Probabilistic Fatigue Life Prediction for Bridges Using System Reliability Analysis and SHM-based Finite Element Model Updating

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ABSTRACT: Fatigue is one of the main causes of bridge failures. A bridge is designed with a particular service life, but after it is constructed, its strength degrades over time. Therefore, to effectively maintain and retrofit a bridge, it is essential to predict its remaining fatigue life. However, doing so is a very challenging task because fatigue life prediction should be based on the current condition of the bridge, and this obviously incurs many uncertainties. In addition, fatigue life prediction should be performed at the system level to take the structural redundancy of a bridge into account. This paper proposes a new approach based on the probabilistic fatigue life prediction of bridges using finite element (FE) model updating based on structural health monitoring (SHM) data. The proposed method involves three steps: (1) identifying the modal parameters of a bridge, such as the natural frequencies and mode shapes, from the ambient vibration under the influence of passing vehicles; (2) updating the structural parameters of an initial FE model using the identified modal parameters; and (3) predicting the probabilistic fatigue life at the system level by employing the updated FE model. The proposed method is applied to a numerical bridge example, and the analysis results are verified by comparing them with the results obtained from a Monte Carlo simulation.

1. INTRODUCTION

Fatigue is one of the major causes of structural failure. Many structural systems are subjected to the risk of fatigue-induced failure caused by repeated loading over their service lives. This issue is especially critical for bridges. For this reason, a bridge is designed to survive for a certain period, but after it is constructed, its condition will change over its service life. It is thus essential to be able to predict the fatigue life to enable the making of decisions about effective bridge maintenance and retrofitting. However, this is a very challenging task because fatigue life prediction for a bridge should be based on its current structural condition, which will incur several sources of uncertainty, including material properties, anticipated vehicle loads, and environmental conditions.

Probabilistic methods of fatigue life prediction for highway bridges have been studied by many researchers. In previous studies, however, fatigue reliability assessments for bridges were based on the initial designs and visual inspection data, and this kind of approach is limited in that the current bridge condition cannot be considered.

This paper presents a new approach for probabilistic fatigue life prediction for bridges using finite element (FE) model updating based on structural health monitoring (SHM) data. Recently, various types of SHM systems have been introduced for civil engineering structures to monitor and evaluate their long-term structural performance. One example involves SHM data being applied to estimate the degradation of an in-service bridge, which makes it possible to update the initial FE model (Yi *et al.* 2007, Yi *et al.* 2012, Lee *et al.* 2014). The proposed method using this technique involves three main steps: (1) identifying the modal parameters for a bridge, such as the mode shapes and natural frequencies, using the ambient vibration caused by passing vehicles; (2) updating the structural parameters of an initial FE model of the bridge using the identified modal parameters; and (3) predicting the probabilistic fatigue life by employing the updated FE model. The performance of the proposed method is demonstrated by applying it to the numerical bridge example addressed by Yi *et al.* 2007 and Lee *et al.* 2014 for SHM-based FE model updating.

2. PROBABILISTIC FATIGUE LIFE PREDICTION USING FINITE ELEMENT ANALYSIS

To calculate the probability of fatigue failure, it is necessary to express the failure event of interest by using the so-called "limit-state function," which is an analytical function based on random variables and deterministic parameters. In this research, the Paris equation (Paris and Erdogan 1963), which is a widely used model for describing fatigue crack growth based on fracture mechanics, is introduced to derive formulations for bridge fatigue life evaluation based on SHM data.

First, let us consider the following crack-growth model using the Paris equation:

$$\frac{da}{dN} = C\left(\Delta K\right)^m \tag{1}$$

where a denotes the crack length, N is the number of load cycles, C and m are the material parameters, and ΔK denotes the range of the stress intensity factor. By applying Newman's approximation (Newman and Raju 1981), the range of the stress intensity factor can be estimated as

$$\Delta K = S \cdot Y(a) \cdot \sqrt{\pi a} \tag{2}$$

where S denotes the range of the stress, and $Y(a)$ is the "geometry" function. By substituting Eq. (2) into Eq. (1) and integrating it from the initial condition to the current time point, the relationship between the time duration and the current crack length can be derived as

$$\int_{a_0}^{a} \frac{1}{\left[Y(a)\sqrt{\pi a}\right]^m} da = C \cdot N \cdot S^m = C \cdot \nu_0 \cdot T \cdot S^m \tag{3}$$

where a_0 is the initial crack length, N is the total number of applications of vehicle loading with frequency ν_0, and T denotes the time duration. Suppose a crack failure is defined as that event whereby the length of a crack exceeds the critical crack length a_c. Then, the time required for the crack to grow from a_0 to a_c, that is, T_0, is described as

$$T_0 = \frac{1}{C\nu_0\left(S_0\right)^m} \int_{a_0}^{a_c} \frac{1}{\left[Y(a)\sqrt{\pi a}\right]^m} da \tag{4}$$

where a_0 and S_0 are the initial crack length and stress range, respectively. Then, the limit-state function for the failure of a member within a given time $[0, T_s]$ is

$$\begin{aligned} g(\mathbf{X}) &= T_0 - T_s \\ &= \frac{1}{C\nu_0\left(S_0\right)^m} \int_{a_0}^{a_c} \frac{1}{\left[Y(a)\sqrt{\pi a}\right]^m} da - T_s \end{aligned} \tag{5}$$

where $\mathbf{X}$ denotes the vector of random variables representing the uncertainties in the parameters of the problem, including the material properties (C, m) and initial crack length (a_0). In structural reliability analysis, $g(\mathbf{X}) \leq 0$ generally indicates the occurrence of a failure event.

After the initial FE model is updated based on the SHM data, however, the stresses change, such that the time required for the fatigue failure also needs to be re-estimated. If the FE model updating is done at T^1_{up}, a recursive formulation of the time duration from that moment to the crack failure is developed, as follows. Consider the auxiliary "damage" function

$$\boldsymbol{\Psi}(a)=\int_{a_0}^{a}\frac{1}{\left[Y(a)\sqrt{\pi a}\right]^m}da \tag{6}$$

From Eq. (4), it can be seen that

$$\begin{aligned}\boldsymbol{\Psi}(a_1)-\boldsymbol{\Psi}(a_0)&=C\cdot\nu_0\cdot T_{up}^1\cdot(S_0)^m\\ \boldsymbol{\Psi}(a_c)-\boldsymbol{\Psi}(a_1)&=C\cdot\nu_0\cdot T_1\cdot(S_1)^m\end{aligned} \tag{7}$$

where a_1 and S_1 denote the crack length and stress at the instant that the FE model is updated, respectively. Eq. (7) represents the crack growth before and after the FE model is updated. Summing up the two equations and solving it for T_1 gives

$$\begin{aligned}T_1&=\frac{1}{C\nu_0(S_1)^m}\left[\boldsymbol{\Psi}(a_c)-\boldsymbol{\Psi}(a_0)\right]-\left(\frac{S_0}{S_1}\right)^m T_{up}^1\\ &=\frac{1}{C\nu_0(S_1)^m}\int_{a_0}^{a_c}\frac{da}{\left[Y(a)\sqrt{\pi a}\right]^m}-\left(\frac{S_0}{S_1}\right)^m T_{up}^1\end{aligned} \tag{8}$$

Note that the ratio of the stresses S_0/S_1 incorporates the effect of the stress change obtained by the FE model updating. Similarly, if there is another FE model updating at T^2_{up}, the time required for the crack failure after the second updating is derived as

$$\begin{aligned}T_2=&\frac{1}{C\nu_0(S_2)^m}\int_{a_0}^{a_c}\frac{da}{\left[Y(a)\sqrt{\pi a}\right]^m}\\ &-\left(\frac{S_0}{S_2}\right)^m T_{up}^1-\left(\frac{S_1}{S_2}\right)^m T_{up}^2\end{aligned} \tag{9}$$

Through mathematical induction, a recursive formulation is derived for multiple FE model updating as

$$\begin{aligned}T_j=&\frac{1}{C\nu_0(S_j)^m}\int_{a_0}^{a_c}\frac{da}{\left[Y(a)\sqrt{\pi a}\right]^m}\\ &-\sum_{i=1}^{j-1}\left(\frac{S_i}{S_j}\right)^m T_{up}^i\end{aligned} \tag{10}$$

where j and T_j denote the number of FE model updating and the duration from the last SHM to the crack failure, respectively. The fatigue failure within a given time $[0, T_s]$ is then described as

$$g(\mathbf{X})=T_{up}^1+T_{up}^2+\cdots+T_{up}^j+T_j-T_s \tag{11}$$

By using Eq. (11), the fatigue life of a bridge can be evaluated by applying multiple SHM and FE model updating. For the sake of simplicity, however, it is assumed in this research that the structural health monitoring is performed only once (i.e., $j = 1$).

To calculate the probabilities of fatigue failure events using Eq. (11), a reliability analysis method should be employed. Numerous methods have been developed and adopted in various engineering disciplines [17]. In this study, the first-order reliability method (FORM) and second-order reliability method (SORM) (Der Kiureghian 2005) are used, both of which are representative methods of structural reliability analysis.

For a complex structural system, a failure may be described as a system event, which requires system reliability analysis. The following example bridge consists of five girders, and the fatigue failure of each girder is defined by one component event, which means there are five component events. In addition to their calculation, it is assumed that the bridge system fails if any one of the five girders fails. Then, the system failure event is described by a series event requiring system reliability analysis. In this paper, the multivariate normal integral method developed by Genz (1992), dedicated to series and parallel system probability calculations, is used.

3. FINITE ELEMENT MODEL UPDATING BASED ON STRUCTURAL HEALTH MONITORING DATA

Yi *et al*. (2007, 2012) recently presented a useful application of an instrumented SHM system for reliable seismic performance evaluation based on measured vibration data under ambient wind and traffic loadings. The procedure consists of (1) constructing an initial FE model of a target bridge, based on its design drawings, (2) measuring the ambient vibration of the bridge under the influence of vehicles passing; (3) identifying the modal parameters, such as natural frequencies and mode shapes, from the measured acceleration data using an output-only modal identification method; (4) updating the linear structural parameters for an initial FE model using the identified modal parameters; and finally (5) performing the probabilistic analysis of interest by employing the updated FE model.

To find the optimal structural parameters for the initial FE model, based on the measured modal properties of a bridge, this study employs the downhill simplex method (Nelder and Mead 1965). The objective function J in the optimization procedure represents the difference between the measured and calculated natural frequencies, while the constraint equations are constructed based on the differences between the measured and calculated mode shapes as

$$J = \sum_{i=1}^{N_m} \left\{ w_i \left(\frac{f_i^c - f_i^m}{f_i^m} \right) \right\}^2 \quad \text{subject to } \left| \phi_{ji}^c - \phi_{ji}^m \right| \le \varepsilon \tag{12}$$

where f_i is the i-th natural frequency, and φ_{ji} denotes the j-th component of the i-th normalized mode shape ϕ_i. w_i and ε are the weighting factor for the i-th mode and the admissible error bound for the mode shape, respectively. Superscripts m and c indicate the measured and calculated data, respectively. Details of the FE model updating procedure can be found in the following section as well as in Yi *et al*. (2007, 2012) and Lee *et al*. (2014).

4. NUMERICAL EXAMPLE

4.1. Example Bridge

To validate the FE model updating method proposed in Yi *et al*. (2007), extensive field tests were conducted on the Samseung Bridge, which was constructed as part of the Jungbu Inland Expressway, Korea, in 2002. In this research, as a numerical example, the Samseung Bridge example is reused to evaluate its probabilistic fatigue life based on both the initial and updated FE models.

The Samseung Bridge is a single-span, steel-plate girder bridge with a span length of 40 m. It is composed of five main steel girders, floor beams, and a concrete slab. Based on the design drawings of the bridge, an initial FE model is constructed using SAP2000 (with a span length of 38.8 m), as shown in Fig. 1.

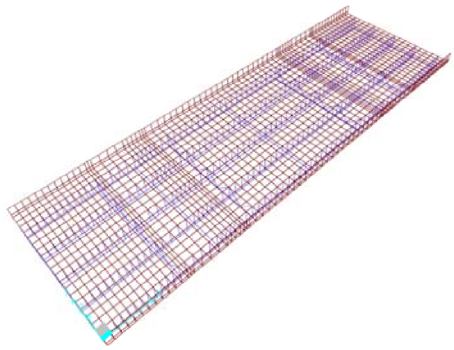


Figure 1: FE model of Samseung Bridge for SAP2000.

In the figure, the shell elements (red) and frame elements (blue) represent the concrete slab and steel girders, respectively. The five girders are labeled Girder 1 to Girder 5, in order from the bottom to the top.

4.2. FE Model Updating

To update the FE model, in Yi *et al*. (2007), a series of tests were carried out on the bridge in each of the four seasons: (1) August 2004, (2) December 2004, (3) July 2005, and (4) February 2006. In each test, the loading tests followed by an ambient vibration test were carried out. The deflections (i.e., vertical displacements) induced by the heavy trucks were measured in the loading tests, while the accelerations were measured in the ambient vibration tests. The measured accelerations in the ambient vibration tests were used to update the initial FE model, while the measured displacements in the loading tests were used to validate the updated FE models for each season. More details of the field tests can be found in Yi *et al*. (2007). This paper uses the updated FE model obtained from the test of the third season (in July 2005) for the proposed method.

In the ambient vibration tests of the third season, 21 accelerometers were installed on the bridge. The ambient vibrations were measured for 30 min at a sampling frequency of 200 Hz with a low-pass filter at 90Hz.. Since the test bridge was not allowed to pass the traffic during the test, the wind and traffic on the adjacent bridge were the sources of vibration. The modal parameters were successfully figured out from the ambient vibration using an output-only modal identification method (Hermans and Van Der Auweraer 1999).

The initial FE model was updated using the identified modal properties. The objective function in Eq. (12) was minimized using the downhill simplex method. After the sensitivity analysis, 31 structural parameters were selected for the model updating: spring stiffnesses for both supports (2), Young's modulus of the concrete slab (1), second moment of inertias and torsional coefficients for five girders (10) and nine floor beams (18). Since the downhill simplex method is significantly affected by the complexity of the objective function space and initial starting point (i.e., initial FE model), the FE model updating was processed in two steps. In the first instance, 31 updating parameters were grouped into nine groups that were updated first to find better starting point of updating. Then the 31 parameters were updated using the updated FE model in the first step using nine grouped parameters.

After updating the FE model, the natural frequencies of the initial FE model and updated FE model were compared to the measured ones, which showed that the natural frequencies of the updated FE model were closer to the measured values than those of the initial FE model. The updated FE model was validated using the deflections measured in the loading test. The weight profile of the loaded trucks in the loading test was applied to the updated FE model, and the deflection simulated from the updated FE model was found to have error less than 5% compared to the measured deflection in the loading tests.

4.3. Statistical Parameters

To evaluate the fatigue life of a bridge, in this research, the fatigue load was modeled by using the vehicle load model (DL-24) of the Korea Highway Bridge Design Specification, KHBDS (Ministry of Construction and Transportation 2005), as shown in Fig. 2.

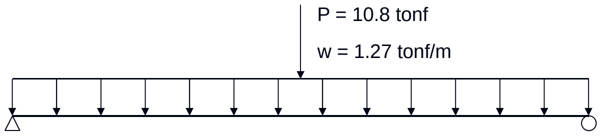


Figure 2: Vehicle load model (DL-24) of KHBDS.

Using this load model, an FE analysis was performed for the initial and updated FE models to determine the maximum stresses in the five girders. Because the stresses are obtained using static analyses, they are multiplied by an impact factor I to account for the dynamic effect of the vehicle loads.

$$I = \frac{15}{40 + L} \leq 0.3 \qquad (13)$$

where L is the length of the span of the bridge, in meters. In the FE models for SAP2000, the span length is assumed to be 38.8 (m). Thus, the impact factor is 1.19.

From the results of the FE analyses using the initial and updated FE models, the maximum stresses in the five girders are obtained. From the results obtained with the initial model, the maximum stresses in Girders 1–5 are determined to be 15.28, 17.68, 16.76, 17.68, and 15.28 (MPa), respectively. In addition, the maximum stresses obtained with the updated FE model for Girders 1–5 are 13.77, 15.46, 14.91, 15.49, and 13.66, respectively. The stress values obtained with the updated model are smaller than those obtained with the initial model. Note that the stress values are symmetrical about the center of Girder 3 in the initial FE model, reflecting its symmetry, but are slightly asymmetrical in the updated model. This is because the structural parameters change during the FE model updating procedure. It is also seen that the stress values from the updated FE model are relatively small compared with those from the initial model. This is because the SHM test was conducted only 2–4 years (in 2004–2006) after the bridge construction (in 2002), when the bridge would be expected to still be in good condition.

4.4. Random Variables and Deterministic Parameters

In this example, the uncertainties of C in the Paris equation and the initial crack length a_0 are considered as being random variables with mean values of 2.18×10^{-13} (mm/cycle/(MPa·mm)^m) and 0.1 (mm), and coefficients of variation (COVs) of 0.2 and 0.1, respectively. The uncertainties in the stresses are also introduced using a load scale factor S, for which the mean and COV are assumed to be 1.0 and 0.1, respectively. It is assumed that the initial crack length a_0 follows an exponential distribution, whereas the others conform to a lognormal distribution.

All of the random variables are assumed to be statistically independent of each other, except in the following cases: (1) between the Paris equation parameter C for the five girders (correlation coefficient: 0.6); and (2) between the initial crack lengths (a_0) of the five girders (correlation coefficient: 0.6). These 0.6 correlations indicate that Girders 1–5 were manufactured by the same manufacturer, and thus their material properties are closely correlated. For more accurate results, however, the correlation values should be obtained from actual tests.

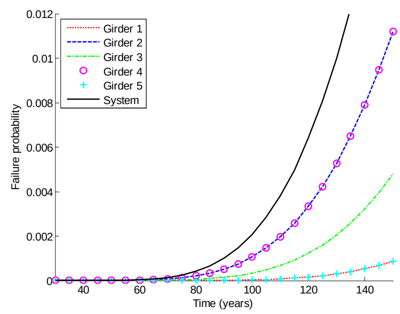
In addition, the following deterministic parameters were used: half flange width (W): 650 (mm), flange thickness (f_{th}): 30 (mm), critical crack length (a_c): 30 (mm), average daily truck traffic (ADTT): 5,351/day, duration of SHM test (T^1_{up}): 4 years. The ADTT value is determined from the actual passing truck data provided by the Korea Expressway Corporation. For the geometry function $Y(a)$ in Eq. (2), the following function for the I-beams was introduced from Wang *et al*. (2006):

$$Y = \frac{1.0 - 0.5\left(\frac{a}{W}\right) + 0.37\left(\frac{a}{W}\right)^2 - 0.044\left(\frac{a}{W}\right)^3}{\sqrt{1 - \frac{a}{W}}} \quad (14)$$

4.5. Analysis Results

First, Fig. 3 show the results obtained from the fatigue life assessment using the initial and updated FE models for Girders 1–5 and the bridge system. As previously mentioned, a system failure event is assumed to have occurred when any of the five girders fails. Obviously, the fatigue failure probability of the bridge increases as its number of years in service increases, and the failure probability of the bridge system is larger than those of the individual girders because of the event definition. It can also be seen that the failure probabilities obtained from the updated FE model are smaller than those obtained from the initial model, at both the component and system levels. This is because the stress values are relatively small for the updated FE model, as described in Section 4.3.

For verification purposes, a Monte Carlo simulation (MCS) was performed using 10^6 samples. Unlike the proposed method employing FORM and SORM, the MCS imposes a limit on the level of probability calculation. Therefore, the MCS is conducted only for those cases in which the probability is larger than 5×10^{-6}. Fig. 4 shows the reliability indices for different service times, as obtained from the proposed method and MCS, for the five girders and the bridge system.



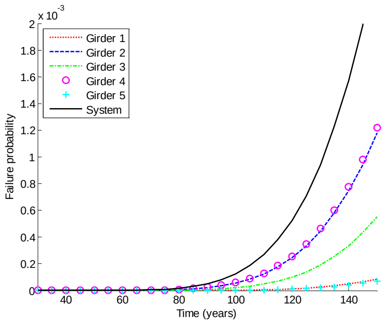
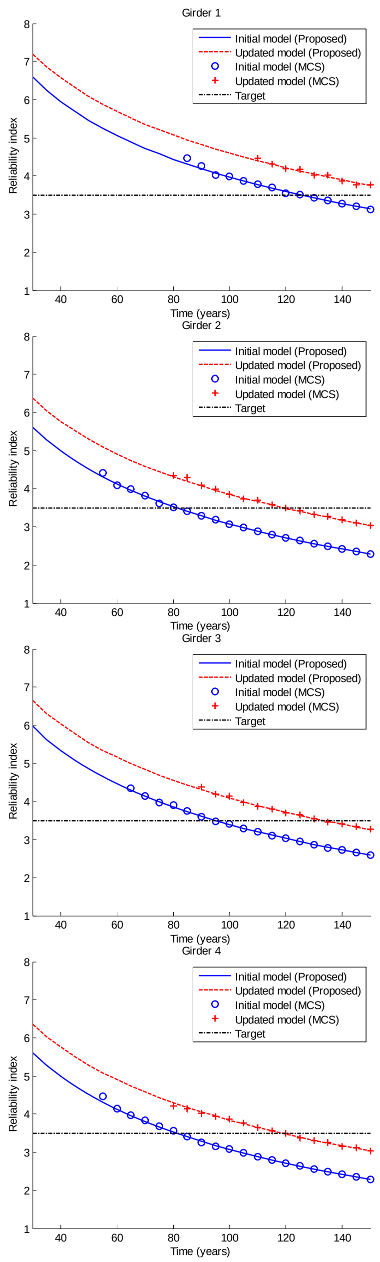


Figure 3: Failure probabilities using the initial (top) and updated (bottom) FE models.



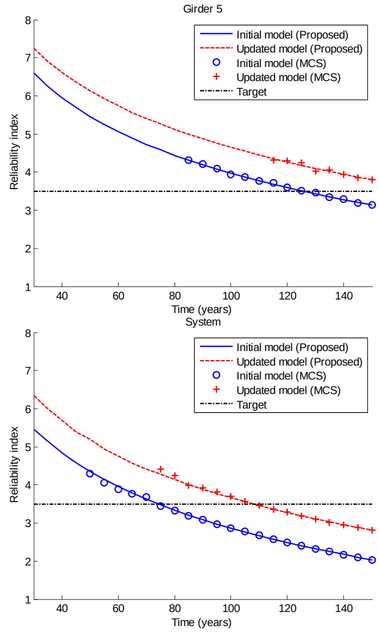


Figure 4: Reliability indices for Girders 1-5 using the initial and updated FE models from the proposed method and MCS.

It can be seen that the results obtained using the proposed method match those determined by the MCS. In addition, this figure shows that the level of the reliability indices is higher for the updated FE models. The American Association of State Highway and Transportation Officials (AASHTO) recommends, in the AASHTO Bridge Design Code, a target reliability index of 3.5 (i.e., a failure probability of 2.33×10^{-4}) with a service life of 75 years for the steel and prestressed concrete components. The fatigue lives of Girders 1–5 and the bridge system were estimated using the target reliability index (i.e., the black lines in Fig. 7). When using the updated FE model, the fatigue lives of the girders and bridge system are found to be longer, and all of them satisfy the AASHTO requirements, with fatigue lives in excess of 75 years.

5. CONCLUSIONS

In this paper, a new approach is proposed for predicting the probabilistic fatigue life of a bridge using an FE model updating method based on SHM data. The proposed method consists of three parts: (1) identifying the modal parameters of a bridge such as the mode shapes and natural frequencies based on the ambient vibrations caused by passing vehicles; (2) updating the structural parameters of an initial FE model using the identified modal parameters; and (3) predicting the probabilistic fatigue life by employing the updated FE model. After the initial FE model of a bridge is constructed based on its design drawings, the optimal values of the structural parameters, which minimize the differences in the natural frequencies from the SHM data and the FE model, were identified using a downhill simplex algorithm to update the FE model. In addition, new limit-state formulations were derived to express the crack failure and predict the probabilistic fatigue life with updated stress values. These formulations allowed us to evaluate the fatigue life of a bridge with multiple SHM and FE model updating. The proposed method was applied to a numerical example of the Samseung Bridge, for which FE model updating has been addressed in a previous study. As a result, the fatigue failure probabilities of the five girders and bridge system, as determined from the updated FE model, were found to be smaller than those obtained from the initial FE model, because the stress levels obtained with the updated model proved to be relatively low. As a result, the fatigue life values for the girders and bridge system were estimated as being longer, which indicates that the bridge is still in good condition. Furthermore, the analysis results using the proposed method were compared with those obtained using MCS, and the results were found to be in good agreement. In conclusion, it was shown that the proposed method enables us to predict the probabilistic fatigue life of a bridge from its current condition by using the SHM data and the corresponding FE model updating.

6. ACKNOWLEDGMENT

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