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On Standardization of the Reliability Basis of Structural Design

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ABSTRACT: The principles of structural reliability are sufficiently advanced to be used extensively to develop design standards even at international level. The concept of limit states is generally accepted and implemented. Present standards are however to a large extent based on past experience, rather than on quantitative reliability modeling. It appears that reliability principles and models can bridge the gap between probabilistic assessment and operational design. Representative theoretical models are presented to indicate how judgment based reliability concepts can be complemented or replaced by the use of such models. The importance of reliability levels, reference period, design working life, specified characteristic values of basic variables and methods to derive their design values from reliability procedures are demonstrated. On this basis a standardized basis of structural design can be formulated to convert the reliability principles (such as provided by ISO 2394) into operational design procedures (such as used by EN 1990 and other standards). It is concluded that reliability principles and models could contribute further to international harmonization of structural design.

1. INTRODUCTION

The implementation of the principles of structural reliability in standards for structural design has advanced beyond the first generation of limit states design procedures. Standards for use in common design practice are essentially based on these concepts.

Advancement includes the standardization of the principles of reliability. The Eurocode principle standard EN 1990 (2002) represents an example of an operational design standard, based on the principles of reliability, as standardized in ISO 2394 (1998). The potential of the reliability approach in achieving optimal structural performance has not yet been fully exploited. Arguably reliability concepts mainly serve as reference for schemes for the basis of design and even for quantitative measures. In many cases reliability procedures are still dominated by judgment.

The premise of this paper is that the quantitative implementation of standardized reliability concepts and models can contribute to further advancement of structural performance.

2. RELIABILITY BASIS OF DESIGN

2.1. Standardization based on Reliability

The Eurocode head standard EN 1990 (2002) is a prime example of an operational design standard expressed in terms of the reliability based partial factor limit states design format. Procedures for the treatment of actions and their combinations and material independent provisions are stipulated. This serves as the common basis for the balance of standards on specific actions, and materials based resistance.

The concepts of reliability from which EN 1990 is derived are summarized in its Annex C and further elaborated by Gulvanessian et al (2002) and the JCSS background documentation (1996). This body of information serves as the model for the reliability basis of structural design.

The general principles on the reliability of structures are standardized at an international level in ISO 2394 (1998). In revision ISO FDIS 2394 (2014) a distinction is made between risk-based, reliability-based and semi-probabilistic design. Alternative versions of the operational semi-probabilistic approach are the design value, partial factor or load and resistance factor design (LRFD) methods. Conversion of the reliability principles and general format for the semi-probabilistic approach presented in ISO FDIS 2394 still requires substantial additional input. This results in various degrees of divergence in the design standards.

At the international level ISO 22111 (2007) is nominally equivalent to EN 1990; but without any guidance on the application of the principles of reliability. Extension of this standard to include an annex to present standardized operational reliability procedures as outlined in this paper could serve as a platform for international unification between suites of standards nationally. The standards AIJ (2004), ASCE-7 (2010), AS/NZS 1170.0 (2011), CSA S408 (2011) and SANS 10160-1 (2011) provide examples of national unification.

The compilation of a set of operational reliability models and procedures could enhance further development of the rational basis of design for structural performance in a harmonized manner.

2.2. Outline of Reliability Procedures

The reliability models under consideration here are limited to those from which quantitative design parameters are derived. The models and design procedures are nevertheless directly coupled to the scheme of measures with the nature of reliability management, such as the prerequisites for competence and quality management; reliability classes, limit states and design situations; general design verification formats for actions, their combinations and resistance.

Taking the target level of reliability as a given starting point, reliability models should provide for its application in terms of reliability classes, design life and reference period, design situations, partitioning it into actions and their combination, resistance and failure modes.

The selection of design variables that need to be expressed probabilistically as basic variables provides another entry point into the process. Typically material properties are based on the specifications for their production. The reliability models for actions are based on a classification system and on the data and procedures for determining action values and their effects. Geometrical properties are typically taken at nominal values. Cases where special treatment is required, such as imperfections or geotechnical material properties need to be identified; also cases where simplifications can be made, such as material properties which are only indirectly related to resistance.

Structural analysis is only used to determine action effects at element level. Therefore it is typically treated generically in terms of various levels of approximation. However, it provides an important link between element reliability and integral structural reliability, specifically when considering robustness.

Finally, probability models of the basic variables are required to be used for the calibration of the set of partial factors required by the set of design situations, in accordance with the design verification format. Provision should also be made for model uncertainty.

This process should be extended from the inclusive basis of design to the partitioned treatment of actions and structural resistance. Allowance can thereby be made for the specific characteristics of the respective actions, materials and structural types, whilst unified consistency is maintained with the overall performance levels derived from the basis of design.

3. RELIABILITY PRINCIPLES

3.1. General

Reliability and risk-based semi-probabilistic design, as presented in ISO 2394 (1989) and implemented in EN 1990 (2002), include the design value and partial factors methods. The principles of reliability are presented in general terms in various references, such as the JCSS Probabilistic Model Code (JCSS 2001).

Due to several "safe" assumptions and simplifications, the operational procedures based on these principles may lead to conservative and uneconomic design (Holický and Schneider 2001). Moreover, these standards provide only vaguely formulated provisions concerning the target reliability level related to failure consequences, relative costs of safety measures, reference periods and to the working design life.

Two equivalent indicators of reliability level are used in general: the failure probability p_f and the reliability index β. Their mutual relationship can be expressed as

$$p_f = \Phi(-\beta) \tag{3.1}$$

Hereafter Φ denotes the standardized normal distribution. The reliability index β is more frequently used in standards, as its numerical values are more convenient than the values of the failure probability p_f.

3.2. Target reliability level

The target reliability level depends primarily on the consequences of failure and the relative costs of safety measures. The last parameter will be further analyzed in this contribution. When discussing the target reliability, the design working life should be distinguished from the reference period. The design working (service) life is understood as the assumed period for which a structure or a structural member is to be used for its intended purpose with anticipated maintenance, but without a substantial repair being necessary. The reference period is used as a basis for reliability verification. It is the period of time to which properties of all the basic variables (particularly the variable and accidental actions) are related (ISO 2394, 1998).

In some cases the reference period coincides with the design working life, but it may not always be so. For example, the reliability of a structure with a design service life of 50 years may be verified using data related to the reference period of 1 year. Then the relevant target reliability index related to 1 year should be greater than the reliability index related to 50 years. If the design service life coincides with the reference period, then the target reliability is almost independent of the number of years (neglecting discounting). The reliability index for the design service life of 5 years would be almost the same as for 50 years. However, the characteristics of time variant basic variables will be dependent on the design working life.

Probabilistic reliability methods are based on the comparison of the failure probability p_f with its target value p_t or the reliability index β with its target value β_t. It is generally required to design the structure in such a way that the determined level of reliability is close to the target values.

The target values of the reliability index β_t given in ISO 2394 (1998) and EN 1990 (2002) were derived mainly from previous reliability studies of structural members made from different materials. However, the obtained reliability indices depend on many factors (the type of component, loading conditions and structural materials) and, consequently, have a great scatter. It is also known that the results of any reliability study are significantly dependent on the assumed theoretical models used to describe the basic variables (Holický 2009).

Another possibility for specifying the target reliability index, or the target failure probability, is the economic optimization (Rackwitz 2000), Holický and Retief (2011) and Holický (2014), or requirement for human safety from the individual or social point of view, when the expected number of fatalities is taken into account (Fisher et al 2012). Commonly, the lethal accident rate of 10^{-6} per year, which corresponds to the reliability index $\beta_{t,1}$ = 4.7, is accepted by some codes. This value corresponds to the target reliability index accepted in EN 1990 (2002) for an ultimate limit state associated with medium consequences of failure, and for the reference period of one year, n = 1.

The reliability index $\beta_{t,n}$ for a reference period of n years and an independence period of k years ($k < n$) may be then approximated as

$$\Phi(\beta_{t,n}) = [\Phi(\beta_{t,1})]^{n/k} \qquad (3.2)$$

Figure 3.1 shows the variation of $\beta_{t,n}$ with $\beta_{t,1}$ for n = 5, 25, 50 and 100. Variation of $\beta_{t,n}$ with n for $\beta_{t,1}$=4 and k =1, 5 and 10 is shown in Figure 3.2.

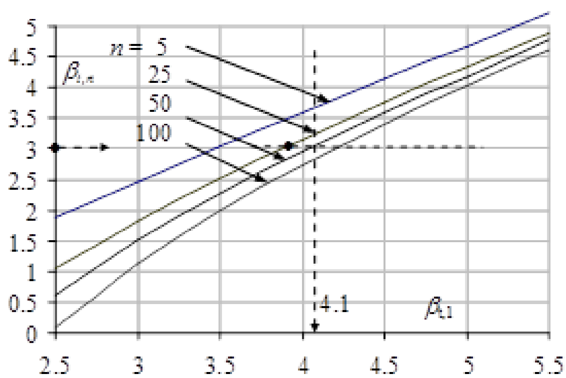

Figure 3.1: Variation of β_n with β_1 for k =1.

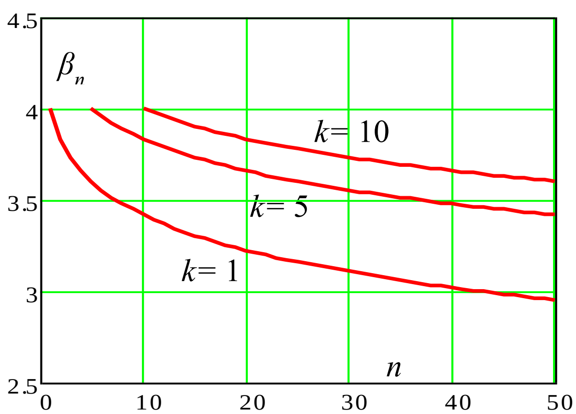

Figure 3.2: Variation of β_n with n for β_1 = 4 0.

It follows from Equation (3.2) and Figures 3.1, 3.2 that for n = 50 and k = 1 year (annual independence) $\beta_{t,50}$ = 3.0 corresponds approximately to $\beta_{t,1}$ = 4.0, if $n=k$ (full dependence) then $\beta_{t,1} = \beta_{t,n}$ is independent of the reference period n.

3.3. The design value method

The design value method, also called the “semi-probabilistic method” (in EN 1990 2002), is a very important step from probabilistic design methods toward operational partial factors methods. The design value method is directly linked to the basic principle, according to which it should be verified that no limit state is exceeded when the design values of all basic variables are used in the models of structural resistance R and action effect S. Thus, if the design values S_d and R_d of S and R are determined by taking into account the design values of all basic variables, then a structure is considered as reliable when

$$S_d < R_d \qquad (3.3)$$

where the design values S_d and R_d are symbolically expressed as

$$S_d = S\{F_{d1},F_{d2},\ldots\ a_{d1},a_{d2},\ldots\ \theta_{d1},\theta_{d2},\ldots\} \qquad (3.4)$$

$$R_d = R\{X_{d1},X_{d2},\ \ldots\ a_{d1},a_{d2},\ldots\ \theta_{d1},\theta_{d2},\ldots\} \qquad (3.5)$$

Here, S denotes a function describing the action effect, R a function describing the structural resistance, F is a general symbol for actions, X for material properties, a for geometrical properties, and θ for model uncertainties. The subscript ‘d’ refers to design values.

If only two variables S and R are considered, then the design values S_d and R_d may be found using the following formulae

$$P(S > S_d) = \Phi(+\alpha_S\beta) \qquad (3.6)$$

$$P(R \le R_d) = \Phi(-\alpha_R\beta) \qquad (3.7)$$

where β is the target reliability index, α_S and α_R, with $|\alpha| \le 1$, are the weight (sensitivity) factors determined by the First Order Reliability Method (FORM). The sensitivity factor α_S is negative for unfavorable actions and action effects, the resistance sensitivity factor α_R is positive. In EN 1990 it is recommended to accept the values α_S = – 0.7; α_R = 0.8.

4. PARTIAL FACTOR METHOD

4.1. Design values of basic variables

The design values of the basic variables, X_d, are commonly expressed in terms of the characteristic values X_k, i.e. values with a prescribed or intended probability of being exceeded. The characteristic values describe the basic variable by quantities independent of the required design working life and reliability level.

The characteristic values X_k are used to derive the design values X_d through division or multiplication by the appropriate partial factors. For material properties X or action F the design values can be expressed as

$$X_d = X_k/\gamma_M \text{ or for actions } F_d = \gamma_F F_k \qquad (4.1)$$

where γ_M denotes the partial factor of material properties and γ_F the partial factor of action. Both partial factors γ_M and γ_F are in most cases greater than 1. The above expressions for the design values of actions F and material properties X may be modified depending on the type of verified structural member and construction material.

Both partial factors γ_M and γ_F should include model uncertainties, which may significantly affect the reliability of a structure. The partial factor γ_F may be split into the load intensity uncertainty factor γ_f and model uncertainty factor γ_{Sd}. Similarly, the partial factor γ_M may be split into the material property factor γ_m and resistance model uncertainty factor γ_{Rd} (EN 1990 2002). Generally, it holds that

$$\gamma_F = \gamma_f\,\gamma_{Sd}, \quad \gamma_M = \gamma_m\,\gamma_{Rd} \qquad (4.2)$$

Numerical values of both the factors of model uncertainty depend on particular conditions and should be derived from previous experience and available experimental data. The load effect factor γ_{Sd} may be expected within the interval from 1.05 to 1.15. The resistance factor γ_{Rd} depends on the construction materials and behaviour of the structural member. For example, the uncertainty of the bending capacity of a steel beam will be lower (about 1.05) than the uncertainty of a welded connection capacity (about 1.15). Similarly, uncertainty of the bending moment capacity of a reinforced concrete beam will be lower than uncertainty of its shear capacity.

4.2. Partial factors for time invariant variables

Partial factor for resistance γ_m is defined in Equation (4.1) by fractiles X_k and X_d. Taking into account the general expression for fractiles of random variables, the factor γ_m may be written as

$$\gamma_m = \frac{X_k}{X_d} = \frac{\mu_X + u_{0.05}\,\sigma_X}{\mu_X + u_p \sigma_X} = \frac{1 + u_{0.05}\,w_X}{1 + u_p w_X} \qquad (4.3)$$

$$p = \Phi(-0.8\beta)$$

where w_X denotes coefficients of variation of X, $u_{0.05}$ or u_p denotes 5%- or p-fractile of the standardised random variable having the same probability distribution as the resistance X (5%-fractile $u_{0.05}$ is commonly accepted for specifying the characteristic values X_k).

Consider a permanent load G (self-weight) with a normal distribution. It is assumed that the characteristic value G_k of G is defined as the mean μ_G : $G_k = \mu_G$. The design value G_d is

$$G_d = \mu_G - \alpha_G \beta \sigma_G = \mu_G + 0.7\beta\sigma_G = \mu_G(1+0.7\beta\, w_G) \qquad (4.4)$$

In Equation (4.4) μ_G denotes the mean, σ_G the standard deviation, w_G the coefficient of variation and $\alpha_G = -0.7$ the sensitivity factor. The partial factor γ_G of G is given as

$$\gamma_g = G_d / G_k = (1 + 0.7\beta\, w_G) \qquad (4.5)$$

4.3. Partial factors for variable loads

A similar procedure as for time-invariant variables can be used for the estimation of the partial factors γ_q for variable loads Q. Assuming the Gumbel distribution, the characteristic value is usually defined as 0.98 fractile of annual extremes (or extremes related to a certain basic reference period) and is given by Equation (4.6) and the design value Q_d related to the design working life or other reference period is given by Equation (4.7)

$$Q_k = \mu_Q\,(1 - w_Q\,(0.45 + 0.78\ \ln(-\ln(0.98)))) \quad (4.6)$$

$$Q_d = \mu_Q\,(1 - w_Q\,(0.45 - 0.78\alpha_T \ln(N) + 0.78\ \ln(-\ln(\Phi^{-1}(-\alpha_S\beta))))) \quad (4.7)$$

In Equations (4.6) and (4.7) commonly assumed Gumbel distribution is considered (Holický 2009), μ_Q denotes the mean, w_Q the coefficient of variation of extreme values of Q determined for the basic reference periods (that is, for example 1 or 5 years), N denotes the ratio of the working design life, for example 50 years (or other reference period), and the basic reference period. As an example, the period ratio N = 10 (= 50/5) is considered below. Finally, α_S = – 0.7 is the sensitivity factor of Q, and α_T is the time sensitivity factor given by the ratio w'_Q / w_Q ,where w'_Q denotes the coefficient of variation of the time-dependent component of Q , and w_Q denotes the coefficient of variation of the total Q. When Q depends on time-dependent components only, w'_Q = w_Q and α_T = 1. Note that the reliability index β in Equation (4.7) is related to the design working life (for example to the reference period of 50 years) and not to the basic reference period (for example to 1 or 5 years).

The partial factor γ_q of Q is given as (EN 1990 2002)

$$\gamma_q = Q_d / Q_k \quad (4.8)$$

The partial factor γ_q of a variable action Q defined by Equation (4.8) depends on the parameters w_Q, α_S, β (used also in the case of time-invariant basic variables), the partial factor of variable actions γ_Q depends also on the period ratio N and on the time sensitivity factor α_T. Figure 4.1 shows the variation of γ_q with the coefficients of variation w_Q for β = 3, assuming a Gumbel distribution of Q, and the period ratio N = 50 (the reference period 50 times greater than the basic reference period) and selected time sensitivity factors α_T.

Consider wind action, with a coefficient of variation of annual intensities w_Q = 0.3: Assume that N = 50 and α_T = 0.6. For the reliability index β = 3.0 it follows from Figures 4.1 that the partial factor γ_q = 1.32; for the model uncertainty γ_{Sd} = 1.05, then from Equation (4.2):

$$\gamma_Q = \gamma_q\,\gamma_{Sd} = 1.32\times1.05 = 1.39 \approx 1.4$$

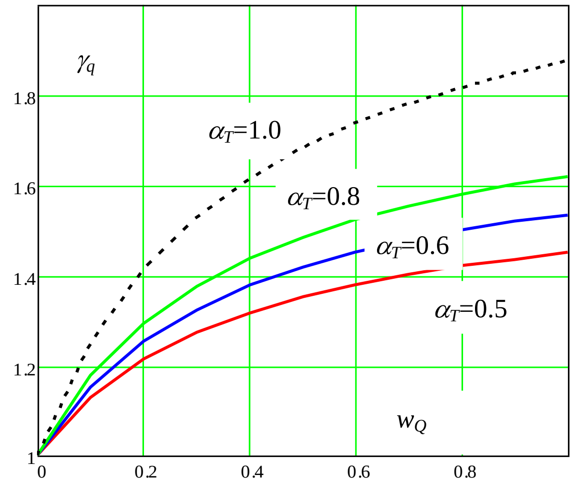

Figure 4.1: Variation of γ_q with the coefficient of variation w_Q for the reliability index β = 3.0, N = 50 and α_T = 0.5, 0.6, 0.8 and 1.0, assuming a Gumbel distribution.

However, Gumbel distribution may be a rather conservative assumption for wind action as it has a relatively high positive skewness (1.14) enhancing occurrence of great values. If other distribution with lower skewness was more appropriate, then the partial factor for wind loads might be lower.

5. APPLIED RELIABILITY BASIS

The principles and models presented above should be used to derive an outline of a standard for the reliability basis of design. This is partly based on experience with adaptation of the comprehensive EN 1990 (2002) to SANS 10160-1 (2011) with its scope limited to buildings.

The entry point for quantitative reliability procedures is the target level of reliability $\beta_{t,n}$ with typical values for $\beta_{t,50}$ ranging between 3.0 and 3.8. The way in which $\beta_{t,n}$ is adjusted for reference period and reliability classes should be given. Separation into target levels for different standards on actions and resistance is based on Equations (3.6) and (3.7). Further differentiation of reliability levels for limit states and design situations should be provided.

Standard practice for the specification of basic variables (X_k) should be taken into account when accounting for the reliability properties of representative (characteristic) values. Specified X_k values should therefore be independent of reliability levels, design working life and only be dependent on the distribution type and variability.

When the characteristic values of basic variables are not specified independently as input to the design process, such a specification is included as part of the design process, e.g. by specifying for variable actions (Q_k) at the p = 0.98 annual maximum fractile. For geotechnical strength properties fractile values for the mean or extreme value should be specified, depending on whether global or local failure modes are considered.

The formulation of action combination schemes for the various design situations represents a key element in the reliability basis of design. This scheme ties together the respective combinations of permanent, variable and accidental actions and the associated expressions of resistance for the specified design situations.

A combination of EN 1990 Equations 6.10(a&b) could serve as the fundamental an action combination scheme. Design formats can be derived by adaptation and simplification in terms of the partial factors ($\gamma_{G,i}$; $\gamma_{Q,i}$) and combination factors (ξ; $\psi_{0,i}$). The three schemes allowed by EN 1990 and one by SANS 10160-1 can all be derived from Equation (5.1) for the permanent ($G_{k,i}$) and variable ($Q_{k,i}$) actions.

$$\sum \xi \gamma_{G,i} G_{k,i} + \gamma_{Q,1} \psi_{0,1} Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \psi_{0,i} Q_{k,i} \tag{5.1}$$

For resistance an equivalent treatment to that of actions is required. The general principles of reliability given in ISO 2394 (1998 & 2914) should be developed to provide a common basis for the various structural materials and the diverse set of failure modes. This should provide improve unification between separate standards for actions and materials; equally important is unification from structural steel to geotechnical design.

Provisions that need to be considered include the specification of characteristic values of basic variables and how these values are related to specification, testing and quality measurement. Differentiation between classes of failure modes such as ductility, stability, composite behavior, connections in terms of reliability classes and design situation should be also reflected.

The systematic development of the standardized reliability basis for structural resistance can be based on how EN 1990 (2002) relates to the Eurocode standards for the respective materials and the way in which the *fib* Model Code 2010 (Bigaj-van Vliet & Vrouwenvelder 2013) treats this topic.

6. CONCLUSIONS

Within the extensive value chain of standardized reliability-based design procedures (JCSS-PMC, ISO 2394, EN 1990/CSA S408/SANS 10160-1), dedicated reliability models will advance the implementation of quantitative reliability based design. This should further complement the extensive role of judgment-based prerequisites and requirements which are conceptually based on principles of reliability.

Standardization of the structural design is unavoidably based on the probabilistic theory of structural reliability. In particularly the First Order Reliability Method (FORM) can be effectively used for estimating the reliability elements used in various forms of the partial factor method. It provides operational techniques for estimating the design points, partial and reduction factors and for calibration procedures.

However, present standards based on partial factor methods are often conservative due to various “safe” assumptions and simplifications. Consequently, application of these standards may lead to rather uneconomical structures. Present standards are also difficult to use for the design of structures having unusual requirements concerning reliability and design working life. Operational procedures applicable by practicing designers are needed.

An important aspect thereby is the suitability of theoretical probabilistic models for time- variant actions. Using the traditional Gumbel distribution to describe variable actions may lead to an overestimation of the resulting design values (particularly for imposed load and climatic actions) and excessive (unrealistic high) partial factors.

Furthermore, the specification of various reliability elements in the standards for structures is always partly based on tradition and past experience. Such an approach is fully justified for climatic actions and traditionally-used construction materials. Consequently, it might lead to considerably diverse reliability elements being used in different countries. That is why a number of reliability elements and parameters in the present standards differ and are open according to national choice. Here, again, the theory of structural reliability may be an extremely useful tool to clarify different views.

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