Model Uncertainty for the Capacity of Strip Footings under Negative and General Combined Loading

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ABSTRACT: This paper investigates the model uncertainty of Eurocode 7 approach for estimating the bearing capacity of shallow foundations under negative and general combined loading. In the first stage, a regression equation f is derived to remove the dependency of the model factor M_s on the input parameters, where M_s is defined as the ratio between the lower bound solution and the calculated capacity from Eurocode 7 approach. The probability distribution of the residual part η of M_s is then determined. Secondly, the model uncertainty of the lower bound limit analysis is characterized by using a loading database. The result is represented as the probability distribution of the model factor M_{LB}, which is defined as the ratio between the measured capacity and the lower bound solution. Finally, the model uncertainty of the modified model factor M' defined as the ratio between the measured capacity and the capacity from Eurocode 7 approach multiplied by the regression equation f is characterized by combining the results for η and M_{LB}. The results are validated by using another independent loading database.

1. INTRODUCTION

The ultimate bearing capacity of shallow strip foundations under combined loading is usually calculated by using the Terzaghi equation with the inclination factors and the effective width rule $B'=B-2e$ (Meyerhof 1953), where e is the loading eccentricity, given by

$$q_{\mathrm{u}} = cN_c i_c + qN_q i_q + 0.5\gamma B' N_\gamma i_\gamma \tag{1a}$$

where c=soil cohesion, $q=\gamma D$=surcharge load at the ground surface, D=embedment depth of foundation; and γ=soil weight.

The bearing capacity factors N_c, N_q and N_γ are given by (CEN 2004)

$$\begin{aligned} N_c &= \left(N_q - 1\right)\cot\varphi \\ N_q &= e^{\pi\tan\varphi}\tan^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right) \\ N_\gamma &= 2\left(N_q - 1\right)\tan\varphi \end{aligned} \tag{1b}$$

The loading inclination factors i_c, i_q, and i_γ are given by

$$\begin{aligned} i_c &= i_q - \frac{1-i_q}{N_c\tan\varphi} \\ i_q &= \left(1-\tan\alpha\right)^2 \\ i_\gamma &= \left(1-\tan\alpha\right)^3 \end{aligned} \tag{1c}$$

where α is the loading inclination and φ is the friction angle of soil.

Phoon & Tang (2015) studied the model uncertainty of Eurocode 7 approach for the bearing capacity of shallow foundations under positive load combination (see Figure 1a). However, the effect of combined loading direction on the model uncertainty is not investigated. Therefore, the objective of the present study is to characterize the model uncertainty of Eurocode 7 approach under negative and general combined load (see Figure

1b) by using the finite element formulation of the lower bound limit analysis (FELA). The framework of Phoon & Tang (2015) will be used in this paper.

2. PROBLEM DEFINITION

In the present study, a strip footing under a general load with eccentricity e and inclination α on granular materials is considered. In this case, the bearing capacity of foundations is related to qN_q and γN_γ only. Note that N_γ usually decreases as the footing width increases, which is generally known as the scale effect. One possible reason for the scale effect is the dependency of the sand friction angle φ on the mean stress σ_m according to (Ueno et al. 1998)

$$\tan\varphi = \tan\varphi_a \left(\sigma_m / \sigma_a\right)^{-\xi} \qquad (2)$$

where σ_a=100 kPa is the atmospheric pressure; φ_a is reference friction angle; and ξ is an empirical parameter varying from 0.02 to 0.12.

FELA with the stress level effect [i.e. Eq. (2)] can also be found in Phoon & Tang (2015).

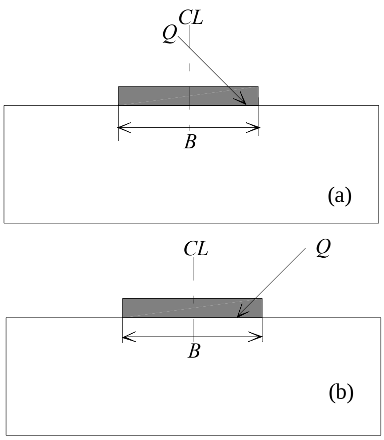

Figure 1: (a) Positive load combination; (b) negative load combination.

3. LOADING DATABASE

Laboratory model tests were conducted recently by Patra et al. (2012a, b) to determine the bearing capacity of eccentrically and obliquely loaded strip foundations. The results were illustrated in Patra et al. (2012a, b), which are not reproduced here. The loading database for positive load combination in Patra et al. (2012a), has been used by Phoon & Tang (2015) to evaluate the model uncertainty of Eurocode 7 approach. The loading database for negative load combination was presented in Patra et al. (2012b) consisting of 72 cases, which will be used to investigate the combined loading direction on the model uncertainty of Eurocode 7 approach. This database will be divided into two parts evenly. Part I and II will be used to characterize the model uncertainty of FELA and Eurocode 7 approach, respectively.

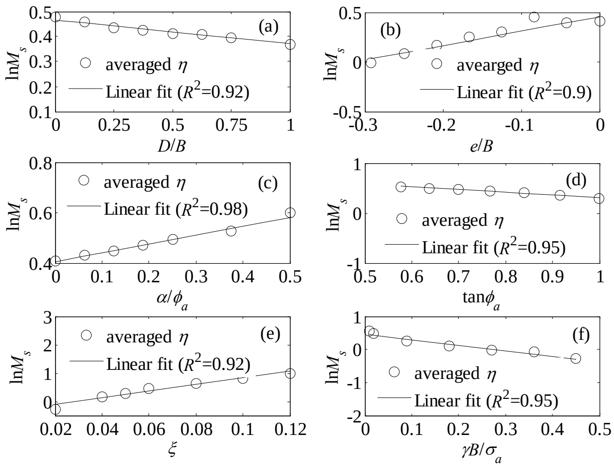

Figure 2: The variation of the averaged value of $\ln M_s$ with input parameters.

Table 1: Coefficients in the regression equation f of the model factor M_s for general load combination.

Coefficients	Positive	Negative	General
b_0	0.28	0.1	0.1
b_1	5.05	4.5	4.5
b_2	11.4	10.4	10.25
b_3	-0.26	-0.25	-0.15
b_4	-0.09	-0.12	0.05
b_5	0.21	-1.03	-0.93
b_6	-1.12	-0.45	-0.05
b_7	-0.98	-1.81	-2.53
R^2	0.9	0.9	0.9

4. RESULTS AND DISCUSSION

This section will focus on the case of negative loading combination. The procedure is similar to the positive loading combination as presented in Phoon & Tang (2015). The correspond results for the general combined loading, which includes the positive and negative load combination, are also summarized to explain the effect of combined loading direction on the model uncertainty.

4.1. Comparison between FELA and Eurocode 7 approach

In this case, the model factor M_s is defined as the ratio between the FELA calculated capacity $q_{\text{u\_calc}}^{\text{LB}}$ and the predicted capacity $q_{\text{u\_calc}}$ from Eurocode 7 approach

$$M_s = q_{\text{u\_calc}}^{\text{LB}} \big/ q_{\text{u\_calc}} \tag{3}$$

Phoon & Tang (2015) have shown that the model factor M_s is a function of the following dimensionless parameters: (1) D/B; (2) $\tan\varphi_a$; (3) ξ; (4) $\gamma B/\sigma_a$; (5) e/B; and (6) α/φ_a. The same ranges for each parameter as the positive load combination (Phoon & Tang 2015) will be used; however, the load eccentricity e is taken as negative. The minus of e denotes the direction of moment. Consequently, a total of 128 orthogonal parameter sets are designed.

Table 2: Spearman rank correlation analysis.

	Negative			General		
	η	M_{LB}	M'	η	M_{LB}	M'
D/B	0.32	0.36	0.4	0.23	0.31	0.37
α/φ_a	0.21	0.25	0.2	0.19	0.2	0.18
$\tan\varphi_a$	0.35	0.4	0.38	0.3	0.37	0.34
e/B	0.18	0.2	0.15	0.12	0.16	0.13
ξ	0.12	0.16	0.17	0.1	0.14	0.15
$\gamma B/\sigma_a$	0.6	0.54	0.58	0.55	0.5	0.45

Table 3: Statistics of the residual part η, M_{LB}, and M' for negative load combination

	Positive		Negative		General	
	Mean	S.D	Mean	S.D	Mean	S.D
η	1.01	0.06	1.01	0.06	1.02	0.09
M_{LB}	1.03	0.09	1.06	0.09	1.04	0.09
M'	1.04	0.11	1.06	0.11	1.06	0.13

The variation of the averaged value of M_s with each parameter is plotted Figure 2. It shows that $\ln M_s$ varies linearly with D/B, e/B, $\ln\gamma B/\sigma_a$, α/φ_a, ξ, and $\tan\varphi_a$ with a coefficient of determination (R^2) that is larger than 0.9. Consequently, these variation trends of $\ln M_s$ can be fitted approximately by a linear function of these input parameters, given by

$$f = e^{b_0} \times e^{b_1 \ln(\gamma B/\sigma_a)} \times e^{b_2 \tan\varphi_a} \times e^{b_3 \xi} \times e^{b_4(D/B)} \times e^{b_5(e/B)} \times e^{b_6(\alpha/\varphi_a)} \times e^{b_7(e/B)(\alpha/\varphi_a)} \tag{4}$$

The model coefficients $\{b_i\}_{i=0,\ldots,7}$ are determined by using the MATLAB function 'regress' to carry out multiple linear regression analysis. The results for positive, negative and general load combination are given in Table 1. It can be seen that the combined loading direction significantly affect the regression equation f. In fact, in the case of the positive load combination, the rotation induced by moment exacerbates the displacement induced by the horizontal load, leading to smaller failure loads. In contrast, for negative load combination, the induced rotations counteract the horizontal displacements leading to higher failure loads. The effect of loading direction on the bearing capacity of shallow foundations under eccentric and inclined loading has been discussed by Tang et al. (2014).

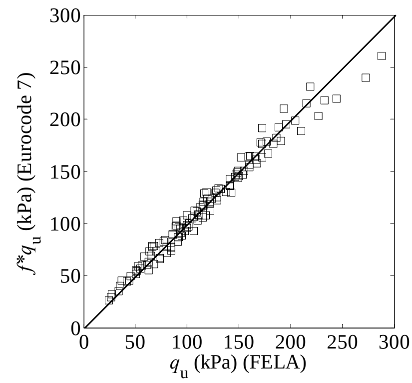

Figure 3: Calculated capacity from Eurocode 7 approach and its value multiplied by the regression equation f (modified Eurocode 7 approach) versus the FELA calculated capacity.

Figure 3 plots the modified capacity $q'_{\text{u\_calc}} = q_{\text{u\_calc}} \times f$ against the FELA results. The discrepancy between $q'_{\text{u\_calc}}$ and $q_{\text{u\_calc}}^{\text{LB}}$ is relatively small, compared to the greater discrepancy between $q_{\text{u\_calc}}$ and $q_{\text{u\_calc}}^{\text{LB}}$. It suggests that the performance of the modified Eurocode 7 approach multiplied by the regression equation f is better than that of the original Eurocode 7 approach.

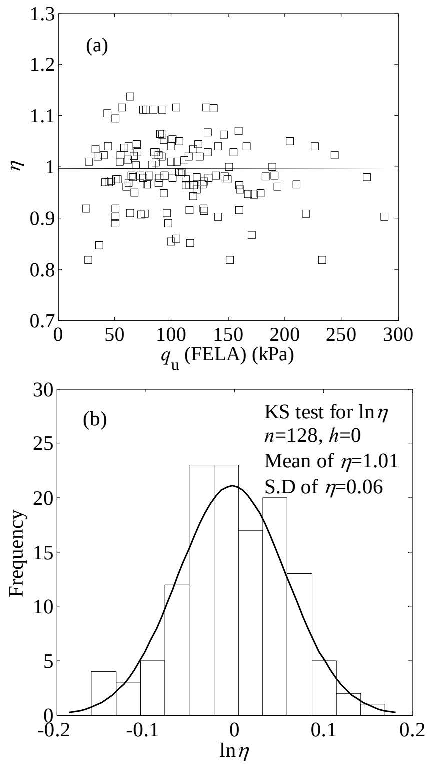

Figure 4: (a) the residual part of the model factor M_s versus the FELA calculated capacity; (b) empirical distribution of $\ln\eta$.

The residual part $\eta = q_{\text{u\_calc}}^{\text{LB}} / q'_{\text{u\_calc}}$ is plotted against the FELA calculated capacity in Figure 4a. In contrast to M_s, η is independent of the input parameters. This can be verified by *p*-values of the Spearman rank correlation as shown in Table 2, which are largely higher than 0.05. Therefore, η can be treated as a random variable. The mean and standard deviation of η is 1.01 and 0.06, as summarized in Table 3. The probability model for η is identified by using Kolmogorov-Smirnov goodness-of-fit hypothesis test (i.e. KS test). This is performed by using the MATLAB function 'h=kstest(**x**)'. It returns a test decision for the null hypothesis that the data in vector **x** comes from a standard normal distribution, against the alternative that it does not come from such a distribution. The KS test for $\ln\eta$ with h=0 indicates $\ln\eta$ is a normally distributed random variable. Therefore, the lognormal distribution model with the above mean and standard deviation is a reasonable model to describe η. An empirical distribution of η is presented in Figure 4b.

4.2. Comparison between FELA and model tests

In this case, the model factor M_{LB} is expressed as the ratio between the measured capacity $q_{\text{u\_exp}}$ and the FELA capacity $q_{\text{u\_calc}}^{\text{LB}}$, namely

$$M_{\text{LB}} = q_{\text{u\_exp}} / q_{\text{u\_calc}}^{\text{LB}} \tag{5}$$

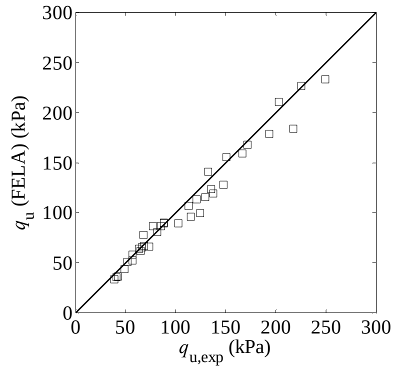

Figure 5: FELA calculated capacity versus the measured capacity

For negative combined load, the 36 bearing capacities calculated from FELA are plotted against the 36 measured capacities from Part I database given by Patra et al. (2012b) in Figure 5. As the mean trend line of the FELA results is quite close to the 45° trend line, it is visually verified that the FELA methodology is unbiased. Figure 6a plots the model factor M_{LB} for each case against the corresponding measured capacity, which appears to be randomly distributed. It is quantitatively validated by Spearman rank correlation *p*-values, which are higher than 0.05, as shown in Table 2. This indicates that M_{LB} is not a function of the input parameters. Therefore, M_{LB} can be treated as a random variable directly.

The same KS test procedure as used for the remaining residual part η is performed to identify the probability distribution of M_{LB}. The h-value for the normality of $\ln M_{LB}$ is 0. It suggests that M_{LB} can be modeled as a lognormal random variable. The mean value and standard deviation of M_{LB} is 1.06 and 0.09, respectively. This can be revised with more exact description of the variation of friction angle with stress level. The empirical distribution of M_{LB} is plotted in Figure 6b.

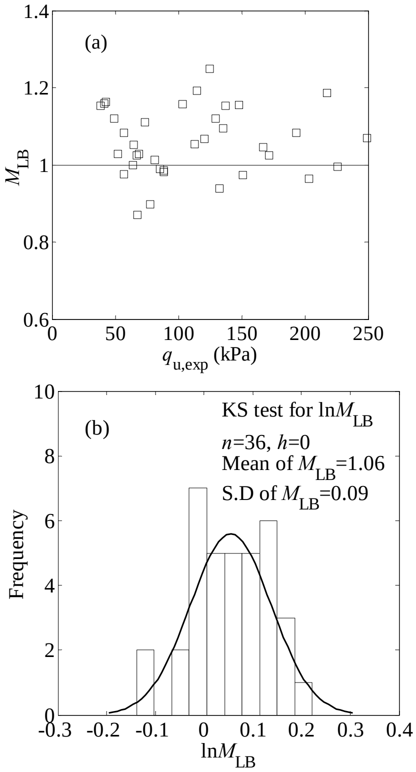

Figure 6: (a) model factor M_{LB} versus the measured capacity; (b) empirical probability distribution model for M_{LB}.

4.3. Comparison between model tests and Eurocode 7 approach

In this case, the model factor is defined as the ratio between the measured capacity q_{u_exp} and the calculated capacity q_{u_calc} from Eurocode 7 approach, given by

$$M = q_{u\_exp} / q_{u\_calc} \tag{6}$$

Introducing the modified capacity $q'_{u\_calc} = q_{u\_calc} \times f$ with Eq. (3) and (5) into Eq. (6), the modified model factor M' is given by

$$M' = M_{LB} \times \eta \tag{7}$$

According to Eq. (7), the modified model factor M' can be fully characterized by using the results for M_{LB} and the residual part η. The residual part η and M_{LB} follow the lognormal distribution. It is known that the product M' of these two statistically independent lognormal random variables M_{LB} and η is also a lognormal variable. According to the equations presented in Phoon & Tang (2015), the mean and standard deviation of M' is 1.04 and 0.09.

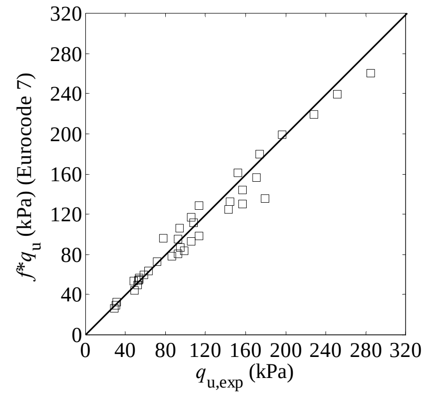

Figure 7: Calculated capacity from Eurocode 7 approach and its value multiplied by the regression equation f (modified Eurocode 7 approach) versus the measured capacity.

On the other hand, the Part II database will be used to validate the probability model of M' obtained before. The modified model factor M' is plotted against the measured capacity in Figure 7a. It appears to be randomly distributed with the measured capacity. The Spearman rank correlation analysis with p-values being greater than 0.05 also confirms that M' is independent of the input parameters. Consequently, M' can also be modeled as a random variable directly. The computed results for the mean value and standard deviation of M' are 1.06 and 0.11. The empirical distribution of M' is illustrated in Figure 8b, which is also lognormal. It indicates that the above framework to characterize the model uncertainty by using the FELA methodology is reasonable.

5. CONCLUSION

The model uncertainty of Eurocode 7 approach for estimating the bearing capacity of shallow foundations on granular materials under negative and general load combination is characterized.

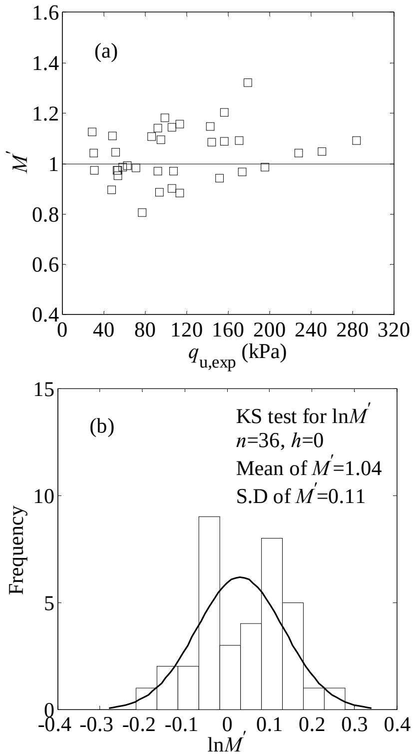

Figure 8: (a) modified model factor M′ versus the measured capacity; (b) empirical probability distribution of the modified model factor M′.

It can be seen that the combined loading direction has a significant effect on the regression equation f expressing the dependency of the model factor M_s on the input parameters, while the effect on the probability models of the residual part η, M_{LB} and M' can be negligible, as shown in Table 3.

However, the model uncertainty arising from the empirical shape and base inclination factors is not investigated, which should be done in future.

6. REFERENCES

CEN (2004). "EN 1997-1: Eurocode 7-Part 1: Geotechnical design-part 1: General rules." *Brussels, Belgium: CEN.*

Meyerhof, G. G. (1953). "The bearing capacity of foundations under eccentric and inclined loads." *In Proceedings of the 3rd International Conference on Soil Mechanics and Foundation Engineering (ICSMFE)*, Zurich, vol. I., pp. 440-445.

Patra, C. R., Behara, R. N., Sivakugan, N., and Das, B. M. (2012a). "Ultimate bearing capacity of shallow strip foundation under eccentrically inclined load, Part I." *International Journal of Geotechnical Engineering*, 6, 343-352.

Patra, C. R., Behara, R. N., Sivakugan, N., and Das, B. M. (2012b). "Ultimate bearing capacity of shallow strip foundation under eccentrically inclined load, Part II." *International Journal of Geotechnical Engineering*, 6, 507-514.

Phoon, K. K., and Tang, C. (2015). "Model uncertainty for the capacity of strip footings under combined loading." *Submitted to Geotechnical Special Publication in honor of Wilson. H. Tang (ASCE)*.

Tang, C., Phoon, K. K., and Toh, K. C. (2014). Effect of footing width on N_γ and failure envelope of eccentrically and obliquely loaded strip footings on sand. *Can. Geotech. J.*, 10.1139/cgj-2013-0378.