

# Quantifying and Accounting for Aftershock Hazard in Performance-Based Earthquake Engineering

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**ABSTRACT:** After a mainshock, the threat to occupant life safety and risk of excessive building damage can be considerably higher than before the occurrence of the mainshock, because of building damage sustained in the mainshock, uncertainty in the time of occurrence, magnitude and location of a potential aftershock. Aftershocks around the world have been observed to collapse damaged buildings. Nevertheless, most of current seismic risk assessment tools only consider mainshock effects without taking into account aftershocks. In this paper a procedure is introduced to investigate and quantify the effect of aftershock hazard on the performance-based design of a four-story woodframe building.

## 1. INTRODUCTION

Different structures are vulnerable to multiple earthquake ground motions including foreshocks, mainshock and aftershocks. The aftershocks are the smaller earthquakes following large earthquakes and have the potential to cause severe damage to buildings even when the damage from the mainshock is moderate. One reason for the high damage potential of aftershocks is that they are unpredictable in terms of their location (site-to-source distance), occurrence time and energy content. The other reason is the lower strength and stiffness of the mainshock-damaged building.

A conceptual framework was proposed by Yeo and Cornell (2005) for including aftershock hazard in performance-based earthquake engineering (PBEE), where they noted that this topic is still “in its infancy at every stage”. The occurrence time of aftershocks is very difficult to predict but basic models for an occurrence rate are available (e.g. see Reasenber and Jones, 1987). A number of basic aftershock models have been proposed including Markov process models (Al Hajjar et al., 1997). Gerstenberger et

al. (2007) investigated the short term probability of an earthquake following a mainshock.

Steel frame buildings were investigated under mainshock plus aftershock (MS+AS) earthquake sequences and it was found that the damage level from the mainshock, the amplitude and the frequency of the aftershock significantly affects the damage pattern due to the aftershocks (Li and Ellingwood, 2007).

Based on the previous earthquake sequences and the evidence such as 2008 China Wenchuan earthquake collapses and several other major earthquakes with aftershocks, it is obvious that the collapses of the mainshock-damaged structures were due to the occurrence of subsequent aftershocks. However, to date, the effect of aftershocks in seismic risk analysis has not been explicitly accounted for in modern design codes, nor in emerging performance-based seismic design methodologies. The objective of this paper is to summarize a methodology for integrating aftershock seismic hazard into PBEE. Although the methodology presented is applicable to any type or suite of

buildings and structures, only a woodframe building is presented herein as example.

## 2. METHODOLOGY

### 2.1. Model Development

The building that is investigated is a woodframe building located in Los Angeles, CA. The building is designed using the simplified Direct Displacement-based Design (DDD) approach presented by Pang et al. (2010) for a specific design drift, a probability of non-exceedance and a performance level. The design consists of computing the required story shears and then selecting seismic force resisting elements by summing the hysteretic backbones to achieve the story shears at the target drift. In the next step, the building is modeled using one degree of freedom at each story. A lumped mass and a spring model with the capability to degrade in both stiffness and strength represent each story of the building. The backbone curve for each story of the building model is derived by pushover analysis of the model using a predefined monotonic displacement protocol in SAPWood software. Once the backbone curves are determined for each story, a hysteretic model is fit to each backbone curve and the parameters of the CUREE (Consortium of Universities for Research in Earthquake Engineering) model are identified for each story. The mainshock collapse fragilities are developed via nonlinear time history analysis of the structural models using a suite of 22 mainshock records. The earthquake ground motion records used in this study were from the Applied Technology Council Project 63 which resulted in the development of FEMA P-695 (FEMA, 2009) and consists of a suite of 22 far field (ordinary) ground motions (FEMA, 2009). Specifically, these were used as mainshock records and the record-to-record variation represents the total earthquake variability for the mainshock. An incremental dynamic analysis (IDA) of the shear building model is used to generate fragility curves for a mainshock. At an inter-story drift of approximately 7%, the IDA curves for the 22

records have a near-flat slope. This flat slope essentially means that the spectral acceleration required to produce larger and larger drifts would not increase substantially and therefore reaching a drift of 7% is herein defined as collapse in this paper. The mainshock hazard curve is determined using the National Seismic Hazard Mapping Project (NSHMP) hazard application software (USGS, 2013). The mainshock fragility curve is then convolved with the location-specific mainshock hazard curve to obtain the probability of collapse of the building, i.e. (P(MS)) following the approach outlined by Luco et al. (2007).

The annual probability of collapse provides a metric for evaluation of risk, and it includes the site specific seismic hazard. This risk integral can then be expressed generally by using the total probability theorem. By taking into account the uncertainty in the collapse capacity of a given model described by Eq. 1 (Luco et al, 2007), the annual collapse probability (risk integral) can be calculated as:

$$P[\text{Collapse}] = \int_0^{\infty} P[Sa > c] \cdot f_{\text{Capacity}}(c) dc \quad (1)$$

### 2.2. Development of Aftershock Hazard

The aftershock hazard curve for the building's location can be determined using the procedure presented by Yeo and Cornell (2005). An estimate of the aftershock hazard curve is obtained by multiplying the mainshock hazard curve at the site for all Peak Ground Acceleration (PGA) values by a constant number of  $\mu^*(t, T, m_m)$  presented in Eq. 2 (Yeo and Cornell, 2005) as:

$$\begin{aligned} \mu^*(t, T; m_m) &= \int_t^{t+T} \mu(\tau; m_m) d\tau \\ &= \frac{10^{a+b(m_m-m_l)} - 10^a}{p-1} [(t+c)^{t-p} \\ &\quad - (t+T+c)^{t-p}] \end{aligned} \quad (2)$$

where  $\mu^*$  is the mean number of aftershocks with magnitudes between  $m_1$  and  $m_m$  in the time interval  $[t, t+T]$  following a mainshock of magnitude  $m_m$ . In addition, parameters for the “generic California” aftershock model proposed by Reasenber and Jones (1989) were used in this study. Time  $t$  following a mainshock of magnitude  $m_m$  was assumed to be  $t=0$  days and  $T=1000$  days. The terms  $m_1$  and  $m_m$  are the minimum and maximum aftershock moment magnitudes of engineering interest, respectively;  $m_1$  is typically assumed to be 5.0 and  $m_m$  is usually set equal to the moment magnitude of the mainshock, which in the present study is assumed to be 8.0. Parameters of  $a=-1.67$ ,  $b=0.91$ ,  $p=1.08$  and  $c=0.05$  from the generic California aftershock sequence are used in this study. Based on the above assumptions the scalar value of  $\mu^*$  is approximately equal to 100. This factor is then multiplied with the mainshock hazard curve which has horizontal axis of PGA. This procedure is valid with PGA horizontal axis for both mainshock and aftershock hazard curves. However, the aftershock hazard curve in terms of Spectral Acceleration ( $S_a$ ) is needed for purpose of convolution with the fragility curve. Therefore, the aftershock hazard curve with PGA horizontal axis is converted to hazard curve with  $S_a$  horizontal axis by using a basic procedure for scaling earthquakes (Nazari et al., 2014). It should be noted that the AS hazard curve produced using the above procedure is the AS hazard curve conditioned on the occurrence of the previous MS. Therefore, it should be convolved with the aftershock fragility curve which is computed only for the building models that survived the mainshock earthquake.

### 2.3. Quantifying the Needed Design Changes

The objective in this study is to make the original building stronger such that its total probability of collapse under mainshock and aftershock is equal to the probability of collapse of the original building under only the mainshock. Obviously the total probability of collapse under the

mainshock and the aftershock is larger than the probability of collapse under the mainshock only. Therefore, the original building is made stronger in order to satisfy the above criteria. To achieve this, the original building which is designed for base shear demand resulting from the DDD of the building for a specific design drift should be re-designed for a larger base shear demand which is equal to the Adjusted Base Shear Demand =  $X_v \times$  Original Base Shear Demand where  $X_v$  is called base shear adjustment factor. For example, if the original building is designed for 4% design drift with a base shear demand of 200 kips for the first story of the building and the stronger building is re-designed for a 3% design drift with a base shear demand of 250 kips by using the simplified DDD approach then, the base shear adjustment factor is calculated to be equal to 1.25. A new adjusted base shear demand is chosen for the stronger building and it is designed using the DDD procedure. Additionally, a new numerical model is calibrated for the stronger building using the same procedure as was used originally.

The next step requires computation of the total probability of collapse of the original and revised building under MS and  $MS_{MCE} + AS$ , respectively. In order to provide aftershock fragility curves  $MS_{MCE} + AS$  sequences are needed for IDA analysis. Back-to-back  $MS_{MCE} + AS$  records are chosen randomly among the 22 earthquake records. Each earthquake in the suite of 22 records has its own magnitude (between M6.5 and 7.6), associated with the historical event. However, a different procedure is used herein to develop MS+AS scenarios. Initially, the MS is scaled to the Maximum Considered Earthquake (MCE) level spectral acceleration of 2.4g for Los Angeles. Natural period of the four-story building which is approximately 0.5 seconds is used for calculation of spectral acceleration. The MS magnitude was selected to be a predetermined magnitude which was not necessarily the magnitude of the original record. This was done in order to enable the scaling of the aftershock intensity by using a

smaller magnitude value in the aftershock equation. The aftershock intensity in terms of spectral acceleration is determined based on the aftershock magnitude using the Next Generation Attenuation (NGA) ground motion relations (Abrahamson and Silva, 2008). This is clearly an approximation since the attenuation equation brings with it epistemic uncertainty. Table 1 presents the spectral acceleration values calculated using NGA relationship. These values were used as scaling factors for mainshocks and aftershocks.

Table 1: Spectral Acceleration values from NGA relationship

Spectral Acceleration (g)	
MS	AS
1.424	0.773

The aftershock fragility is determined using nonlinear time history analysis. Similar to the MS analysis, a story drift of 7% was defined as the collapse criteria. The AS hazard curve is conditioned on the occurrence of the previous MCE level MS. Therefore, the AS fragility curve is produced by using only the data points that have survived from the MS earthquake. For example, if 10 out of 22 MCE level MS records cause the collapse of a building that is designed for 4% design drift, then the AS fragility curve is produced by using only 22-10=12 data points instead of 22 data points.

Then, the AS fragility curve is convolved with the AS hazard curve and the annual collapse probability for MS<sub>MCE</sub>+AS case ( $P(AS|\overline{MS_{MCE}})$  or  $P(AS)$ ) is calculated. It is assumed that  $P(AS)$  and  $P(AS|\overline{MS_{MCE}})$  are equal in this study since the occurrence of AS is conditioned on the survival from an MCE level MS.

The total probability of collapse under the occurrence of mainshock and aftershock is the summation of the collapse probability under the mainshock ( $P(MS)$ ), and the annual probability of collapse under the occurrence of both mainshock and aftershock ( $P(AS \cap MS)$ ) which is presented mathematically using the total probability theorem can be expressed as:

$$\begin{aligned}
 &P(MS) + P(AS \cap MS) \\
 &= P(MS) + P(AS) * P(MS) \\
 &= P(MS) + P(AS|\overline{MS_{MCE}}) \\
 &\quad * P(MS) \qquad \qquad (3)
 \end{aligned}$$

In the end, the total annual collapse probability for the MS<sub>MCE</sub>+AS case is compared with the annual MS collapse probability of the original building and the difference is calculated in percent. The building design is then revised such that its total collapse probability is equal to the annual MS collapse probability for the original building.

### 3. ILLUSTRATIVE EXAMPLE

An example is provided to illustrate the methodology using a four-story woodframe building. A four-story woodframe building is designed using the simplified DDD approach for a probability of non-exceedance of 50% is used. The MCE level design accelerations associated with the building's location are calculated based on ASCE 7-10 standard. It is assumed that the building is located in Los Angeles, CA with soil category D. Table 2 presents design spectral accelerations for 5% damping and natural period of 0.5 seconds. It is assumed that the building is designed for MCE intensity.

Table2: MCE level Spectral Acceleration values

Spectral Acceleration (g)	
$S_{XS}$	$S_{XL}$
2.448	1.288

Fig. 1 presents the plan view of each story for the four-story woodframe building; all stories are architecturally identical.

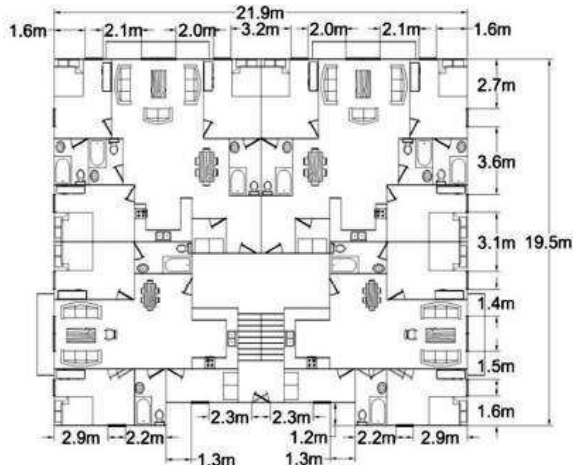


Figure 1: Plan view for each story of the four-story woodframe building (Excerpted from Jennings et al., 2014)

Using the above spectral accelerations, base shear demands are calculated by simplified DDD for a design drift of 4%. Calculated base shear values are presented in Table 3.

Table 3: Base shear demands for four-story building designed for 4% design drift

Story	$V_s(KN)$
4	512
3	894
2	1152
1	1277

Fig. 2 shows the design points versus the hysteretic backbone curves for the four stories of the building. In order for the building designed by simplified DDD approach to be satisfactory, the design points should be just below the backbone curves for each story. As can be seen in Fig. 2, this criterion is satisfied and each story's design point falls below the associated backbone curve. A 10-parameter CUREE model is fitted to the backbone curves of each story and the parameters are determined using SAPWood (Pei and van de Lindt, 2008). The parameters of the CUREE model are presented in Table 4.

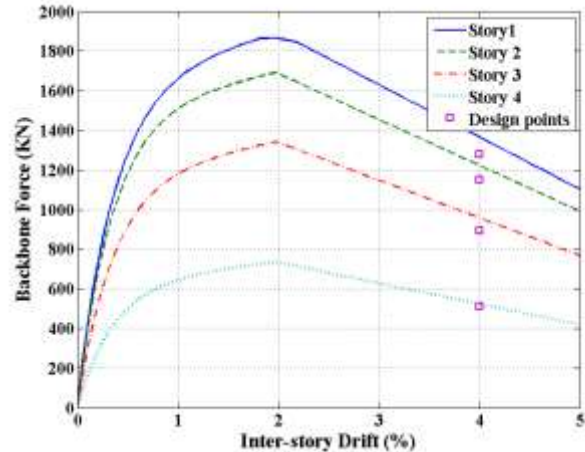


Figure 2: Design points vs. backbone curves for the building with 4% design drift

Fig. 4 presents the Cumulative Distribution Function (CDF) form of a fragility curve for the four-story building designed for 4% drift. The fragility curve shows the probability of exceedance ( $P[S_a > c]$ ) versus collapse capacity ( $c$ ) parameter which in this study is selected as spectral acceleration ( $S_a[g]$ ). A lognormal curve is fitted to data points generated from the analysis and the parameters for the lognormal fit are presented in Fig. 3.

The mainshock hazard curve is then generated based on the NSHMP hazard application software (USGS, 2013). Fig. 4 presents an example of a MS hazard curve for Los Angeles, CA (Latitude: 34.05, Longitude: -118.24).

Table 4: Parameters of the CUREE model for the building designed for 4% design drift

Parameters (KN, mm)	Story 1	Story 2	Story 3	Story 4
$K_0$	83.34	74.04	60.63	31.84
$F_0$	672.57	594.73	504.43	271.83
$F_1$	0.16	0.15	0.09	0.1
$R_1$	0.04	0.03	0.02	0.02
$R_2$	-0.06	-0.06	-0.06	-0.06
$R_3$	1	1	1	1
$R_4$	0.02	0.02	0.02	0.02
$X_u$	48.26	47.5	46.99	46.99
$\alpha$	0.75	0.75	0.75	0.75
$\beta$	1.1	1.1	1.1	1.1

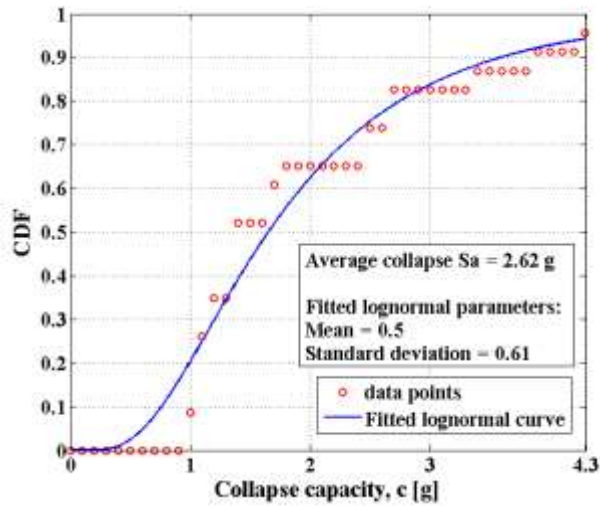


Figure 3: Mainshock fragility curve for the building designed for 4% design drift

Based on the procedure mentioned in the previous section, the CDF fragility curve is transformed to its Probability Density Function (PDF) form. The convolution of the MS fragility curve with the MS hazard curve results in the MS annual collapse probability ( $P(\text{MS})$ ) of 0.000285. Fig. 5 presents the MS hazard curve, the fragility curve and the product of these two curves. The area under the third curve is equal to the annual collapse probability as shown earlier in Eq. 1.

Similar to the mainshock case, the aftershock fragility curve is provided for the four-story building. The aftershock hazard curve is provided using the procedure explained in the previous section. The value of  $\mu^*$  is calculated to be 100 using Eq. 2 and the mainshock magnitude of M8. Fig. 4 shows aftershock hazard curve versus mainshock hazard curve in this example.

The aftershock fragility is convolved with the aftershock hazard curve and the aftershock annual collapse probability ( $P(\text{AS})$ ) is calculated to be 0.091572. The total annual collapse probability under mainshock and aftershock is 0.00031 using Eq. 3. The difference of the total annual collapse probability with mainshock annual collapse probability is approximately 9%. Therefore the building should be revised and

strengthened in order to make these annual collapse probabilities approximately equal.

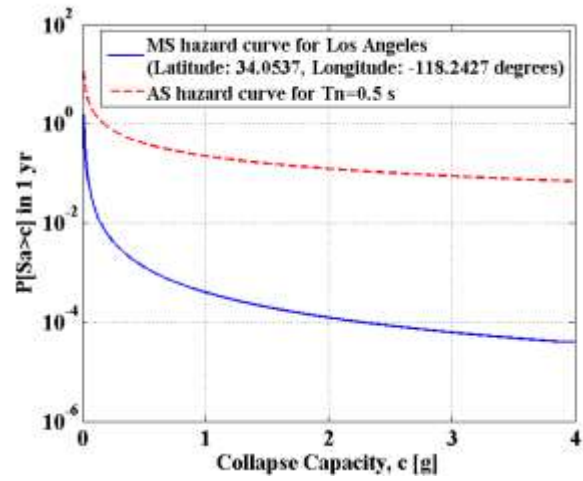


Figure 4: Aftershock hazard curve vs. mainshock hazard curve

The four-story building is re-designed for a design drift of 3% using the DDD procedure. The total annual collapse probability for mainshock and aftershock is calculated to be 0.00026 for the stronger building using a similar procedure. This value results in approximately -8% difference with mainshock annual collapse probability of the building designed for 4% design drift. This means that the 0% difference associated with the correct revised building's base shear demand lies between the base shear demands of the buildings with 4% and 3% design drift. Interpolation between base shear demands and the difference percent results in approximately a 24% increase in base shear demand for the building. In other words, a base shear adjustment factor ( $X_v$ ) of 1.24 is needed to account for the effect of the aftershock in this example. The base shear demands for the first story of the four-story model with 4% and 3% design drifts that were used in the interpolation are 1277 KN and 1868 KN, respectively.



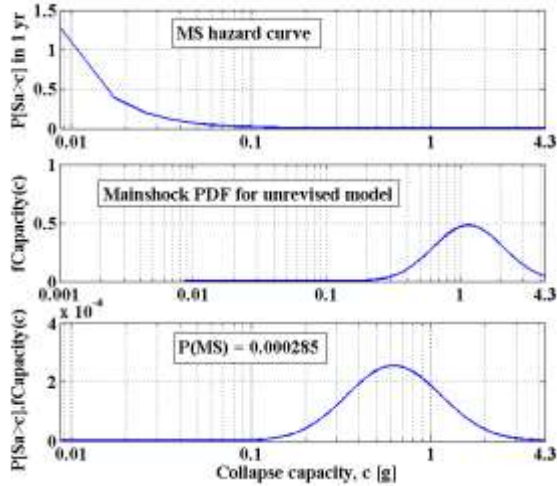


Figure 5: Calculation of the mainshock annual collapse probability of the four-story model

It should be noted that the base shear adjustment factor,  $X_v$  of 1.24 was calculated for the MS+AS scenario with mainshock of M8 and the largest possible AS of M7 (see Bath 1965). Therefore, a 24% increase in the design base shear due to the effect of aftershock hazard is conservative. Since, for a mainshock of M8, the probabilities of occurrences of aftershocks are different (Reasenber, 1994). Table 5 presents the probabilities of occurrence of aftershocks of different magnitudes. These values are excerpted from Reasenber (1994) with assumption of  $t=0$  and  $T=1000$  as mentioned earlier in this paper.

Table 5: Probability of occurrence of aftershocks with different magnitudes (Reasenber, 1994)

AS magnitude	Probability
$M > 7.5$	0.019
$M > 7$	0.048
$M > 6.5$	0.115
$M > 6$	0.264
$M > 5.5$	0.536
$M > 5$	0.854

In order to find a less conservative base shear adjustment factor, first the adjustment factors are calculated for different MS(M8)+AS scenarios with different aftershock magnitudes. Table 6

presents the  $X_v$  values computed using the procedure proposed in this paper.

Table 6: Base shear adjustment factors calculated for different AS magnitudes

AS magnitude	$X_v$
M7.5	1.27
M7	<b>1.24</b>
M6.5	1.13
M6	1.07
M5.5	1.05
M5	1.03

The probability values presented in Table 5 could be used as weights in order to compute a single base shear adjustment factor for  $X_v$  values presented in Table 6 with a specific MS magnitude. Consider the base shear adjustment factors in Table 6 which is associated with MS magnitude of M8 and different MS+AS scenarios with aftershocks of magnitudes M7.5, M7, M6.5, M6, M5.5 and M5. A weighted base shear adjustment factor can be calculated using the probabilities presented in Table 5 which takes into account the probability of occurrence of aftershocks with different magnitudes. The weighted base shear adjustment factor for the MS of M8 is calculated to be approximately 1.06 for the four-story building with 4% design drift and 50% non-exceedance probability.

#### 4. CONCLUSIONS

A methodology for identifying the changes in design to account for aftershock hazard was presented and illustrated herein. For a four-story woodframe building it was determined that approximately 24% more base shear would be needed to account for the scenario in this study. To extend this approach and generalize it one would need to develop suites of buildings representative of a typical community design space and combine them with an array of realistic scenarios to determine the statistical distribution of the needed design change.

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