flows) can cause significant economic loss and endanger human lives. Due to these reasons the reliable and effective procedures which are easy to apply are needed in order to predict these natural phenomena (i.e. applying early warning systems), which is often a difficult task. Furthermore, a post-event analyses and modelling of these events is also not a straightforward task (Grillakis et al., 2010; Norbiato et al., 2008; Rusjan et al., 2009), however this should be done in order to improve the knowledge about the hydro-meteorological conditions connected with these events.

Detailed analysis of extreme events has also a pedagogical value. Integration of detailed post-event analyses in the hydrological or any similar lessons is desired, that students can gain new knowledge using the real measured data of extreme events, which are mostly the main focus of numerous geo-hydrological studies around the world. In such a way, students obtain the insight into the practical examples and at the same time learn to use a variety of empirical, mathematical and statistical methods (tools) used in hydro-meteorology.

Several rainfall induced extreme events happened in Slovenia (south-central Europe) in the last 25 years, as e.g., flash flood in Železniki 2007 (Rusjan et al., 2009), flood in Ljubljana 2010, three major landslides (Slano Blato landslide in 2000, Stogovce landslide in 2010, and Macesnik landslide in 1990, a combination of flood and several shallow landslides in the southeastern (SE) part of Slovenia in 2005 and a debris flow in the Log pod Mangartom village in 2000. These extreme events caused 17 casualties and about 500 million Euro of economic losses; furthermore numerous houses, industrial objects and other infrastructure were damaged. All these floods and landslides were induced by extreme rainfall conditions.

Floods and landslides, mainly triggered by heavy rainfall, can be predicted using empirical rainfall thresholds or with the use of physically based numerical models (Vennari et al., 2014). Also the use of intensity-duration-frequency (IDF) curves is very common. In order to construct the IDF relationship copula functions, which are a useful mathematical tool, can be used (Ariff et al., 2012; Singh and Zhang, 2007) as an alternative to the mostly used univariate approach (e.g. Koutsoyiannis et al., 1998; Maidment, 1993) where only one variable (rainfall intensity) is considered in the analysis (Singh and Zhang, 2007). Copula functions enable multivariate approach where rainfall duration and intensity can be considered in the analysis. In the last decade copula functions have become more popular among hydrologists and have been used in several geophysical applications (e.g. Ariff et al., 2012; Bezak et al., 2014; Grimaldi and Serinaldi, 2006; Kao and Govindaraju, 2007; Singh and Zhang, 2007; Šraj et al., 2014). The relationship between rainfall duration and intensity is also used in the empirical rainfall threshold curves, which can be used as a part of early warning systems for triggering shallow landslides and debris flows (Aleotti, 2004; Caine, 1980; Guzzetti et al., 2008).

The main focus of the study was on the Železniki 2007 flash flood event; however a brief comparison with the previously mentioned extreme events was also made. The specific aims of the study were as follows: (i) to construct the intensity-duration-frequency (IDF) relationship using copula functions for several rainfall stations in Slovenia; (ii) to compare the estimated return period values using copula functions and ordinary univariate approach for the rainfall event, which caused the Železniki 2007 flash flood; (iii) to evaluate the adequacy of several empirical rainfall thresholds.

2. DATA AND METHODS

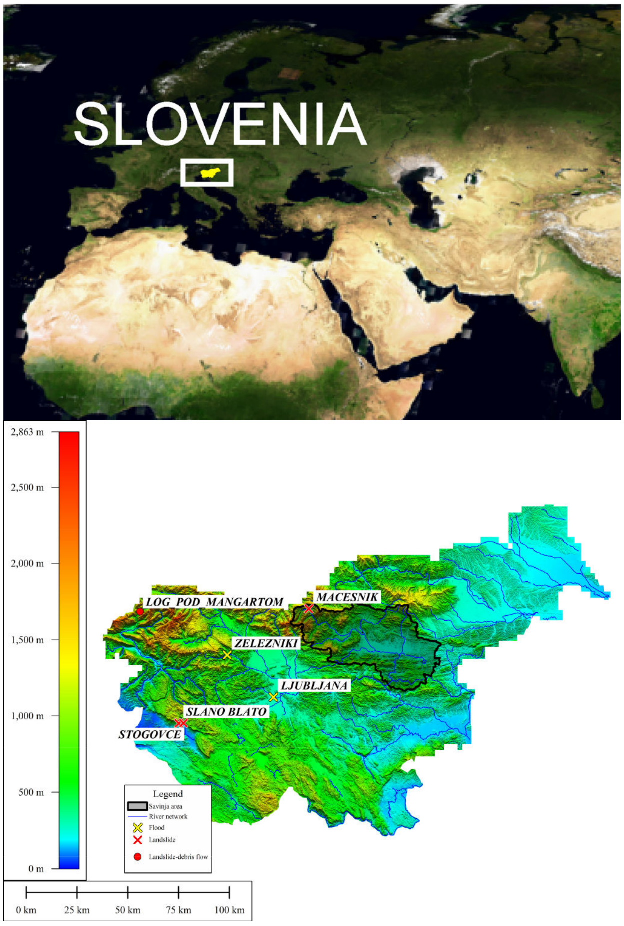

Figure 1: Location of Slovenia (upper) and location of selected case studies in Slovenia (lower).

The flash flood in Železniki village, which occurred in September 2007, was the main focus of the study. Location of the Železniki village on the Slovenia map is shown in Fig. 1, which also shows the location of the other extreme events considered in this study (Stogovce, Slano Blato and Macesnik landslides; combination of flood and several shallow landslides in SE part of Slovenia, flood in Ljubljana and a debris landslide in Log pod Mangartom). Four pluviographic rainfall stations (Fig. 2), which are located relatively close to the Železniki village, were selected to construct the IDF curves using copula functions. Table 1 shows the basic characteristics of the selected rainfall stations. The rainfall data with 5-minute time step was used in this study where snowfall events were not considered in the analysis due to the problems connected with snowfall observations with pluviographs. Unfortunately relatively short series are available for some stations (Table 1).

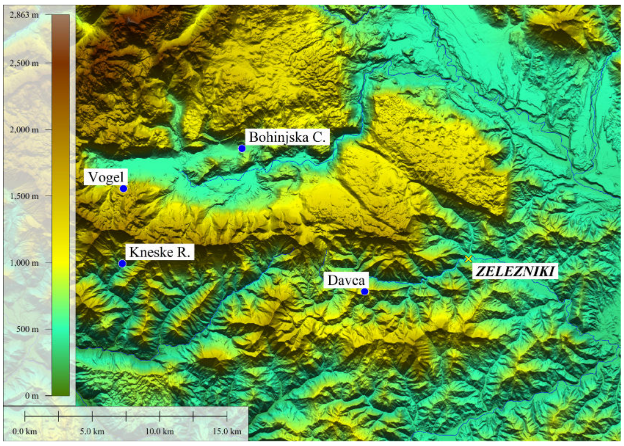

Figure 2: Location of the rainfall stations around the Železniki village.

In order to construct the IDF curves using copula function approach, the first step was the sample determination. The actual rainfall durations (from start to the end of the rainfall event with 5-minutes time step) and corresponding rainfall intensities were used. The inter-event time, which is a minimum duration without rain between two consecutive rainfall events (dry spell period), was selected as 6 hours, which is often the case for small catchments (Ariff et al., 2012). However, the selection of the inter-event time can have significant influence on the final results (Segoni et al., 2014). Then the sample was defined using the annual maximum series (AM) method where the AM events were defined based on the maximum storm intensities (*I*). However, one should note that the definition of AM event in univariate case is different from the bivariate case (Kao and Govindaraju, 2007). Gumbel (extreme-value type I) and Gamma distributions were selected as marginal distributions for modelling rainfall intensities (*I*) and rainfall duration (*D*), respectively. The parameters of marginal distributions were estimated with the method of L-moments (Hosking and Wallis, 2005) and the Kolmogorov-Smirnov test was used to check the adequacy of the selected marginal distribution functions.

Table 1: Basic characteristics of the selected pluviographic rainfall stations.

Station	*Analyzed period*	*Mean annual precipitation [mm]*	*Station elevation [m.a.s.l.]*
Bohinjska Češnjica	*2003-2013*	*2,203*	*595*
Davča	*1999-2013*	*1,818*	*960*
Kneške Ravne	*1982-2013*	*2,834*	*752*
Vogel	*1982-2013*	*3,077*	*1,535*

The Frank copula from the Archimedean family of copulas was used to connect the univariate marginal distribution functions with the multivariate probability distribution:

$$C_\theta(u,v) = -\frac{1}{\theta}\ln\left\{1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right\} \quad (1)$$

where u and v are marginal distribution functions (I and D) and θ is the Frank copula parameter. The copula function parameter was estimated with the inversion of Kendall's tau (Ariff et al., 2012; Salvadori et al., 2007). More information about copula functions was provided by Salvadori et al. (2007). The adequacy of the Frank copula was tested with the use of Cramer-von Mises test (Genest et al., 2009). Then the relationship between conditional copula and return period was defined:

$$C_\theta(u|V=v) = \frac{\partial}{\partial v} C_\theta(u,v)|_{V=v} = 1 - 1/T \quad (2)$$

where T is the return period.

Empirical rainfall thresholds are often used as part of early warning systems for shallow landslides and debris flows (e.g. Segoni et al., 2014).

$$I = \alpha D^\beta \quad (3)$$

where α and β are the intercept and the slope parameters, which are generally determined based on the previously recorded shallow landslide and debris flow events. One of the first studies dealing with empirical rainfall thresholds was conducted by Caine (1980) where α and β parameters were 14.82 and -0.39, respectively. More recently Guzzetti et al. (2008) proposed a new empirical rainfall threshold curve with α and β parameters 2.2 and -0.44, respectively. Both rainfall threshold curves were constructed based on the shallow landslides and debris flow global database. The later curve is applicable for rainfall durations between 1 and 1000 hours, while the first curve was developed using the data with rainfall durations between 0.167 and 500 hours (Caine, 1980; Guzzetti et al., 2008). Furthermore, antecedent moisture (rainfall) conditions can have significant influence on the landslide and debris flow triggering; therefore Aleotti (2004) proposed an empirical rainfall threshold, which also considers antecedent rainfall conditions:

$$NCR = 11.5\mathrm{e}^{-0.08NAR} \quad (4)$$

where NCR is the normalized critical rainfall [%], which triggered the event, and NAR is the normalized antecedent rainfall for 10 days before the event [%] (Aleotti, 2004). A long term mean annual precipitation values [mm] are used to normalize both accumulated rainfall values [mm], namely the antecedent and critical rainfall amounts.

3. RESULTS AND DISCUSSION

On 18.9.2007 a weather front passed over the large part of Slovenia, where extreme rainfall caused high Selška Sora River flows, which is flowing through the Železniki municipality, and consequently severe flash flood in the Železniki village (Rusjan et al., 2009). Six people were killed during the Železniki flash flood in September 2007 (Rusjan et al., 2009): 2 people were killed by a landslide, one died due to the electricity shock during water pumping and other 3 casualties were caused directly by high water. One of these fatalities was vehicle related. Recent studies in USA showed that more than 60% of fatalities and injuries connected with flash floods is vehicle related (Špitalar et al., 2014). Furthermore, the most of the fatalities happened in the night time (between 10 pm and 6 am).

3.1. IDF curves determination using copula functions

Extreme rainfall amounts in several rainfall stations around the Železniki municipality were measured during the flash flood in September 2007 by the Slovenian Environment Agency (ARSO) (Fig. 2). Table 2 shows some basic characteristics of the extreme rainfall event, which caused the Selška Sora River flash flood, for four selected rainfall stations. Values shown in Table 2 represent the complete rainfall event; however maximum short duration rainfall rates were even more extreme (Rusjan et al., 2009). The maximum 120 min rainfall rate for the Kneške Ravne station was 157 mm, which corresponds to the mean rainfall intensity of 78.5 mm/h. Furthermore, for the Bohinjska Češnjica rainfall station the maximum 60 min rainfall rate was 95 mm, which corresponds to the mean intensity of 95 mm/h (Rusjan et al., 2009). Rusjan et al. (2009) also found out that the meteorological radar underestimated the measured ground rainfall rates up to 50 %.

Based on the measured data from four rainfall stations (Fig. 2), the intensity-duration-frequency (IDF) curves were constructed using the copula approach. First the AM samples were defined based on the maximum storm intensities (I) where inter-event time 6 hours was selected to separate precipitation events. Fig. 3 shows the AM series samples, which were used for the IDF curves determination using copula approach. Based on the rainfall intensity (*I*) and rainfall duration (*D*) values the parameters of the marginal distribution functions were determined using the method of L-moments where Gumbel and Gamma distributions were used to model *I* and *D*, respectively. For all four stations none of the selected marginal distribution functions could be rejected by the Kolmogorov-Smirnov test with the chosen significance level of 0.05. Frank copula parameters were estimated with the method of moments (inversion of Kendall's tau). The Kendall's correlation coefficients for the AM samples presented in Fig. 3 were -0.42, -0.18, -0.23 and -0.47 for the Davča, Kneške Ravne, Vogel and Bohinjska Češnjica rainfall stations, respectively. The Frank copula from the Archimedean family could not be rejected by the Cramer-von Mises test with the chosen significance level of 0.05 for any of four selected rainfall stations.

Table 2: Characteristics of the complete rainfall event, which triggered the Selška Sora River flash flood in 2007.

Station	*Duration [h]*	*Accumulated rainfall [mm]*	*Mean intensity [mm/h]*
Bohinjska Češnjica	*25*	*287.4*	*11.5*
Davča	*18*	*220.1*	*12.2*
Kneške Ravne	*20*	*318.6*	*15.9*
Vogel	*31*	*315.8*	*10.2*

Using Eq. 2 the relationship among rainfall intensity, rainfall duration and frequency was defined. Fig. 4 shows the constructed IDF relationships, which were determined using the copula approach, for the Bohinjska Češnjica, Davča, Kneške Ravne and Vogel rainfall stations. One can notice that the constructed IDF points greatly depend on the basic characteristics of the rainfall stations, as on the station location, mean annual precipitation (Table 1) and consequently also on the calculated Kendall's correlation coefficients for the selected AM samples shown in Fig. 3. The presentation of IDF points instead of actual IDF curves was selected to improve visualization of the results.

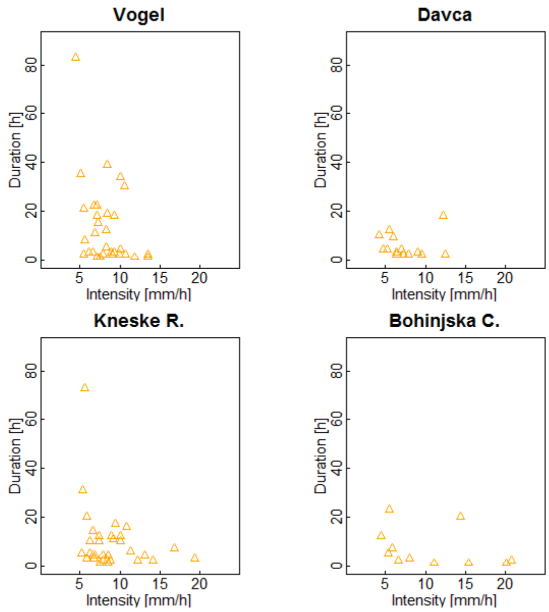

Figure 3: Annual maximum (AM) series samples used for the construction of the intensity-duration-frequency curves using copula approach.

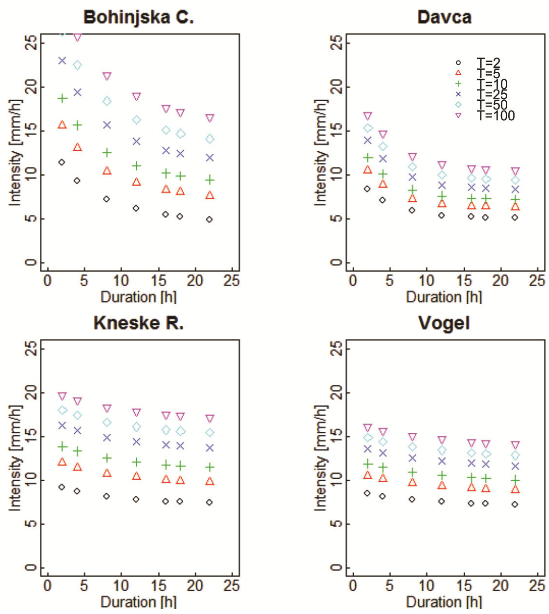

Figure 4: Intensity-duration-frequency (IDF) relationship constructed using copula function approach for the four rainfall stations.

In the next step the return periods of the complete rainfall event, which caused the Selška Sora River flash flood, were determined using the classical univariate and multivariate copula approach. The classical univariate approach is described in hydrological textbooks (e.g. Maidment, 1993). The Gumbel distribution was used to construct the IDF curves with the use of univariate approach (rainfall durations are fixed; ARSO, 2014) (Table 3).One can notice that the estimated return periods are between 10 and 100 years, depending on the selected rainfall station. However, as stated by Rusjan et al. (2009) the calculated return period values can contain error due to the relatively short data series. Different periods were used to construct the IDF curves, namely the period 1999-2012, 2002-2012, 1975-1978/1982-2012 and 1982-2012 for the station Davča, Bohinjska Češnjica, Kneške Ravne and Vogel, respectively. As we can see for some stations relatively short series were available for the construction of the IDF curves.

The return periods were also determined with the use of the multivariate copula approach. The estimated return periods are shown in Table 3. They were between 10 and 250 years (Table 3). The results demonstrate significant difference among the estimated return periods (Table 3) depending on the selected approach, which can be attributed to the fact that different methodology is used for sample definition in both cases. For the Davča rainfall station almost all selected AM events, which were used for the construction of copula based IDF relationship, are smaller than 18 hours (Fig. 3), which was the duration of the extreme rainfall event in September 2007 (Table 2). Therefore, the calculated return period values are overestimated due to the fact that only relatively short duration events are included in the AM series sample used for the IDF curves determination. Similar conclusions can be made for other results presented in Table 3. The copula function and classical univariate approach gave the most similar results for the Vogel rainfall station, which has the longest data series available (Table 1). We can conclude that in case of short data series the classical univariate approach is more appropriate as the copula methodology for the IDF curves determination, because in the first case different samples (defined based on the predefined duration values) are used for the IDF construction and in the latter case the IDF curves are constructed based on the n pairs of variables where n is the length of data series. However, if longer data series are available copula function approach could be more appropriate because the actual durations and intensities are used (and not predefined duration values). Furthermore, additional analyses are needed (with emphasis on stations with longer series) in order to confirm or reject the findings, which were drawn based on the rainfall data for four stations presented in Table 3.

Table 3: Estimated return period values using classical univariate approach (Gumbel distribution) and multivariate copula methodology.

Station	*Copula approach*	*Univariate approach*
Bohinjska Češnjica	*59*	*25*
Davča	*248*	*25-50*
Kneške Ravne	*23*	*50-100*
Vogel	*13*	*10-25*

3.2. Empirical rainfall thresholds evaluation

Rainfall duration and intensity values are also part of the empirical rainfall threshold curves. Empirical rainfall threshold curves can be used as part of the early warning system (e.g. Guzzetti et al., 2008). Fig. 5 shows the adequacy of the empirical rainfall threshold curve proposed by Aleotti (2004), which also considers antecedent rainfall conditions, for the extreme rainfall event, which caused high Selška Sora River flows and eventually catastrophic flash flood. One can notice that the Vogel station lies below the curve suggested by Aleotti (2004); however other three stations are located above the previously mentioned curve. Fig. 6 shows the adequacy of the empirical rainfall thresholds proposed by Caine (1980) and Guzzetti et al. (2008) for the same extreme rainfall event in September 2007. All four stations are located above the curve suggested by Caine (1980) and even more significantly above the curve proposed by Guzzetti et al. (2008). Both these empirical rainfall threshold curves were developed based on the shallow landslides and debris flows data sets. However in this study both curves were also applied to the Železniki flash flood event where also few shallow landslides were triggered but were not of primary importance with regards to the extreme flash flood.

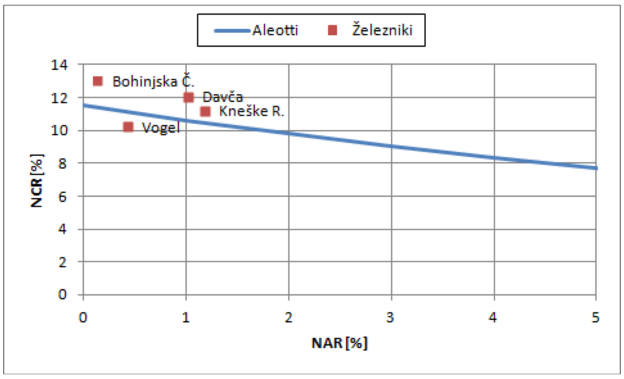

Figure 5: Evaluation of the empirical rainfall threshold curve proposed by Aleotti (2004) for the extreme rainfall event, which caused the flash flood in September 2007.

Furthermore, the adequacy of the different selected empirical rainfall thresholds was also tested for other extreme events, which happened in Slovenia in the last 25 years (Stogovce, Slano Blato, and Macesnik landslides; combination of flood and several shallow landslides in SE part of Slovenia, flood in Ljubljana and a debris landslide-debris flow in Log pod Mangartom). The results indicate that in some cases proposed rainfall threshold curves are appropriate and in other cases only some of the curves would yield a critical state if used as part of the early warning systems. We can conclude that the use of empirical rainfall threshold curves requires a good spatial density of rainfall stations, especially because flash floods and debris flows mostly occur in areas with complex topography, where spatial rainfall variability is significant. Furthermore, we also suggest using a combination of different curves, such as curves suggested by Aleotti (2004) and Guzzetti et al. (2008).

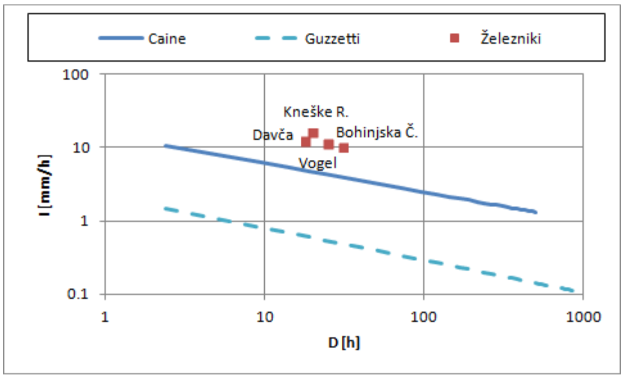

Figure 6: Evaluation of the empirical rainfall threshold curves proposed by Caine (1980) and Guzzetti et al. (2008) for the extreme rainfall event, which caused the flash flood in September 2007.

4. CONCLUSIONS

This study presents the methodology for the determination of intensity-duration-frequency (IDF) curves using the copula approach, which differs from the classical univariate approach where the rainfall durations are fixed. Furthermore, different empirical rainfall thresholds are also evaluated for several Slovenian extreme events, which happened in the last 25 years (landslides, flash floods, shallow landslides, debris flows) and caused 17 casualties and about 500 million Euros of economic loss.

The results indicate that adequate measuring rainfall network with good spatial resolution is needed if empirical rainfall threshold curves are used as part of the early warning system. Furthermore, at least two different empirical threshold curves should be used, where at least one should include antecedent rainfall conditions, which are most often important factor in extreme event initiation.

In case of relatively short data series classical univariate approach for the construction of IDF curves might have an advantage over the multivariate copula methodology, however in opposite situations copula approach should be preferred. Furthermore, additional analyses using more rainfall stations with longer data series are desires to validate the results presented in this study.

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