A Fully Parametric Non-Stationary Spectral-Based Stochastic Ground Motion Model

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ABSTRACT: A novel strong ground motion stochastic model is formulated in association with physically interpretable parameters that are capable of efficiently characterizing the complex evolutionary nature of the phenomenon. A multi-modal, analytical, fully non-stationary spectral version of the Kanai-Tajimi model is introduced achieving a realistic description of the evolutionary spectral energy distribution of the seismic ground motions. The functional forms describing the temporal evolution of the model parameters can effectively model complex highly non-stationary power spectral characteristics. The analysis space, where the analytical forms describing the evolution of the model parameters are established, is the energy domain instead of the typical use of the time domain. The Spectral Representation Method facilitates the simulation of sample model realizations.

1. INTRODUCTION

The increased and constantly rising interest in performance-based earthquake engineering, in parallel to modern code requirements, has enhanced the need for reliable, diverse and realistic ground motion time-histories. Naturally, the availability of actual seismic records pertaining to certain earthquake scenarios can be proven of quite limited size. Engineers are thus frequently forced to scale and/or modify the spectral content of actual records. It is obvious that this procedure, mainly motivated by necessity, is fraught with specific concerns regarding the resulting representation of the ground motions. Structural response is sensitive to input loading characteristics and extreme care should be given to an astute input modeling. Apart from overcoming the record scaling and/or spectral matching concerns, simulated earthquake waveforms based on stochastic representations have the extra advantage that can be directly used for stochastic dynamic analyses. In parallel to that, a description of the seismic hazard in the form of simulated waveforms provides a meticulous characterization of the seismic risk. Various stochastic ground motion models have been formulated in the past, ranging from time-domain approaches, e.g. Rezaeian and Der Kiureghian (2008), to spectral-based, e.g. Deodatis and Shinozuka (1988); Conte and Peng (1997), and wavelet-based, e.g. Spanos and Failla (2004); Yamamoto and Baker (2013).

In this paper, a new analytical non-stationary spectral stochastic ground motion model is presented in association with physically interpretable parameters. The model formulation is based on a novel multi-modal, fully non-stationary spectral version of the Kanai-Tajimi model (Kanai (1957); Tajimi (1960)). In relation to all previous works in the literature, the presented model is the first fully analytical model to date that is capable of directly and efficiently describing multi-modal evolutionary power spectral densities, allowing for a realistic description of the spectral energy distribution over time. The introduced model and the developed functional forms describing the temporal evolution of its parameters are capable of efficiently representing complex non-stationarities. The analysis space, where the model's analytical forms are established, is the energy domain, instead of the typical use of the time domain. The model is completely defined in parametric form and supports ground motion simulations through the Spectral Representation Method (Shinozuka and Deodatis (1991); Deodatis (1996); Liang et al. (2007)). The parametric character of the model allows also its straightforward implementation in random vibration problems. By treating the model parameters as random variables and by examining a large ground motion database, the associated seismic risk can be effectively quantified through identification of the distribution patterns of the model parameters, in addition to estimating their correlation structure. In this way, the natural variability of the database ground motions' stochastic nature can be effectively captured. Such an analysis for a subset of the NGA West database can be found in detailed form in the under review paper by Vlachos et al. (2015).

2. NON-STATIONARY SPECTRAL ESTIMATION METHOD

The fully non-stationary spectral estimation of the earthquake ground motion records is performed by employing the Short-Time Multiple-Window estimation technique as formulated and presented in great detail by Conte and Peng (1997). Based on the assumption of local stationarity in the underlying stochastic process, multiple orthogonal leakage-resistant moving time-windows are employed to extract the spectral content of each signal segment. The resulting multiple local spectra are then averaged in a weighted sense to produce the non-stationary spectral estimate of the target process. The resulting non-stationary power spectrum is a consistent estimator of the true spectrum, not hampered by the usual trade-off between bias and spectral leakage.

3. EVOLUTIONARY MULTI-MODAL SPECTRAL-BASED MODEL

3.1. *Evolutionary Bimodal Kanai-Tajimi Model*

The presented parametric, multi-modal, fully non-stationary model is based on the superposition of classical unimodal Kanai-Tajimi expressions. Without any loss of generality, a bimodal version is presented in this paper, which is deemed adequate for the objectives of this work. The employment of a high-pass filter is also necessary since it appropriately remedies the signal contamination due to long-period noise in the simulated waveforms, e.g. Liao and Zerva (2006). The employed bimodal evolutionary model is thus expressed as:

$$S_{XX}(f,t) = |HP(f)|^2 \sum_{k=1}^{K=2} S_o^{(k)}(t) \times \frac{1+\left(2\zeta_g^{(k)}(t) f/f_g^{(k)}(t)\right)^2}{\left(1-\left(f/f_g^{(k)}(t)\right)^2\right)^2+\left(2\zeta_g^{(k)}(t) f/f_g^{(k)}(t)\right)^2} \quad (1)$$

where $S_{XX}(f,t)$ is the model evolutionary power spectrum, $HP(f)$ is the deterministic Butterworth high-pass filter and $\left\{f_g^{(k)}(t), \zeta_g^{(k)}(t), S_o^{(k)}(t)\right\}$ are the time-varying dominant modal frequency, modal apparent damping ratio and modal participation factor with respect to the k^{th} mode respectively. The modal numbering is directly associated with the numerical value of the respective modal dominant frequencies, being ordered in an ascending way, i.e. $f_g^{(1)}(t)$ is always the numerically smallest modal frequency.

3.2. *High-Pass Butterworth Filter*

The high-pass filter used in this study is a deterministic high-pass (low-cut) Butterworth filter of 4^{th} order. The energy content of the transfer function of the Butterworth filter is expressed as:

$$|HP(f)|^2 = \frac{(f/f_c)^{2N}}{1+(f/f_c)^{2N}} \quad (2)$$

where $N = 4$ indicates the order of the filter and f_c its corner frequency. Following Liao and Zerva (2006), the corner frequency is selected in a purely numerical way such that most of the low-frequency components of the ground motion, corresponding to periods longer than the duration T_d of the seismic record, are practically eliminated by specifying a low amplitude energy threshold for the frequency $f_d = 1/T_d$. The adopted energy threshold H_d is set to 10^{-3} uniquely defining the cut-off frequency as:

$$f_c = \frac{1}{T_d \left(H_d^2 / \left(1 - H_d^2\right) \right)^{1/2N}} \tag{3}$$

where $f_c \in [0.10, 0.25]\, Hz$.

3.3. *Model Parameter Identification*

Employing the Short-Time Multiple-Window non-stationary spectral estimation technique as outlined in Section 2, in conjunction with the bimodal evolutionary Kanai-Tajimi model as in Eq. (1) and the deterministic Butterworth filter as in Eqs. (2)-(3), allows for the estimation of the model parameters as shown below. The 6-parameter set $\widehat{\mathbf{x}}_t$ defined as:

$$\widehat{\mathbf{x}}_t = \left\{ \widehat{f_g^{(1)}}(t),\ \widehat{\zeta_g^{(1)}}(t),\ \widehat{S_o^{(1)}}(t),\ \widehat{f_g^{(2)}}(t),\ \widehat{\zeta_g^{(2)}}(t),\ \widehat{S_o^{(2)}}(t) \right\} \tag{4}$$

is identified as the best estimator of the true parameter time-varying set $\mathbf{x}_t$ of the seismic record in the non-linear least-squares sense. The minimization is performed for every time instant over the entire frequency domain and can be expressed as:

$$\widehat{\mathbf{x}}_t = \arg\min_{\mathbf{x}_t} \int_{-\infty}^{+\infty} \left(S_{XX}^{(s\text{-}t)}(f,t) - S_{XX}(f,t,\mathbf{x}_t) \right)^2 df \tag{5}$$

where $S_{XX}^{(s\text{-}t)}(f,t)$ is the evolutionary spectral estimate of the signal as obtained by the Short-Time Multiple-Window technique and $S_{XX}(f,t,\mathbf{x}_t)$ is the model power spectrum as in Eqs. (1)-(3). It is important to be noted here that the analytical forms describing the temporal evolution of the identified modal characteristics are presented in the following pages without any sort of hat accents.

3.4. *Transformation of Time-Domain to Energy-Domain*

One of the innovative characteristics of the presented study is that the analysis space, where the analytical forms describing the temporal evolution of the previously identified spectral characteristics are established, as shown later, is the energy domain instead of the typically employed time domain. Based on this choice, the modeling of the evolving spectral content becomes more accurate in the strong shaking part of the ground motion, where the greatest interest for engineering purposes lies. The adopted non-dimensional energy definition is analogous to the Arias Intensity by Arias (1970) and is given by:

$$\varepsilon(t) = \frac{\int_0^t x^2(\tau)\, d\tau}{\int_0^{T_d} x^2(\tau)\, d\tau} \tag{6}$$

where $t \in [0, T_d]$ and $\varepsilon(t) \in [0,1]$ is the non-dimensional cumulative energy contained in the $[0,t]$ fraction of the seismic record $x(t)$. In essence, Eq. (6) provides a non-linear mapping between the time domain and the non-dimensional energy domain, allowing higher resolution in the high amplitude part of the seismic ground motion.

3.5. *Analytical Forms Expressing Temporal Evolution of Model Parameters*

3.5.1. *Dominant Modal Frequencies*

The evolutionary nature of the frequency content of seismic ground motion records arises due to the non-concurrent intermingling of the different types of seismic waves at the site, as a direct result of their different propagating velocities. The body waves, P and S waves, arrive faster compared to the surface waves, Rayleigh and/or Love waves, with the S-waves typically comprising the dominant portion of the strong motion part of the seismic records. Body waves are typically characterized by higher frequency content as compared to the surface waves, and are attenuated more as the source-to-site distance increases. Furthermore, local site conditions can significantly amplify the long period spectral energy. As a result of this apparent complexity, the modal dominant frequencies cannot be accurately characterized by only one specific temporal pattern, e.g. linear. Thus, the usually decaying nature of the dominant frequencies needs to be described by a versatile parametric expression being able of effectively describing different patterns of temporal variation. The analytical form in this paper describing the evolution of the two identified modal dominant frequencies in the non-dimensional cumulative energy domain is given as:

$$f_g^{(k)}(\varepsilon) = Q_k \left(\frac{1}{2} + \varepsilon \right)^{\alpha_k} \left(\frac{3}{2} - \varepsilon \right)^{\beta_k} \tag{7}$$

for $k = 1,2$ and $\{Q_k, \alpha_k, \beta_k\}$ being the parameters of the analytical form with respect to the k^{th} mode, where $\alpha_k, \beta_k \in \Re$ and $Q_k > 0$.

In Fig. 1, the aforementioned fitted expression is provided in colored thick line for both modes with respect to the Parkfield seismic record, along with the associated record identified values $\left\{\widehat{f_g^{(1)}}(\varepsilon), \widehat{f_g^{(2)}}(\varepsilon)\right\}$ represented in thin black line. The used Parkfield seismic record is the component 320 of the Cholame-Shandon Array #8 recording (NGA Sequence No: 31) of the Parkfield 1966 earthquake ($M_w = 6.19$). It is important to be noted here that the entirety of the fitted functional form parameter values pertaining to the Parkfield record can be found in the Appendix.

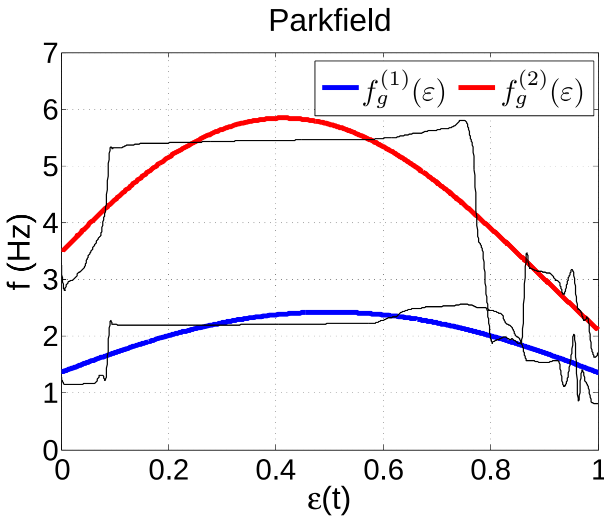

Figure 1: Dominant modal frequencies fitting for the Parkfield record.

3.5.2. *Modal Apparent Damping Ratio*

Considering the damping ratio as the signal bandwidth, there are observations according to which the signals exhibit a trend of having broader bandwidth at their beginning and end. On the other hand, one may approach the apparent damping as the equivalent viscous damping of a complex system experiencing material damping, which is known to be amplitude dependent, therefore resulting in higher apparent damping during the strong motion part of the record. Furthermore, considering the high sensitivity of the damping ratio identification process and aiming to a sophisticated but also concise model, the modal apparent damping ratio is considered a constant, however different for each mode. It is eventually calculated as the averaged identified damping ratio in the strong shaking part of the seismic record, with the latter being defined as the part of the ground motion between 5% and 95% of the seismic record energy, given as:

$$\zeta_g^{(k)}(\varepsilon) = \zeta_g^{(k)} = \overline{\left\{\widehat{\zeta_g^{(k)}}(\varepsilon) \middle| \varepsilon \in [0.05, 0.95]\right\}} \quad (8)$$

for $k = 1,2$, where $\overline{\{\cdot\}}$ denotes the averaging operator.

3.5.3. *Modal Participation Factors*

The identified modal participation factors $\left\{\widehat{S_o^{(1)}}(\varepsilon), \widehat{S_o^{(2)}}(\varepsilon)\right\}$ are not directly associated to the spectral energy carried by each mode and thus their numerical values mainly characterize the identified linear combination of the modes, as in Eq. (1). Consequently, their relative values are far more important compared to their absolute ones, resulting in their normalization by the first modal participation factor. Accordingly, the logarithmic normalized modal participation factors are defined as follows:

$$\begin{aligned} \overline{\overline{S_o^{(k)}}}(\varepsilon) &= \widehat{S_o^{(k)}}(\varepsilon) / \widehat{S_o^{(1)}}(\varepsilon) \\ \widehat{R^{(k)}}(\varepsilon) &= log_{10}\left[\overline{\overline{S_o^{(k)}}}(\varepsilon)\right] \end{aligned} \quad (9)$$

for $k = 1,2$. It is apparent that the above mentioned definition leads to $\overline{\overline{S_o^{(1)}}}(\varepsilon) = 1$ and $\widehat{R^{(1)}}(\varepsilon) = 0$. Due to the oscillatory character of the second logarithmic normalized participation factor, its modeling needs to be performed with a flexible parametric expression. The chosen analytical form is a mixture of two bell-shaped Gaussian functions, as follows:

$$\begin{aligned} R^{(2)}(\varepsilon) =& F^{(I)} \exp\left[-\left(\frac{\varepsilon - \mu^{(I)}}{\sigma^{(I)}}\right)^2\right] + \\ & F^{(II)} \exp\left[-\left(\frac{\varepsilon - \mu^{(II)}}{\sigma^{(II)}}\right)^2\right] - 2 \end{aligned} \quad (10)$$

where $\{F^{(I)}, F^{(II)}\}$ are the scaling factors, $\{\mu^{(I)}, \mu^{(II)}\}$ the peak locations and $\{\sigma^{(I)}, \sigma^{(II)}\}$

parameters controlling the width of the 'bells'. The scaling factors F and the width-controlling parameters σ are not allowed to take negative values. The labeling of this parametric set is directly associated with the numerical values of $\{\mu^{(I)}, \mu^{(II)}\}$, assigning always the numerically smaller of the two identified peak locations to $\mu^{(I)}$ and the larger to $\mu^{(II)}$. In Fig. 2, the fitted $R^{(2)}(\varepsilon)$ expression is provided for the Parkfield record, along with the associated record identified values, $\widehat{R^{(2)}}(\varepsilon)$, represented in thin black line.

3.5.4. *Energy Accumulation and Amplitude Modulating Function*

The use of the normalized form of the modal participation factors, as described in Eq. (9), results in the non-uniform temporal modulation of the power spectral intensity. In order to rectify the modulated signal intensity, the evolutionary power spectrum is converted to unit-variance at each time instant, through equating its integral over the entire frequency domain to unity, subsequently followed by the introduction of an amplitude modulating function $z(t)$. The analytical form of the resulting fitted bimodal evolutionary power spectrum is then given as:

$$S_{XX}(f,t) = z^2(t) \times \underbrace{|HP(f)|^2 \sum_{k=1}^{K=2} S_o^{(k)}(\varepsilon(t)) \frac{1+\left(2\zeta_g^{(k)} f/f_g^{(k)}(\varepsilon(t))\right)^2}{\left(1-\left(f/f_g^{(k)}(\varepsilon(t))\right)^2\right)^2+\left(2\zeta_g^{(k)} f/f_g^{(k)}(\varepsilon(t))\right)^2}}_{\text{unit-variance}} \tag{11}$$

In order to generally express the parameters of Eq. (11) in the time domain, a parametric form is required to describe the non-dimensional energy accumulation over time, as defined in Eq. (6). The following non-dimensional analytical form is proven to effectively portray this measure, pertaining to the seismic records of the selected NGA West database:

$$\varepsilon(t) = \frac{e^{-\left(\frac{t/T_d}{\gamma}\right)^{-\delta}}}{e^{-\left(\frac{1}{\gamma}\right)^{-\delta}}} \tag{12}$$

where $\{\gamma, \delta\}$ are positive scale and shape parameters respectively.

It is also demonstrated that there is a remarkable connection between the amplitude modulating function introduced in Eq. (11) and the energy accumulation function described in Eq. (12). Consider the following slightly different definition of the accumulated energy, adopting now a dimensional form, as follows:

$$\frac{\varepsilon(t)}{I_x} = \frac{e^{-\left(\frac{t/T_d}{\gamma}\right)^{-\delta}}}{e^{-\left(\frac{1}{\gamma}\right)^{-\delta}}} \tag{13}$$

where $\varepsilon(t) \in [0, I_x]$ and I_x is the total energy content of the seismic record $x(t)$ defined as:

$$I_x = \int_0^{T_d} x^2(\tau)\, d\tau \tag{14}$$

Equating the signal energy accumulation at any arbitrary time instant $t \in [0, T_d]$ with the respective cumulative evolutionary spectral energy as follows:

$$\varepsilon(t) = \int_0^t \int_{-\infty}^{+\infty} S_{XX}(f,\tau)\, df\, d\tau \tag{15}$$

and considering Eq. (11) results to:

$$\varepsilon(t) = \int_0^t z^2(\tau)\, d\tau \tag{16}$$

Finally, Eqs. (13) and (16) lead to the following definition of the amplitude modulating function $z(t)$ as:

$$\frac{z^2(t)}{I_x} = \frac{\delta/\gamma}{T_d} \cdot \frac{e^{-\left(\frac{t/T_d}{\gamma}\right)^{-\delta}} \left(\frac{t/T_d}{\gamma}\right)^{-1-\delta}}{e^{-\left(\frac{1}{\gamma}\right)^{-\delta}}} \tag{17}$$

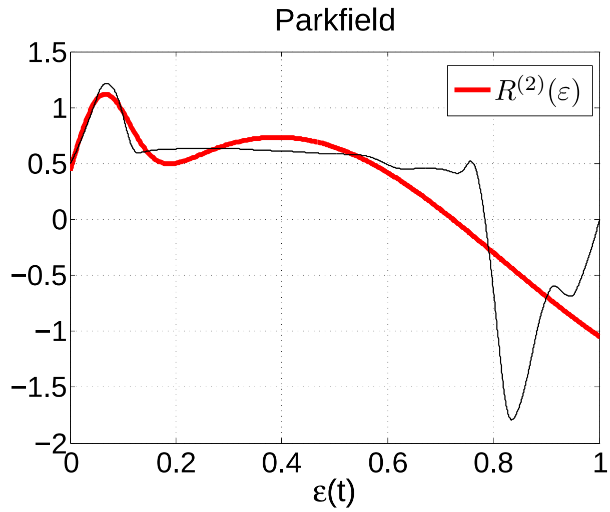

Figure 2: Logarithmic normalized modal participation factor $R^{(2)}(\varepsilon)$ fitting for the Parkfield record.

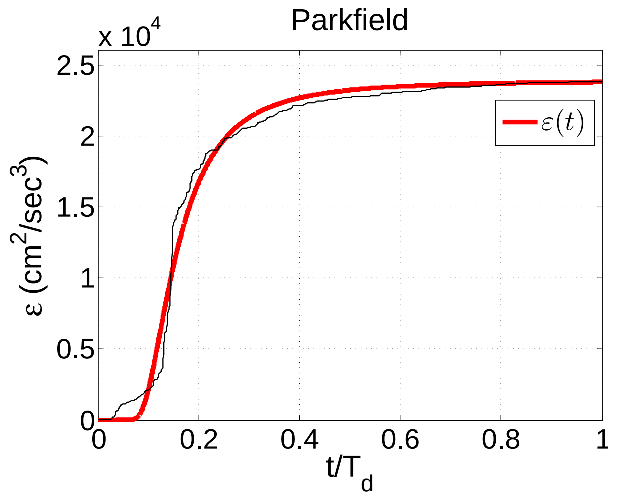

Figure 3: Energy accumulation function $\varepsilon(t)$ fitting for the Parkfield record.

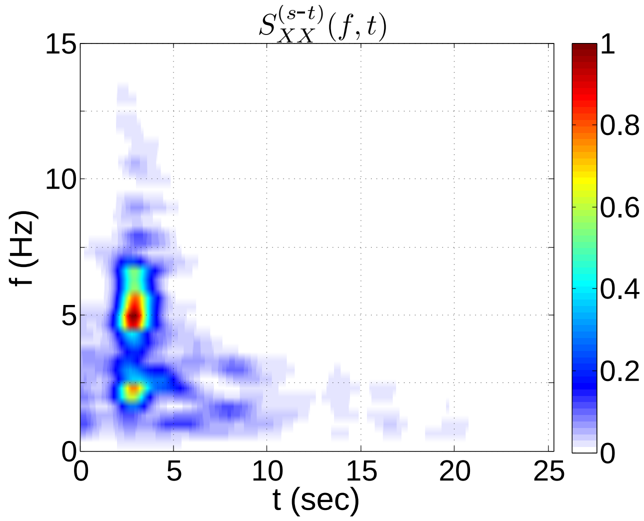

Figure 4: Estimated power spectrum $S_{XX}^{(s\text{-}t)}(f,t)$ for the Parkfield record.

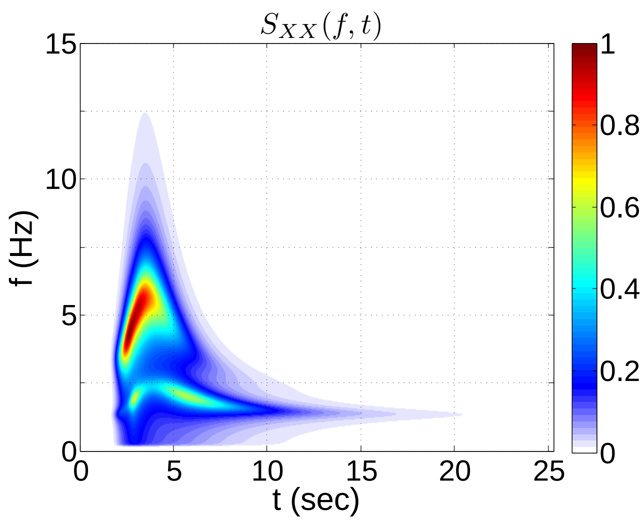

Figure 5: Model evolutionary power spectrum $S_{XX}(f,t)$ for the Parkfield record.

Hence, the modeling of the energy accumulation function serves two distinct and very important goals; the first one being the mapping between the time and energy domains and the second one being the simultaneous formation of the required amplitude modulating function $z(t)$, responsible for the time modulated signal intensity. In Fig. 3, the fitted dimensional version of the energy $\varepsilon(t)$ expression of Eq. (13) is provided with respect to the Parkfield record, along with the associated record cumulative energy represented in thin black line. In Figs. 4–5, the evolutionary spectral estimate $S_{XX}^{(s\text{-}t)}(f,t)$ of the Parkfield record is provided, along with the associated fitted model evolutionary spectrum $S_{XX}(f,t)$ of Eq. (11) respectively. The evolutionary power spectra in the two figures are plotted in a non-dimensional normalized form for visualization purposes. The absolute magnitudes of the spectra are directly associated with the energy accumulation function $\varepsilon(t)$ fit, provided in Fig. 3

4. SIMULATION OF THE DEVELOPED EVOLUTIONARY BIMODAL KANAI-TAJIMI POWER SPECTRUM

Following the complete mathematical description of the model, the Spectral Representation Method, Shinozuka and Deodatis (1991); Deodatis (1996); Liang et al. (2007), can be engaged in a straightforward manner for the stochastic ground motion simulations. Consider a zero-mean, real-valued non-stationary stochastic process $x_0(t)$ with two-sided evolutionary power spectrum $S_{x_0x_0}(f,t)$, as in Eq. (11). It has been shown, Liang et al. (2007), that the stochastic process $x_0(t)$ can be represented by the following series as $N \to \infty$:

$$x(t) = 2\sum_{n=0}^{N-1}\left[2S_{x_0x_0}(f_n,t)\,\pi\Delta f\right]^{1/2}\cos\left(2\pi f_n t + \Phi_n\right) \tag{18}$$

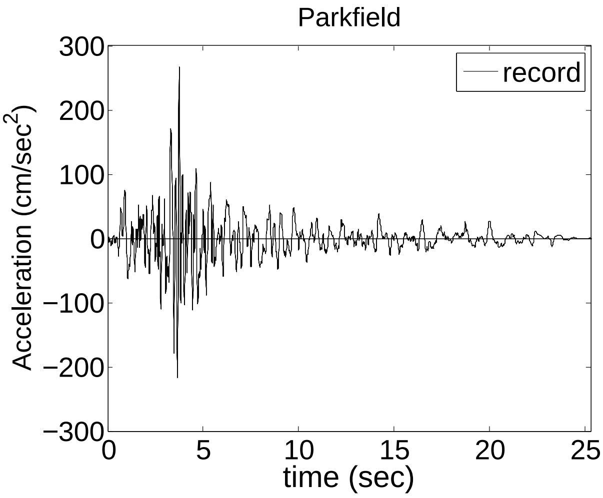

Figure 6: Parkfield 1966, Cholame - Shandon Array #8 (NGA No: 31), component 320.

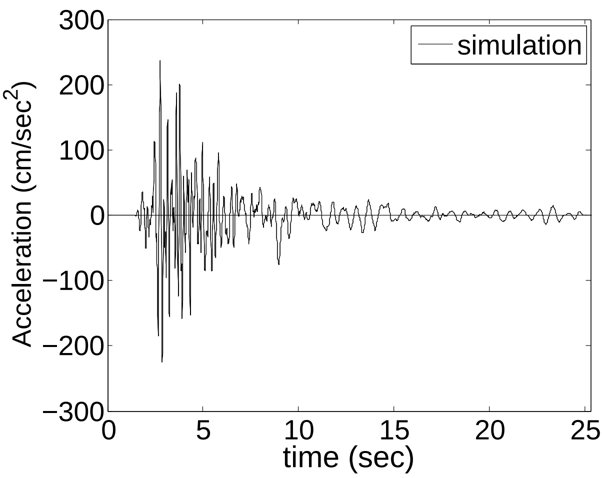

Figure 7: Sample acceleration of the Parkfield record.

where a suitably discretized version of the evolutionary power spectrum is needed and:

$$\Delta f = f_u/N\ , \quad f_n = n\Delta f\ , \quad S_{x_0x_0}(f_0 = 0,t) = 0 \quad (19)$$

In Eq. (19), f_u represents an upper cut-off frequency beyond which the evolutionary power spectrum $S_{x_0x_0}(f,t)$ may be assumed to possess negligible amount of power for either mathematical or physical reasons. The $\{\Phi_0, \Phi_1, \Phi_2, ..., \Phi_{N-1}\}$ in Eq. (18) are N independent random phase angles distributed uniformly over the interval $[0, 2\pi]$. The simulated stochastic process $x(t)$ is asymptotically Gaussian as $N \to \infty$ due to the central limit theorem.

Following the simulation of seismic acceleration time-histories, velocity and subsequently displacement time-histories can be obtained via numerical integration. However, the simulated acceleration time-histories can be numerically contaminated, containing some amounts of low-frequency waveform components, resulting in unrealistic velocities and displacements, both exhibiting increasing drifts as time increases and overestimating the structural response in the low-frequency (long-period) range. Therefore, a simple post-processing filtering technique is employed by causally applying the same high-pass Butterworth filter, as described in Section 3.2. A more detailed discussion about the used filtering scheme can be found in the under review paper by Vlachos et al. (2015).

In Figs. 6–7, the Parkfield seismic record is provided, along with one filtered sample acceleration time-history, respectively. The pseudo-acceleration (5% damped) elastic response spectrum of the Parkfield record is provided (in thick line) in Fig. 8, accompanied by the response spectra from 100 realizations of the associated stochastic ground motion model. As seen in Fig. 8, the actual response spectrum of the Parkfield record is very well contained within the range defined by the response spectral ordinates of the associated sample realizations, evincing the efficiency of the suggested evolutionary spectral modeling.

5. CONCLUSIONS

A new analytical non-stationary spectral-based stochastic ground motion model is presented in association with physically interpretable parameters. The new model is based on a novel multi-modal, non-stationary spectral version of the well known Kanai-Tajimi model and is the first fully analytical model to date that is capable of directly and efficiently describing multi-modal evolutionary power spectral densities, allowing for a realistic description of the spectral energy distribution over time. The model, along with the functional forms describing its time-varying parameters, depict high versatility in portraying distinctive highly non-stationary power spectral characteristics. The energy domain is preferred over the time domain as the analysis space. Among others, the para-

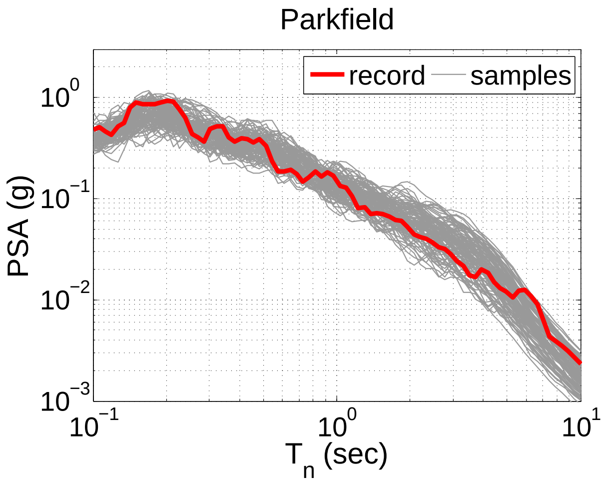

Figure 8: Pseudo-acceleration (5% damped) elastic response spectrum for the Parkfield record together with 100 response spectra model realizations.

metric character of the model allows its straightforward implementation in random vibration problems. By treating the model parameters as random variables and by examining a large ground motion database, the associated seismic risk can be effectively quantified, capturing the natural variability of the database seismic motions. Furthermore, the fully analytical nature of the model is fitting for future development of appropriate predictive models that can eventually link the complete evolutionary power spectra to very specific descriptions of 'site-based' earthquake scenarios. The clear theoretical connection of the power spectra to different seismological scenarios encourages this direction further and future works by the authors will be also concentrated on this effort that can offer new knowledge and resources in the broad context of earthquake engineering and seismic hazard analysis.

APPENDIX

Table 1: Model parameter values of Parkfield record.

γ	δ	Q_1	α_1	β_1	Q_2	α_2	β_2
0.14	2.77	2.43	2.00	2.00	5.73	2.38	2.82

$\zeta_g^{(1)}$	$\zeta_g^{(2)}$	$F^{(I)}$	$\mu^{(I)}$	$\sigma^{(I)}$	$F^{(II)}$	$\mu^{(II)}$	$\sigma^{(II)}$
0.16	0.21	1.11	0.06	0.08	2.74	0.39	0.59

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