Age- and State-Dependent Seismic Reliability of Structures

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ABSTRACT: Life-cycle analysis of civil structures requires stochastic modeling of degradation. Phenomena causing structures to degrade are typically categorized as aging and point-in-time overloads. The former refers to deterioration of material characteristics and/or small yet frequent shocks, while earthquake effects are the members of the latter category this study deals with. Earthquake damage is usually modeled as dependent on the state of the structure at the time of each seismic shock only, while increments of deterioration due to aging are typically assumed to depend on the age of the structure at the most. While several studies deal with stochastic modeling of degradation they neglect, in an attempt to obtain easy-to-compute equations for the life-cycle reliability, to account at the same time for both forms of dependency. The presented study explicitly addresses this issue via a Markov-chain-based approach. Indeed, the model, which is practically in closed-form even if approximate, is able to describe a generic age- and state-dependent degradation process. The model also relies on the homogeneous Poisson process assumption for earthquake occurrence, a common case in seismic hazard analysis. An illustrative application shows the potential of the model.

1. INTRODUCTION

Mainly because of the importance of sustainability-related issues, there is an increasing interest in the life-cycle analysis of civil constructions. The latter requires modeling structural performance across the service life. Indeed, structures are generally subjected to degradation, the evaluation of which may aid in design of maintenance policies. It is common to distinguish phenomena causing degradation in two main categories: aging that may reflect on structural performance, and instantaneous (with respect to the lifespan of the structure) and observable overloads, such as earthquakes. Both cause damage accumulation on structures and both are random in nature. Aging is related to an aggressive environment, which worsens the mechanical features of structural elements (e.g., corrosion of reinforcing steel due to chloride attack, carbonation in concrete, etc.) or shocks the occurrence of which may be difficult to observe (e.g., ambient vibrations, traffic loads, fatigue, etc.). Degradation due to aging is typically described assuming that it takes place gradually over time. Earthquake shocks potentially cumulate damage on the hit structure during its lifetime. In general, mainly because earthquake occurrences can be treated as instantaneous with respect to structural life, that is safety-treating point-in-time events, it is advantageous to model the cumulative seismic damage process separately from aging (i.e., gradual deterioration).

A number of studies has recognized the stochastic nature of structural degradation, and approached modeling for reliability assessment; e.g., Pandey and van Noortwijk (2004). Some of the proposed models specifically address one of the two causes of deterioration, while more recently there is an increasing attention in accounting for both of them in the evaluation of the time-variant failure probability of structures; e.g., Sanchez-Silva et al. (2011) and Iervolino et al. (2013).

On these premises, the study presented in the following, which is, for the seismic damage part, a brief summary of Iervolino et al. (2015), tackles stochastic modeling of degrading structures in which both aging and cumulating seismic damage are considered. The issues specifically addressed are: (i) aging may be represented by a stochastic process with age- and state-dependent increments (Giorgio et al., 2011); (ii) seismic fragility depends on the state of the structure (Yeo and Cornell, 2009; Giorgio et al., 2010), and seismic response may change because of aging (this point implies that aging and damage are not independent processes).

The model set-up relies on a Markov-chain-based description of degradation, which enables almost closed-form with evident computational appeal, yet sound probabilistic and mechanical basis. In this context, random occurrence of seismic shocks is described by a homogenous Poisson process (HPP); McGuire (2004). A unified sketch of the issues tackled is given in Figure 1.

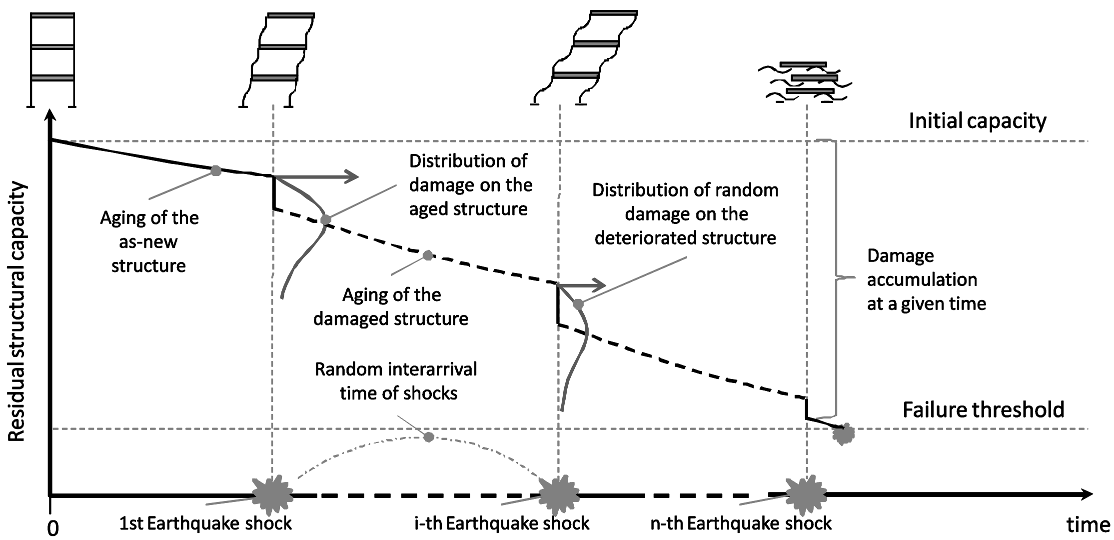

Figure 1. Sketch of the deterioration process for a structure subject to cumulative earthquake damage and aging.

The paper is structured such that the formulation of the considered reliability problem is given first. Then, the damage accumulation is addressed modeling both the process of occurrence of seismic shocks and the effect they produce, and the issues related to stochastic modeling of aging are discussed. Subsequently, the Markov-chain-based solution for the reliability problem is derived. Finally, a purely illustrative application, consistent with principles of performace-based earthquake engineering (PBEE; Cornell and Krawinkler, 2000), is exploited to show the potential and the implications of the proposed model.

2. BACKGROUND

The objective of this study is to enable the computation of the failure probability of structures that deteriorate due to progressive aging and seismic damage. The effect of degradation is measured in terms of residual seismic capacity. From the analytical point of view, the degradation process is that in Equation (1), where μ_0 is the initial capacity, and $D(t)$ is the total deterioration at the time *t*; the initial time is assumed to be zero, that is, $t_0 = 0$.

$$\mu(t) = \mu_0 - D(t) \tag{1}$$

$D(t)$ can be seen as the sum of two effects, one due to aging and one due to accumulation of seismic damage, as in Equation (2), where the first term at the right hand side is the loss of capacity at time *t* due to aging, $\mu_C(t)$, and the second one is the cumulated loss of resistance due to all earthquake events, $N(t)$, occurring until time *t*. Note that $\mu_C(t)$, $\Delta\mu_i$ (damage in a single seismic shock), and $N(t)$, all are random variables (RVs).

$$D(t) = \mu_C(t) + \sum_{i=1}^{N(t)} \Delta\mu_i \qquad (2)$$

Given this formulation, the probability the structure fails within *t*, $P_f(t)$, or the complement to one of the structural reliability $R(t)$, is the probability that the structure passes a threshold related to a certain limit state, μ_{LS}, at any time before *t*, Equation (3). In other words, it is the probability that in $(0,t)$ the capacity reduces travelling the distance, $\bar{\mu}$, between the initial value and the threshold. Note that, by definition, Equation (3) also provides the cumulative probability function of structural lifetime, $F_T(t)$.

$$\begin{aligned} P_f(t) &= 1 - R(t) = F_T(t) = \\ &= P\left[D(t) \geq \mu_0 - \mu_{LS}\right] = P\left[D(t) \geq \bar{\mu}\right] \end{aligned} \qquad (3)$$

A reliability problem with a formulation similar to that in Equation (3) was addressed in Iervolino et al., (2013). With respect to that study, the one herein relaxes some assumptions that may be considered strong in the structural engineering context. In particular, it is explicitly considered that:

1. the damage increment RV in each shock is dependent on the age and state of the structure at the time of the shock;
2. this will also enable to relax the hypothesis that aging and seismic damage are independent processes, as gradual deterioration contributes to determine the structural state at the time of the seismic shock, while they are conditionally independent given the age and the state of the structure.

In the following two sections, stochastic modeling for seismic damage and aging are addressed separately, and then they will be combined in a single process.

3. SEISMIC DAMAGE

To model degradation in structures with possible energy dissipation during seismic shaking (e.g., hysteretic behavior) an index measuring accumulating damage, and its effect on structural performance, is needed. This has been, and currently is, a relevant topic in the earthquake engineering literature. According to the review in Cosenza and Manfredi (2000), damage indices may be grossly categorized in two main classes labeled as *displacement-related* and *energy-related*. Hybrid damage indices, accounting for both damage phenomena in a single metric, also exist; Park and Ang (1985).

The main issue in modeling the stochastic evolution of accumulating seismic damage is that the increment of deterioration in a generic earthquake shock may be dependent on the history prior to its occurrence. In fact, this may be because of two different causes: (1) the hysteretic behavior of the structure does not remain the same in subsequent earthquakes; and (2) the way damage is measured introduces a dependency on history, even if the structural behavior remains the same. The case of (1) refers to evolutionary or degrading hysteretic behaviors. An example of (2) is the maximum-displacement-based criterion; i.e., damage accumulation in an earthquake occurs only once the maximum displacement reached in it is larger than the maximum in those previous, which makes the damage increment dependent on the distance between the residual displacement of the damaged structure at the time of the shock and the recorded maximum.[1]

In the remainder of the paper, the damage measure is based on the maximum displacement demand (or an equivalent of it, such as the drift ratio). In particular, the *kinematic ductility*, μ (i.e., the maximum displacement demand, when the yielding displacement is the unit) is chosen as the displacement-based damage proxy. The collapse is assumed to occur when kinematic ductility reaches some capacity value. This displacement-based criterion introduces a form of dependency on the structural damage history which will be described via a Markovian process.

3.1. Evolution of deterioration in multiple earthquake shocks

In this study damage accumulation is of concern, that is, it is assumed that failure may also be produced by multiple partially-damaging seismic shocks, and not only in a single catastrophic event. An approach to address this issue was proposed in some studies as Iervolino et al. (2013, 2014a, 2014b). A remarkable limitation of this kind of models stays in the assumption of independent and identically distributed damage increments.

An alternative approach presented herein is based on modeling the damage process via a Markov chain (i.e., a discrete-time and discrete-state Markovian process). The time *t* is discretized in intervals of fixed width equal to Δ, which may be considered to be the time unit (e.g., one year), then $\Delta = 1$. The domain of the damage index is partitioned to have a finite number of damage states (DS'). The transition probabilities between the *i-th* and the *j-th* damage states, in the $[k, k+1[$ time interval and given the occurrence of an earthquake, are indicated as $P_{E,ij}(k)$. In this context, the DS' are factually intermediate limit states, identifying intervals of the damage measure considered, between as new conditions and failure. A similar approach was originally pursued by Cornell and co-workers (e.g., Luco et al., 2004), which, however, neglects the effect of aging on life-cycle.

Considering a structure for which the damage increment in a generic seismic shock depends on the age and the state, the damage increment may be susceptible of the representation by the Markovian transition matrix in Equation (4).

The matrix, the rows and columns of which are labeled with the discrete damage states of the structure, collects the $P_{E,ij}(k)$ probabilities. The first state is, therefore, the *as new* (or initial) condition and *collapse* is represented by the *n-th* state. The lower triangle of the matrix is comprised of zeros because of the monotonic nature of deterioration. Note that the matrix is, in general non-stationary, as the transition probabilities may depend on the age of the structure. This happens for example, in a structure where damage is measured by a strain-based criterion and aging affects seismic response.

$$\left[P_E(k)\right] = \begin{bmatrix} 1-\sum_{j=2}^{n} P_{E,1j}(k) & P_{E,12}(k) & \cdots & P_{E,1n}(k) \\ 0 & 1-\sum_{j=3}^{n} P_{E,2j}(k) & \cdots & P_{E,2n}(k) \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \quad (4)$$

In the PBEE framework, the individual elements of the matrix can be computed via the probability density function (PDF) of the intensity in a seismic shock (IM), and the response of the structure conditional to the occurrence of an earthquake event, *E*, as per a simple application of the total probability theorem in Equation (5).

$$\begin{aligned} P_{E,ij}(k) &= \\ &= P\left[\, j-th\ state\ at\ (k+1) \middle| i-th\ state\ at\ k \cap E \,\right] = \\ &= \int_{im} P\left[\, j-th\ state\ at\ (k+1) \middle| i-th\ state\ at\ k \cap IM = z \,\right] \cdot \\ &\cdot f_{IM|E}(z) \cdot dz \end{aligned} \quad (5)$$

[1] In fact, the calibration procedure (Iervolino et al., 2015), not shown herein for the sake of brevity, allows to model transition probabilities as a function of maximum displacement only (i.e., the current state).

Note that, as per classical probabilistic seismic hazard analysis or PSHA (McGuire, 2004), the random variables representing ground motion intensities of different earthquakes are independent and identically distributed. Note also that the $P\left[j-th\ state\ at\ (k+1) \middle| i-th\ state\ at\ k \cap IM=z \right]$ term is factually equivalent to an age- and state-dependent fragility of the damaged structure.

3.2. Occurrence of seismic shocks and damage process

PSHA typically refers to the HPP as the process modeling the occurrence of earthquakes at a seismic source. HPP is such that the counting process for earthquakes is completely defined by one parameter, or the rate of occurrence, ν_E. Indeed, in PSHA, also the process of occurrence of events causing exceedance of a ground motion intensity threshold at a site of interest is described by a HPP, whose rate, λ_{im}, is obtained via Equation (6).

In the equation, usually referred to as the *hazard integral*, $f_{M,R|E}$ is the distribution of magnitude and source-to-site distance of earthquakes from the source of interest, and $P\left[IM > im \middle| M=x, R=y \right]$ is the probability of exceeding *im* in an earthquake of known magnitude (M) and source-to-site distance (R), or a *ground motion prediction equation*.

$$\begin{aligned} \lambda_{im} &= \nu_E \cdot P\left[IM > im \middle| E \right] = \\ &= \nu_E \cdot \int_r \int_m P\left[IM > im \middle| M=x, R=y \right] \cdot \\ &\cdot f_{M,R|E}(x,y) \cdot dx \cdot dy \end{aligned} \quad (6)$$

On the premises of Equation (6), and if the unit-time rate of occurrence of earthquake shocks is small enough, such that the probability of observing more than one event in the unitary interval is negligible, the transition probability is given in Equation (7) for $i \neq j$. In the equation, it is assumed that the ground motion intensity measure is a sufficient one (Luco and Cornell, 2007).

$$\begin{aligned} & P\left[j-th\ state\ at\ (k+1) \middle| i-th\ state\ at\ k \right] = \nu_E \cdot P_{E,ij}(k) = \\ & = \nu_E \cdot \int_{im} \int_r \int_m P\left[j-th\ state\ at\ (k+1) \middle| i-th\ state\ at\ k \cap IM=z \right] \\ & \times f_{IM|M,R}(z|x,y) \cdot f_{M,R|E}(x,y) \cdot dx \cdot dy \cdot dz \end{aligned} \quad (7)$$

Then, transition matrix over the unitary time interval is given by Equation (8). In the equation: $[I]$ is the identity matrix representing the certitude that the structure remains in the same state if no earthquakes occur in the time-interval; and $(1-\nu_E)$ is the probability of not observing an earthquake. Equation (8) assumes that the transition probabilities in the $[k, k+1[$ time interval only depend on the conditions of the structure at the beginning of the time step.

$$\left[P(k,k+1) \right] = \nu_E \cdot \left[P_E(k) \right] + (1-\nu_E) \cdot [I] \quad (8)$$

In these conditions, the Markov chain describing degradation due to earthquakes is completely characterized, as time varies, by the transition matrix in Equation (8). This means that the probabilities of transition in *m* time units, $\left[P(k,k+m) \right]$, are given by simply taking the product as in Equation (9).

$$\left[P(k,k+m) \right] = \prod_{i=1}^{m} \left[\nu_E \cdot \left[P_E(k+i-1) \right] + (1-\nu_E) \cdot [I] \right] \quad (9)$$

In particular, if the transition matrix is not time-variant, that is $\left[P_E(k) \right] = [P_E]$, this result specializes as in Equation (10). In this case the transition probabilities in *m* time units are obtained simply taking the *m-th* power of the unit time transition matrix that characterizes the process.

$$\left[P(k,k+m) \right] = \left[\nu_E \cdot [P_E] + (1-\nu_E) \cdot [I] \right]^m \quad (10)$$

4. AGING

Models typically used to describe the effect of aging have a key difference with respect to those for earthquake damage: they describe deterioration as a gradually increasing process if compared to the effect of point-in-time seismic shocks. The stochastic representation of aging in engineering systems, for the sake of mathematical tractability, is typically based on *age-dependent* processes, that is, processes with independent increments, even if possibly non-stationary. A well-known example of this class of models is the *gamma process* (Çinlar, 1980; Van Noortwijk et al., 2007), in which increments are independent and gamma distributed. Clearly, the gamma process is not the only option for stochastic modeling of aging, and other models are available. For example, a model with similar characteristics is a process with inverse-Gaussian-distributed increments (Wang and Xu, 2010).

It is to note that both gamma and inverse-Gaussian process are not able to model degradation in the case it has state-dependent increments (this is the case of the damage index discussed in section 3 for earthquake damage accumulation). Therefore, to model a deterioration process in which the increment is dependent both on the age and the state of the structure (e.g., Giorgio et al., 2011), an alternative approach has to be pursued. To address this, the next section discusses a transition matrix to stochastically model aging effects similar to that developed for earthquakes.

4.1. Markov chain approximation of aging

The objective of this work is to combine earthquake-based damage accumulation and degradation due to aging, when the former is described via the Markov chain illustrated in section 3. The first step to get the sought model is to consider the same set of states used to define damage progression of the structure in the seismic damage accumulation process. In that context, the $\{1 \cdots i \cdots n\}$ states have to be defined such as the first represents initial structural conditions, the *n-th* represents collapse, and the *i-th* state represents a degradation level (DS_i) intermediate between state $i-1>0$ and state $i+1<n$. This set of states is used to discretize the degradation variable, which is usually continuous in the case of aging, and transition probabilities between these states are used to describe the effect of aging process in a unit-time interval. The aim is to define the non-stationary transition matrix, $[P_A(k,k+1)]$, in Equation (11). The generic $P_{A,ij}(k,k+1)$ element of this matrix represents the probability of the system to be in damage state *j* at $k+1$ given that it was in DS_i at *k*. Then, the failure probability in *m* time units, due to aging only, $[P(k,k+m)]$, may be obtained by Equation (12).

$$[P_A(k,k+1)] = \begin{bmatrix} 1-\sum_{j=2}^{n} P_{A,1j}(k,k+1) & P_{A,12}(k,k+1) & \cdots & P_{A,1n}(k,k+1) \\ 0 & 1-\sum_{j=3}^{n} P_{A,2j}(k,k+1) & \cdots & P_{A,2n}(k,k+1) \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \tag{11}$$

$$[P(k,k+m)] = \prod_{i=1}^{m} [P_A(k+i-1,k+i)] \tag{12}$$

In the case the gradual deterioration increments depend on the state only, then the matrix in Equation (11) is not a function of time, that is $[P_A(k,k+1)] = [P_A]$. In this case Equation (12) becomes Equation (13), which characterizes a homogeneous Markov chain, and therefore is perfectly analogous to Equation (10).

$$[P(k,k+m)] = [P_A]^m \tag{13}$$

5. MARKOV-CHAIN-BASED MODEL FOR THE LIFE-CYCLE

Consider now that the structure is subjected to both degradation phenomena. In a unitary time interval, two cases are possible: (i) no earthquakes occur, then degradation is due to aging only; (ii) an earthquake occurs, then the structure can travel to a worse DS because of both phenomena. Applying the total probability theorem with respect to the occurrence of earthquakes, the transition matrix across the unitary interval, $[P(k,k+1)]$, is given by Equation (14).

Consequently, the transition probabilities over *m* intervals are given by Equation (15).

$$\begin{aligned}\left[P(k,k+1)\right] &= \nu_E \cdot \left[P_E(k)\right] \cdot \left[P_A(k,k+1)\right] + \\ &+ (1-\nu_E) \cdot \left[P_A(k,k+1)\right]\end{aligned} \tag{14}$$

$$\begin{aligned}\left[P(k,k+m)\right] &= \prod_{i=1}^{m} \nu_E \cdot \left[P_E(k+i-1)\right] \cdot \\ &\cdot \left[P_A(k+i-1,k+i)\right] + (1-\nu_E) \cdot \left[P_A(k+i-1,k+i)\right]\end{aligned} \tag{15}$$

If earthquake damage and aging are represented by means of stationary-increments processes, then the unit-time transition matrix is time-invariant and the more simple Equation (16) applies.

$$\begin{aligned}\left[P(k,k+m)\right] &= \\ &= \left[\nu_E \cdot [P_E] \cdot [P_A] + (1-\nu_E) \cdot [P_A]\right]^m = [P]^m\end{aligned} \tag{16}$$

It is now clear that the one step (e.g., annual) transition matrix $[P]$ completely characterizes the degradation process, and allows simple computations of reliability in the life-cycle. The generic element of $[P]^m$ represents the probability of finding the structure in the state *j* after *m* years, given that it was in state *i* at the beginning of the period.

It is to note that, the combination of the aging and cumulative earthquake damage matrices as in Equation (16) implies assuming that aging stochastically changes the DS of the structure analogously to seismic shocks. In other words, given the state of the structure, the future evolution of deterioration stochastically depends only on the state and age, while it is independent of the causes (aging or earthquakes) that have taken it there.

6. ILLUSTRATIVE APPLICATION

The example developed in this section refers to a simple non-evolutionary elastic-perfecly-plastic (EPP) single-degree of freedom (SDOF) system featuring an elastic period of 0.5s; however, the model can be applied, virtually, to any hysteretic loop and structure. Weight is 100 kN and the yielding force is equal to 12.25 kN, which corresponds to a strength reduction factor equal to 4 when the mass acceleration is equal to 0.49g; drift at the yielding is equal to 0.76%. The structure is supposed to be located in Sulmona (close to L'Aquila, in central Italy).

At this point, it is to recall that, in general, aging affecting material characteristics could have an effect on the seismic response; i.e., it may modify the hysteretic loop in time. However, because this application, for simplicity, refers to the non-evolutionary EPP-SDOF system, aging should not be (conceptually) ascribed to material deterioration, while rather to those small shocks, which produce a similar effect of earthquakes, yet whose occurrence cannot be monitored.

6.1. Hazard, state-dependent fragility curves and transition probabilities for earthquakes

As discussed with respect to Equation (5), the earthquake damage accumulation process requires the distribution of earthquake intensity given the occurrence of a seismic shock. This means to carry out PSHA for the site of the construction. The chosen ground motion intensity measure is the spectral acceleration at the elastic period of the system, $Sa(0.5s)$. PSHA for the Sulmona site was carried out via the FORTRAN code also used in Convertito et al. (2009) and Iervolino et al. (2011). A relevant information for this study is that the rate of occurrence of earthquakes within the magnitude bounds of the considered seismic sources is $\nu_E = 1.95\ events/year$.

Damage herein is related to a maximum displacement criterion. In fact, the considered engineering demand parameter (EDP) is the maximum drift (mass displacement divided by the height of the SDOF, which is equal to 1 m) ratio during the event (or *transient drift*). Damage states are defined in compliance with the recommendations of FEMA (2000) for concrete structures. In particular, three intermediate states are identified: *immediate occupancy* (IO), *life safety* (LS), and *collapse prevention* (CP). Thresholds indicating the limit to exit each DS in terms of considered EDPs are reported in Table 1. Moreover, two additional limit states have to be considered: *As New* (AN) and *Collapse* (COL) conditions; transient drift indicating the end of AN is equal to the yielding one and collapse is identified by the drift to *exit* CP.

To develop state-dependent transition probabilities, state-dependent fragility curves are needed first. To get those the method suggested in Luco et al. (2004), which relies on Equation (17), was followed. In the equation Sa_{cap}^{ij} is the (capacity) spectral acceleration (IM) needed to let the structure to travel from DS_i to DS_j or worse (Yeo and Cornell, 2005). It is to note that the procedure in Luco et al. (2004) allows to define the state in terms of maximum transient recorded response only, while in principle, the damage increment depends on the maximum recorded and the current residual drifts.

$$P\left[\, j-th\ state\ or\ worse \,\middle|\, i-th\ state \cap IM \ = z \right] = \\ = P\left[Sa_{cap}^{ij} \le z \right] \qquad (17)$$

The method requires incremental dynamic analysis, or IDA (Vamvatsikos and Cornell, 2002), which was carried out using ninety records; see Iervolino et al. (2015) for further details.

Table 1. Limit state thresholds in terms of transient drift according to FEMA (2000).

IO	LS	CP
0.01	0.02	0.04

Once $Sa_{cap,kh}^{ij}$ are computed for a set of records, a lognormal distribution may be fitted through them. These distributions, integrated with the hazard curve, provide the transition probabilities. The obtained probabilities are given in Table 2, which represents $\left[P_E\right]$ of section 3, for the considered structure. Note that such a matrix is dominated by the principal diagonal, a feature, which will be recalled in the following.

Table 2. Transition probabilities given earthquake.

	AN	IO	LS	CP	COL
AN	0.9807	0.0080	0.0079	0.0021	0.0013
IO	0	0.9856	0.0107	0.0023	0.0014
LS	0	0	0.9945	0.0040	0.0016
CP	0	0	0	0.9977	0.0024
COL	0	0	0	0	1

At this point, the probability of getting any damage state in a given number of years may be computed according to Equation (10).

Note that, in this case the annual rate of earthquakes is much larger than one, being 1.95 events per year (section 6). Therefore, in principle, Equation (10) should not be applied as the rate cannot be confused with the probability of occurrence of one earthquake. The problem can be easily solved obtaining, for example, the monthly earthquake rate dividing 1.95 by 12 and then considering the unit-time to be the month, while keeping the same $\left[P_E\right]$ matrix. However, because the latter is dominated by the principal diagonal, which renders it very similar to an identity matrix, the approximation in Equation (10) works also if the rate is kept yearly.

6.2. Unit-time transition matrix for aging

To represent aging in the application, which refers to a non-evolutionary system, a stationary transition matrix, $\left[P_A\right]$, is adopted. For this kind of system, aging has to be regarded as the effect of non-monitored, yet frequent, small shocks. On these premises, and to ideally represent a deterioration process mildly affecting structural reliability if compared to earthquakes, the calibration of $\left[P_A\right]$ is carried out dividing the elements off the diagonal of the unit-time transition matrix in Equation (10) by ten. The obtained matrix is reported in Table 3.

Table 3. Unit-time transition probabilities from a state to another in the gradual deterioration case.

	AN	IO	LS	CP	COL
AN	0.9962	0.0016	0.0015	0.0004	0.0003
IO	0	0.9972	0.0021	0.0005	0.0003
LS	0	0	0.9989	0.0008	0.0003
CP	0	0	0	0.9995	0.0005
COL	0	0	0	0	1

6.3. Life-cycle reliability assessment

To characterize the whole degradation process the annual transition matrix, $\left[P\right]$, in Equation (14) has to be assigned. This corresponds to Equation (18), which again takes advantage of the dominance of the principal diagonals in the $[P_E]$ and $[P_A]$ matrices.

$$[P] = \nu_E \cdot [P_E] \cdot [P_A] + (1 - \nu_E) \cdot [P_A] \quad (18)$$

This matrix completely characterizes the degradation process, as it is the annual transition matrix in the case of earthquake damage accumulation and aging, and its power raised to *m* provides the reliability assessment over *m* years. Consequently, Figure 2 reports the probability that at a certain time the structure is found in a specific DS given that it was in another DS at t = 0.

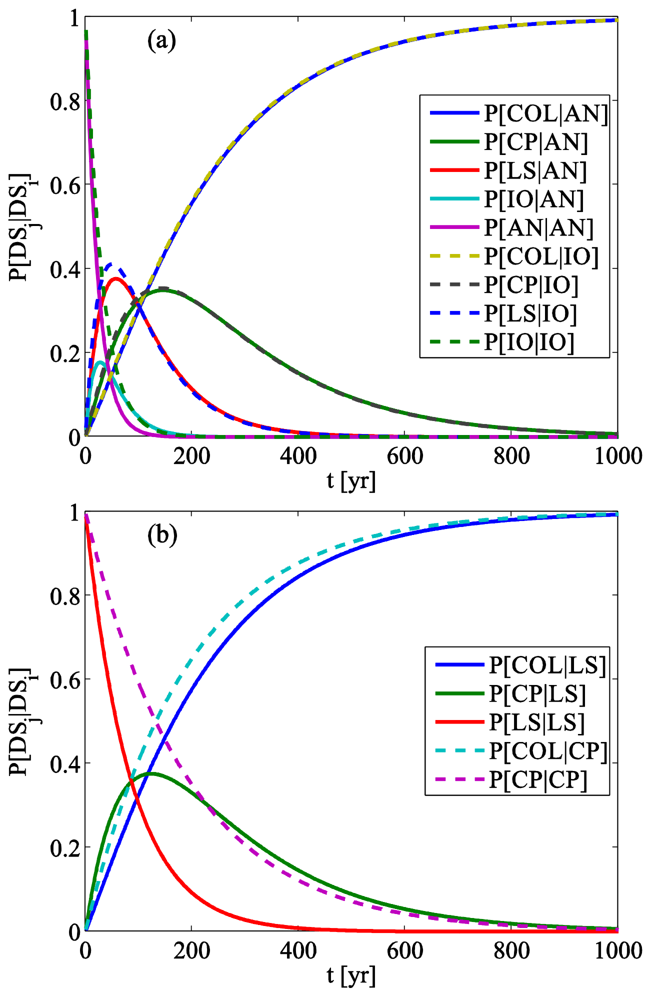

Figure 2. Life-cycle state probabilities due the whole degradation process including aging and seismic damage accumulation.

In particular, Figure 2a shows the time-variant probabilities for the structure which is AN or IO conditions at t = 0. Figure 2b is analogous, yet for the structure with an initial DS equal to LS or CP. In all cases, the probability of remaining in a given DS decreases monotonically with time, while the probabilities of reaching collapse tend to one as time increases. Finally the probability of moving to an intermediate DS have non-monotonic trends.

7. CONCLUSIONS

In the study, a life-cycle reliability model for structures subjected to aging and cumulative earthquake damage was presented. The model is based on a non-homogeneous Markov chain representation of the degradation process, thus it relies on two non-stationary matrices: one referring to the unit-time transitions due to the aging phenomenon and one referring to the state transitions due to the occurrence of an earthquake.

The developed model may virtually describe any kind of age- and state-dependent degradation process. Moreover, even if strictly of non-closed form, it allows the reliability assessment in the life-cycle with a very low computational demand (simply taking products, or only powers of the annual transition matrix over the time interval of interest). Finally, it enables to remove the conventional hypotheses of previous work and of a quite large deal of literature, which assumes independent, and often identically distributed, degradation increments.

To illustrate the potential of the model, and to address its calibration with respect to a specific (yet generic) structure, an elastic perfectly plastic single degree of freedom system, ideally located in central Italy, was considered. The reliability of this simple, non-evolutionary, system is formulated via a homogeneous Markov chain, that is, with a stationary transition matrix. On the other hand, it allows to show how the use of a displacement-based damage criterion introduces a dependency of deterioration increments, at least on the state of the structure, which requires explicit modeling. In fact, for this structure, state-dependent fragility curves were computed and combined with hazard for the considered site, resulting in transition probabilities in the event of an earthquake shock. Regarding gradual deterioration, it was assumed that aging is a milder degradation process, yet of similar type, with respect to earthquakes. Results of the reliability assessment for the illustrative application, allow appreciating the generality of the developed approach for life-cycle analysis of deteriorating earthquake-resistant structures.

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