# Efficient Post-hazard Probabilistic Flow Analysis of Water Pipe Networks

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ABSTRACT: This paper proposes a reliability based flow analysis framework for water pipe networks after an earthquake event. The first part of the framework is an automated leakage and breakage modeling of water pipe segments in a network damaged by an earthquake event. The second part is an efficient system-level probabilistic flow analysis based on the Matrix-based System Reliability (MSR) formulation and the branch-and-bound method. It estimates the system-level flow quantities of network pipes and the system damage state probabilities, and significantly reduces the computational costs by sequentially prioritizing the system states according to their likelihoods and by selecting their partial sets. The proposed framework is demonstrated by a water pipe network consisting of 11 pipe segments under an earthquake event, in which the statistical moments and the probability of the water flows of several pipes are estimated based on the proposed framework.

## 1. INTRODUCTION

Water pipe networks are one of the largest infrastructure assets among lifeline networks and substantially contribute to the economic services, industrial activities, quality of life, and the environment. Their primary duty is to provide ample amounts of water at a sufficient pressure level in order to meet all demands from consumers. Their reliable water supply is an essential service for communities and is part of the so-called service of general interest being vital to the general welfare, public health, and the collective security of populations, as well as economic activities (Alegre et al. 2006). When natural or man-made disasters occur, however, water pipe networks are often vulnerable to structural failures and may lose a significant amount of water or even become non-operational due to the leakage and breakage of pipe segments. This prevents them from carrying the specified quantities of water and the required pressure heads, and post-disaster disruption of the water supply obviously causes considerable public inconvenience. In this context, it is imperative to immediately predict the post-hazard flow of a water network, considering possible leakage and breakage scenarios for prompt risk-informed decision making on hazard mitigation and disaster management.

Many research attempts have been made to estimate the seismic performance of water supply networks using Monte Carlo simulation (MCS) or non-simulation-based methods, which include Dueñas-Osorio et al. (2007), Adachi & Ellingwood (2009), Zolfghari & Niari (2009), Lee et al. (2011), Kim et al. (2012), Torri & Lopez (2012), and Kang & Kliese (2014). These studies mainly focused on connectivity analyses or non-quantity-based analyses, and some of the MCS-based methods conducted probabilistic estimation of quantities, but most of the non-simulation methods are limited in quantity estimations.

This paper proposes a system-level probabilistic flow analysis framework for water pipe networks after an earthquake event. The first part of the proposed framework is an automated modeling of a water pipe network considering its leaked and broken components due to earthquake shaking. Multiple software packages including ArcGIS (ESRI 2011), EPANET (Rossman 2000), and GIRAFFE (Cornell University 2008) are integrated for this purpose. The second part of the proposed framework is an efficient algorithm for a probabilistic post-earthquake water flow analysis of the modeled network considering system-level damage scenarios such as the leakage and breakage in multiple pipes using the Matrix-based System Reliability (MSR) analysis framework (Lee et al. 2011, Kang & Song 2011) and the branch-and-bound method. The branch-and-bound method sequentially identifies critical damage scenarios according to their likelihoods, and reduces the size of the vectors/matrices in the MSR method by considering only important scenarios. The proposed approach is demonstrated by a benchmark water pipe network example having 11 pipelines.

## 2. PROPOSED FRAMEWORK

### *2.1. Network modeling for hydraulic analysis*

The first part of the framework is a post-hazard network modeling. First, a topology of a water pipe network is prepared in Google Earth, and the coordinate information for network junctions is stored in a Keyhole Markup Language (KML) file. This file is then imported to ArcGIS, Geographic Information System (GIS) based software developed by ESRI, through GPS Visualizer, web-based free software. In ArcGIS, additional parameter information such as pipe diameter, pipe roughness, elevation of junctions, and water demand locations and levels is added, and graphically unsnapped junctions in the Google Earth model are fully snapped using the snapping tool in ArcGIS. Finally, the model is stored in a Drawing Interchange Format (DXF) file and imported to EPANET, through a proper file conversion using EPACAD. In EPANET, the remaining parameters such as the locations of reservoirs, tanks, and pumps, the required demand at junctions are defined, and a hydraulic analysis is carried out to estimate the water flow of each pipe segment.

### *2.2. System damage modeling for multiple pipe leakage and breakage*

To model leakage and breakage in multiple pipe segments due to a seismic event, a modeling scheme in GIRAFFE is adopted and simplified such that a pipe segment damage has only two states, i.e., leakage and breakage, for computational simplicity. The modeling details for pipe breakage and leakage are illustrated in Figures 1 and 2, respectively. Figure 1 shows the modeling of a pipe breakage through the removal of the broken pipe element. The corresponding water discharge is modeled by installing two empty reservoirs at the two end points of the removed element. To model the water discharge from the broken pipe into the atmosphere, one-way check valves are installed.

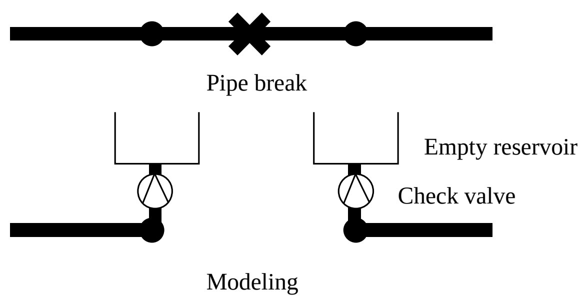



*Figure 1 Hydraulic model for a pipe break (Cornell University, 2008)*

Figure 2 shows the modeling of pipe leakage, where an empty reservoir is installed at the mid-point of the leaked pipe, and the leaked pipe and the reservoir are connected through a fictitious pipe and a one-way check valve. The diameter of the fictitious pipe determines the leakage rate, and in this study, it is assumed to be 1/6 of that of the original pipe. All of these modeling procedures are automated in MATLAB® using an EPANET MATLAB Toolkit (Eliades, 2009) to model random breakages and leakages of multiple pipelines after an earthquake event.

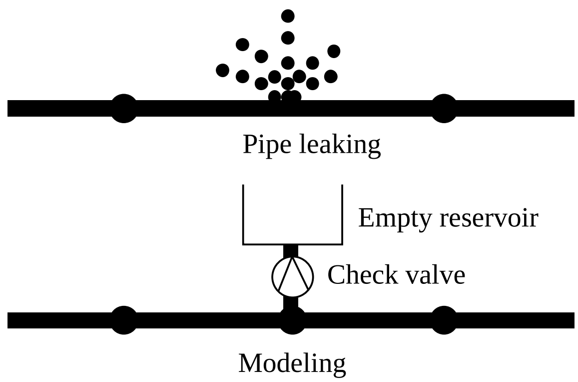



*Figure 2 Hydraulic model for a pipe leak (Cornell University, 2008)*

### *2.3. MSR-based post hazard flow analysis*

In the proposed framework, the MSR-based uncertainty quantification method (Lee et al. 2011) is employed to carry out post-hazard water pipeline network flow analysis, in which the automated modeling procedure introduced in the previous section is used to estimate water flow quantities. Consider a water pipe network system consisting of $n$ pipeline components. The $i^{\text{th}}$ component has $d_i$ prescribed damage states, $i = 1,\ldots,n$. Thus, the system has a total of $d_1 \times d_2 \times \ldots \times d_n$ system states determined by component damage states. In this study, all $d_i$'s are assumed to be 3, representing no damage, leakage, and breakage, and the system has $3^n$ damage states accordingly. Let $P_{i,(j)}$, $i = 1,\ldots,n$, $j = 1,\ldots,d_i$, denotes the probability of the $i^{\text{th}}$ component in the $j^{\text{th}}$ damage state. By assuming that all components are statistically independent, the probability of each system state can be obtained as the product of the corresponding component probabilities, i.e.

$$\mathbf{p} = \begin{bmatrix} P_{(1,1,\ldots,1)} \\ P_{(2,1,\ldots,1)} \\ \vdots \\ P_{(d_1,d_2,\ldots,d_n)} \end{bmatrix} = \begin{bmatrix} P_{1,(1)} \times P_{2,(1)} \times \cdots \times P_{n,(1)} \\ P_{1,(2)} \times P_{2,(1)} \times \cdots \times P_{n,(1)} \\ \vdots \\ P_{1,(d_1)} \times P_{2,(d_2)} \times \cdots \times P_{n,(d_n)} \end{bmatrix} \tag{1}$$

where $\mathbf{p}$ is termed the probability vector (Lee et al. 2011) showing all possible system states, and $P_{(\ldots)}$ denotes the probability of a system state determined by the component damage states shown in the subscript. For example, (2,1,…,1) indicates that all the components are in the first damage state except that the first component is in the second damage state.

For each of the system state, any corresponding system quantity such as the post-hazard flow of each pipeline can be estimated using the water flow analysis model proposed in the previous section. For each of the system states in Equation 1, the system quantities are evaluated as follows:

$$\mathbf{q} = \begin{bmatrix} Q_{(1,1,\ldots,1)} \\ Q_{(2,1,\ldots,1)} \\ \vdots \\ Q_{(d_1,d_2,\ldots,d_n)} \end{bmatrix} = \begin{bmatrix} f(q_{1,(1)}, q_{2,(1)}, \ldots, q_{n,(1)}) \\ f(q_{1,(2)}, q_{2,(1)}, \ldots, q_{n,(1)}) \\ \vdots \\ f(q_{1,(d_1)}, q_{2,(d_2)}, \ldots, q_{n,(d_n)}) \end{bmatrix} \tag{2}$$

where $\mathbf{q}$ is termed the quantity vector (Lee et al. 2011), $Q_{(\ldots)}$ denotes the system quantity of the system state determined by the component damage state shown in the subscript, and $f(\cdot)$ denotes the post-hazard flow analysis model proposed in the previous section.

Using the probability vector $\mathbf{p}$ in Equation 1 and the quantity factor $\mathbf{q}$ in Equation 2, the statistical parameters and the probability functions of the system quantities such as the mean, the variance, and the cumulative distribution function (CDF) of the system quantity can easily be estimated by the following matrix calculations (Lee et al. 2011):

$$\begin{aligned} \mu_Q &= \mathbf{q}^{\mathrm{T}}\mathbf{p} \\ \sigma_Q^2 &= \mathbf{p}^{\mathrm{T}}(\mathbf{q}.*\mathbf{q}) - \mu_Q^2 \\ F_Q(q) &= P(Q \le q) = \sum_{\forall k: q_k \le q} p_k \end{aligned} \tag{3}$$

where ".*" denotes the element-wise product of two vectors, and $p_i$ and $q_i$ are the $k^{\text{th}}$ elements of the vectors $\mathbf{p}$ and $\mathbf{q}$, respectively.

### *2.4. Branch-and-bound method for efficient probabilistic flow analysis*

The system-level probabilistic flows analysis described in the previous section may require a huge computational costs for a large system. This is because the number of the system states (i.e. $d_1 \times d_2 \times \ldots \times d_n$) increases exponentially with the number of components. In this study, we propose an efficient method employing the branch and bound method (Murotsu 1984, Guenard 1984).

The branch-and-bound method is used to sort the system state probability vector in Equation 1 as follows. We starts from the probability vector of the first component only, i.e., $[P_{1(1)},\cdots,P_{1(d_1)}]^{\mathrm{T}}$. We first find the element with the maximum value in this vector and multiply it by the probability vector of the next component,$[P_{2(1)},\cdots,P_{2(d_2)}]^{\mathrm{T}}$ . Then, the total size of the probability vector increases to $(d_1+d_2-1)$. Next, we repeat finding the element with the maximum value in this increased size vector and check if this element has already been branched out in the previous process. If so, we multiply it by the vector of the next component, $[P_{3(1)},\cdots,P_{3(d_3)}]^{\mathrm{T}}$ to increase the vector size to $(d_1+d_2+d_3-2)$. If not, we multiply the element by the vector of the second component, $[P_{2(1)},\cdots,P_{2(d_2)}]^{\mathrm{T}}$ to increase the vector size to $(d_1+2d_2-2)$. These processes are repeated to prioritize important system states and to evaluate their probabilities.

At each process, we check if the element with the maximum value contains all $n$ network components in the product. If that is the case, the maximum value and the corresponding system state are stored, and the element of the second highest system probability is branched out to continue the process. Critical system states and their probabilities are collected until the sum of the system state probabilities reaches a value close to 1.0 or a targeted value. The size of this partial probability vector is significantly smaller than the full vector in Equation 1 because many of the system states have negligible likelihoods. Then a normalizing process dividing the stored values by their sum is conducted so that the sum of the probabilities in the partial vector becomes 1.0. By replacing the complete probability vector in Equation 1 by the normalized partial probability vector, one can perform probabilistic flow analysis with significantly reduced computer memory requirement and improved efficiency.

## 3. APPLICATION

### *3.1. Application network*

The proposed framework is applied to the water pipe network in Figure 3, which is modeled based on a network example in Vitkovsky et al. (2000). It has 11 pipes (indexed by the numbers in parentheses), seven nodes (indexed by the numbers in circles), two inflows, and one outflow at a demand node. A constant demand at a fixed time point is assumed with a specified demand value. The pipe diameters are equally 254 mm for all pipes, and the lengths of the 11 pipes are 1,372 m, 762 m, 762 m, 1,067 m, 762 m, 762 m, 762 m, 914 m, 1219 m, 762 m, and 762 m, respectively. The Hazen-Williams coefficient is chosen as W = 120.

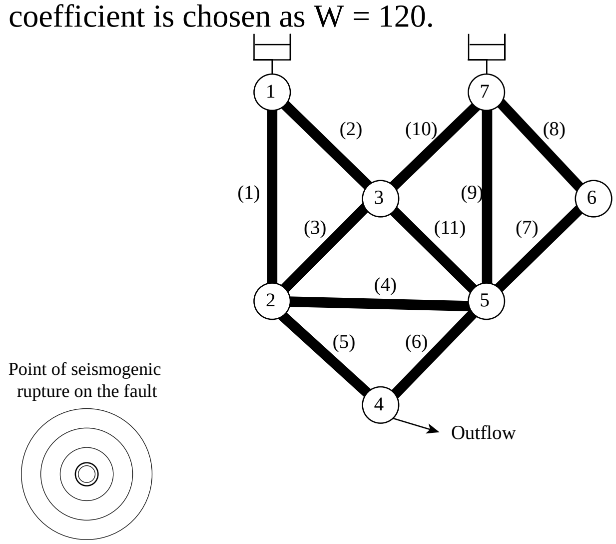



*Figure 3 Example water pipe network with 11 pipe segments*

When an earthquake event occurs, it is assumed that each pipeline in the network will have one of the following three damage states: undamaged, leakage, or breakage. The failure probabilities of the pipes are estimated using the following repair rate given as a function of the peak ground velocity (PGV) in the HAZUS technical manual (FEMA 2003), which is defined by the average number of failures per unit length (km) of a pipe:

$$\text{repair rate} \cong 0.0001 \times (\text{PGV})^{2.25} \tag{4}$$

The failure probability of each pipe is computed using a Poisson process along a dimension of length. This paper deals only with failures induced by ground shaking and uses the repair rate model in Equation 4; the ground failure is ignored.

The PGV is computed from the following attenuation relationship (Campbell 1997):

$$\begin{aligned}\ln(\mathrm{PGV}) = {} & \ln(\mathrm{PGA}) + 0.26 + 0.29M \\ & -1.44\ln\left[r + 0.0203\exp(0.958M)\right] \\ & +1.89\ln\left[r + 0.361\exp(0.576M)\right] \\ & +(0.0001 - 0.000565M)r - 0.12F \\ & -0.15S_{SR} - 0.30S_{SR} \\ & -0.75\tanh(0.51D)(1 - S_{HR}) - f_V(D)\end{aligned} \tag{5}$$

where

$$\begin{aligned}\ln(\mathrm{PGA}) = {} & -3.512 + 0.904M \\ & -1.328\ln\{\sqrt{r^2 + [0.149\exp(0.647M)]^2}\} \\ & +[1.125 - 0.112\ln r - 0.0957M]F \\ & +[0.440 - 0.171\ln r]S_{SR} \\ & +[0.405 - 0.222\ln r]S_{HR}\end{aligned} \tag{6}$$

where PGA is the peak ground acceleration, $M$ denotes the earthquake magnitude assumed to vary within the range 6.5~8.0 in this study, $F$ represents the fault type, assumed to be 0 for strike-slip type faulting, $S_{SR}$ and $S_{HR}$ define the local site conditions, assumed to be alluvium or firm soil ($S_{SR} = S_{HR} = 0$), $D$ denotes the depth to bedrock, assumed to be 0.45 km, and $r$ is the distance between the center of each pipe and the epicenter. The distances between the pipes and the epicenter are 3.9 km, 4.2 km, 3.8 km, 3.9 km, 3.5 km, 4.0 km, 4.1 km, 4.7 km, 4.9 km, 4.5 km, and 4.5 km (from pipes 1 to 11). For D < 1 km, $f_V(D)$ is given as

$$f_V(D) = -0.30(1 - S_{HR})(1 - D) - 0.15(1 - D)S_{SR} \tag{7}$$

When a pipe segment is damaged by a ground shaking in an earthquake, it can have either one of the two states: leakage or breakage. Their respective likelihoods of occurrence are assumed to be 0.2 and 0.8 modifying the distribution used in Zolfaghari & Niari (2009). Then, the probabilities of the leakage and breakage of a pipe in an earthquake event are computed as the product of the failure probability obtained by use of the repair rate in Equation 4 and the probabilities of occurrence of either leakage or breakage when the pipe is damaged. The undamaged state is calculated as one minus the failure probability of the pipe by leakage or breakage.

### *3.2. Results and discussions*

Using the proposed flow based reliability analysis framework, the mean, the standard deviation, and the coefficient of variance (c.o.v.) of the flow rate in pipe 5 for given earthquake magnitudes are estimated in Table 1. In this table, the earthquake magnitudes 6.0, 7.0, and 8.5 are selected, and also unknown earthquake magnitude is considered by using the PDF of the earthquake magnitude used in Kang et al. (2008). It is first seen from the results that the flow in pipe 5 increases with the earthquake magnitude. This is because the pipes in the network are more likely to be damaged as the earthquake magnitude increases and will have more chances of water losses, and the flow in pipe 5 should increase to fill up the water lost to maintain the required outflow demand at node 4. In Figure 4, the probability of the water flow is plotted.

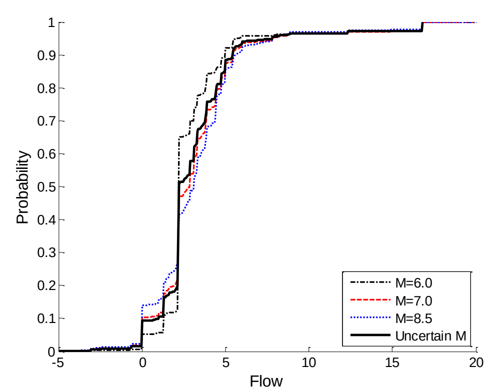


*Figure 4 Probability of water flow in pipe 5*

*Table 1. Mean, standard deviation, c.o.v. of water flow from node 1 to pipe 5*

| | *Mean ($ft^3/s$)* | *Standard deviation ($ft^3/s$)* | *c.o.v.* |
|---|---|---|---|
| *M = 6.0* | *3.0602* | *4.6059* | *1.5051* |
| *M = 7.0* | *3.3920* | *8.2345* | *2.4276* |
| *M = 8.5* | *3.4470* | *11.0899* | *3.2172* |
| *Uncertain M* | *3.3147* | *7.5096* | *2.2488* |

The results for pipe 6 are shown in Table 2. In this table, all mean values are negative, which means that the assumed flow direction is opposite to reality. For increasing earthquake magnitudes, the mean flow rates decrease due to water loss by leakage and breakage. Note that if we add the mean flow rate of pipes 5 and 6, they almost have a constant value close to the demand flow at node 4, for all earthquake magnitudes considered. This means that they try to maintain the required outflow demand at node 4. In Figure 5, the probability of water flow in pipe 6 is plotted.

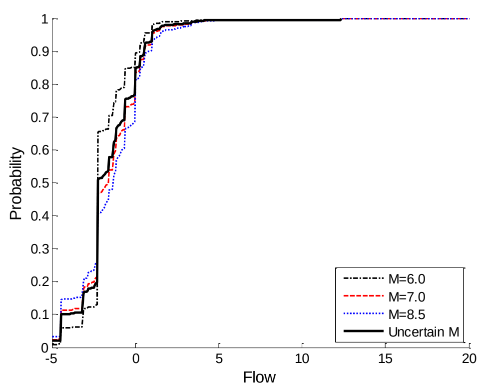


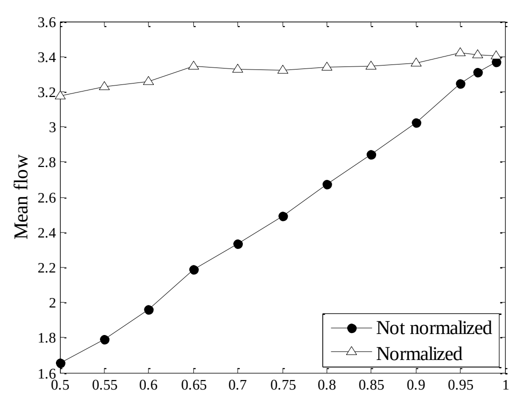

*Figure 5 Probability of water flow in pipe 6*

*Table 2. Mean, standard deviation, c.o.v. of water flow in pipe 6*

| | *Mean ($ft^3/s$)* | *Standard deviation ($ft^3/s$)* | *c.o.v.* |
|---|---|---|---|
| *M = 6.0* | *-1.7689* | *3.8992* | *2.2043* |
| *M = 7.0* | *-1.4388* | *7.8287* | *5.4413* |
| *M = 8.5* | *-1.2668* | *11.1008* | *8.7631* |
| *Uncertain M* | *-1.5103* | *7.0480* | *4.8678* |

It should be noted that, for all results and plots, the quantity vector **q** in Equation 2 was evaluated only once, and the change of the earthquake magnitudes and the corresponding changes in the pipe damage probabilities were handled only by updating the probability vector in Equation 1, which significantly saves computational costs as most computational efforts are needed in the evaluation of the quantity vector **q** requiring repeated evaluations of flow analyses.

To further reduce computational costs, the branch-and-bound method introduced in Section 2.4 is employed. The mean flow rates of pipes 5 and 6 are calculated as shown in Figure 6 for the earthquake magnitude $M$ = 7.0. The analysis is repeated for increasing number of branches in the branch-and-bound method represented by the sum of the probabilities in the probability vector, **p**. The results are represented as two cases such that the vector **p** is normalized by the sum of the probabilities and the vector **p** is not normalized. It is seen that the mean flow predictions converge to the values in Tables 1 and 2. The number of the identified branches and the computational costs are provided in Figure 6 and Figure 7 as normalized values by those for the full system states. The computational costs for the analysis for the full system states requires about 30 hours using MATLAB on a computer with Intel I7 CPU (2.80 GHz each) and 3GB of RAM.



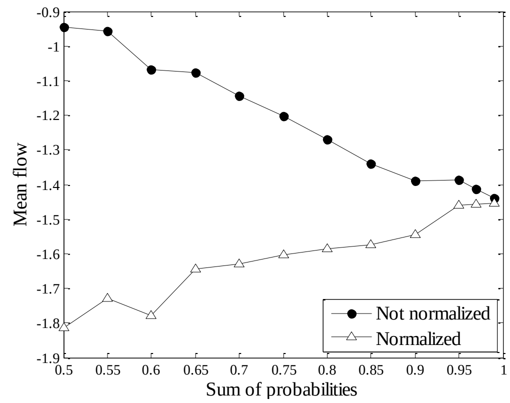



*Figure 6 Mean flow estimation for pipes 5 and 6 according to the sum of probabilities in a probability vector*

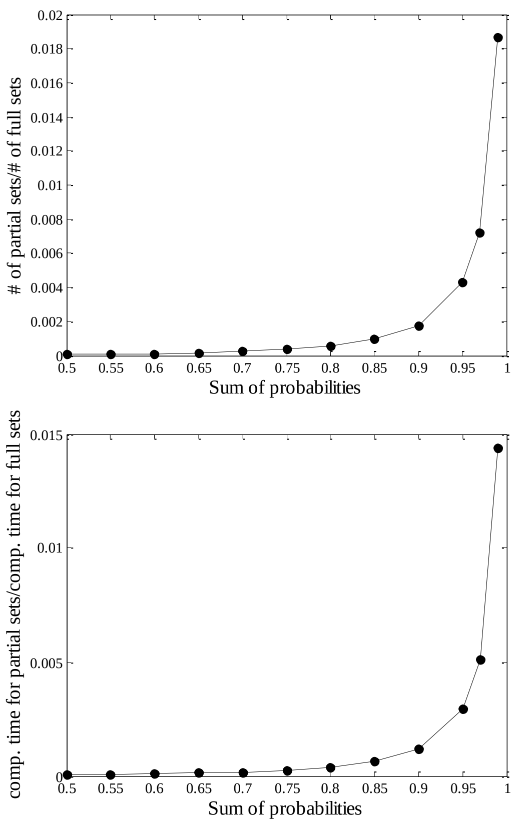



*Figure 7 The normalized number of identified system states (upper) and the normalized computation time (lower) according to the sum of probabilities in a probability vector*

It should be noted that the branch-and-bound method in the proposed framework is purely based on the order of the probabilities of system states without the consideration of system quantities, although a proper ordering should be made considering both the probabilities and the associated quantities because we are interested in the prediction of the quantities in terms of their partial descriptors. Hence, the convergence of the branch-and-bound method cannot be robust, and a sudden step change can occur even after a large number of branches are identified, which makes the determination of truncation point in the branch-and-bound method difficult. However, if the branching process is performed by considering both probabilities and quantities, it will significantly decelerate the computation because the order of the branches cannot be determined until each branch is fully expanded.

To avoid sudden quantity changes during branching processes, any expected extreme quantities with low probability should be pre-identified if possible, and they should be manually incorporated in the reliability analysis to avoid such convergence issues.

## 4. CONCLUSION

This study proposed an efficient system-level probabilistic flow analysis framework to estimate the post-hazard performance of water pipe networks in a probabilistic way. The framework is consisting of the two parts: (1) a post-hazard network modeling procedure; and (2) an efficient system-level probabilistic flow analysis algorithm developed based on the MSR method and the branch-and-bound method. The framework was demonstrated by an 11 pipe network after an earthquake event. For various seismic damage scenarios, the system performance was probabilistically measured in terms of the water flows of the network pipes. The analysis results showed that the flow rate of pipes are determined to maintain the required demand amount at a specified node. A computational efficiency was achieved using the branch-and-bound method by sequentially prioritizing system states according to their likelihoods and considering only important scenarios. This computational efficiency was demonstrated through the same example, and a discussion for convergence issues in the branch-and-bound method was drawn.

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