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Bayesian Assessment of the Compressive Strength of Structural Masonry

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ABSTRACT: The application of hierarchical models for assessing the compressive strength of structural masonry is investigated. Based on current codified models the distribution of compressive strengths within an ensemble of masonry wall specimens is related to the statistical properties of the populations of brick units and mortar used. The parameters of this relation are calibrated with test data acquired at ETH Zürich. This approach allows for heterogeneous material modeling, consistent uncertainty management and optimal information processing. Costly compression tests of full-size masonry and inexpensive tests of brick and mortar samples are jointly utilized for learning about the masonry wall characteristics.

1. INTRODUCTION

Structural masonry is a composite material that consists of brick units and mortar. The mechanical key characteristic of masonry is the compressive strength perpendicular to the bed joints. Estimating or predicting this material property are thus issues of central importance to assessing the reliability of masonry structures. These problems are therefore addressed in current standards (EN 1996-1-1, 2005; JCSS, 2001) and numerous enhancements (Dymiotis and Gutlederer, 2002; Glowienka and Graubner, 2006; Schueremans and Van Gemert, 2006; Mojsilovic and Faber, 2009; Sýkora and Holický, 2010; Garzón-Roca et al., 2013; Sykora and Holicky, 2014; Sykora et al., 2014).

The motivation of this research study is twofold. Firstly, we observe a systematic discrepancy between measured data and predictions of the masonry compressive strength according to EN 1996-1-1 (2005). This suggests a recalibration of the model code parameters. Secondly, it is noticed that current approaches either suffer from their semi-probabilistic character or their unsatisfactory treatment of the emerging uncertainties. Thus the goal of this paper to develop a fully probabilistic extension of current codified models for assessing the compressive strength of unreinforced masonry. We will rely on hierarchical models (Nagel and Sudret, 2015, 2014) and Bayesian networks (Sankararaman et al., 2012; Urbina et al., 2012). This approach will allow for heterogeneous modeling of structural masonry, quantification of various types of uncertainty and acquisition of information from diverse sources.

More specifically it is aimed at analyzing the compressive strength of structural masonry with system-level data, i.e. measurements that are taken from full-scale masonry specimens, component-level data, i.e. results from testing brick units and mortar samples individually, and prior or expert knowledge. Compression tests of masonry specimens are rather costly, whereas data associated to component-specific material characteristics are relatively inexpensive to acquire. Hierarchical models enable the joint processing of information from different levels of the overall system. This way the information is optimally utilized. Moreover a predictive relationship is established that connects the masonry compressive strength with the component-level compressive strengths.

The remainder of this document is organized as follows. Previous approaches of assessing the compressive strength of structural masonry will be reviewed in Section 2. Hierarchical models will be introduced in Section 3. In Section 4 the acquired data will be discussed and Section 5 will show the results of Bayesian updating. Lastly we will summarize and conclude in Section 6.

2. CURRENT MODELS

In EN 1996-1-1 (2005) it is tried to relate the compressive strength of masonry to the resistances of its brick and mortar components. The relationship is realized as a power function

$$f_w = k' f_b^{\alpha'} f_m^{\beta'}. \tag{1}$$

On the one hand, the compressive strength of masonry is summarized by the characteristic value f_w, i.e. a 5%-quantile. On the other hand, f_b denotes the normalized mean compressive strength of the units and f_m denotes the mean compressive strength of the mortar. Estimates of the constants (k', α', β') are given for different types of masonry. In JCSS (2001) the empirical relation Eq. (1) is interpreted similarly. Here f_w, f_b and f_m represent the mean values of the corresponding distributions. Different prior estimates of the coefficients (k', α', β') are provided. The coefficients are often set so that they (approximately) satisfy $\alpha' + \beta' = 1$. This choice can be justified for reasons of the physical dimension in Eq. (1).

Ensuing from these semi-probabilistic models, a variety of extensions have been proposed in the literature. There are probabilistic reinterpretations of Eq. (1) based on lognormal distributions (Schueremans and Van Gemert, 2006; Sýkora and Holický, 2010; Sykora and Holicky, 2014; Sykora et al., 2014). In other studies the model uncertainty of Eq. (1) is quantified (Dymiotis and Gutlederer, 2002; Glowienka and Graubner, 2006). A conjugate Bayesian updating approach based on Gaussian distributions is presented in Mojsilovic and Faber (2009). Another idea is to establish a connection between the compressive strengths of masonry and its components via artificial neural networks (Garzón-Roca et al., 2013).

These previous approaches suffer from the fact that they either do not clearly distinguish between epistemic and aleatory shares of uncertainty or they neglect material heterogeneity. Fitting the parameters of a probabilistic extension of Eq. (1) is a problem that has hardly been satisfactorily solved as yet.

3. HIERARCHICAL MODELS

In the following hierarchical Bayesian modeling is introduced as a tool for distinguishing and handling uncertainty in codified models of the form Eq. (1). The aim of this section is to establish a Bayesian model and updating strategy for the following experimental situation. The compressive strength is measured for a number of clay block masonry specimens. Specimens can be grouped according to the ensembles of brick units and mortar that were used for their construction. Here ensembles of clay bricks are characterized by the same ingredients used and the same manufacturing procedure. Similarly in every ensemble of mortar samples identical constituents were used for mixing. In this modeling approach material heterogeneity is accounted for by distinguishing between brick and mortar samples used in constructing the masonry wall systems. The final goal is the assessment and prediction of the compressive capacity of structural masonry by utilizing system- and component level information.

3.1. *Aleatory Model*

Within an ensemble of masonry wall specimens, the compressive strength of the masonry wall is represented as a random variable

$$F_w = k F_b^{\alpha} F_m^{\beta}. \tag{2}$$

This is a probabilistic extension of the codified model in Eq. (1). We remark that the coefficients (k, α, β) of the relation Eq. (2) are not immediately identified with the ones of Eq. (1).

The compressive strengths of the bricks and the mortar are modeled as lognormal random variables $F_b \sim \mathscr{LN}(f_b|\mu_b, \sigma_b^2)$ and $F_m \sim \mathscr{LN}(f_m|\mu_m, \sigma_m^2)$. Their distributions are determined by hyperparameters $\boldsymbol{\theta}_b = (\mu_b, \sigma_b)$ and $\boldsymbol{\theta}_m = (\mu_m, \sigma_m)$ that are the mean and standard deviation of $\log(F_b)$ and $\log(F_m)$, respectively. Consequently the masonry wall compressive strength in Eq. (2) is a random variable that follows a lognormal distribution

$$F_w \sim \mathscr{LN}\left(f_w|\mu_w, \sigma_w^2\right), \tag{3a}$$

$$\text{with } \mu_w = \alpha\mu_b + \beta\mu_m + \log k, \tag{3b}$$

$$\text{and } \sigma_w^2 = \alpha^2\sigma_b^2 + \beta^2\sigma_m^2. \tag{3c}$$

The distribution Eq. (3a) represents the variability, i.e. the frequency distribution, of the masonry compressive strengths within the population of specimens. It is parametrized by hyperparameters $\boldsymbol{\theta}_w = (\mu_w, \sigma_w)$ that are determined by the statistical properties of component populations due to Eqs. (3b) and (3c).

The mean value and the variance of the distribution $\mathscr{LN}\left(f_w|\mu_w, \sigma_w^2\right)$ in Eq. (3) are simply given as $\mathbb{E}[F_w] = \exp(\mu_w + \sigma_w^2/2)$ and $\mathrm{Var}[F_w] = (\exp(\sigma_w^2) - 1)\exp(2\mu_w + \sigma_w^2)$, respectively. The 5 %-quantile of $\mathscr{LN}\left(f_w|\mu_w, \sigma_w^2\right)$, e.g. for comparison with Eq. (1), follows as $Q_{w,5\%} = \exp(\mu_w - 1.645\sigma_w)$.

3.2. *Epistemic Model*

If the coefficients (k, α, β) of Eqs. (2) and (3) are not perfectly known, one can represent their epistemic uncertainty as prior random variables $(K, A, B) \sim \pi(k, \alpha, \beta)$. In the following we will confine the analysis to the case $\beta = 1 - \alpha$. We consider mutually independent prior random variables

$$K \sim \pi(k), \quad A \sim \pi(\alpha). \tag{4}$$

Their joint prior uncertainty $\pi(k, \alpha) = \pi(k)\,\pi(\alpha)$ can be reduced by Bayesian data analysis of experimental measurements. In the following two different updating approaches are outlined for experimental situations where the assumption of known hyperparameters, i.e. the distributional parameters of the ensembles of masonry wall components, is either justified or rather unfounded.

3.3. *Known Hyperparameters*

Let us consider experiments of the following type. In each batch of experiments $i = 1, \ldots, n$ the masonry compressive strength $f_{w,ij}$ is measured for a number of different specimens $j = 1, \ldots, J_i$ from an ensemble. We use $\langle f_{w,ij} \rangle = (f_{w,11}, \ldots, f_{w,nJ_n})$ to denote the set of these measurements. The hyperparameters $\boldsymbol{\theta}_{b,i}$ and $\boldsymbol{\theta}_{m,i}$ are measured for the bricks and the mortar used in experiment i, too. This can be accomplished by a statistical analysis of data $\langle f_{b,ik} \rangle = (f_{b,11}, \ldots, f_{b,nK_n})$ and $\langle f_{m,il} \rangle = (f_{m,11}, \ldots, f_{m,nK_n})$ with $k = 1, \ldots, K_i$ and $l = 1, \ldots, L_i$. These data must be numerous and they must be observed for the ensembles of brick units and mortar used. The Bayesian multilevel model for this scenario can be written as

$$(F_{w,ij}|k, \alpha) \sim \pi(f_{w,ij}|k, \alpha), \tag{5a}$$

$$(K, A) \sim \pi(k)\,\pi(\alpha). \tag{5b}$$

Here the conditional distributions Eq. (5a) are given by Eq. (3), where batch-specific knowns $\boldsymbol{\theta}_{b,i}$ and $\boldsymbol{\theta}_{b,i}$ are plugged in. The epistemic prior uncertainty of the coefficients (k, α) is encoded in Eq. (5b). As long as not indicated otherwise, all random variables in Eq. (5) are assumed to be (conditionally) independent. A directed acyclic graph (DAG) as in Fig. 1 serves as an intuitive visualization of the model Eq. (5).

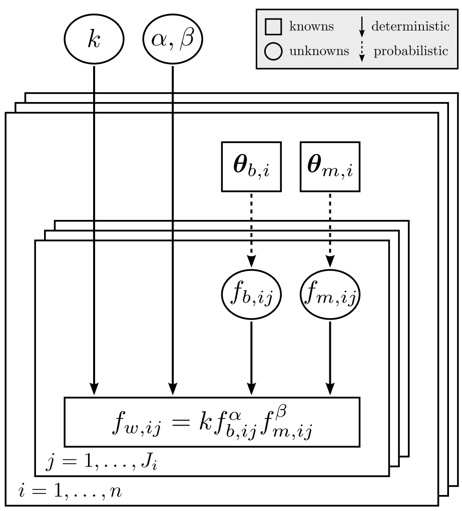

*Figure 1: **Known hyperparameters**. Nodes symbolize known (□) or unknown (○) quantities. Arrows represent deterministic (⟶) or probabilistic (⇢) relations.*

As usual, Bayesian updating is accomplished by conditioning the prior distribution $\pi(k,\alpha) = \pi(k)\,\pi(\alpha)$ on the acquired data $\langle f_{w,ij} \rangle$. One obtains

$$\pi(k,\alpha|\langle f_{w,ij}\rangle) \propto \pi(k)\,\pi(\alpha) \prod_{i=1}^{n}\prod_{j=1}^{J_i} \pi(f_{w,ij}|k,\alpha). \tag{6}$$

Note that Eq. (6) is based on exact values the hyperparameters $\boldsymbol{\theta}_{b,i}$ and $\boldsymbol{\theta}_{m,i}$ for every batch i.

3.4. Unknown Hyperparameters

The requirement of known hyperparameters $\boldsymbol{\theta}_{b,i}$ and $\boldsymbol{\theta}_{m,i}$ restricts the applicability model Eq. (5) to situations that are rarely met in practice. Therefore we consider the situation when only prior knowledge $\pi(\boldsymbol{\theta}_{b,i},\boldsymbol{\theta}_{m,i}) = \pi(\boldsymbol{\theta}_{b,i})\,\pi(\boldsymbol{\theta}_{m,i})$ about the hyperparameters is available. Additionally in each batch of experiments i a variable number of measurements $f_{b,ik}$ and $f_{m,il}$ for $k = 1,\ldots,K_i$ and $l = 1,\ldots,L_i$ are taken of the brick unit and the mortar compressive strength, respectively. The corresponding hierarchical Bayesian model reads

$$\begin{aligned}
(F_{w,ij}|k,\alpha,\boldsymbol{\theta}_{b,i},\boldsymbol{\theta}_{m,i}) &\sim \pi(f_{w,ij}|k,\alpha,\boldsymbol{\theta}_{b,i},\boldsymbol{\theta}_{m,i}),\\
(F_{b,ik}|\boldsymbol{\theta}_{b,i}) &\sim \pi(f_{b,ik}|\boldsymbol{\theta}_{b,i}), \qquad (7a)\\
(F_{m,il}|\boldsymbol{\theta}_{m,i}) &\sim \pi(f_{m,il}|\boldsymbol{\theta}_{m,i}),
\end{aligned}$$

$$\begin{aligned}
(\boldsymbol{\Theta}_{b,i},\boldsymbol{\Theta}_{m,i}) &\sim \pi(\boldsymbol{\theta}_{b,i})\,\pi(\boldsymbol{\theta}_{m,i}),\\
(K,A) &\sim \pi(k)\,\pi(\alpha).
\end{aligned} \tag{7b}$$

While Eq. (7a) summarizes the aleatory uncertainties, Eq. (7b) contains the epistemic uncertainties. The model Eq. (7) is visualized as the DAG in Fig. 2. We remark that the observations $\langle f_{b,ik} \rangle$ and $\langle f_{m,il} \rangle$ inform about the statistical properties $\boldsymbol{\theta}_{b,i}$ and $\boldsymbol{\theta}_{m,i}$ of the component ensembles. This way they give information about the unobservable properties of the brick and mortar samples used for constructing the masonry wall i.

Bayesian analysis proceeds by updating the joint prior $\pi(k,\alpha,\langle\boldsymbol{\theta}_{b,i}\rangle,\langle\boldsymbol{\theta}_{m,i}\rangle) = \pi(k)\,\pi(\alpha)\prod_{i=1}^{n}\pi(\boldsymbol{\theta}_{b,i})\,\pi(\boldsymbol{\theta}_{m,i})$. Conditioned on the data $(\langle f_{w,ij}\rangle,\langle f_{b,ik}\rangle,\langle f_{m,il}\rangle)$ one obtains for

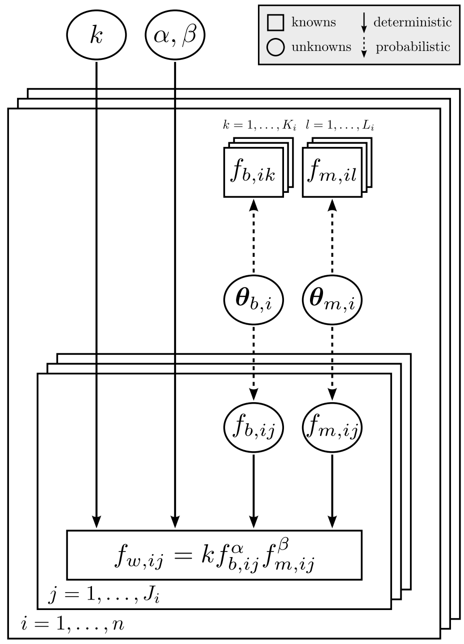

*Figure 2: **Unknown hyperparameters**. The batch-specific hyperparameters $\boldsymbol{\theta}_{b,i}$ and $\boldsymbol{\theta}_{m,i}$ are unknown. They can be inferred from the data $\langle f_{b,ik}\rangle$ and $\langle f_{m,il}\rangle$.*

the joint posterior

$$\begin{aligned}
&\pi(k,\alpha,\langle\boldsymbol{\theta}_{b,i}\rangle,\langle\boldsymbol{\theta}_{m,i}\rangle|\langle f_{w,ij}\rangle,\langle f_{b,ik}\rangle,\langle f_{m,il}\rangle)\\
&\quad\propto \pi(k)\,\pi(\alpha)\prod_{i=1}^{n}\pi(\boldsymbol{\theta}_{b,i})\,\pi(\boldsymbol{\theta}_{m,i})\\
&\quad\cdot \prod_{j=1}^{J_i}\pi(f_{w,ij}|k,\alpha,\boldsymbol{\theta}_{b,i},\boldsymbol{\theta}_{m,i})\\
&\quad\cdot \prod_{k=1}^{K_i}\pi(f_{b,ik}|\boldsymbol{\theta}_{b,i})\prod_{l=1}^{L_i}\pi(f_{m,il}|\boldsymbol{\theta}_{m,i}).
\end{aligned} \tag{8}$$

Notice that the posterior Eq. (8) gathers information from both system- and component-level data.

4. EXPERIMENTAL DATA

In the years 2009-2012 and 2014 the compressive strength of clay block masonry was measured for a variable number of specimens in a series of compression tests. In addition, the compressive strengths of bricks and mortar were recorded for realizations from the same ensembles that were later used for the construction of the masonry wall. The tests were performed at the laboratories of the Department of Civil, Environmental and Geomatic Engineering of ETH Zürich. In Table 1 the experimen-

Table 1: ***Experimental data.*** *Data are shown for tests of clay block masonry that were performed in the years 2009-2012 and in 2014. Blocks from the same ensemble were used in 2009 and 2010. Thus the corresponding rows show duplicate data entries.*

2009	**batch 1**												
f_w [MPa]	9.41	5.53	7.98	8.86	6.67	7.17	7.92						
f_b [MPa]	33.58	34.55	37.1	39.21	39.63	36.1	35.46	37.61	35.6	36.26	35.2	32.7	36.6
f_m [MPa]	15.8	16.1	14.4	14.8	16.1	15.4	13.9	14.6	14.6	14.4	16.8	16.1	
2010	**batch 2**												
f_w [MPa]	6.67	6.12	5.91	8.3	6.44	5.32	6.7						
f_b [MPa]	33.58	34.55	37.1	39.21	39.63	36.1	35.46	37.61	35.6	36.26	35.2	32.7	36.6
f_m [MPa]	12.5	12.94	12.43	13.33	12	12.32							
2011	**batch 3**												
f_w [MPa]	4.32	3.71	6.06	4.95	4.29	2.8	6.28	4.22	5.23				
f_b [MPa]	23.8	26.8	25.7										
f_m [MPa]	14.9	14.7	14.9	15.4	14.7	14.6							
2012	**batch 4**												
f_w [MPa]	8	7.87	8.1	7.53	8.14	6.99	7.82	9.13	5.87	7.71			
f_b [MPa]	37	39.9	38										
f_m [MPa]	26.9	28.1	17	16.2	18.7	21.1							
2014	**batch 5**												
f_w [MPa]	6.53	7.01	6.12	5.94	7.14	5.69	5.82	6.34	5.96				
f_b [MPa]	28.15	27.74	28.05	27.20	26.25	23.15	26.69	28.04	27.69	26.73			
f_m [MPa]	11.73	12.19	12.13	10.49	10.34	10.44							

tal data are summarized. Five batches of experiments were performed in total. At the system- and the component level the available data is generally scarce. Especially in the years 2011 and 2012 the number of component-level tests was very limited. Moreover, in the years 2009 and 2010 brick units from the same ensemble were used.

We observe that the empirical relation Eq. (1) generally overpredicts the masonry wall compressive strength. In Fig. 3 the actually acquired data for $i = 1$, i.e. for the year 2009, is shown together with the correspondingly predicted characteristic value. The values $k' = 0.45$, $\alpha' = 0.7$, $\beta' = 0.3$ provided in EN 1996-1-1 (2005) were used. Brick unit data $\langle f_{b,ik} \rangle$ have been normalized according to their geometry. Moreover a lognormal distribution of the form Eq. (2) is shown, where $\alpha = \alpha'$ and $\beta = \beta'$ have been identified with the corresponding coefficients from EN 1996-1-1 (2005). The remaining coefficient $k = k' \cdot \exp(1.645\sqrt{\alpha_i^2 \sigma_{b,i}^2 + \beta_i^2 \sigma_{m,i}^2} + \alpha_i[\sigma_{b,i}^2/2] + \beta_i[\sigma_{m,i}^2/2])$ has been set in order that the 5 %-quantile equals Eq. (1). Note that the above-mentioned identification/transformation of the coefficients establishes another way of extending Eq. (1) and comparing it to Eq. (2). In this paper we do not pursue this approach, though.

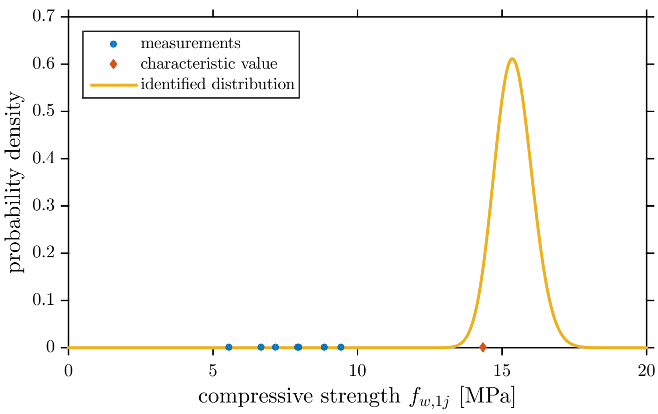

Figure 3: ***Data & predictions for 2009****. The data, its expected 5 %-quantile and a corresponding lognormal distribution are shown. The data are overpredicted.*

Of course, the unexpected code/measurement discrepancy raises important questions. Anticipating our results it is said that we will not able to satisfactorily explain this discrepancy. Instead we will calibrate the coefficients k and α in a way that leads to better predictions. Those predictions are valid for the testing machine and the materials used in our laboratory. Using the predictions outside their scope of applicability is questionable and should only be done with utmost caution.

5. BAYESIAN ANALYSIS

The Bayesian framework discussed in Section 3 is now applied to analyze the experimental data that was presented in Section 4. More specifically we use the first two batches of experiments that were conducted in 2009 and 2010 to calibrate the unknown coefficients of the model Eq. (5). For those batches the amount of component-level data is deemed sufficient to fit the hyperparameters and to treat them as knowns subsequently. Moreover the first four batches will be analyzed with the model Eq. (7) that allows to treat the hyperparameters as unknowns. Especially in the years 2011 and 2012 the small amount of component-level data does not allow to proceed in another way. The fifth batch of experiments from 2014 will be used as an independent test set.

Since the coefficients in Eqs. (2) and (3) cannot be identified with those of Eq. (1), it is not possible to elicit informative priors about the former by exploiting expert knowledge or code information about the latter. Hence uninformative priors are used. Specifically we assign uniform prior distributions $\pi(k) = \mathscr{U}(0,1)$ and $\pi(\alpha) = \mathscr{U}(0.5,1)$. Due to $\beta = 1-\alpha$ the latter assignment enforces $\alpha \geq \beta$. This reflects the intuition that, regarding the masonry compressive strength, the brick units are more influential than the mortar. For the Bayesian model in Eq. (7) priors $\pi(\boldsymbol{\theta}_{b,i}) = \pi(\mu_{b,i})\,\pi(\sigma_{b,i})$ and $\pi(\boldsymbol{\theta}_{m,i}) = \pi(\mu_{b,i})\,\pi(\sigma_{b,i})$ have to be elicited for the unknown hyperparameters. We use independent uniform hyperprior distributions with reasonable bounds for the means and standard deviations.

The posteriors Eqs. (6) and (8) can be sampled by means of Markov chain Monte Carlo (MCMC) techniques (Brooks et al., 2011). In Figs. 4 and 5 the resulting posterior marginals of k and α are depicted. It can be seen that $\pi(k,\alpha|\langle f_{w,ij}\rangle,\langle f_{b,ik}\rangle,\langle f_{m,il}\rangle)$ contains a higher degree of posterior uncertainty than $\pi(k,\alpha|\langle f_{w,ij}\rangle)$. Since more data has entered the former posterior, at first sight this seems to be surprising. This fact can be attributed to the differences of the models Eqs. (5) and (7) in treating the hyperparameters and their uncertainties, though.

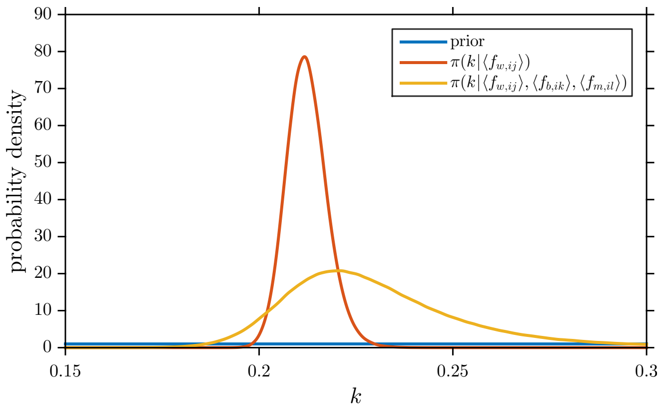

*Figure 4: **Posterior of** k. The posteriors $\pi(k|\langle f_{w,ij}\rangle)$ and $\pi(k|\langle f_{w,ij}\rangle,\langle f_{b,ik}\rangle,\langle f_{m,il}\rangle)$ are shown. It can be seen that the latter is broader than the former.*

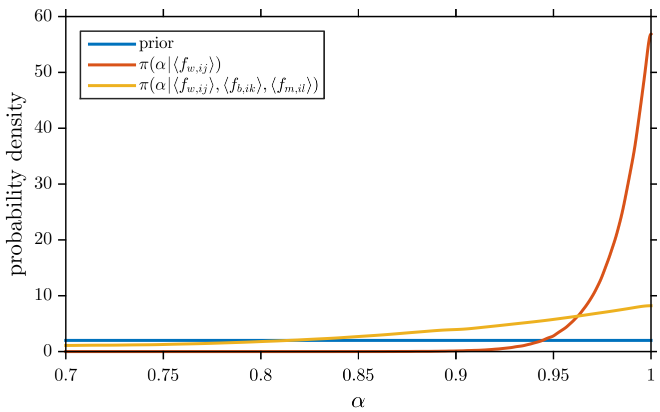

*Figure 5: **Posterior of** α. Both the posterior marginals $\pi(\alpha|\langle f_{w,ij}\rangle)$ and $\pi(\alpha|\langle f_{w,ij}\rangle,\langle f_{b,ik}\rangle,\langle f_{m,il}\rangle)$ peak at their upper boundary.*

Specifically the modes $\hat{k} = 0.21$ and $\hat{\alpha} = 1$ are found for the posterior $\pi(k,\alpha|\langle f_{w,ij}\rangle)$ that represents the situation that hyperparameters are assumed to be known. The posterior $\pi(k,\alpha|\langle f_{w,ij}\rangle,\langle f_{b,ik}\rangle,\langle f_{m,il}\rangle)$, for the scenario that hyperparameters are treated as unknowns, features the modes $\hat{k} = 0.22$ and $\hat{\alpha} = 1$.

The fact that the posterior of α in Fig. 5 peaks at the upper bound of its prior is somewhat surprising. As a consequence of $\hat{\beta} = 1 - \hat{\alpha} = 0$, the influence of mortar occurs to be negligible. Moreover, such a behavior may indicate that the inverse problem is improperly solved, e.g. the true parameter value was accidentally excluded a priori. It was therefore tried to relax the assumption $\beta = 1 - \alpha$ by permitting arbitrary values $\alpha > 0$ and $\beta > 0$. To that end independent priors $\pi(\alpha)$ and $\pi(\beta)$ were assigned. We had to conclude that the limited amount of available data is not sufficiently informative in order to calibrate this extended model.

Plugging the point estimates $\hat{k}$ and $\hat{\alpha}$ in Eq. (3) establishes a predictive relation of the frequency distribution of structural masonry. For that purpose one has to specify the values or estimates of the hyperparameters $\boldsymbol{\theta}_b$ and $\boldsymbol{\theta}_m$ for the ensembles of bricks and mortar used in the construction of the masonry wall. The predicted distributions, that are obtained this way for the actually analyzed batches of experiments, describe the masonry wall resistances adequately well. Since the estimations of the coefficients were informed by the very same data, this does not seem to be very surprising. Yet this signifies that the representation Eq. (3) is adjustable enough to match the data. In turn this may indicate that Eq. (3) is indeed a suitable representation of the masonry wall compressive strength.

When applied to the fifth batch of experiments the procedure described above can serve as a validation test, i.e. the data collected in 2014 are used as an independent test set. In Fig. 6 the measured masonry wall compressive strengths are shown together with the their predicted distribution. The plot is supplemented with the corresponding 5 %-quantile. Here the point estimates $\hat{k} = 0.22$ and $\hat{\alpha} = 1$ that were obtained by analyzing the previous four batches are used on one side. On the other side component-level data $\langle f_{b,5j} \rangle$ and $\langle f_{m,5j} \rangle$ for the fifth batch are used to estimate $\boldsymbol{\theta}_{b,5}$ and $\boldsymbol{\theta}_{m,5}$. The predictive distribution captures the data fairly well. Obviously it is of higher quality than the poor code-forecast shown in Fig. 3.

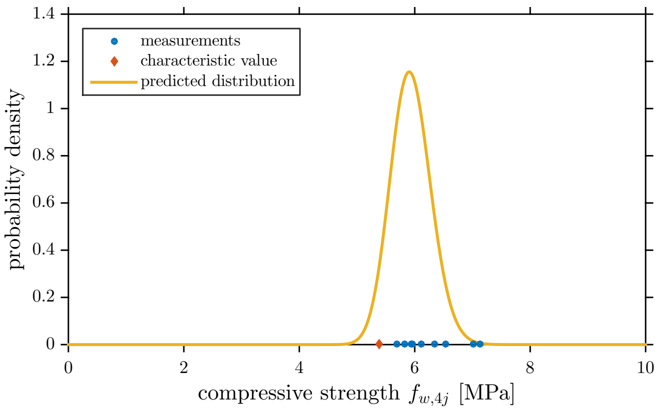

*Figure 6: **Data & predictions for 2014**. The gathered data, the predicted distribution and its 5 %-quantile are shown. Predictions conform to data tolerably well.*

6. SUMMARY & CONCLUSION

It was demonstrated how hierarchical Bayesian models can serve the purpose of assessing the compressive strength of structural masonry. This establishes a fully probabilistic alternative to the existing semi-probabilistic approaches. The hierarchical framework offers versatile and powerful tools of uncertainty quantification and information aggregation at multiple system levels. Different types of uncertainty, i.e. ignorance and variability, are thoroughly managed, while heterogeneous types of information, e.g. data and expert knowledge, are consistently utilized. This way the analysis of the masonry wall resistance can be based on large-scale compression tests as well as on inexpensive tests of brick unit and mortar samples.

Our hope is that this possibility will encourage experimenters in entirely publishing their collected data. In fact it seems to be commonplace to quote statistical data summaries only, e.g. sample means or characteristic values. The proposed methodology, however, allows to process the acquired data as a whole.

A number of questions have arisen. It is queried if Eq. (3) is an adequate representation of the distribution of masonry compressive strength in terms of distributional parameters of the components. With regard to the complexity of structural masonry, its failure modes and their dependency on the quality of workmanship, the relations Eqs. (1) and (2) are oversimplifying. They were inspired by the structure of current models but lack a solid physical foundation. For future studies this motivates the introduction of model uncertainty in addition to the emerging parameters uncertainties. Beyond that future work will also involve the construction and objective selection of better system-level models of aleatory variability. A more fundamental question concerns the general suitability of empirical relations for any probabilistic extension whatsoever. Another raised issue relates to the observed mismatch between measurements and code-predictions. We were not able to explain this discrepancy.

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