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Serial Correlation of Withdrawal Properties from Axially-Loaded Self-Tapping Screws

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ABSTRACT: Previous investigations outline the applicability of a two-level hierarchical stochastic material model combined with equicorrelation for the description of timber strength and elasticity, by explicit differentiation in variation within and between timber elements. Consequently, as far as withdrawal of primary axially-loaded self-tapping screws is concerned, the load bearing capacity of screw groups in laminated timber products depends on their positioning relative to the product layup. We analyse the first time the applicability of a two-level hierarchical model on withdrawal strength, stiffness properties and density. By testing a saturated data set, the hypothesis of equicorrelated withdrawal properties could not be rejected. Test setup, examination and accompanied epistemic uncertainties in analysing the stiffness properties are seen as general reason for their relatively high variation and consequently low correlation, whereas the high equicorrelation of withdrawal strength is explained by the homogeneous test material. However, in reality screw groups are influenced by unavoidable flaws which provoke higher variation and lower correlation. In view of previous investigations on timber strengths, an equicorrelation for withdrawal strength in the range of 0.40 to 0.50 (0.60) appears more reasonable.

1. INTRODUCTION

Timber is a hierarchical material, structured in five scales (Speck and Rowe 2006). On each scale, properties' variation, e.g. of strength, depends on variations in dimension, distribution and interaction of flaws which are representative for each scale. Previous investigations (e.g. Isaksson 1999, Köhler 2007, Brandner 2013) show that aleatory (natural) variation within scales can be classified in variation within and between individuals, samples or even between different material batches or proveniences.

We aim on the scale of timber, with dimensions 10 ± 2 m, as basis for laminated engineered timber products. With focus on timber engineering, within the last few decades two important developments have been achieved: one by establishing unidirectional and orthogonal layered linear and laminar engineered timber products, e.g. duo- and trio-beams, glued laminated timber (glulam; GLT), cross laminated timber (CLT) and laminated veneer lumber (LVL). The second development addresses innovations in connection technique, which allow almost utilizing the high load bearing and stiffness capacity of engineered timber products also at joints between structural components; this by recent achievements in bonding technology and in regard to self-tapping screws. Both connection techniques provide high utilization degrees even if the stresses are transferred in grain direction, the main direction of timber, and are additionally useable for reinforcing timber's weak material properties, e.g. stresses perpendicular to grain and shear.

We concentrate on partly- or fully-threaded self-tapping screws inserted in laminated timber products, e.g. glulam. These screws are optimized for primary axial loading. Joints of up to a few hundreds screws show an enormous load-bearing and stiffness potential with possible failure scenarios: (i) steel failure, (ii) head-pull through, (iii) withdrawal failure, and (iv) block shear failure of a group of screws. Although such joints are often designed to fail by steel fracture, the resistance against withdrawal has to be considered as well. With focus on (iii), the withdrawal strength f_{ax} based on tests and the characteristic withdrawal strength $f_{ax,k}$ according to EN 1995-1-1 (2006), respectively, are given as

$$f_{ax} = \frac{F_{ax,max}}{d\,\pi\,l_{ef}}\,;\;\; f_{ax,k} = \frac{0.52\,d^{-0.5}\,l_{ef}^{-0.1}\,\rho_k^{0.8}}{\pi} \qquad (1)$$

with $F_{ax,max}$ as the maximum withdrawal resistance, d and l_{ef} as nominal diameter and effective penetration length of the screw, respect-tively, $\pi = 3.41\ldots$ and ρ_k as the characteristic density of the timber product. Thus, density is the only timber property indicating the withdrawal resistance of screws in timber. Due to the high stiffness combined with limited plastic deformation till the ultimate load, load sharing between common acting screws in a group is limited. Current design procedures take this into account by a reduced chargeable number of screws in calculating the group resistance, e.g. by $n_{ef} = n^{0.9}$ according to EN 1995-1-1 (2006), with n_{ef} and n as chargeable and common acting number of screws, respectively, in a joint.

In contrast to the use of screws in timber elements, in laminated timber products the withdrawal capacity of a single screw depends on the number and properties of the penetrated elements composing the timber product (Brandner 2013, Ringhofer et al. 2014a). Furthermore, the interaction of screws in a group inserted in such timber products depends additionally on the positioning of the group relative to the layup of the laminated timber product, see Figure 1. However, all these influences are currently not considered in the design process.

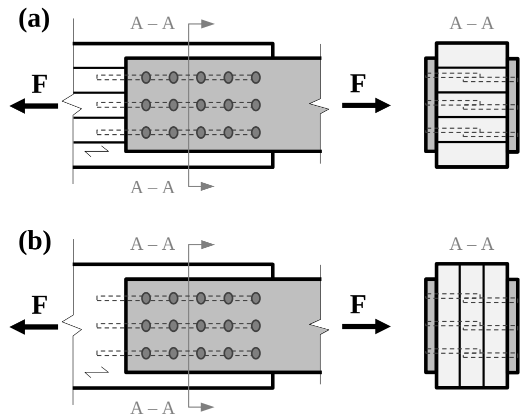

Figure 1: Positioning of screws in a butt joint with outer steel plates relative to the layup of a laminated timber product: (a) parallel, (b) perpendicular to lamination's face

In view of these circumstances, analysing the allocation of the variation of withdrawal properties to variation within and between timber elements appears worthwhile. In fact, the magnitude of stochastic system action or homogenization, as consequence of the reduced variation in cases where more than one element are penetrated by a single screw as well as by the group action, is directly a function of the property's variation and of the correlation of properties between sub-elements.

For the description of the spatial variation of properties within and between timber elements, recent and earlier results on timber, e.g. from Riberholt and Madsen (1979), Taylor (1988), Williamson (1992), Lam et al. (1994), Källsner et al. (1997), Isaksson (1999), Köhler (2007) and Brandner (2013), demonstrate the applicability of a hierarchical model combined with equicor-relation rather than a serial correlation with Markov property. In probability theory, equicorrelation constitutes the simple stochastic case where all coefficients in the correlation matrix $\boldsymbol{\rho}$ of size $M \times M$ of M variables in a set are equal, with $\rho_{ij} = \rho_{\text{equi}}$, for $i, j = 1, \ldots, M$, $\forall\, i \neq j$, and ρ_{ij} and ρ_{equi} as pairwise correlation and equicorrelation coefficient, respectively.

We focus further on the simplest case of a two-level hierarchical model and define

$$Z_{ik} = Y_k + X_i | k \,, \tag{2}$$

with $i = 1, \ldots, M$, $k = 1, \ldots, N$, M as the number of random variables (e.g. number of segments per board), N as the number of elements or realizations per variable (e.g. number of boards), and with Y_k as mean value of the kth element, $X_i \,|\, k$ as ith independent and identically distributed (iid) deviation of sub-element i from Y_k given element k, with expectation $\mu_X = \mathrm{E}[X_i \,|\, k] = 0$ and variance $\mathrm{Var}[X_i \,|\, k] = \sigma_X^2$, and covariance $\mathrm{CoVar}[Y_k + X_i \,|\, k, Y_k + X_l \,|\, k] = \mathrm{Var}[Y_k] = \sigma_Y^2$, expectation $\mathrm{E}[Z_{ik}] = \mathrm{E}[Y_k] = \mu_Y$, variance $\mathrm{Var}[Z_{ik}] = \sigma_X^2 + \sigma_Y^2$ and with

$$\rho_{\text{equi}} = \frac{\sigma_Y^2}{\sigma_X^2 + \sigma_Y^2} \,. \tag{3}$$

In fact, this hierarchical model, introduced by Källsner and Ditlevsen (1994), Källsner et al. (1997) and Ditlevsen and Källsner (1998, 2005), can be directly inferred from the hierarchical material structure of wood.

The focus is on the correlation between segments (sub-elements) within one element. This information constitutes the basis for a more realistic consideration of the variation in withdrawal properties and serves as input parameter for capacity modelling of common acting groups of screws, by taking into account their positioning relative to the layup of laminated timber products.

2. MATERIAL AND METHODS

2.1. Withdrawal tests

Withdrawal of self-tapping screws was tested on 20 boards of Norway spruce (*Picea abies*), conditioned to $u = 12$ % moisture content, with final (planed) dimensions $w \times t \times l = 120 \times 40 \times 3{,}000$ mm³, with w, t, and l as width, thickness and length, respectively. These boards were taken from an ungraded batch of 900, at reference moisture content $u_{\text{ref}} = 12$ % with average batch density $\rho_{\text{batch,12,mean}} = 461$ kg/m³ and coefficient of variation $\mathrm{CV}[\rho_{\text{batch,12}}] = 8.3$ %. Subsequent grading limited the density of boards to 440 to 500 kg/m³, with $\rho_{\text{board,12,mean}} = 468$ kg/m³ and $\mathrm{CV}[\rho_{\text{board,12}}] = 4.1$ %. The unusual low variation consequences also from further reject criteria: compression wood, bark and cracks.

Before testing, each board was divided in 19 to 23 segments with and without knots, in total 421. Due to the fact that withdrawal tests shall only be conducted without penetrating knots, this amount reduced to nine to 14 segments per board, in total 248. Consequently, the distance between the segments' centres is not constant but their order is known. This approach appears adequate as long as the hypothesis of equi-correlation can be confirmed which corresponds to a serial correlation between segments' properties within elements which is independent of their in-between distance, also known as lag-distance.

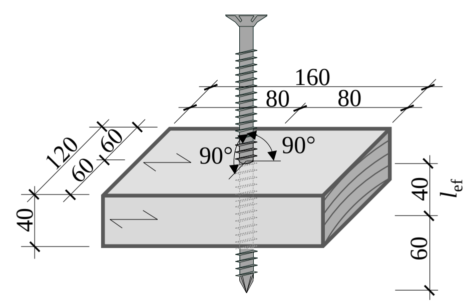

Figure 2: Specimen's dimensions and screw positioning

The withdrawal tests were conducted according to EN 1382 (1999) by using fully-threaded self-tapping screws (ETA-12-0373 2012) of nominal diameter $d = 8$ mm. All fasteners were screwed through the segments in radial direction of the timber (thread-fibre angle $\alpha = 90°$), eliminating any influences of the screw tip, without pre-drilling and with anchoring length $l_{\text{ef}} = 40$ mm, see Figure 2. Testing was done by using a push-pull configuration and a pre-load of 150 N. After testing, specimens with

$w \times t \times l = 40^3$ mm³ for determination of the local density and moisture content according to EN 13183-1 (2002) were taken centrically to the screw hole. More details concerning the tests can be found in Bratulic (2012).

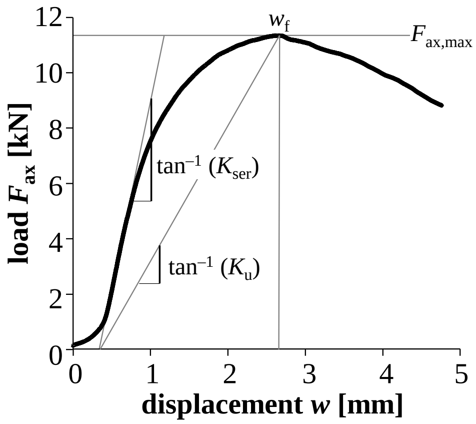

Figure 3: Typical load-displacement curve of an axially loaded self-tapping screw: denomination of $F_{ax,max}$, w_f, K_{ser} *and* K_u

The definitions of $F_{ax,max}$, K_{ser} and K_u, corresponding to the maximum withdrawal force, the slip modulus and the instantaneous slip modulus for ultimate limit states, respectively, as used in further data processing, are illustrated in Figure 3. The displacement was measured only globally, thus influences from the testing device and test setup are included in K_{ser} and K_u leading to an underestimation. These properties can consequently only considered as "apparent". However, focusing on the inference of the spatial correlation correct absolute values are not required. By considering the additional displacements in steel as approximately deterministic the realizations of K_{ser} and K_u are still usable.

With timber as hygroscopic material, deviations from $u_{ref} = 12$ % conditions corrections in the physical properties: 0.5 % for density (EN 384 2010), 3 % for f_{ax} (Ringhofer et al. 2014b) and 2 % for K_{ser} and K_u per 1 % Δu. Correction factors for the mechanical properties are motivated by considering that withdrawal primary causes shear stresses and failure in timber.

2.2. Statistical methods

Statistical analysis and inference was primary performed in R (R Core Team 2012). After calculation of basic statistics, the focus was on testing the pairwise correlations ρ_{ij} between segments within elements, with

$$Z_{ik} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1M} \\ Z_{21} & Z_{22} & \cdots & Z_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NM} \end{bmatrix}, \tag{4}$$

given as

$$\rho_{ij} = \begin{bmatrix} 1 & \cdots & \rho(Z_{k1};Z_{kM}) \\ \rho(Z_{k1};Z_{k2}) & \cdots & \rho(Z_{k2};Z_{kM}) \\ \vdots & \ddots & \vdots \\ \rho(Z_{k1};Z_{kM}) & \cdots & 1 \end{bmatrix}. \tag{5}$$

This was done for all five properties: the density of the tested segments $\rho_{seg,12}$, the local density around the screw hole $\rho_{loc,12}$, and the withdrawal properties $f_{ax,12}$, $K_{ser,12}$ and $K_{u,12}$.

A statistic for testing the hypothesis H0: $\rho_{ij} = \rho = \rho_{equi}$ vs. H1: $\rho_{ij} \neq \rho_{equi}$, for $\forall\, i, j = 1, 2, \ldots, M$ and $i \neq j$, is given as (Lawley 1963)

$$T_N = \frac{N-1}{\left(1-\bar{R}\right)^2}\left[\sum\sum_{i<j}\left(R_{ij}-\bar{R}\right)^2 - A\sum_{j=1}^{M}\left(R_j-\bar{R}\right)^2\right], \tag{6}$$

with

$$R_j = \frac{1}{M-1}\sum_{i\neq j}^{M} R_{ij}\ ,\ \bar{R} = \frac{2}{M(M-1)}\sum\sum_{i<j} R_{ij} \tag{7}$$

and

$$A = \frac{(M-1)^2\ \bar{R}\left(2-\bar{R}\right)}{\left[M-(M-2)\left(1-\bar{R}\right)^2\right]}, \tag{8}$$

which follows asymptotically a χ^2 distribution

$$T_N \overset{\text{asym.}}{\sim} \chi^2_{f,\alpha}\text{, with } f = \frac{1}{2}(M+1)(M-2) \tag{9}$$

degrees of freedom. Hereby M variables are assumed to follow a multivariate normal distribution. Estimates R_{ij} confirm to the Pearson's sample correlation coefficients between properties of segments at varying lag-distances. The null hypothesis has to be rejected whenever T_N exceeds the $(1 - \alpha)$-quantile of the χ^2 distribution. The best estimator for the equi-correlation coefficient ρ_{equi} is represented by the weighted averaged correlation coefficient $\bar{R}$ gained from the $M \times M$ correlation matrix $\boldsymbol{R} = R_{ij}$ (N realizations of M variables). We use Lawley's test because of three reasons: (i) a likelihood ratio test for equicorrelation is not available in closed form (Lawley 1963), (ii) the asymptotic null distribution of T_N is independent from ρ_{equi} (Gleser 1968), and (iii) the test procedure can be easily implemented even in standard software packages like Microsoft® Excel.

3. RESULTS AND DISCUSSION

3.1. Basic statistics and outcomes

Table 1 shows basic statistics of withdrawal properties and local densities. Nine data sets were excluded; one because of a knot touching the screw and eight because of statistically outliers, classified via box plots according to Tukey (1977). Analysis was performed on the untransformed and logarithmized data set; reasoning therefore is indicated later in section 3.2.

Table 1: Basic statistics of withdrawal properties ($K_{\text{ser},12}$ & $K_{\text{u},12}$ as apparent values)

	$\rho_{\text{seg},12}$ [kg/m³]	$\rho_{\text{loc},12}$	$f_{\text{ax},12}$ [MPa]	$K_{\text{ser},12}$ [N/mm]	$K_{\text{u},12}$
no. [–]	239				
mean	456	430	5.61	5,074	3,074
CV [%]	5.0	5.0	7.2	9.0	15.5

In contrast to previous investigations which outline CV[f_{ax}] between 10 and 20 % (Ringhofer et al. 2014a), in our data a much lower variation is found. Also contrary to other timber properties the variations in stiffness properties K_{ser} and K_{u} are higher than in the corresponding strength. Reasoning is seen in the examination procedure and by additional displacements caused by the test setup. Although not provable from this data set it is assumed that a relevant amount of variation in K_{ser} and K_{u} is epistemic and not aleatoric. Overall, the low variations identified for all five properties underline the homogeneity of the test material. This is in particular related to the use of clear wood samples, a circumstance which necessitates further discussions on the practical applicability of the outcome.

3.2. Tests on equicorrelation

Lawley's (1963) test presumes multivariate normal variables. Although density is frequently described as normal distributed, the same model can be physically hardly argued for strength and stiffness properties. Kowalski (1972) outlines the vulnerability of the distribution of equicorrelation on nonnormality for all $\rho_{\text{equi}} \neq 0$. JCSS (2006), Brandner (2013) and others confirm a lognormal distribution (2pLND) as representative for strengths, elastic properties and even for density. In fact, the lognormal distribution can be directly inferred from multiplicative processes (Gibrat 1930), or reverse, according to Brandner (2013) from subsequent (cascade) fracture processes which are typical for hierarchically organised materials.

With the definition of $U \sim$ 2pLND, $V = \ln(U)$ is normal distributed. Consequently, Lawley's test can be applied after logarithmic transformation of presumed lognormal variables. The coefficients $R_{\text{equi},V}$ can be re-transferred to $R_{\text{equi},U} = R_{\text{equi}}$ by (Law and Kelton 2000)

$$\rho_{\text{equi},U} = \frac{\exp\left(\rho_{\text{equi},V}\ \text{Var}[V]\right) - 1}{\exp\left(\text{Var}[V]\right) - 1}. \tag{10}$$

Setting the significance level α to 5 %, by testing the complete set of 239 segments (nine to 14 per board), for all five variables ($\rho_{\text{seg},12}$, $\rho_{\text{loc},12}$, $f_{\text{ax},12}$, $K_{\text{ser},12}$, $K_{\text{u},12}$) the hypothesis of equicorrelation (see section 2.2) was rejected at a realized significance of $p < 0.001$ ($p < \alpha$). Reasoning is the considerable decrease of the number of realizations for $m > 9$. As the number of segments per board is not in relation to the timber quality and in particular to the clear wood tested, Lawley's test and the statistical analysis were repeated on a reduced, saturated data set which contains only the realizations of the first nine segments per board. The basic statistics, average coefficients of variations within boards and the estimated equicorrelations are shown in Table 2.

Table 2: Basic statistics of withdrawal properties ($K_{ser,12}$ & $K_{u,12}$ as apparent values) and estimated equicorrelations (reduced data set)

	$\rho_{seg,12}$	$\rho_{loc,12}$	$f_{ax,12}$	$K_{ser,12}$	$K_{u,12}$
	[kg/m³]		[MPa]	[N/mm]	
no. [–]	180				
mean	457	430	5.62	5,055	3,048
CV[Z_{ik}] [%]	4.8	4.8	7.2	9.3	15.6
CV[$X_i \mid k$] [%]	2.2	1.9	4.1	7.6	13.5
$r_{equi,LN}$ [–]	0.80 $p<0.001$	0.85 $p=0.001$	0.71 $p=0.559$	0.35 $p=0.166$	0.26 $p=0.951$
r_{equi} [–]	0.80	0.85	0.71	0.34	0.25

The comparison between the basic statistics of Table 1 and 2 indicates the reduced data set as equally representative. Values of CV[$X_i \mid k$] already imply the proportions of variation within and between boards and corresponding equicorrelation coefficients.

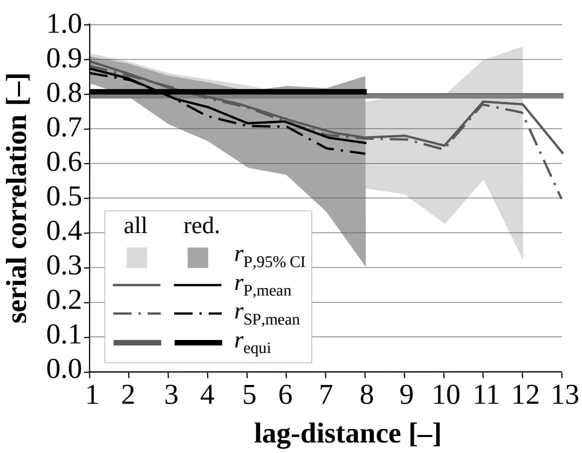

Figure 4: Serial (pairwise) correlation of $\rho_{seg,12}$

Figures 4 to 8 visualize pairwise correlations in dependency of the lag-distance for the complete and the reduced, untransformed data sets. Both, Pearson's (P) correlation coefficient (together with a 95 % confidence interval; CI) and Spearman's (SP) rank correlation coefficient are included for comparison. The increasing bandwidths in the confidence interval with increasing lag-distance consequence from the decreasing amount of data pairs.

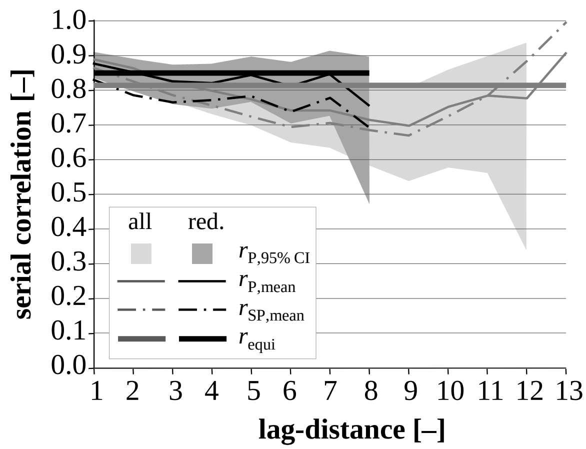

Figure 5: Serial (pairwise) correlation of $\rho_{loc,12}$

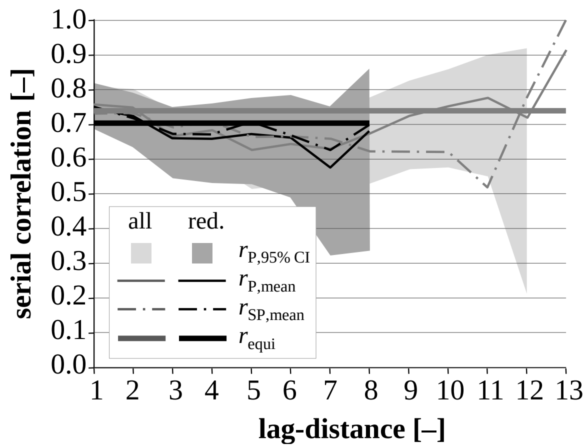

Figure 6: Serial (pairwise) correlation of $f_{ax,12}$

The equicorrelation of density agrees well with results in Brandner and Schickhofer (2014). Consequently, the variation of (clear wood) densities within boards is widely negligible. However, the null hypothesis was rejected with $p \leq 0.001$.

In fact, equicorrelation within timber elements, which is motivated by the hierarchical material structure, is especially obvious by observing the regular pattern of knot clusters of a standing coniferous tree in combination with the radial differentiation in juvenile and adult timber zones. However, in sawn timber this and the formation of different knot zones in radial and longitudinal direction, as consequence of changing environmental conditions and needs of the living tree, together with the cutting process, which is usually oriented parallel to the pith, impose gradually changes in longitudinal properties' profiles. These circumstances may to some extend explain the decreasing pairwise correlations in densities (Figure 4 and 5) and other timber properties with increasing lag-distance.

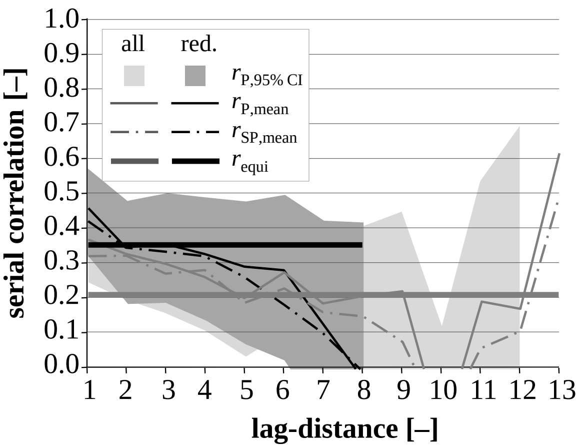

Figure 7: Serial (pairwise) correlation of $K_{ser,12}$

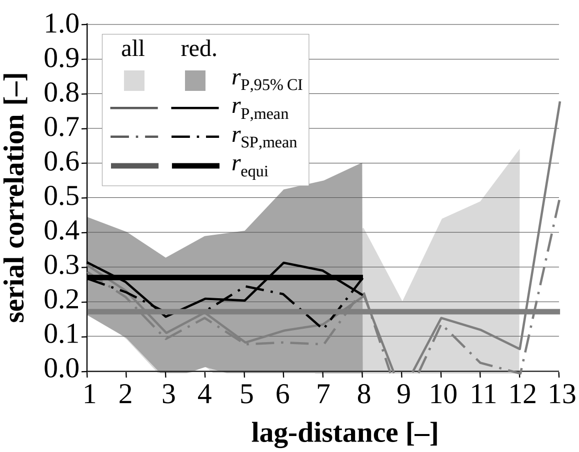

Figure 8: Serial (pairwise) correlation of $K_{u,12}$

In contrast to density, the hypothesis of equicorrelation could not be rejected for all tested withdrawal properties, $f_{ax,12}$, $K_{ser,12}$ and $K_{u,12}$. Figures 6 to 8 visualise the corresponding serial pairwise correlations.

Whereas the results for $f_{ax,12}$ appear quite homogenous, the serial pairwise correlations of the stiffness properties are very heterogeneous and in case of $K_{ser,12}$ even with a distinctive decreasing tendency. Reasons for the heterogeneous results and the minor equicorrelation of the stiffness properties are again supposed to be related to the examination procedure, whereas $r_{equi}\,[K_{u,12}] < r_{equi}\,[K_{ser,12}]$ consequences from additional variation caused by variation of the displacement at ultimate load.

Brandner (2013) summarized previous investigations on equicorrelation of timber properties and concluded $\rho_{equi}[E] = 0.50$ to 0.60 and $\rho_{equi}[f] = 0.40$ to 0.50 for elastic moduli and strengths, respectively, with higher values for timber of higher quality and grade. In comparison to these ranges the coefficients for $K_{ser,12}$ and $K_{u,12}$ appear rather low whereas $r_{equi}[f_{ax,12}] = 0.71$ appears rather high. By considering that $f_{ax,12}$ bases on testing of already very homogenous clear wood samples the outcome can be judged as reasonable.

However, in timber engineering placing fasteners near or through knots and other growth characteristics, like checks, reaction wood, bark inclusions and resin pockets, cannot be prevented. The additional variation caused by unavoidable timber properties directly influences the variation of withdrawal properties and thus their spatial variation within timber elements.

4. SUMMARY AND CONCLUSIONS

We investigated the serial correlation of withdrawal properties and densities based on a data set of Bratulic (2012). Motivated by the hierarchical structure of timber and successful previous investigations, we analysed the applicability of a two-level hierarchical model combined with equicorrelation. Using the test statistic of Lawley (1963) the hypothesis of equicorrelated properties could not be rejected for $f_{ax,12}$, $K_{ser,12}$ and $K_{u,12}$. However, equicorrelation could not be confirmed for the densities of segments and small clear wood samples. In view of the stochastic modelling of the group action of common acting and primarily axially loaded self-tapping screws in dependency of the insertion of fasteners relative to the layup of laminated timber products and the failure mode "withdrawal", for general applications in timber and unavoidable influences of flaws, e.g. knots, checks and reaction wood, $\rho_{equi}[f_{ax,12}] = 0.71$ is judged as too high. In view of previous investigations on equicorrelation of timber strength properties (e.g. Brandner 2013) $\rho_{equi}[f_{ax,12}]$ in the range of 0.40 to 0.50 (0.60) appears more adequate; modelling the influence of equicorrelation on the group action is envisaged.

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