# Optimization of Inspection Plans for Structures Submitted to Non-stationary Stochastic Degradation Processes

Rodrigue Décatoire
*Graduate Student, PHIMECA Engineering, Cournon d'Auvergne, France*

Sidi Mohammed Elachachi
*Associate Professor, GCE Department I2M, Univ. of Bordeaux, Bordeaux, France*

Thierry Yalamas
*CEO, PHIMECA Engineering, Cournon d'Auvergne, France*

Franck Schoefs
*Professor, Research Institute of Civil Engineering and Mechanics, UMR CNRS 6183*
*LUNAM Université, Nantes, France*

ABSTRACT: The inspection plan optimization of civil engineering structures became a true economical challenge in context of limited budget in the last two decades. It is therefore mandatory to optimize the inspection, maintenance and repair plans depending on the evolution of degradation indexes in order to ensure the reliability of these structures. Based on simulation of a predictive carbonation model which are used to identify the degradation index evolution, this paper presents a new methodology which helps to design an optimal inspection plan, taking into account the spatial variability of the degradation process. The position of the measurements points derives from an adaptive design of experiments built with the degradation predictions. The optimal time between inspections is thus determined with respect to the inspection, maintenance and failure costs.

## 1. INTRODUCTION

Since the 19$^{th}$ century, the expansion of developed countries and the fast increase of their population has lead to the construction of numerous structures and infrastructures, in order to answer to the needs in energy, in accommodations or in road and railway infrastructures. Those buildings, mostly built in reinforced concrete, are submitted to several degradation phenomena that impair their integrity. However their importance implies that their safety is to be insured all the time, on account of economic and social matters. In order to ensure the reliability level of those structures, they are regularly inspected and maintained according to IMR (Inspection, Maintenance and Repair) plans previously defined by the stakeholders. However by considering the global economic context, the costs induced by these operations call for an optimized procedure in order to reduce the long run monitoring cost. Many methodology have been developed to solve this problem (e.g. Barone and Frangopol (2014); Bucher and Frangopol (2006); Kallen (2007); Straub (2004); Breysse et al. (2009)), by using Markov chain (e.g. Frangopol et al. (2004); Orcesi and Cremona (2010)), decision tree in Straub and Faber (2005) or performance criteria in Okasha and Frangopol (2010), at different scales : component, structure, or a set of structures scales. Some consider different maintenance methods depending on the component (e.g. Okasha and Frangopol (2009); Straub and Faber (2005)). However concerning reinforced concrete, few of them directly address the spatial variability of a component degradation (e.g. Schoefs (2007); Schoefs et al. (2009); Sudret (2008)) which is due to the spatial variability of concrete properties and to the environmental aggressiveness. This paper thus presents the first steps of a new methodology based on decision trees for optimizing an inspection plan, able to deal with such kind of variability. This methodology aims at being:

1. Simple to implement,
2. Accurate,
3. Fast.

It helps in determining when and where a structure should be inspected, considering that it is submitted to carbonation process. The future work will be dedicated to introduce the effect of measurement uncertainties in the process.

## 2. OPTIMIZATION OF INSPECTIONS PLANS

### *2.1. The methodology*

#### *2.1.1. Degradation index*

The first step of the methodology is to determine a degradation index based on $N_{sim}$ probabilistic simulations of the degradation process. For the sake of simplicity let us consider a 1D structure indexed by $x_p \in X$. Concerning the carbonation process, we want to avoid the carbonation depth $X_c(x_p)$ from being greater than the concrete cover $b$. Yet, it seems logical that a structure will not be maintained if only one point reaches this criterion since the carbonation process itself is not critical regarding the serviceability of a structure. Let thus defined another criterion which considers that a maintenance is to be applied if a critical length threshold $L_{crit}\%$ of the structure reaches the previous criteria. Since we are in a probabilistic context, the criterion thus reads:

- Failure of a point means $X_c(x_p) >= b$,
- If the failed length $L_c$, representing the percentage of the structure considered as failed exceeds the given threshold $L_{crit}$, the structure is to be maintained,
- The probability that the structure has to be maintained is supposed to be driven by the condition state

$$P_{L_c>=L_{crit}} = N_{L_c>=L_{crit}}/N_{sim}, \qquad (1)$$

  where $N_{L_c>=L_{crit}}$ is the number of simulated trajectories which present a percentage $L_c$ of the structure considered as failed higher than $L_{crit}$

#### *2.1.2. Adaptive design of experiments*

The second step of the methodology is to determine, at a given inspection date, where to inspect in order to obtain the best evaluation of the degradation index. It could have been interesting to put the inspection location in an optimization process, however it may lead to an intractable optimization problem. These locations are thus found with an adaptive design of experiments (ADoE). Since the methodology is based on multiple realizations of the degradation process, the location of interest for an inspection are to be defined based on a statistical quantity. In order to be conservative, we chose to work with the 95% quantile of the degradation simulations. Let us defined the maximum number of locations $N_l$ that can be inspected (knowing a given inspection budget). As the methodology is based on a degradation model, it could be dangerous to entrust this model entirely by only inspecting the locations where a high degradation is predicted. We propose to split the inspections locations in two parts :

- one for the maximum values of the 95% quantile $Q(x_p)$ which verifies

$$P(X_c(x_p) \leq Q(x_p)) = 0.95 \qquad (2)$$

- the other for the minimum values of this quantile, in order to avoid a false evaluation of the probability index due to an overconfidence in the model.

For the sake of simplicity, it is defined that half of the $N_l$ locations will be chosen in the maximum values, the other half in the minimum values, and the effect of this distribution is not studied. To ensure a good identification of the degradation statistical properties, the autocorrelation between each selected point has to be lower than 0.3 in order to suppose pseudo-independent measurement (see Schoefs et al. (ress)).

The selected points are then extracted from the simulations, and between them a linear interpolation is performed.

However, if this proposal ensures that the space will be satisfactorily explored, it does not prove that the incomplete spatial trajectories will permit to accurately estimate the degradation index.

An adaptive loop is therefore realized:

1. Evaluate the relative difference between the degradation index derived from the simulations and the one derived by the incomplete trajectories,

2. If this difference is higher than a given threshold $\varepsilon_{P_{L_c>=L_{crit}}}$, and the maximum number of locations $N_l$ has not been reached, a new inspection point has to be chosen.

   - Thus for each point compute the criterion

     $$I(x_i) = \varepsilon_\mu(x_i) + \varepsilon_\sigma(x_i), \quad x_i \in X, \quad (3)$$

   - The point which maximizes this criterion is added to the inspection plan,

   - Start again

$\varepsilon_\bullet(x_i)$ is the absolute error made at point $x_i$ between the estimate of $\bullet$ derived from the incomplete trajectories and from the simulations.

From this ADoE an estimate $\tilde{P}_{L_c>=L_{crit}}$ of the empirical degradation index $P_{L_c>=L_{crit}}$ is derived.

### *2.1.3. Decision tree*

Since we assume an inspection gives a perfect measure, after an inspection a stakeholder only have two choices :

1. to maintain the structure,
2. or to wait for the next inspection.

The probability that the structure will be maintained is $\tilde{P}_{L_c>=L_{crit}}$, the complementary probability is the probability to do nothing. An example of this decision tree is shown figure 1 for a plan with two inspections.

Something hidden in this tree, yet obvious, is that between two inspections the degradation is progressing. If a maintenance is applied, the degradation goes back to the initial state after a delay called maintenance time $t_{Ma}$. With the degradation comes a failure probability: the probability that corrosion

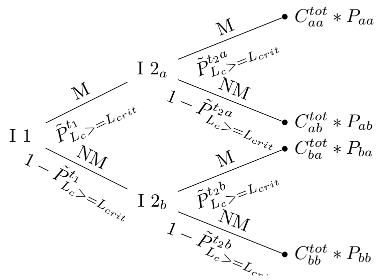


*Figure 1: Decision tree with two inspections. M corresponds to a maintenance action, NM to no maintenance, I$\bullet$ to inspection number $\bullet$ performed at time $t_\bullet$. $C^{tot}_\circ$ is the total cost induced by the branch $\circ$*

appears. Since no value is available in the literature, the failure probability per year is computed by

$$P_f(level) = 1 \times 10^{-6+level} * P(L_c \in Int_{level}), \quad (4)$$

with *level* goes from 0 to 4, $Int = ([0\%, 10\%[, [10\%, 20\%[, [20\%, 30\%[, [30\%, 40\%[, [40\%, 100\%[)$ is a vector of intervals of the degradation index possible values and $P(L_c \in Int_i)$ is the probability that the degradation index is within $Int_i$.

### *2.1.4. Modeling costs*

**Inspection cost** According to the tree (see figure 1) the expected inspection costs writes :

$$E\left[C^{In}\right] = C_{I_n} \sum_{i=1}^{m} P_i \sum_{j=1}^{t_{In}} \frac{n_{I_j}}{(1+r)^{t_j}} \quad (5)$$

where

- $C_{In}$ is the cost of an inspection,

- $t_{In}$ is the number of inspection date,

- $n_{I_j} < N_l$ is the number of measures performed at year $t_j$,

- $r$ is the discount rate of money,

- $m = 2^{t_{In}}$ is the number of possible branches,

- $P_i$ is the probability of the branch $i$.

**Maintenance cost** The expected maintenance cost writes:

$$E\left[C^{Ma}\right] = C_{Ma}\sum_{i=1}^{m} P_i \sum_{j=1}^{t_{In}} \frac{\mathbb{1}_{ij}}{(1+r)^{t_j+t_{Ma}}} \tag{6}$$

where

- $C^{Ma}$ is the maintenance cost,
- $\mathbb{1}_i j$ is equal to 1 if a maintenance action is decided on branch $i$ at time $t_j$, 0 otherwise.

**Failure cost** The expected failure cost writes

$$E\left[C^F\right] = C_F\sum_{i=1}^{m} P_i \sum_{j=1}^{t_{tot}} \frac{\prod_{l=1}^{5} P_f(l) * P(L_c \in Int_l)}{(1+r)^{t_j}} \tag{7}$$

### 2.1.5. *Optimization problem*

The aim of the methodology is therefore to minimize the expected total cost which writes

$$E\left[C^{tot}\right] = E\left[C^{In}\right] + E\left[C^{Ma}\right] + E\left[C^F\right] \tag{8}$$

To act on this cost, the only variables are the inspection dates. For the sake of simplicity, and since it is in a better agreement with the stakeholders habits, the step between two inspections is set as a constant. Therefore there is only one variable left to optimize: the time between two inspections $\Delta I$.

A global scheme of the methodology is presented figure 2.

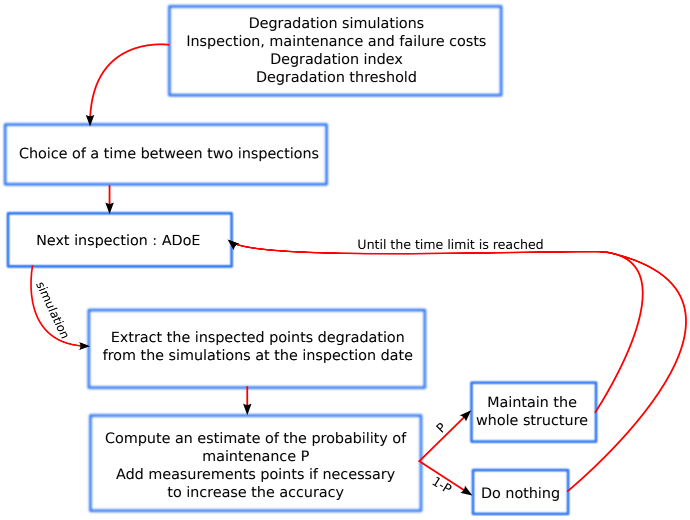


*Figure 2: Global scheme of the methodology*

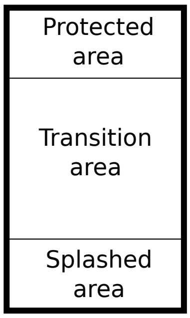


*Figure 3: Basic illustration of the column*

## 3. Application

### 3.1. *Description*

### 3.1.1. *The bridge*

The application uses a model of a reinforced concrete bridge column located on the edge of a road, in a rainy environment. We assume that the base of the column is regularly splashed by the cars passing on the soaked road. The top of the column is protected from the rain by the deck here supposed to be perfectly waterproof. A basic illustration of the column is shown figure 3. We only consider the saturation rate of the concrete cover as a one-dimensional non-stationary random field since it is directly correlated to the water present in the concrete cover which is not homogeneously splashed by the cars over the column height. This random field of the saturation rate is simulated with the Karhunen-Loeve expansion with the following parameters:

- the mean is equal to 100% from the base of the column up to 1 meter,
- the mean is equal to 60% from the top of the column down to 3 meters,
- the mean decreases linearly from 100% to 60% from 1 meter to 3 meters,
- the variance is a constant equal to 5%,
- the autocorrelation is modeled by a quadratic exponential function with a correlation parameter of 1 meter.

Ten trajectories belonging to this stochastic field are plotted on figure 4.

### *3.1.2. The degradation*

The degradation considered in this paper is the carbonation. To simulate the carbonation process we chose to use the DuraCrete model (see DuraCrete (2000)) since its use is recommended by the European Union. Based on the DuraCrete report, the parameters of the model are set in order to simulate a C25 concrete. This model reads

$$X_c(t) = \sqrt{\frac{k_e k_c k_t C_s t}{R_{carb}}} * \left(\frac{t_0}{t}\right)^n \qquad (9)$$

where $k_e$ is derived from the relative humidity determined by the saturation rate (thanks to a realistic isotherm BSB of a C25 concrete). An illustration of the resulting fields of the carbonation depth after 20 years of natural carbonation is presented figure 5.

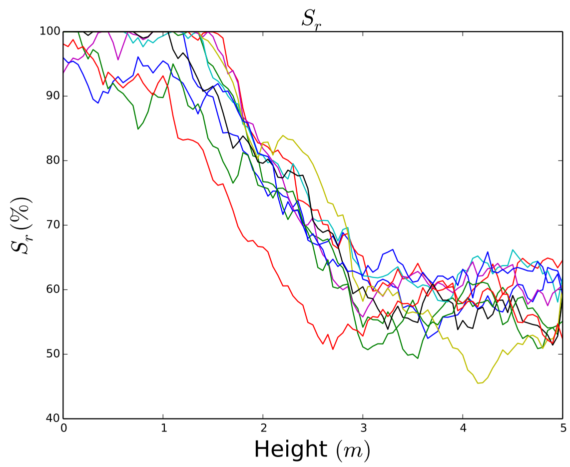


*Figure 4: 10 sample paths of the saturation rate.*

### *3.2. Data*

The data used to simulate the carbonation process are summarized in the table 1. They are chosen as deterministic variables in order to isolate the effect of the spatial variability of the saturation rate on the inspection plan. 1000 simulations have been launched to minimize the statistical bias. The carbonation depth progression is computed every year. The different costs, the maximum number of points

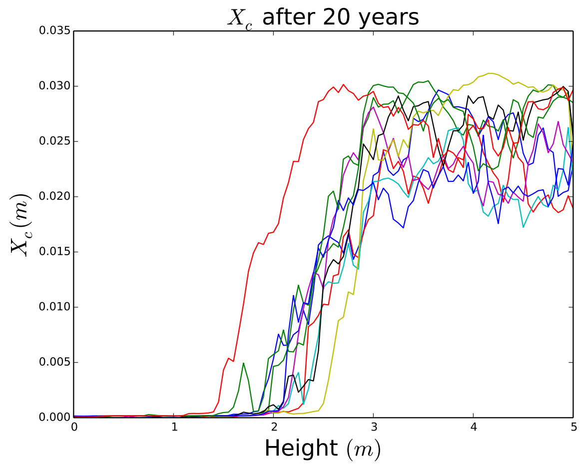


*Figure 5: 10 sample paths of carbonation depth computed with the DuraCrete model.*

| | | |
|---|---|---|
| $n$ | = | 0.1 |
| $t_0$ | = | 1 *year* |
| $k_c$ | = | 4.94 |
| $k_t$ | = | 0.983 |
| $R_{carb}$ | = | $7.05e-5$ |

*Table 1: Parameters value for the carbonation DuraCrete model (see DuraCrete (2000)). They correspond to a mean C25 concrete.*

that can be measured at an inspection date and the threshold for the ADoE are referenced in the table 2.

| | | |
|---|---|---|
| $b$ | = | $1cm$ |
| $r$ | = | 0.02 |
| $N_l$ | = | 25 |
| $L_{crit}$ | = | 30% |
| $t_{tot}$ | = | 60*years* |
| $\varepsilon_{P_{L_c >= L_{crit}}}$ | = | 5% |
| $E\left[C^{In}\right]$ | = | 5 |
| $E\left[C^{Ma}\right]$ | = | 100 |
| $E\left[C^{F}\right]$ | = | 500 |

*Table 2: Parameters of the inspection plan optimization*

### *3.3. Results*

#### *3.3.1. Analysis of the autocorrelation*

The autocorrelation of the carbonation depth illustrated figure 6 appears to be non-stationary even though it was not the case for the autocorrelation of the saturation rate. It obviously affects the ADoE, however since there is no proof that this model correctly transfers spatial correlation, we will not draw any conclusion on this point and will assume that it represents the reality. Yet, the use of this matrix proves that the methodology is able to deal with complex non-stationarity.

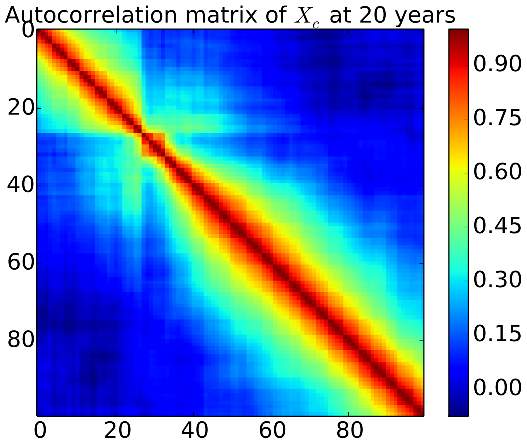


*Figure 6: Autocorrelation matrix of the carbonation depth after 20 years. A distance of 20 points is equal to one meter.*

#### *3.3.2. An example of ADoE*

The figure 7 illustrates what would be the ADoE if the first inspection was run after 10 or 20 years. From the original design of experiment, only 6 measurements are performed. Yet, to ensure a good quality in the estimate of the degradation index $L_c$, the adaptive part adds 4 points at 10 years, and 1 point at 20 years. Without any optimization procedure, it proves to be efficient since only 10 and 7 points are inspected when the maximum allowed number of inspections was around 25 points. This ADoE is then computed for each inspection date of every branch of the decision tree.

#### *3.3.3. Optimization of the inspection plan*

The predictive part of the methodology is the minimization of the expected total cost, with $\Delta I$ as the only optimization variable. Therefore, the objective function has been computed for every $\Delta I$ from 5 years (meaning 4096 branches in the decision tree) to 25 years (respectively 4 branches) for a total time prediction $t_{tot}$ of 60 years. The results took two hours to be computed and are presented figure 8. The different costs present a classical behavior (e.g. Sheils et al. (2010)), many inspections lead to a higher maintenance cost and a low failure cost when a long time without inspection leads to the opposite. The variations observed in the inspection cost are an effect of the ADoE which helps to introduce the effect of the spatial variability. In this example, the optimum time between two consecutive inspections is 30 years. This methodology therefore proves itself to be usable in the case of a non-stationary degradation process.

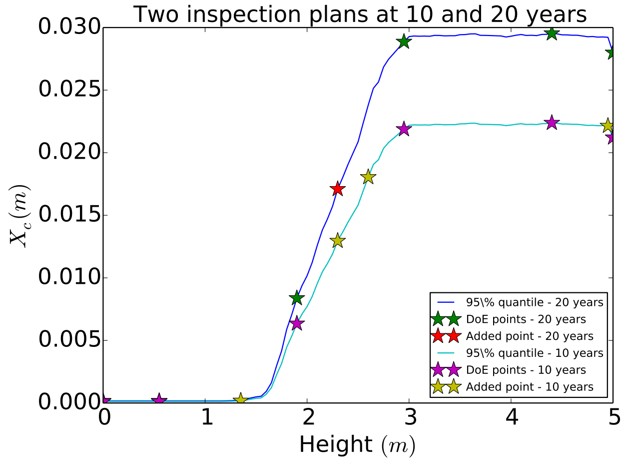


*Figure 7: Illustration of the adaptive design of experiment.*

## 4. CONCLUSIONS

The optimization of inspection plan is a wide subject which has been intensely studied throughout the last decade. Many methodologies, based on probabilistic studies, markov chains, decision trees, bayesian framework, ..., have been proposed, yet only a few of them deal with the spatial variability of the degradation a stakeholder can observe on its structure. The methodology proposed in this paper is a first step to address for this issue. As simple as possible, it takes into account the spatial variability without increasing the complexity nor the com-

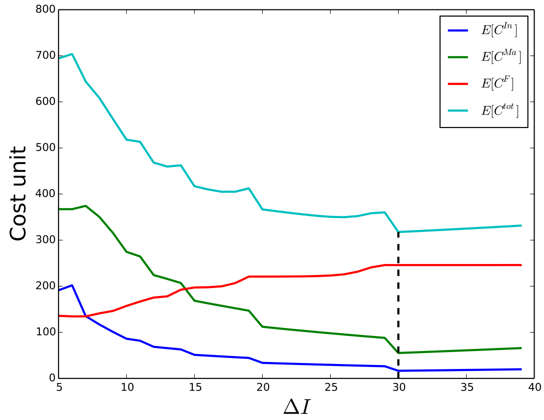


*Figure 8: Evolution of the inspection, maintenance, failure and total costs with respect to the time between two inspections. The minimum total cost is obtained for a time of 30 years.*

putational time of the cost optimization problem commonly solved in the non-spatial case. Yet it remains a first step. The next one will be to take account for the measurement error closely related to the inspection cost, and for real costs of maintenance and failure.

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