Determination of Soil Property Characteristic Values from Standard Penetration Tests

Yu Wang
Associate Professor, Department of Architecture and Civil Engineering, Shenzhen Research Institute, City University of Hong Kong, Hong Kong

Tengyuan Zhao
Graduate Student, Department of Architecture and Civil Engineering, Shenzhen Research Institute, City University of Hong Kong, Hong Kong

Zijun Cao
Associate Professor, State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, Wuhan, China

ABSTRACT: Characteristic values of soil property is a key element in geotechnical design guidelines, particularly for probability-based design codes, and it is usually defined as a pre-specified quantile, such as a lower 5% quantile in Eurocode 7, of the probability distribution of soil property. Such a probabilistic characterization requires a large number of data points from laboratory and/or in situ tests, which are usually not available for most geotechnical projects, especially for those with medium or small sizes. For most projects, only a limited number of standard penetration test (SPT) N values are generally available. It is therefore rather challenging to determine the probability distribution and characteristic values for geotechnical design. To address this challenge, this paper presents a Bayesian equivalent sample approach that determines the probability distribution and characteristic value of effective friction angle and Young's modulus using only a limited number of SPT N values. Two case histories are used to illustrate the approach.

1. INTRODUCTION

Characteristic values of soil property is a key element in geotechnical design guidelines, particularly for probability-based design codes, and it is usually defined as a pre-specified quantile, such as a lower 5% quantile in Eurocode 7, of the probability distribution of soil property. Such probabilistic characterization requires a large number of data points from laboratory and/or in situ tests, which are usually not available for most geotechnical projects, especially for those with medium or small sizes. For most projects, only a limited number of SPT N values are available. SPT is relatively quick, inexpensive, and it is one of the most common in-situ geotechnical tests in the world (e.g., Kulhawy and Mayne 1990). Since the number of data points from a specific project/site is relatively limited, it is rather challenging to determine the probability distribution and characteristic values for geotechnical design.

This paper aims to address this challenge by presenting a Bayesian equivalent sample approach. It integrates the limited number of SPT N values from a specific site with the engineering experience, which exists prior to the undergoing project, in a rational and consistent manner under a Bayesian framework. Then, the probability density function (PDF) of the soil property of interest is derived. Using this derived PDF, Markov Chain Monte Carlo (MCMC) (Wang and Cao 2013; Cao and Wang 2014) is subsequently adopted to transform the PDF into a large number of equivalent samples. Based on these equivalent samples, conventional statistical analysis is performed, and the characteristic values are determined accordingly.

For illustration, SPT N values are used to derive the PDFs of a strength parameter (i.e., effective friction angle, ϕ') and a deformation parameter (i.e., undrained Young's modulus E_u), respectively. For these two cases, field data from a site in Atlanta, Georgia and the clay site of the National Geotechnical Experimentation Sites (NGES) at Texas A&M University are used for demonstration and validation.

2. PROBABILISTIC FRAMEWORK FOR SOIL PROPERTIES

2.1. Modeling of the variability of soil properties

Properties of natural soil vary spatially. Previous studies (e.g., Lumb 1966; Lacasse and Nadim 1996) showed that this variability can be properly modelled by a random variable, X, which follows normal or lognormal distribution. For both scenarios, X can be defined as:

$$X_T = \mu_{X_T} + \sigma_{X_T} z \tag{1}$$

where X_T is a transformation of X, i.e., $X_T = X$ if it follows normal distribution and $X_T = \ln(X)$ if it follows lognormal distribution; μ_{X_T} and σ_{X_T} are the respective mean and standard deviation of X_T; z is a standard Gaussian variable. Note that Eq. (1) shows that no matter X follows normal or lognormal distribution, the transformation of X, X_T is a Gaussian random variable. The PDF of X_T therefore can be expressed as:

$$f(X_T \mid \mu_{X_T}, \sigma_{X_T}) = \left(\sqrt{2\pi} \times \sigma_{X_T}\right)^{-1} \times \exp\left[-\frac{\left(X_T - \mu_{X_T}\right)^2}{2\sigma_{X_T}^{\ 2}}\right] \tag{2}$$

As defined in Eqs. (1) and (2), the determination of the PDF of X_T relies on the determination of the parameters of μ_{X_T} and σ_{X_T}. In this study, μ_{X_T} and σ_{X_T} are determined through the integrated knowledge of limited site-specific SPT N values and engineering experience that exists prior to the project. Given the integrated knowledge, there are many possible combinations of μ_{X_T} and σ_{X_T}. Then, the conditional PDF of X_T, $f(X_T \mid Data, Prior)$, can be expressed as Eq. (3) using the theorem of total probability:

$$f(X_T \mid Data, Prior) = \int_{\mu_{X_T}, \sigma_{X_T}} f(X_T \mid \mu_{X_T}, \sigma_{X_T}) \times f(\mu_{X_T}, \sigma_{X_T} \mid Data, Prior) d\mu_{X_T} d\sigma_{X_T} \tag{3}$$

where *Data* is the site-specific SPT N values; *Prior* is the engineering experience that exists prior to the project; $f(\mu_{X_T}, \sigma_{X_T} \mid Data, Prior)$ is the posterior PDF of μ_{X_T} and σ_{X_T}. Note that the posterior PDF, $f(\mu_{X_T}, \sigma_{X_T} \mid Data, Prior)$, reflects the updated knowledge of μ_{X_T} and σ_{X_T} based on a limited number of project-specific SPT N values and engineering experience that exists prior to the project. It can be determined using a Bayesian framework shown in the following subsection.

2.2. Bayesian determination of model parameters

In the context of Bayesian framework, $f(\mu_{X_T}, \sigma_{X_T} \mid Data, Prior)$ is usually simplified as $f(\mu_{X_T}, \sigma_{X_T} \mid Data)$, and it is expressed as:

$$f(\mu_{X_T}, \sigma_{X_T} \mid Data) = K f(Data \mid \mu_{X_T}, \sigma_{X_T}) \times f(\mu_{X_T}, \sigma_{X_T}) \tag{4}$$

where K is a normalizing constant which doesn't depend on μ_{X_T} and σ_{X_T}; $f(Data \mid \mu_{X_T}, \sigma_{X_T})$ is the likelihood function which reflects that the model fit with the *Data*; $f(\mu_{X_T}, \sigma_{X_T})$ is the prior distribution of μ_{X_T} and σ_{X_T} reflecting the engineering experience prior to the observation data in the project.

It is worthwhile to note that, the likelihood function $f(Data \mid \mu_{X_T}, \sigma_{X_T})$ in Eq. (4) is crucial in the Bayesian framework, and it depends on how the site-specific SPT N values are observed and related to the soil properties of interest. Suppose that site-specific SPT N values are related to the soil properties, X_T, as below:

$$N_T = aX_T + b + \varepsilon \quad (5)$$

where N_T is the transformation of SPT N values, i.e., N_T can be ln(N), $(N_1)_{60}$ and so on; a and b are the respective regression coefficients; ε is the transformation uncertainty or modelling scatterness associated with Eq. (5), and it is usually modelled as a Gaussian random variable with a zero mean and a standard deviation of σ_ε.

Combing Eqs. (1) and (5) leads to:

$$N_T = (a\mu_{X_T} + b) + a\sigma_{X_T} z + \varepsilon \quad (6)$$

Because z and ε are from different sources, they can be treated as independent of each other. Then, N_T is a Gaussian variable with a mean of $(a\mu_{X_T} + b)$ and standard deviation of $\sqrt{(a\sigma_{X_T})^2 + \sigma_\varepsilon{}^2}$. Since SPT is usually performed discretely with a depth interval of 1m or above, which is similar to the vertical correlation length reported in the literatures (e.g., Phoon and Kulhawy 1999a; El-Ramly et al. 2003), the site-specific SPT N can be taken as weakly correlated or even independent of each other. $Data$= $\{(N_T)_i$, i= 1, 2, …, $n_s\}$ is therefore a realization of n_s independent Gaussian random variable, and the likelihood function, $f(Data \mid \mu_{X_T}, \sigma_{X_T})$, is expressed as:

$$f(Data \mid \mu_{X_T}, \sigma_{X_T}) = \prod_{i=1}^{n_s} \left(\sqrt{2\pi} \sqrt{(a\sigma_{X_T})^2 + \sigma_\varepsilon{}^2} \right)^{-1} \times \exp\left\{ -\frac{\left[\left(N_T\right)_i - (a\mu_{X_T} + b) \right]^2}{2\left((a\sigma_{X_T})^2 + \sigma_\varepsilon{}^2 \right)} \right\} \quad (7)$$

In the absence of prevailing knowledge on μ_{X_T} and σ_{X_T}, $f(\mu_{X_T}, \sigma_{X_T})$ can be taken as a joint uniform distribution:

$$f(\mu_{X_T}, \sigma_{X_T}) = \begin{cases} \left(\mu_{X_T,\max} - \mu_{X_T,\min} \right)^{-1} \; for\, \mu_{X_T} \in [\mu_{X_T,\min}, \mu_{X_T,\max}] \\ \times \left(\sigma_{X_T,\max} - \sigma_{X_T,\min} \right)^{-1} \; and\, \sigma_{X_T} \in [\sigma_{X_T,\min}, \sigma_{X_T,\max}] \\ 0 \qquad for\ others \end{cases} \quad (8)$$

where $\mu_{X_T,\max}$, $\mu_{X_T,\min}$, and $\sigma_{X_T,\max}$, $\sigma_{X_T,\min}$ are the respective upper and lower bound for the mean μ_{X_T} and standard deviation σ_{X_T}. Respective ranges of μ_{X_T} and σ_{X_T} can be found from the literatures, such as Phoon (1995), Phoon and Kulhawy (1999a, b) and Baecher and Christian (2003).

Combing Eqs. (4), (7) and (8) leads to the posterior PDF of model parameters of μ_{X_T} and σ_{X_T}, $f(\mu_{X_T}, \sigma_{X_T} \mid Data, Prior)$. With the posterior PDF of model parameters available, the PDF of X_T is then determined using Eqs. (2) and (3). Since the combined equation is generally too complicated and difficult to express analytically and explicitly, Markov Chain Monte Carlo (MCMC) is adopted to transform the PDF into a large number of equivalent samples of X_T (Wang and Cao 2013; Cao and Wang 2014). Using these equivalent samples, conventional statistical analysis can be carried out to construct the histogram and cumulative distribution function (CDF), and characteristic values can be determined accordingly.

For illustration, two important parameters in geotechnical design are used to illustrate how to determine their probability distribution and characteristic values using a limited number of SPT N values: (1) a strength parameter, effective friction angle, ϕ' in Section 3, and (2) a deformation parameter, undrained Young's modulus, E_u, in Section 4.

3. STRENGTH PARAMETER

3.1. PDF of effective friction angle

Section 2 shows that to determine the PDF of soil property of interest, three elements are necessary: the probability distribution type of soil property of interest, the likelihood model, and the possible ranges of μ_{X_T} and σ_{X_T} (i.e., Eq. (1) or (2), Eq. (7) and (8)).

In this section, the strength parameter, effective friction angle, ϕ', is of interest (i.e., $X=\phi'$). Lumb (1966) and Lacasse and Nadim (1996) reported that the ϕ' follows normal distribution. Therefore, $X_T=X=\phi'$. The first element of the PDF of ϕ' is obtained using Eq. (1) or (2).

To obtain the second element, a correlation between ϕ' and SPT N is necessary. Consider, for example, a correlation between ϕ' and SPT N from Mayne et al. (2002) with data from Hatanaka and Uchida (1996):

$$\phi' = \sqrt{15.4(N_1)_{60}} + 20 \tag{9}$$

For the development of the proposed approach, the original ϕ' vs $(N_1)_{60}$ data points from Mayne et al. (2002) and Hatanaka and Uchida (1996) are used to obtain a regression between $\sqrt{15.4(N_1)_{60}}$ and ϕ', so that $\sqrt{15.4(N_1)_{60}}$ can be estimated from ϕ'. The new regression is given as:

$$\sqrt{15.4(N_1)_{60}} = 0.923\phi' - 17.847 + \varepsilon_1 \tag{10}$$

where ε_1 is the transformation uncertainty associated with Eq. (10), with $\mu_{\varepsilon_1} = 0$ and. $\sigma_{\varepsilon 1}=2.11$ Comparing Eq. (5) with (10) leads to $a=0.923$, $b=-17.847$, $\sigma_\varepsilon=\sigma_{\varepsilon 1}=2.11$ and $\sqrt{15.4(N_1)_{60}} = N_T$. Then, the likelihood function for $Data = N_T = \sqrt{15.4(N_1)_{60}}$ is expressed using Eq. (7).

Phoon (1995) reported that the mean of ϕ' generally falls between [20°, 40°], while the standard deviation of ϕ' falls between [1°, 6°]. Therefore, the prior distribution of μ_{X_T} and σ_{X_T} is defined by Eq. (8), with $\mu_{X_T,\min}=20°$, $\mu_{X_T,\max}=40°$ and $\sigma_{X_T,\min}=1°$, $\sigma_{X_T,\max}=6°$.

With the three elements available shown above, the PDF of ϕ' is determined accordingly. For further illustration, the proposed approach is applied to a case history in Atlanta, USA.

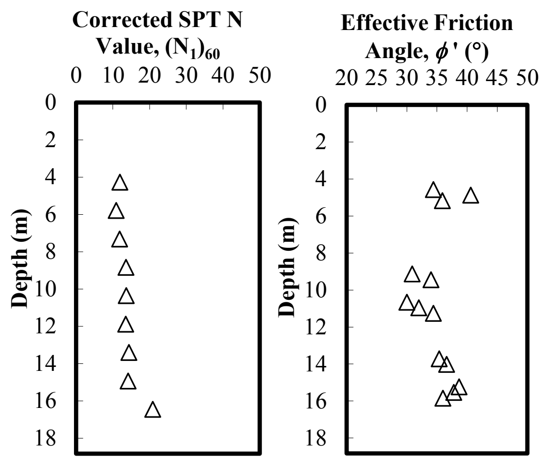

Figure 1: Corrected SPT N values and effective friction angle by triaxial compression tests from a site in Atlanta, USA. (after Mayne and Harris 1993)

3.2. Illustrative example

The proposed approach is applied to characterize probabilistically the effective friction angle, ϕ', of sand using a limited number of SPT N values from a site on the west side of Georgia Institute of Technology campus, in Atlanta, Georgia (Mayne and Harris 1993). In this site, the target layer is a silty sand layer. It is about 14.5 m thick, extending from depth 3.7m to 18.2m under the ground. 9 corrected SPT N values, $(N_1)_{60}$ and 13 ϕ' values by triaxial compression test results from this silty sand layer are shown in Figure 1. The 9 $(N_1)_{60}$ values are used herein as input, while 13 ϕ' values are used for comparison and validation purpose.

Using the PDF of ϕ' obtained from subsection 3.1, a MCMC simulation run is performed with 30,000 equivalent samples of ϕ' as output (Wang and Cao 2013). Using these samples, the histogram of ϕ' is constructed as shown in Figure 2. The histogram peaks at a value of about 36° and 26,942 equivalent samples (i.e., around 90% of the equivalent samples) fall within the range of [31.7°, 38.5°]. Figure 2 also includes the 13 triaxial compression test results, ϕ', by open triangles. It shows that these 13 ϕ' values roughly fall within the 90% inter-quantile range, i.e., [31.7°, 38.5°].

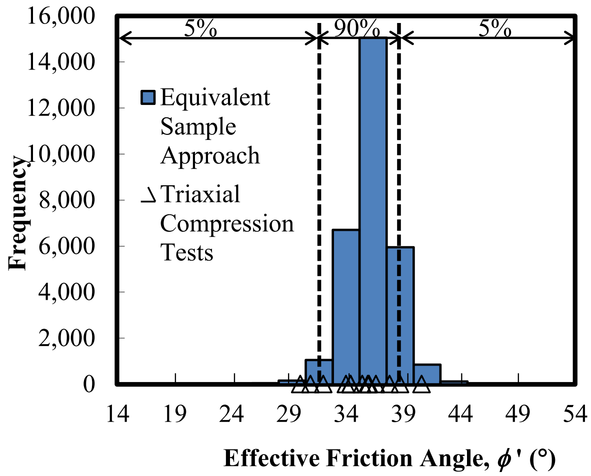

Figure 2: Histogram of the equivalent samples for effective friction angle.

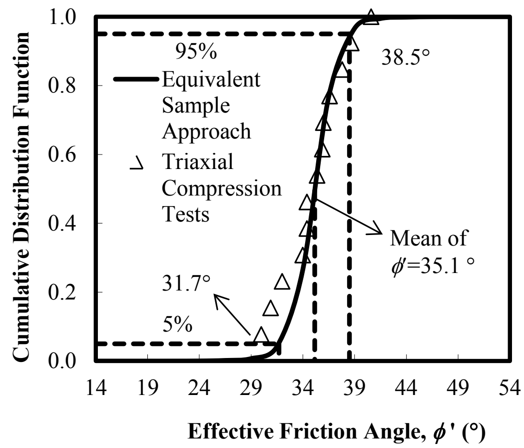

Figure 3: Cumulative distribution function (CDF) of effective friction angle.

Figure 3 plots the two CDFs of ϕ': one is estimated from cumulative frequency diagrams of the 30,000 equivalent samples by a solid line and the other one from the 13 ϕ' values by open triangles. The CDF of ϕ' from the equivalent samples compares favorably with that obtained from the open triangles. Such a good agreement shows that the Bayesian equivalent sample approach provides a reasonable estimate of the probability distribution of ϕ' using a limited number of SPT N values. In addition, using the CDF of ϕ', the quantiles of ϕ' can be determined easily. For example, the 5% (i.e., 31.7°) and 95% (i.e., 38.5°) quantile of ϕ' are determined and shown in Figure 3, respectively, together with the mean of ϕ' (i.e., 35.1°).

Table 1 summarizes the mean (i.e., $\mu_{\phi'}^*$) and standard deviation (i.e., $\sigma_{\phi'}^*$) of ϕ' estimated from 30,000 equivalent samples and those from 13 triaxial compression tests, respectively. The $\mu_{\phi'}^*$ and $\sigma_{\phi'}^*$ from the equivalent samples are calculated as 35.1° and 2.1°, respectively. On the other hand, the $\mu_{\phi'}^*$ and $\sigma_{\phi'}^*$ from the triaxial compression tests are 35.1° and 3.0°, respectively. The absolute differences between $\mu_{\phi'}^*$ and $\sigma_{\phi'}^*$ estimated from these two different approaches are also summarized in the fourth column in Table 1. The absolute difference

Table 1: Summary of the estimated statistics of effective friction angle

Approaches	*Equivalent Sample Approach*	*Triaxial Compression Tests*	*Absolute Difference*
$\mu_{\phi'}^*$ (°)	*35.1*	*35.1*	*0*
$\sigma_{\phi'}^*$ (°)	*2.1*	*3.0*	*0.9*

between the estimated $\mu_{\phi'}^*$ is 0°, while that of $\sigma_{\phi'}^*$ is 0.9°. Such small differences further suggest that the Bayesian equivalent sample approach can properly characterize the effective friction angle using a limited number of SPT N values. The characteristic values of ϕ' can then be properly estimated using a limited number of SPT N values and the Bayesian equivalent sample approach. For example, if the characteristic value is defined as the lower 5% quantile, it is 31.7°. In contrast, if the characteristic value is defined as the mean value, it is about 35.1°.

4. DEFORMATION PARAMETER

4.1. PDF of undrained Young's modulus

In this section, the deformation parameter, undrained Young's modulus, E_u, of clay is of interest (i.e., $X=E_u$). Previous studies by Lacasse and Nadim (1996), and Baecher and Christian

(2003) suggested that E_u follows lognormal distribution. Therefore, X_T=ln(X)=ln(E_u). Eq. (1) or (2) then determines the probability distribution type, i.e., the first element of the PDF of ln(E_u).

To determine the likelihood model, i.e., the second element of the PDF of ln(E_u), correlation between E_u and SPT N values is crucial. Consider, for example, a correlation between E_u of clay and SPT N from Ohya et al. (1982) and Kulhawy and Mayne (1990) is expressed as:

$$E_u / P_a = 19.3N^{0.6} \quad (11)$$

where P_a is the atmospheric pressure (i.e., 0.1 MPa). For the development of the proposed approach, Eq. (11) is rewritten as

$$\ln(N) = 1.587\ln(E_u) - 1.044 + \varepsilon_2 \quad (12)$$

where ε_2 is the transformation uncertainty associated with Eq. (12), and μ_{ε_2} =0 and σ_{ε_2} =1.352.

Comparing Eq. (5) with (12) yields a=1.587, b= -1.044, σ_ε=$\sigma_{\varepsilon 2}$=1.352 and ln(N)=N_T. Then, the likelihood function of $Data = N_T = \ln(N)$ is expressed using Eq. (7) accordingly.

For the possible ranges of μ_{X_T} and σ_{X_T} , i.e., the third element of the PDF of ln(E_u), the following set of prior knowledge of E_u is used: the mean of E_u, μ_{E_u} , falls within the range [5MPa, 15MPa], with a coefficient of variation (COV) varying between 10% and 90%. This set of prior knowledge is consistent with the typical ranges of E_u reported in the literatures, such as Kulhawy and Mayne (1990), Phoon and Kulhawy (1999a, b). Because E_u follows lognormal distribution, i.e., X_T=ln(X)=ln(E_u), the prior knowledge on μ_{E_u} and COV are then transformed to the ranges of μ_{X_T} and σ_{X_T} . Using $\sigma_{X_T} = \sqrt{\ln(1+COV^2)}$ and $\mu_{X_T} = \ln(\mu_{E_u}) - 0.5\sigma_{X_T}{}^2$, the range of μ_{X_T} and σ_{X_T} are calculated as $\mu_{X_T} \in [1.2, 2.7]$ and $\sigma_{X_T} \in [0.1, 0.77]$, respectively.

With these three elements defined above, the PDF of ln(E_u) is determined accordingly. Because the PDF of E_u is of interest, simple mathematical transformation is needed (i.e., E_u=exp(ln(E_u)). For further illustration, the proposed approach is applied to a clay site at Texas A&M University, USA.

4.2. Illustrative example

The Bayesian equivalent sample approach is applied to probabilistically characterize the E_u of one stiff clay layer at the clay site of the NGES at Texas A&M University (Briaud 2000). In this site, the stiff clay layer is about 5.5 m thick, extending from the ground surface to the depth of 5.5 m. Figure 4 shows the 5 SPT N values and 42 E_u values from pressuremeter tests performed within this layer. The 5 SPT N values are used as input, while the 42 E_u values are used for comparison and validation.

Using the PDF of ln(E_u) obtained in subsection 4.1, 30, 000 equivalent samples of ln(E_u) are generated (Wang and Cao 2013). Because E_u is of interest, a simple transformation for these samples is necessary, i.e., E_u=exp(ln(E_u)). Based on the transformed equivalent samples of E_u, the histogram and CDF are constructed as shown in Figure 5 and Figure 6, respectively.

Figure 5 shows the histogram of these samples of E_u. It peaks at a value around 10 MPa, and 27,010 equivalent samples (i.e., around 90% of the 30,000 equivalent samples) fall within the range of [4.0 MPa, 21.6 MPa]. Figure 5 also includes the 42 pressuremeter tests results by open triangles. It shows that 36 out of 42 falls within the 90% inter-quantile range, i.e., [4.0 MPa, 21.6 MPa].

Figure 6 shows the CDFs of E_u estimated from the cumulative frequency diagrams of the 30,000 samples and 42 E_u values, represented by a solid line and open triangles, respectively. It shows that the solid line plots closely to the open triangles. Such a good agreement suggests that the proposed approach provides a reasonable estimate of the probability distribution of E_u using a limited number of SPT N values. In addition, the quantiles (e.g., 5% and 95% quantile) can be determined easily using the CDF of E_u as shown in Figure 6.

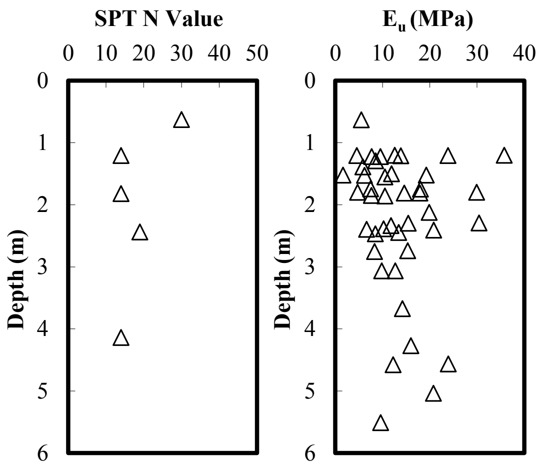

Figure 4: Standard penetration test (SPT) N values and undrained Young's modulus by pressuremeter test at the clay site of the NGES at Texas A&M University (after Briaud 2000).

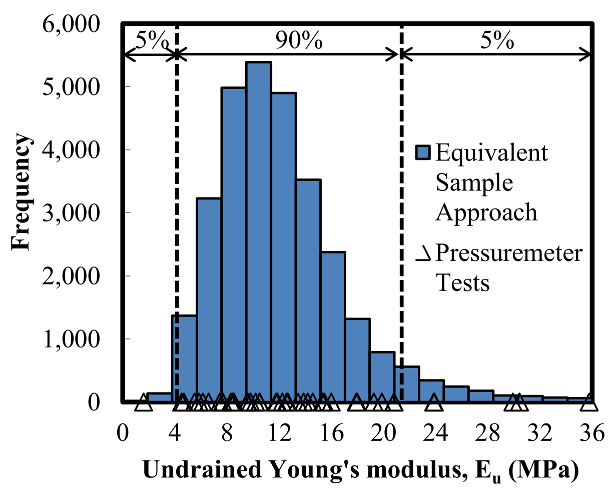

Figure 5: Histogram of the equivalent samples for undrained Young's modulus.

Table 2 summarizes the mean (i.e., $\mu_{E_u}{}^*$) and standard deviation (i.e., $\sigma_{E_u}{}^*$) of E_u estimated from 30,000 equivalent samples and those from 42 pressuremeter tests, respectively. The $\mu_{E_u}{}^*$ and $\sigma_{E_u}{}^*$ from equivalent samples are calculated as 11.1 MPa and 6.7 MPa, respectively (see the second column in Table 2). In contrast, he $\mu_{E_u}{}^*$ and $\sigma_{E_u}{}^*$ from the pressuremeter tests are estimated as 13.5 MPa and 7.5 MPa, Respectively (see the third column in Table 2). The absolute difference between

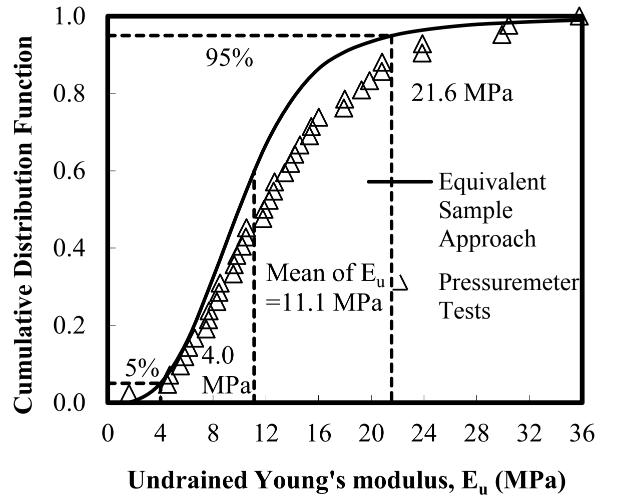

Figure 6: Cumulative distribution function (CDF) of undrained Young's modulus.

Table 2: Summary of the estimated statistics of undrained Young's modulus

Approach	*Equivalent Sample Approach*	*Pressuremeter Tests*	*Absolute Difference*
$\mu_{E_u}{}^*$ (MPa)	*11.1*	*13.5*	*2.4*
$\sigma_{E_u}{}^*$ (MPa)	*6.7*	*7.5*	*0.8*

$\mu_{E_u}{}^*$ and $\sigma_{E_u}{}^*$ estimated from these two approaches are also summarized in the fourth column in Table 2. The absolute difference between the estimated $\mu_{E_u}{}^*$ is 2.4MPa, while that of $\sigma_{E_u}{}^*$ is 0.8 MPa. Compared with the $\sigma_{E_u}{}^*$ values (e.g., 7.5 MPa from the 42 pressuremeter tests), 2.4 MPa is relatively small. This again suggests that the Bayesian equivalent sample approach can properly characterize the E_u in this site using a limited number of SPT N values, which further leads to reasonable estimation of characteristic values of E_u, such as the lower 5% quantile (i.e., 4.0 MPa) as shown in Figure 6.

5. SUMMARY AND CONCLUSION

This study presented a Bayesian equivalent sample approach to probabilistically characterize the soil property of interest from a limited number of SPT N values. It provides a rational method to determine the characteristic values of soil property when extensive testing cannot be carried out, which is usually the case for the geotechnical projects, especially for those with medium or small sizes. In this study, equations were derived for the proposed approach, and it was illustrated through the determination of the probability distribution and characteristic values of the effective friction angle, and that of undrained Young's modulus. For both cases, the probability distribution and characteristic values obtained from the proposed approach compared well with those from extensive lab data or field observation at the same sites.

6. ACKNOWLEDGEMENT

The work described in this paper was supported by a grant from the National Natural Science Foundation of China (Project No. 51208446). The financial support is gratefully acknowledged.

7. REFERENCES

Baecher, G. B., and Christian, J. T. (2003). *Reliability and statistics in geotechnical engineering*, Wiley, Hoboken, NJ.

Briaud, J.L. (2000). "The national geotechnical experimentation sites at Texas A&M University: clay and sand." *National Geotechnical Experimentation Sites*, Geotechnical Special Publication, Vol. 93, ASCE, 26–51.

Cao, Z., and Wang, Y. (2014). "Bayesian Model Comparison and Characterization of Undrained Shear Strength." *J. Geotech. Geoenviron. Eng*.,140 (6), 04014018.

El-Ramly, H., Morgenstern, N.R., and Cruden, D.M. (2003). "Probabilistic stability analysis of a tailings dyke on presheared clay-shale". *Can. Geotech. J.*, 40, 192–208.

Hatanaka, M., and Uchida, A. (1996). "Empirical correlation between penetration resistance and internal friction angle of sandy soils." *Soils Found.*, 36(4), 1–9.

Kulhawy, F. H., and Mayne, P. W. (1990). "Manual on estimating soil properties for foundation design." *Rep. EL-6800*, Electric Power Research Institution, Palo Alto, CA.

Lacasse, S., and Nadim, F. (1996). "Uncertainties in characterizing soil properties." *Uncertainty in the Geologic Environment: From Theory to Practice*, Geotechnical Special Publication, Vol. 58(I), ASCE, Reston, VA, 49–75.

Lumb, P. (1966). "The variability of natural soils." *Can. Geotech. J.*, 3(2), 74–97.

Mayne, P. W., Christopher, B. R., and DeJong, J. (2002). "Subsurface investigations—geotechnical site characterization." *No. FHWA NHI-01-031*, Federal Highway Administration, DOT, Washington, DC.

Mayne, P. W., and Harris, D. E. (1993). "Axial load-displacement behavior of drilled shaft foundations in Piedmont residuum." *Tech. Rep. No. 41-30-2175*, Federal Highway Administration, Washington, D.C.

Ohya, S., Imai, T., and Matsubara, M. (1982). "Relationships between N value by SPT and LLT pressuremeter results." *Proc.*, *2nd European Symposium on Penetration Testing*, Vol. 1, Amsterdam, Dutch, 125–130.

Phoon, K. K. (1995). "Reliability-based design of foundations for transmission line structures". Ph. D. thesis. Cornell University, Ithaca, NY., 208-209.

Phoon, K. K., and Kulhawy, F. H. (1999 a). "Characterization of geotechnical variability." *Can. Geotech. J.*, 36(4), 612–624.

Phoon, K. K., and Kulhawy, F. H. (1999 b). "Evaluation of geotechnical property variability." *Can. Geotech. J.*, 36(4), 625–639.

Wang, Y., and Cao, Z. (2013). "Probabilistic characterization of Young's modulus of soil using equivalent samples." *Eng. Geol.*, 159, 106–118.