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# Stochastic Modeling of Recovery from Seismic Shocks

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ABSTRACT: One of the earthquake engineering buzzwords nowadays is *resilience*, which is related to the bulk of issues related to the recovery of a system of interest (e.g., a civil structure, an infrastructure, or a community) from a seismic shock. Resilience, in fact, includes and is more important than vulnerability itself, which has been the target of earthquake engineering in the last decades. From a probabilistic standpoint, the recovery path for a system should be treated as a stochastic process, as it is a function of time and affected by random variables at each point in time. The study discusses stochastic modeling of resilience to retrieve solutions for the probability of the system's time-to-recovery. In particular, the non-monotonic Gaussian process and the monotonic gamma and inverse-Gaussian processes, typically used in the reliability assessment of deteriorating systems, are considered first to discuss their capabilities in representing the restoration path. Once the limits of these independent increments processes are acknowledged, a more powerful time- and state-dependent increments model is proposed. It is able to account for the situation in which the recovery is possibly slowed-down by aftershocks occurring in a seismic sequence, when the latter is stochastically modeled by means of the results of probabilistic aftershock hazard analysis. For this case, the recovery process is modeled as the combination of two Markov chains: one modeling the recovery effort and the second one modeling the aftershock disruption.

## 1. INTRODUCTION

For a quite long time earthquake engineering research has been focused on the modeling of the impact of earthquakes on structures, infrastructures, and communities. The state-of-the art in this respect is certainly that of the performance-based earthquake engineering, or PBEE, (Cornell and Krawinkler, 2000) and its most recent and advanced declensions. PBEE is a quantitative framework to assess and manage the issues related to the occurrence of the mainshock and its potential in terms of failing the system of interest.

Currently, resilience of communities to natural hazards, including earthquakes, is the tag attached to a large deal of research; e.g., Franchin et al. (2014) and Cavallaro et al. (2014). Resilience is the characteristic of an object, or a system, which measures its capability to rapidly recover from a shock. In the transposition to the case of communities hit by a major loss-causative event, it is considered related to the path towards re-establishment of pre-event conditions, which may be grouped with respect to four categories: *physical* (i.e., the community infrastructure, that is lifelines and structures), *organizational* (i.e., management of the community and its infrastructure), *social* (i.e., related to people's response and dynamics), and *economic* (i.e., directly related to resource loss).

The recovery path, for a system with abrupt performance loss due to a disruptive seismic event, may be sketched as in Figure 1. In the same figure the area between the target performance level (not necessarily the same as in pre-event conditions) and the recovery process is shaded, as it has been proposed as quantitatively related to resilience. According to the quantitative and comprehensive framework for seismic resilience of complex systems given by the seminal work of Bruneau et al. (2003), the losses in a disruptive event may be related to just two of the attributes of resilience (i.e., the *robustness* and *redundancy* of the system), which in turn includes also the modeling and assessment of the recovery from such a loss and its key factors (i.e., *rapidity* and *resourcefulness*).

It is apparent from Figure 1 that, in assessing resilience, it is especially important to probabilistically describe the recovery path.

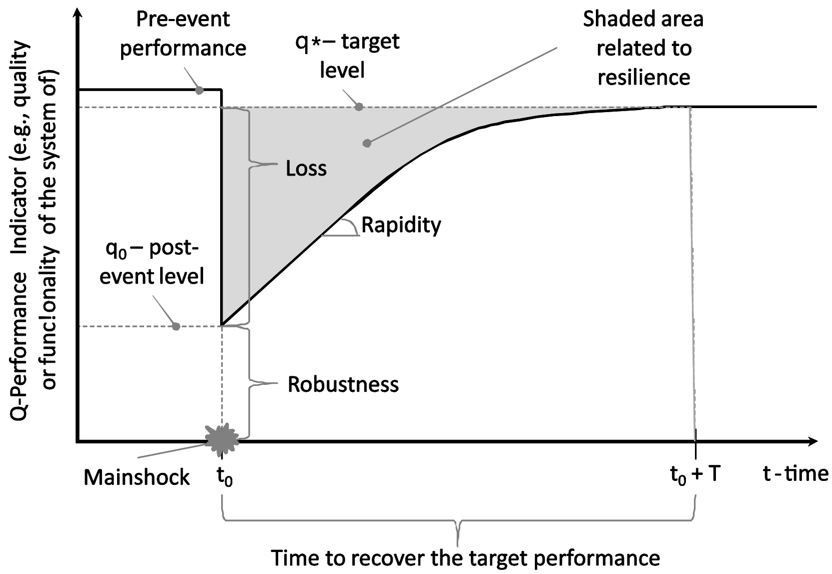


*Figure 1. Sketch of recovery to an earthquake and quantitative definition of resilience (Bruneau et al., 2003).*

In the last decades, it has been widely recognized that dealing with seismic risk necessarily requires a probabilistic approach due to the large uncertainties inherent to earthquake engineering. From the stochastic point of view, the most of PBEE models refer to time-invariant approaches, mainly because common hazard models rely on memory-less Poisson processes for earthquake occurrence (Cornell, 1968), and system's response is referred to failure in one event only (i.e., the *fragility*), without possible damage accumulation. In fact, resilience has to be treated as a *stochastic process*.

On these premises, this paper, starting from the background of reliability analysis of deteriorating systems: (i) it investigates stochastic modeling of the time-to-recovery (TTR), when the system is possibly affected by the occurrence of disruptive aftershocks, by means of independent increments processes such as the *Gaussian* (Itô, 1964), the *gamma* (Çinlar, 1980), and the *inverse-Gaussian* (Ye and Chen, 2013) models, showing capabilities and limits of this approach; (ii) it addresses the TTR via a non-homogenous *Markov chain*, which has state- and time-dependent increments.

The paper is structured such that the addressed problem is formalized in analytical terms to highlights the issues related to stochastic modeling, first. In particular, the recovery effort is described by a stochastic accumulation process, and point-in-time disruptions caused by aftershocks, are also considered. Then, a critical review of classical options of stochastic processes, commonly adopted to describe structural reliability, is given. Finally, a solution based on a *Markov chain*, allowing to overcome most of the limitations of the classical solutions, is proposed for the case when aftershocks follow the *modified Omori law* (e.g., Yeo and Cornell, 2009).

## 2. STOCHASTIC MODELING: GENERALITIES

Even if a quantitative framework of resilience in the context of earthquake engineering has been preliminarily established in the last years, modeling accounting for the large uncertainties involved is not yet consolidated. Indeed, it is clear that the only approach possible to resilience is the probabilistic one, and it is not only that. In fact, resilience, being related with time, is a *time-variant* problem. Modeling recovery from earthquakes (Figure 1) as a stochastic process, is the first goal of this study. In fact, the latter is seen as a particular case of the more general situation of Figure 2, in which aftershocks may affect the recovery path.

From the stochastic point of view, the main issues to be addressed are that: (i) the recovery process may be seen as a gradual process with random increments over any time interval; (ii) abrupt changes due to subsequent events are of the type of a *shock model* (Nakagawa, 2007), that is number of aftershocks, their time of occurrence, and effects they produce, are all random.

The analytical formulation of the sketch of Figure 2 is given in Equation (1), where $Q(t)$ is the performance (quality or functionality) level gained at time *t* during the recovery and $q_0$ is the starting performance level right after the mainshock occurred at $t_0$.

$$Q(t) = q_0 + q_C(t) - \sum_{i=1}^{N(t_0,t)} \Delta q_i \tag{1}$$

The time-variant part at the right hand side of Equation (1) is the sum of two effects, one due to continuous recovery and one due to accumulation of sudden losses due to aftershock events, $N(t_0,t)$, occurring in $(t_0,t)$. Following what discussed: $q_C(t)$, $\Delta q_i$ (damage in a single aftershock), and $N(t_0,t)$ all are random variables. In fact, the possible randomness of $q_0$ may be also accounted for (i.e., via the fragility of the system); however, for simplicity, it will be considered given in the following.

With this formulation, the probability that the system recovers the target performance within *t*, $P[TTR \le t]$, may be seen as *first-passage time* problem. In general, the recovery process is non-monotonic due to the aftershock disruptive effect, then $P[TTR \le t]$ has to be formulated as in Equation (2).[1]

$$P[TTR \le t] = 1 - P\left[Q(t_0,x) < q^*, \forall x \in (t_0,t)\right] \tag{2}$$

It is important to note, for stochastic modeling purposes, that $Q(t)$ should be equal or larger than zero (or any other lower bound) for any *t*, because the system cannot be damaged beyond a minimum functionality level.

## 3. STOCHASTIC MODELING: CLASSICAL OPTIONS

Possible solutions to the problem formulated in the previous section, based on stochastic processes usually adopted to describe the reliability of wearing systems, are briefly summarized in the following.

### *3.1. The gamma process*

One of the most common in this sense is the gamma process (Çinlar, 1980), which is of wide application in stochastic modeling of deteriorating systems; e.g., Iervolino et al. (2013 and 2014). This process, $\{X(t), t \ge 0\}$, has increments, $\Delta X(t,t+\Delta t) = X(t+\Delta t) - X(t)$, which are independent and following the gamma distribution in Equation (3). In the equation $\{\gamma, \eta(t)\}$ is the pair of scale parameter and the shape function, while $\Gamma(\bullet)$ is the gamma function.

$$f_{\Delta X(t,t+\Delta t)}(x) = \frac{\gamma \cdot (\gamma \cdot x)^{\eta(t+\Delta t)-\eta(t)-1}}{\Gamma\left[\eta(t+\Delta t)-\eta(t)\right]} \cdot e^{-\gamma \cdot x} \tag{3}$$

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[1] Note that, due to how the equation defines TTR, it may be that after the recovery aftershocks impair the functionality again.

The mean and variance function of this process are $\eta(t)/\gamma$ and $\eta(t)/\gamma^2$. Therefore, it is apparent how it is an option to model any kind of trend of the recovery process. In fact, if the shape function is set linear $\eta(t) = s \cdot t$ then the process is also of stationary increments (i.e., identically distributed).

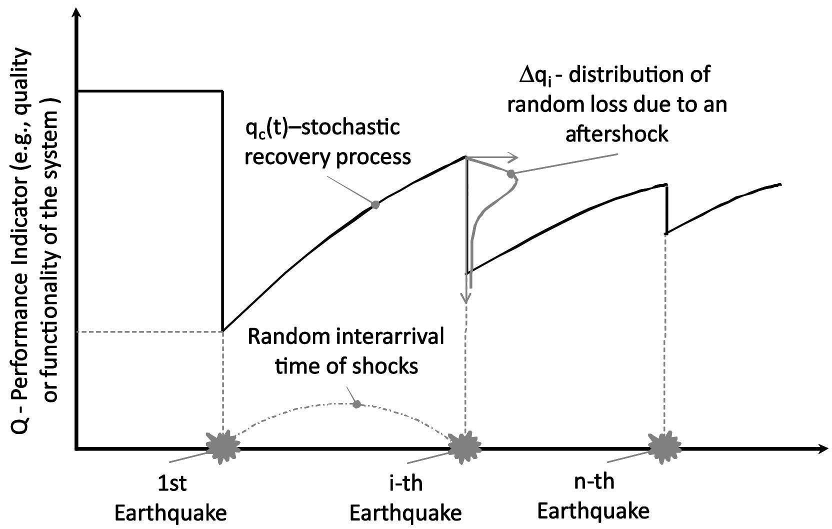


*Figure 2. Sketch of recovery to an earthquake when the aftershock sequence may affect the process.*

In the case the gamma process is considered, the probability of the system passing a threshold, $\bar{x}$, within *t* is given by Equation (4).

$$P[TTR \le t] = \int_{\bar{x}}^{+\infty} \frac{\gamma \cdot (\gamma \cdot x)^{\eta(t)-1}}{\Gamma[\eta(t)]} \cdot e^{-\gamma \cdot x} \cdot dx \qquad (4)$$

It is to note that for this case, and for the following inverse-Gaussian process, due to the non-negative increments, the first-passage of the threshold is also the only one, and Equation (5) results; see also Equation (2).

$$P[TTR \le t] = 1 - P[X(t_0, t) < \bar{x}] \qquad (5)$$

*3.2. The inverse-Gaussian process*

An analogous option with respect to the gamma process, is a process with inverse-Gaussian increments (Ye and Chen, 2013), Equation (6). This RV has a probability density function (PDF) that has mean and variance equation $\varphi(t)/\nu$ and $\varphi(t)/\nu^2$, respectively.

$$f_{\Delta X(t,t+\Delta t)}(x) = \\ = \frac{\varphi(t,t+\Delta t) - \varphi(t)}{\sqrt{2 \cdot \pi \cdot \nu \cdot x^3}} \cdot \mathrm{e}^{-\frac{1}{2} \cdot \frac{(\nu \cdot x - (\varphi(t,t+\Delta t) - \varphi(t)))^2}{\nu \cdot x}} \qquad (6)$$

Therefore, the CDF of TTR, in the case of a process with independent inverse-Gaussian-distributed increments, is that of Equation (7).

$$P[TTR \le t] = \int_{\bar{x}}^{+\infty} \frac{\varphi(t)}{\sqrt{2 \cdot \pi \cdot \nu \cdot x^3}} \cdot \mathrm{e}^{-\frac{1}{2} \cdot \frac{(\nu \cdot x - \varphi(t))^2}{\nu \cdot x}} \cdot dx \qquad (7)$$

*3.3. Independent increments Gaussian process*

This stochastic process has independent and identically distributed Gaussian increments (Itô, 1974). The main difference with respect to the others discussed is that, being Gaussian, the increment is non-monotonic. This means that the process can also have negative increments.

If the accumulation process is defined as in Equation (8); i.e., a deterministic function, $\mu(t)$, plus a process $\varepsilon(t)$, the increments of which are defined in Equation (9). Then, the probability to be above a threshold at a certain point in time is given by Equation (10).

$$X(t) = \mu(t) + \varepsilon(t) \tag{8}$$

$$f_{\varepsilon(t,t+\Delta t)}(x) = \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma^2 \cdot \Delta t}} \cdot e^{-\frac{1}{2} \cdot \frac{x^2}{\sigma^2 \cdot \Delta t}} \tag{9}$$

$$P\left[X(t) \geq \bar{x}\right] = \int_{\bar{x}}^{+\infty} \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma^2 \cdot t}} \cdot e^{-\frac{1}{2} \cdot \frac{(x-\mu(t))^2}{\sigma^2 \cdot t}} \cdot dx \tag{10}$$

In this case, to obtain the CDF of the first-passage time, is not easy to derive; see Equation (2). Nonetheless, it is possible to show that, for example, if the mean function is linear, according to the above assumptions, the PDF of the first-passage time (i.e., the time to be above the threshold), follows an *inverse Gaussian* distribution. In this case, the mean of such a distribution can be interpreted as the mean time to pass the threshold.

### *3.4. Discussion*

The models reviewed could be used to describe $Q(t)$. As an example, a realization of the recovery path to an event bringing the performance indicator down to 20% after a hypothetical disruptive event, when it follows any of the three processes, is given in Figure 3. The parameters of the processes were arbitrarily set so as the mean and the coefficient of variation are always $0.008 \cdot t$ and $0.1/\sqrt{t}$ respectively.

It has to be noted that only the Gaussian process is virtually able to model the effect of aftershocks on $Q(t)$. It may be seen from the plot that it is the only one with possibly negative increments. In fact, also this solution would not be satisfactory in this respect, as decrements would not be explicitly related to the occurrence of seismic events. Therefore, none of these three process, even if mathematically attractive, appears suitable to directly model $Q(t)$ as it is formulated in Equation (1).

As an alternative, they may be an option to only model $q_c(t)$. However, there is more than one reason to consider them not a viable solution for the stochastic modeling of recovery during aftershocks sequences.

(i) To combine $q_c(t)$ with the process describing the cumulative effect of aftershock would increase drastically the mathematical complexity of the resulting model (because, for example, it should be: $q_0 + q_c(t) - \sum_{i=1}^{N(t_0,t)} \Delta q_i \geq 0, \forall t$ ).

(ii) Recalling that these are independent increments models, they assume that the recovery increment in $(t, t+\Delta t)$ are independent of the recovery history up to *t*. In fact, the recovery processes is likely to be *time- and state-dependent*. For example, it is reasonable to consider that the resources deployed in the next time interval depend on the observed restoration level, which creates a stochastic relationship between the *rapidity* and the state of recovery. A simple model, which, in turn, is able to overcome these limitations, is proposed in the next section.

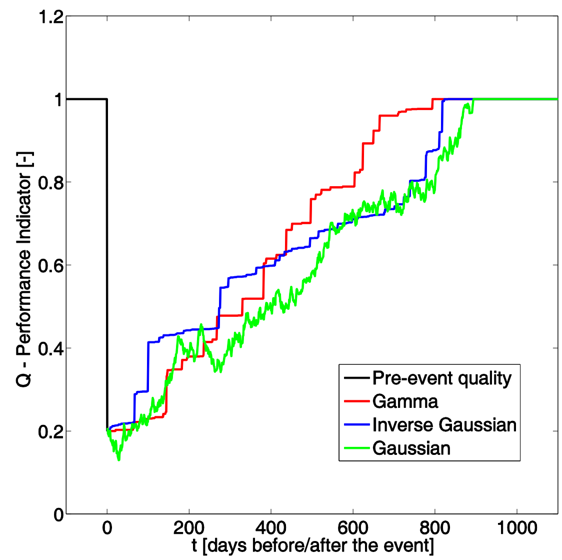


*Figure 3. Realization of the recovery process for some continuous-time stochastic processes.*

## 4. MARKOV CHAIN MODELING

Major earthquakes (i.e., mainshocks) typically trigger a sequence of lower-magnitude events clustered both in time and space. Resilience management in the post-event emergency phase has to deal with this short-term seismicity as aftershocks' disruptive effect may impair the recovery process (Figure 2). It is not easy to deal with this issue when the recovery is modeled via one of the processes above. Indeed, it is addressed in this section discretizing the recovery process via a Markov chain; however, the process of occurrence of aftershocks is discussed first.

### *4.1. Aftershock occurrence process*

For modeling the random occurrence of aftershock events, $N(t)$, the approach of Yeo and Cornell (2009) to aftershock probabilistic hazard analysis (APSHA) will be considered. In APSHA, at time $t$ (assuming that the mainshock occurred at $t=0$), the daily rate of aftershocks, $\lambda(t)$, is provided in Equation (11).

$$\lambda(t)=\left(10^{a+b\cdot(m_m-m_l)}-10^{a}\right)\Big/(t+c)^{p} \tag{11}$$

Aftershock magnitude is bounded between a minimum value of interest, $m_l$, and that of the mainshock, $m_m$. Coefficients *a* and *b* are from a suitable Gutenberg-Richter (1944) relationship, while *c* and *p* are from the modified Omori law.

### *4.2.Markovian approximation of recovery*

To describe the recovery process via a Markov chain (i.e., a discrete-time and discrete-state Markovian process) it is required to discretize the domain of the performance function in an arbitrary set of states $\{1\ldots i\ldots n\}$. The time *t* is discretized in intervals of fixed width equal to $\Delta$, which may be considered to be the time unit (e.g., one day or one week), then $\Delta=1$.

In this context, the $\{1\ldots i\ldots n\}$ states have to be defined such as the first may represent the final quality (e.g., pre-event functionality), while the last is the lowest possible performance level. Initial conditions, $q_0$, may be any of the intermediate states.

At this point, the transition probabilities between these states, in a unit-time interval, have to be defined to stochastically describe the process. The aim is to define the non-stationary recovery transition matrix, $\left[P_R(k,k+1)\right]$, in Equation (12). The generic element of this matrix represents the probability of the system to be in state *j* at time *k+1* given that it was in $Q_i$ at *k*. (The probabilities on each row sum up to one.)

$$\left[P_R(k,k+1)\right]= \\ =\begin{bmatrix} 1 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ P_{R,(n-1)1}(k,k+1) & P_{R,(n-1)2}(k,k+1) & \cdots & 0 \\ P_{R,n1}(k,k+1) & P_{R,n2}(k,k+1) & \cdots & P_{R,nn}(k,k+1) \end{bmatrix} \tag{12}$$

The zero elements above the principal diagonal represent the situation in which the process can only lead to a recovery. Note also that this representation has, as opposite to the discussed processes, an absorbing state, that is, it cannot recover more than the first state.

Then, the probability of the system to travel from any state to any other one in *m* time units, $\left[P(k,k+m)\right]$, is given by Equation (13).

$$\left[P(k,k+m)\right]=\prod_{i=1}^{m}\left[P_R(k+i-1,k+i)\right] \tag{13}$$

If the recovery transition matrix does not change in time, the Markov chain of Equation (13) becomes homogeneous; i.e., Equation (14).

$$\left[P(k,k+m)\right]=\left[P_R\right]^{m} \tag{14}$$

It is to underline that, even if Equation (14) reduces the computational effort required for model calibration and probability calculations, a model of the type in Equation (13) is more interesting, as it may describe a condition-based recovery strategy of the type discussed at the end of section 3.4. Indeed, a non-stationary matrix allows to reshape, as a function of time, the transition probabilities between the states on the basis of the gap analysis between the original plan and the actual recovery.

### *4.3.Damage accumulation due to aftershocks*

In this section it is assumed that the recovery process may be impaired by aftershocks, which may worsen the functionality conditions during the restoration period. In fact, this is one of the reasons why the recovery policy has to be adjusted on the go. The proposed approach relies again on a Markov chain. The domain of the performance index is partitioned in the same way as per the previous section. It is assumed that an aftershock event can lead the system to move from the *i-th* to the *j-th* (same or worse) state. This requires to formulate the transition matrix, $P_{E,ij}(k)$, given the occurrence of an earthquake in the $[k,k+1[$ time interval, Equation (15). The lower triangle of the matrix is comprised of zeros because of the monotonic nature of deterioration assumed for this process.

$$\left[P_E(k)\right] = \begin{bmatrix} P_{E,11}(k) & P_{E,12}(k) & \cdots & P_{E,1n}(k) \\ 0 & P_{E,22}(k) & \cdots & P_{E,2n}(k) \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \tag{15}$$

The individual elements of the matrix can be computed, for example, via the PDF of the intensity measure (IM) in a seismic shock, and the response of the system conditional to the earthquake event, *E*, as per a simple application of the total probability theorem in Equation (16).

$$P_{E,ij}(k) = \int_{im} P\left[state\ j\ at\ (k+1) \middle| state\ i\ at\ k \cap IM = z\ \right] \cdot f_{IM|E}(z) \cdot dz \tag{16}$$

Note that, as per the APSHA, the random variables representing ground motion intensities of different aftershocks are independent and identically distributed. Indeed, in APSHA, the process of occurrence of events causing exceedance of a ground motion intensity threshold at a site of interest is described by a non-homogeneous Poisson process, whose rate, $\lambda_{im}$, is obtained via Equation (17).

$$\lambda_{im}(k) = \lambda(k) \cdot \int_r \int_m P\left[IM > im \middle| M = x, R = y\right] \cdot f_{M,R|E}(x,y) \cdot dx \cdot dy \tag{17}$$

In the equation, $f_{M,R|E}$ is the distribution of magnitude (M) and source-to-site distance (R) of earthquakes from the source of interest, and $P\left[IM > im \middle| M = x, R = y\right]$ is the probability of exceeding *im* in an earthquake of known magnitude and distance.

On the premises of Equation (17), and if the unit-time rate of occurrence of earthquake shocks is small enough, such that the probability of observing more than one event in the unitary interval is negligible, the transition probability is given in Equation (18) for $i \neq j$.

$$\begin{aligned} &P\left[ j-th\ state\ at\ (k+1) \middle| i-th\ state\ at\ k \right] = \\ &= \lambda(k) \cdot P\left[ j-th\ state\ at\ (k+1) \middle| i-th\ state\ at\ k \cap E \right] \end{aligned} \tag{18}$$

Then, the transition matrix over the unitary time interval is given by Equation (19).

$$\left[P(k,k+1)\right] = \lambda(k) \cdot \left[P_E(k)\right] + \left(1-\lambda(k)\right) \cdot [I] \tag{19}$$

In the equation: $[I]$ is the identity matrix representing the certitude that the system remains in the same state if no earthquakes occur in the time-interval; and $\left(1-\lambda(k)\right)$ is the probability of not observing an earthquake. Equation (19) assumes that the transition probabilities in the $[k,k+1[$ time interval only depend on the conditions of the system at the beginning of the time step.

It follows that the probabilities of transition in *m* time units, $\left[P(k,k+m)\right]$, are given by simply taking the product as in Equation (20).

$$\left[P(k,k+m)\right] = \prod_{i=1}^{m} \lambda(k+i-1) \cdot \left[P_E(k+i-1)\right] + \left(1-\lambda(k+i-1)\right) \cdot [I] \tag{20}$$

*4.4. Recovery affected by aftershocks*

Consider now the case the system is subjected to both the recovery and degradation phenomena; i.e., Equation (1). In a unitary time interval, two cases are possible: (a) no earthquakes occur, then the the recovery progresses; (b) an earthquake occurs, then the system can recover and/or travel to a worse state because of both phenomena. Applying the total probability theorem with respect to the occurrence of aftershocks, the total transition matrix across the unitary interval, $\left[P(k,k+1)\right]$, is given by Equation (21).

$$\begin{aligned}\left[P(k,k+1)\right] &= \lambda(k)\cdot\left[P_E(k)\right]\cdot\left[P_R(k,k+1)\right]+\\ &+\left(1-\lambda(k)\right)\cdot\left[P_R(k,k+1)\right]\end{aligned} \tag{21}$$

Consequently, following the same approach used to describe the degradation process on structures in the life-cycle (Iervolino et al., 2015), the transition probabilities over *m* intervals are given by Equation (22).

$$\begin{aligned}\left[P(k,k+m)\right] &= \prod_{i=1}^{m}\lambda(k+i-1)\cdot\left[P_E(k+i-1)\right]\cdot\\ &\cdot\left[P_R(k+i-1,k+i)\right]+\\ &+\left(1-\lambda(k+i-1)\right)\cdot\left[P_R(k+i-1,k+i)\right]\end{aligned} \tag{22}$$

The totally ideal application in Figure 4, with transition matrices in Table 1 (assumed time-invariant) and Table 2 served solely to show the potential of the considered models. For it the quality was discretized in five states, parameters of Equation (11) are $\{a=-1.66, c=0.95, b=0.03, p=0.93, m_l=4.5, m_l=6.5\}$.

*Table 1. Unit-time recovery transition matrix.*

| | 0.8-1 | 0.6-0.8 | 0.4-0.6 | 0.2-0.4 | 0-0.2 |
|---|---|---|---|---|---|
| 0.8-1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.6-0.8 | 0.5 | 0.5 | 0.00 | 0.00 | 0.00 |
| 0.4-0.6 | 0.2 | 0.3 | 0.5 | 0.00 | 0.00 |
| 0.2-0.4 | 0.05 | 0.15 | 0.2 | 0.5 | 0.10 |
| 0-0.2 | 0.05 | 0.05 | 0.10 | 0.3 | 0.5 |

*Table 2. Transition matrix given the occurrence of an aftershock.*

| | 0.8-1 | 0.6-0.8 | 0.4-0.6 | 0.2-0.4 | 0-0.2 |
|---|---|---|---|---|---|
| 0.8-1 | 0.9 | 0.05 | 0.01 | 0.03 | 0.01 |
| 0.6-0.8 | 0 | 0.85 | 0.075 | 0.05 | 0.025 |
| 0.4-0.6 | 0 | 0 | 0.8 | 0.15 | 0.05 |
| 0.2-0.4 | 0 | 0 | 0 | 0.6 | 0.4 |
| 0-0.2 | 0 | 0 | 0 | 0 | 1 |

## 5. CONCLUSIONS

This paper preliminary discussed some options to address stochastic modeling of recovery, during aftershock sequences, starting from some models used in the field of reliability of deteriorating systems.

Two issues were addressed: (i) the recovery is modeled via continuous-time independent increments processes; (ii) the recovery is addressed based on a discrete-time-discrete-state Markov chain, which is the result of the combination of two independent Markov processes related to the recovery and to the aftershock damage.

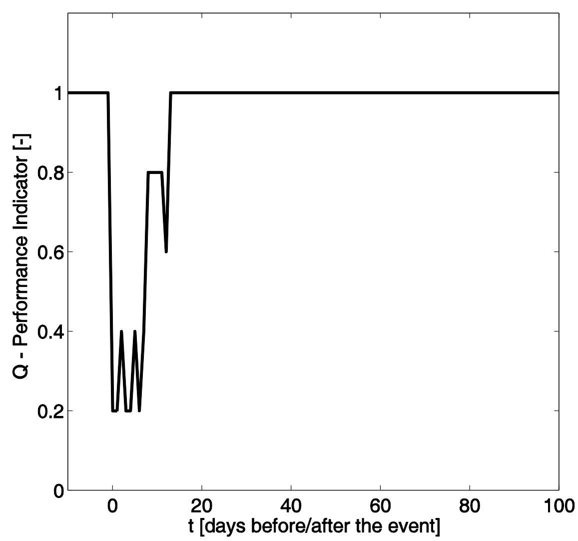


*Figure 4. Realization of a Markov chain describing resilience affected by aftershocks.*

## 6. ACKNOWLEDGEMENTS

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## 7. REFERENCES

Bruneau, M., Chang, S.E., Eguchi, R.T., Lee, G.C., O'Rourke, T.D., Reinhorn, A.M., Shinozuka, M., Tierney, K., Wallace, W.A., and von Winterfeldt, D. (2003). "A framework to quantitatively assess and enhance the seismic resilience of communities" *Earthq. Spectra*, 19(4), 733-752.

Cavallaro, M., Asprone, D., Latora, V., Manfredi, G., and Nicosia, V. (2014). "Assessment of urban ecosystem resilience through hybrid social‐physical complex networks" *Comput.-Aid. Civ. and Inf*. DOI: 10.1111/mice.12080

Çinlar, E. (1980). "On a generalization of gamma processes" *J. Appl. Prob.*, 17, 467-480.

Cornell, C.A. (1968). "Engineering seismic risk analysis" *B. Seismol. Soc. Am.*, 58(5), 1583-1606.

Cornell, C.A., and Krawinkler, H. (2000). "Progress and challenges in seismic performance assessment." *Peer Center Newletter*, 3(2).

Franchin, P., and Cavalieri, F. (2014). "Probabilistic assessment of civil infrastructure resilience to earthquakes" *Comput.-Aid. Civ. and Inf*. DOI: 10.1111/mice.12092

Gutenberg, B., and Richter, C. F. (1944). "Frequency of earthquakes in California" *B. Seismol. Soc. Am.*, 34(4), 185-188.

Iervolino, I., Giorgio, M., and Chioccarelli, E. (2013). "Gamma degradation models for earthquake-resistant structures" *Struct. Saf.*, 45, 48-58.

Iervolino, I., Giorgio, M., and Chioccarelli, E. (2014). "Closed-form aftershock reliability of damage-cumulating elastic-perfectly-plastic systems" *Earthq. Eng. Struct. Dyn.*, 43(4), 613-625.

Iervolino, I., Giorgio, M., and Chioccarelli, E. (2015) "Age- and State-Dependent Seismic Reliability of Structures" *12th International Conference on Applications of Statistics and Probability in Civil Engineering*, ICASP12, Vancouver, Canada, July 12-15, 2015.

Itô, K. (1974). "*Diffusion Processes*" Wiley.

Nakagawa, T. (2007). "*Shock and damage models in reliability theory*" Springer.

Ye, Z.S., and Chen, N. (2013). "The inverse Gaussian process as a degradation model" *Technometrics*, 56(3), 302-311.

Yeo, G.L., and Cornell, CA. (2009). "A probabilistic framework for quantification of aftershock ground-motion hazard in California: Methodology and parametric study." *Earthq. Eng. Struct. Dyn.*, 38(1), 45-60.